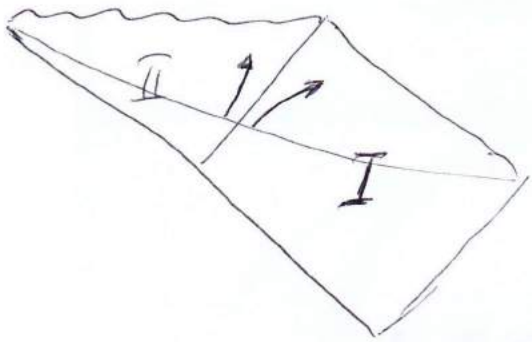
Lecture 6

(1)

Refs. Hawking, PRD, 14, 2460 (1976)
Page, PRL, 71, 3743 (1993)
Susskind, Thorlacius, Uglum, PRD, 48, 3743 (1993)
Almheiri et al., JHEP, 1302, 062 (2013)

1) First formulation of the information paradox



$$ds^2 = -\left(1-\frac{2M}{r}\right) du\, dv =$$

$$= -\frac{2M}{r} e^{-\frac{r}{2M}} d\bar{u}\, d\bar{v}$$

$$\bar{u} = -4M e^{-u/4M}$$

$$\bar{v} = 4M e^{v/4M}$$

$$|0\rangle_u \leftrightarrow \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega\bar{u}}, \quad \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$$

$$|0\rangle_B \leftrightarrow \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega u}, \quad \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$$

$$a_\omega^{(B)} = \alpha_{\omega\omega'} a_{\omega'}^{(u)} + \beta^*_{\omega\omega'} a_{\omega'}^{+(u)}$$

to form complete basis must be supplemented by operators ~~creating~~ corresponding to modes inside the BH

$|0\rangle_u$ is a pure state.

The state of the field outside BH is obtained by tracing over the inside modes

$\Rightarrow$ mixed state

thermal $\Rightarrow$ maximally mixed density matrix

$$\rho = \prod_\omega \left(1 - e^{-\frac{\omega}{T}}\right) e^{-\frac{\omega a_\omega^+ a_\omega}{T}} \propto$$

$$\propto \exp\left[-\frac{1}{T}\int d\omega\, a_\omega^+ a_\omega \omega + \int d\omega \ln\left(1 - e^{-\omega/T}\right)\right]$$

outside modes are entangled with the BH interior and completely uncorrelated among each other

Imagine we form a BH ~~b~~ in a collapse of a pure quantum state. When it evaporates, we are left with a maximally mixed state

$\Rightarrow$ non-unitary evolution

2) Page's scenario

Above reasoning does not take into account back-reaction, which can introduce correlations between the outside modes and restore unitarity.

Consider a model:

large system with $\mathcal{N} \gg 1$ degrees of freedom in a pure state $|\Psi\rangle$

take the degrees of freedom as spins $|\downarrow\rangle$, $|\uparrow\rangle$

$\Rightarrow$ dimensionality of the Hilbert space $2^{\mathcal{N}}$

Divide the system into parts

$$|\Psi\rangle = \mathcal{U}_{AB}\, |\Psi_A\rangle \otimes |\Psi_B\rangle$$

$$\cancel{\sum_B}\ \rho(A) = Tr_B |\Psi\rangle\langle\Psi| = \sum_B \mathcal{U}_{AB}\, \mathcal{U}^*_{A'B}\, |\Psi_A\rangle\langle\Psi_{A'}|$$

Entanglement entropy:

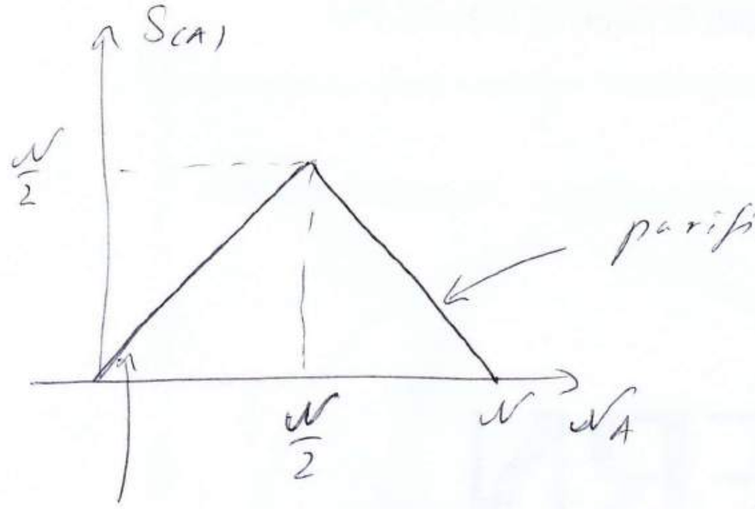
$$S_{(A)} = Tr_A\left(\rho_{(A)} \ln \rho_{(A)}\right)$$

