Financing the Litigation Arms Race*

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October 23, 2021

Abstract: Using a dynamic model of litigation, we show that the increasingly popular practice of third-party litigation financing has ambiguous welfare implications. A defendant and a plaintiff bargain over a settlement payment. The defendant takes costly actions to avoid deadweight losses associated with large transfers to the plaintiff. Litigation financing bolsters the plaintiff, leading to larger deadweight losses. However, by endogenously deterring the defendant from taking costly actions, litigation financing can nonetheless improve the joint surplus of the plaintiff and the defendant. In contrast to popular opinion, litigation financing does not necessarily encourage high-risk frivolous lawsuits.

Keywords: litigation financing, dynamic bargaining, real options
JEL: C73, C78, G23, K41

*We thank John Batter, Lauren Cohen, Jason Roderick Donaldson (SFS discussant), Benjamin Esty, Joseph Grundfest, and Kathryn Spier for very helpful comments. We are grateful for feedback from participants at the 2021 SFS Cavalcade and seminar participants at Boston University and Harvard Business School.

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1 Introduction

Miller UK Ltd. sued Caterpillar, a Fortune 500 company, in 2010 for misuse of trade secrets. Throughout the case, Miller accused Caterpillar of using its superior resources to suppress the lawsuit; Caterpillar's revenue exceeded Miller's revenue by over 50,000%.[1] To fight back, Miller relied on litigation financing, a practice in which a third-party firm provides capital in exchange for claims on future trial awards and settlement payments. Properly financed, Miller ultimately won a $74 million award in trial.[2] Corporate plaintiffs like Miller have increasingly relied on litigation financing to afford rising legal costs. Litigation-financing firms spent $2.3 billion on U.S. cases in 2019, roughly 10% of total spending by large U.S. corporations on outside counsel for litigation.[3]

Legal practitioners disagree on the societal impact of the growth of litigation financing. Proponents argue that allowing plaintiffs and their attorneys access to capital corrects the traditionally unfair corporate litigation system, in which deep-pocketed defendants bully underfunded plaintiffs. As New York Supreme Court Justice Eileen Bransten wrote,[4] "litigation funding allows lawsuits to be decided on their merits, and not based on which party has deeper pockets." In contrast, critics warn that an infusion of third-party capital serves only to increase the volume of litigation and the concomitant increase in payouts to lawyers. Similarly, the US Chamber of Commerce claims that litigation financing encourages frivolous lawsuits that waste the time and resources of the defendant.[5]

In order to assess the merits of these competing arguments, we develop a model to determine the impact of litigation financing on the joint surplus of plaintiffs and defendants. We assume that litigation financing itself provides no direct social benefit and exacerbates an underlying negative externality imposed on defendants by plaintiffs. Surprisingly, we show that litigation financing nonetheless can improve the joint surplus of plaintiffs and

[1] Miller "claimed throughout the four-year case that Caterpillar has deliberately dragged out the legal proceedings in the hope that Miller would simply run out of money to fund its case." https://www.thetimes.co.uk/article/millers-dig-in-for-long-legal-fight-for-their-rights-w0rpgtnsw3r.

[2] See https://www.reuters.com/article/us-caterpillar-lawsuit-idUSKBN0U424I20151221.

[3] See https://www.nortonrosefulbright.com/en-us/knowledge/publications/5ed4f834/2019-litigation-trends-annual-survey and https://www.law360.com/articles/1199902/outside-counsel-litigation-spending-has-soared-clients-say.

[4] See Lawsuit Funding LLC v. Lessoff at https://law.justia.com/cases/new-york/other-courts/2013/2013-ny-slip-op-33066-u.html.

[5] See https://thehedgefundjournal.com/the-emerging-market-for-litigation-funding/.

defendants.

In our model, a plaintiff and a defendant decide whether to settle a lawsuit or wait for trial. We assume that a trial entails a deadweight loss proportional to the size of the trial award. This loss represents legal costs, the defendant's potential financial distress, and the defendant's costs associated with future lawsuits encouraged by a large award. However, parties receive a fixed reputation benefit from going to trial. For example, a litigant might benefit in future lawsuits from a demonstrated willingness to wait for trial. Compared to settlement, a trial thus improves total surplus if the expected trial award is small relative to the fixed reputation benefit.

