**Abstract:** This talk will discuss recent innovations that use operations on polynomials to solve combinatorial problems in linear algebra. The most notable of these innovations is the "method of interlacing polynomials" which (as far as we know) is the only available technique for proving the existence of small probability eigenvalue events. I will then discuss recent applications of these techniques to problems in spectral graph theory, random matrix theory, and electrical engineering.