<u>Ex. 24</u> Prove $S_{(A)} = S_{(B)}$

Theorem. Assume U_{AB} is a column in $2^N \times 2^N$-dimensional random unitary matrix (physically : $|\Psi\rangle$ takes a random direction in $|\Psi_A\rangle \otimes |\Psi_B\rangle$ basis)

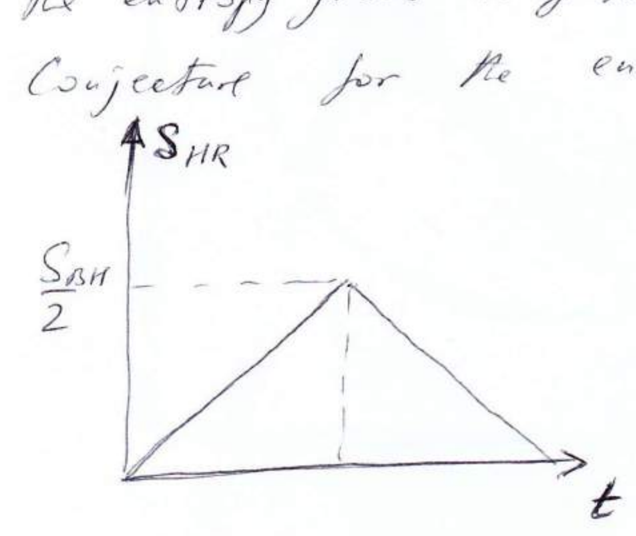
Then on average

$$\overline{S}_{(A)} = \begin{cases} N_A , & \text{if } N_A < N_B \\ N - N_A , & \text{if } N_A > N_B \end{cases}$$



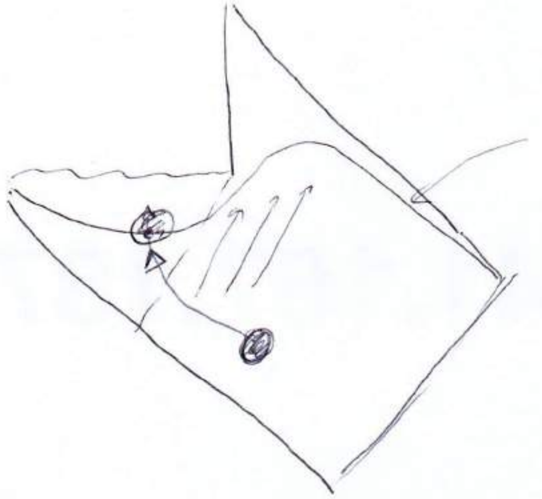
the entropy grows as if the state is maximally mixed

$\Rightarrow$ Conjecture for the entropy of Hawking radiation



This would be very much like a burning fire. The final state of Hawking radiation is pure and all information escaped from BH.

3) Xerox paradox and complementarity



nice slices: start from interior, their curvature is bounded and they cover $(1-\varepsilon)$ fraction of Hawking radiation

Imagine throwing a book into BH. Information about it is imprinted in the HR. But also on the interior portion of the nice slice

$\Rightarrow$ copying of information

$$|\Psi\rangle \mapsto |\Psi_{in}\rangle \otimes |\Psi_{out}\rangle$$

for any state $|\Psi\rangle$

Impossible for a unitary ~~and~~ linear evolution.

Resolution: complementarity:

Considerations involving simultaneously BH interior and exterior do not make sense.

Physical questions must be formulated only in terms of what an observer (asymptotic or infalling) can measure.