We model the settlement negotiation between the plaintiff and the defendant as a continuous-time stochastic-bargaining game (Antill and Grenadier, 2019). Settlement represents a real option to avoid large deadweight losses if the stochastic trial award grows too large relative to the trial reputation benefit. The plaintiff and the defendant bargain over when to jointly exercise the settlement option and how to share the settlement surplus.

Prior to the bargaining game, the defendant can choose a costly legal strategy that reduces the expected trial award. We model litigation financing as an opportunity for the plaintiff to similarly choose a costly legal strategy that *increases* the expected trial award. Since higher trial awards mechanically imply higher deadweight loss, litigation financing has a direct negative effect on joint surplus.

While litigation financing directly increases one deadweight loss in our model, we find that it endogenously alleviates another negative externality: wasteful defense spending. The defendant internalizes the transfer it must pay the plaintiff in a trial. This incentivizes the defendant to choose the costly legal strategy, even when doing so reduces joint surplus, if a trial is likely. Litigation financing endogenously reduces the likelihood of a trial because it increases the expected trial award, encouraging agents to exercise the option to settle. Intuitively, the defendant is less interested in reducing the expected trial award if the starting level is high, because such a trial award will likely be avoided through settlement. Thus, our first main result is that litigation financing may be socially beneficial because it deters wasteful bullying: a strategy in which a defendant incurs large litigation costs simply to secure negotiating advantages over an underfinanced plaintiff. Formally, we show that the net effect of litigation financing on the joint surplus of a defendant and a plaintiff can be positive or negative.

Our second main result examines the potential for litigation financing to encourage the filing of costly frivolous lawsuits. Specifically, we characterize *marginal* lawsuits: those that are filed if and only if litigation financing is available. By definition, marginal lawsuits are the types of lawsuits that become more common as litigation financing increases in prevalence. We show that marginal lawsuits can have larger expected trial awards and less volatility than lawsuits that do not require financing. This finding challenges the view that litigation financing leads to frivolous lawsuits, at least to the extent that frivolous lawsuits are risky lawsuits with poor trial prospects.

This result follows from two observations. First, if the plaintiff has a promising lawsuit, then it will file the lawsuit even if a lack of external financing precludes a strong expensive legal strategy. Thus, marginal lawsuits are those that are less attractive to plaintiffs than lawsuits that are filed without financing. Second, increases in trial-award volatility can benefit the plaintiff. Through bargaining, the plaintiff extracts a fraction of the value of the option to settle. This option value increases with volatility, because settling avoids the large deadweight losses associated with large trial awards. Marginal lawsuits thus tend to have less volatility than lawsuits that are filed without financing, since the latter are more attractive to plaintiffs. By the same logic, we show that a plaintiff can prefer one lawsuit to another lawsuit with a higher expected trial award but lower volatility. The latter lawsuit, with low volatility and strong trial prospects, can thus be marginal while the former is not.

Interestingly, increases in volatility do not always benefit the plaintiff. This counterintuitive comparative static follows from our assumption that the plaintiff's bargaining position varies stochastically. Settlement delays can thus harm a plaintiff with a strong initial bargaining position, because the plaintiff will likely be in a worse bargaining position by the time settlement occurs. Increases in trial-award volatility endogenously delay settlement because of the option to wait and see if the expected deadweight loss will decline. It follows that an increase in volatility can harm or help the plaintiff. We further show that an increase in volatility can increase or reduce the equilibrium cost of litigation financing.

We obtain these results using assumptions that are standard in the real-options literature. The expected trial award follows a geometric Brownian motion. The costs and benefits of a trial are affine functions of the trial award. These simplifying assumptions allow us to solve for closed-form solutions to our stochastic-bargaining game. However, the general intuition behind our results is more general. If settlement allows defendants to avoid the worst trial outcomes, defendants will be less willing to invest in improved trial outcomes when their ex-ante trial prospects are poor. Litigation financing enables a plaintiff to pursue costlier and better strategies, worsening the defendant's trial prospects and discouraging hefty defense spending. We confirm that this intuition, which implies an ambiguous effect of litigation financing on joint surplus, applies in both our full model and a simple illustrative model.

1.1 Contribution to the Literature

We contribute two main results. First, litigation financing deters wasteful defense spending. As a result, litigation financing can improve the joint surplus of a defendant and a plaintiff, even in a model in which financing entails direct social costs and no direct social benefits. Second, while litigation financing encourages more lawsuits, these lawsuits are not necessarily more frivolous than lawsuits that are viable without financing. Both of these results are novel contributions to the literature.

There are few formal theories of litigation financing.[6] Daughety and Reinganum (2014), Kidd (2015), Landeo and Nikitin (2018), and Spier and Prescott (2019) develop static and two-period models in which litigation financing has no real effect on the exogenous trial payoff (i.e., there is no choice of legal or bargaining strategies).[7] We make two fundamentally different assumptions from these papers. First, we assume that litigation financing enables the plaintiff to choose a stronger legal strategy that would otherwise have been unaffordable. Second, we assume that good outcomes for the plaintiff (settlements and high trial awards) increase deadweight losses for the defendant. Importantly, these assumptions imply that litigation financing carries a direct social cost. We nonetheless show that litigation financing can (i) improve the joint surplus of the plaintiff and defendant and (ii) encourage lawsuits that are not necessarily less meritorious than existing lawsuits. We thus contribute to the sparse theoretical literature on litigation financing by showing that common criticisms of litigation financing do not imply that the practice leads to negative outcomes.

Our paper contributes to the literature on stochastic-bargaining games by providing

[6] In empirical work, Esty (2001) studies publicly traded securities that paid out claims in a litigation case. Abrams and Chen (2012) empirically study litigation financing in Australia.

[7] Avraham and Wickelgren (2013) verbally sketch a signalling model in which courts should update their beliefs about the validity of a lawsuit after learning it obtained third-party funding. While litigation funding affects the trial payoff in their model, it does so through a channel that the authors acknowledge does not exist in practice.

the first application to litigation and the first analytical solution to a stochastic-bargaining game.[8] We also contribute to the theoretical literature modeling litigation. In an early example, Bebchuk (1984) models the decision to settle a lawsuit in a setting with asymmetric information. Bebchuk (1984) compares the implications of the "American rule," in which the plaintiff and defendant each pay their own respective legal costs, and the "British rule," in which the losing party pays all the costs.[9] Dana Jr and Spier (1993) show that contingent-fee arrangements arise as optimal contracts if the plaintiff has private information regarding the profitability of the lawsuit. However, these papers, like the rest of the literature,[10] do not study the impact of third-party litigation financing on litigation outcomes. We thus contribute by showing how the novel practice of litigation financing affects litigation outcomes.

Cornell (1990) and Grundfest and Huang (2005) precede this paper in modeling litigation as a real option. Plaintiffs and defendants have an option to settle to avoid particularly costly trials. As we will show, the real-option nature of litigation is critical for our results. We contribute by developing a formal continuous-time stochastic-bargaining model of litigation. We show that the stochastic-bargaining framework yields a novel implication: increases in lawsuit-outcome volatility do not always benefit plaintiffs. Additionally, as described above, neither Cornell (1990) nor Grundfest and Huang (2005) model the role of litigation financing.

1.2 Paper Structure

In Section 2, we provide relevant institutional details. In Section 3, we present an illustrative two-period model to provide intuition for our main results. In Section 4, we augment this model using a continuous-time stochastic bargaining framework to more realistically model the settlement process. We show that our main results continue to hold in this richer setting, and we find two additional novel results. We provide concluding remarks in Section 5.

[8] See Antill and Grenadier (2019); Merlo and Wilson (1995); Merlo (1997); Merlo and Wilson (1998). While Antill and Grenadier (2019) provide a closed-form solution to a continuous-time stochastic-bargaining game, the solution involves the confluent hypergeometric function, which must be evaluated numerically.

[9] Bebchuk and Chang (1996) develop a theory in which neither rule successfully prevents plaintiffs from filing frivolous lawsuits.

[10] For example, Bebchuk (1988, 1996); Bebchuk and Klement (1998); Bebchuk (1998) model lawsuits that are filed solely to extract a settlement. Reinganum and Wilde (1986), Nalebuff (1987), and Spier (1992) model pretrial negotiations under asymmetric information.