4) Firewall issue

Entanglement entropy obeys inequalities

$S_{AB} + S_{BC} \geq S_A + S_C$

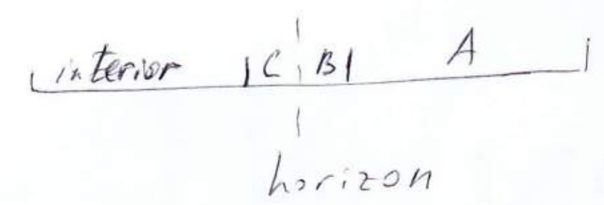
A	B	C

$S_{AB} + S_{BC} \geq S_B + S_{ABC}$

~~for~~ Take B - newly created Hawking particl

C - ~~interior~~ region just inside the horizon

A - previously emitted HR



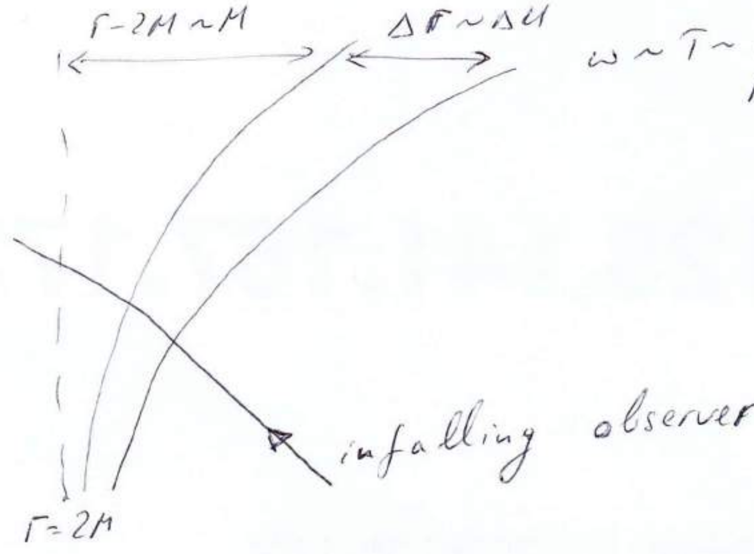
Unruh vacuum $\Rightarrow$ $S_{BC} = 0$

$\Rightarrow$ $S_{AB} \geq S_A + S_C = S_A + S_B$ $\Rightarrow$ entropy of HR cannot decrease

Conversely: If the entropy of HR decreases according to Page: $S'_{AB} = S_A - 1$ $\Rightarrow$

$\Rightarrow$ fields are <u>not</u> in the Unruh vacuum near horizon $\Rightarrow$ ~~there are real particle~~ an infalling observer will detect real particles

How strong is the effect



$$\omega \sim T \sim \frac{1}{M}, \quad n \sim T^3 \sim \frac{1}{M^3}$$

$$\varphi_\omega^{(B)} \sim e^{i\omega u}$$

$$\bar{\omega} \sim \frac{d\varphi_\omega^{(B)}}{d\bar{u}} \frac{1}{\varphi_\omega^{(B)}} = \omega \frac{du}{d\bar{u}} = \omega \frac{2M}{\bar{u}} \sim \omega \frac{M}{\ell} \sim \frac{1}{\ell}$$

proper distance from the horizon

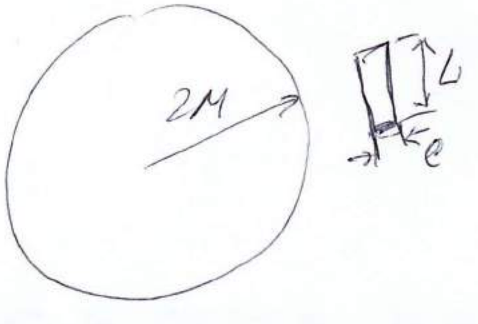
$$\bar{n} \approx \frac{N}{M^2 \ell} \approx \frac{n \cdot M^2 \cdot \Delta u}{M^2 \cdot \ell} \sim \frac{1}{M^3} \frac{2M \Delta\bar{u}}{\bar{u}\,\ell} \sim \frac{1}{M^2 \ell}$$

$$\bar{\varepsilon} \sim \bar{\omega}\bar{n} \sim \frac{1}{M^2 \ell^2}$$

When $\ell \sim \ell_p \Rightarrow \tilde{\varepsilon} \sim \frac{1}{M^2 \ell_p^2} \gg \frac{1}{M^4}$ but $\ll \frac{1}{\ell_p^4}$

Number of particles seen by a detector with cross-section L^2

$$N \sim \frac{L^2 \cdot \ell}{M^2 \ell} \sim \frac{L^2}{M^2} \ll 1$$



⇒ A detector sees less than one particle with energy $\frac{1}{\ell}$ on average

Probably not as dramatic ⇒ difference of the ~~state~~ near-horizon state from vacuum can be established with confidence only by the asymptotic observer who can record the data from many detectors thrown in ?

<u>N.B.</u> Almheiri et al. argues for a much stronger result:

$$\bar{\varepsilon} \sim \frac{1}{\ell_p^4}$$

by including into consideration also partial waves with higher angular momenta that do not escape from the BH.