2 Background

This section provides a brief background of litigation financing and explains some features of litigation that are relevant for our model.

2.1 Litigation financing and contingency fees

Litigation financing refers to the practice in which a third-party financier provides funding to a plaintiff, and its law firm, in exchange for a contingent claim on any settlement payment or trial award associated with the plaintiff's lawsuit. This practice was illegal, under United States champerty laws,[11] until the late twentieth century (Martin, 2004, 2008). Martin (2004) argues that champerty laws were motivated by a desire to discourage frivolous litigation. Champerty laws were first challenged in 1963 with NAACP v. Button,[12] in which the U.S. Supreme Court overturned a Virginia champerty law (Cohen, Malloy, and Powley, 2017). Similar rulings near the turn of the century further weakened champerty laws: for example, the Massachusetts prohibition on champerty ended in 1997 with Saladini v. Righellis.[13] Largely due to these legal developments, litigation financing became increasingly common in the 1990's and 2000's. By 2015, dozens of firms (including at least three publicly traded companies) provided over $3 billion in litigation financing (Cohen, Malloy, and Powley, 2017).

While champerty laws historically prevented third-party financing of lawsuits, lawyers have financed plaintiffs' lawsuits for decades through contingency fees. In a contingency-fee arrangement, the plaintiff pays its lawyer a fraction of any payoff associated with the lawsuit instead of a lump sum or hourly payment. Spier (2007) report that "the typical contract involves a fixed percentage, often 33%." Plaintiffs "universally" rely on these contingency-fee arrangements (Martin, 2004). Kakalik and Pace (1986) find that 96% of individual plaintiffs

[11] Martin (2004) notes that litigation financing agreements have typically satisfied usury laws because the plaintiff's obligation to pay the financier is contingent on a successful lawsuit.

[12] See NAACP v. Button, 371 U.S. 415 (1963) at https://supreme.justia.com/cases/federal/us/371/415/.

[13] See the Massachusetts Supreme Judicial Court ruling in Saladini v. Righellis, 687 N.E.2d 1224 (Mass. 1997) at https://casetext.com/case/saladini-v-righellis. While the Ohio Supreme Court ruled in 2003 that litigation financing constituted champerty, a 2008 Ohio law officially legalized litigation financing. See Rancman v. Interim Settlement Funding Corp., 99 Ohio St.3d 121, 2003-Ohio-2721, at http://www.supremecourt.ohio.gov/rod/docs/pdf/0/2003/2003-Ohio-2721.pdf and https://legiscan.com/OH/bill/HB248/2007 and https://publicintegrity.org/inequality-poverty-opportunity/states-are-battleground-in-drive-to-regulate-lawsuit-funding/ for further details.

and 86% of corporate plaintiffs in tort litigation paid their attorneys on a contingent-fee basis. In contrast, Kakalik and Pace (1986) report that defendants almost never use contingency fees.

Contingency fees encourage corporate plaintiffs to pursue litigation when they otherwise would refuse to expend time and resources on a risky lawsuit.[14] However, law firms are often unable to assume all of the risk associated with a large lawsuit, as described by Shepherd and Stone (2015): "law firms sufficiently sophisticated to handle major business litigation rarely will, or even can, accept contingency-fee cases. Law firms are notoriously illiquid and leveraged business entities." Shepherd and Stone (2015) report that litigation financiers target cases that are "prohibitively expensive or have a payoff too temporally remote for law firms to carry these potentially profitable suits themselves." Nonetheless, financiers "commonly require law firms to accept some risk through a partial contingency-fee arrangement" to align incentives (Shepherd and Stone, 2015).

2.2 Trials and pre-trial settlements

Both litigation-financing arrangements and contingency fees involve contingent claims on payoffs associated with a lawsuit. If the lawsuit ends in a trial, this payoff take the form of a trial award. However, many lawsuits are settled prior to trial with a settlement payoff for the plaintiff. Using a sample of federal cases in two districts, Eisenberg and Lanvers (2009) report that 67% of lawsuits end in a settlement. The rate varies across types of lawsuits, with 87% of tort cases ending in a settlement.

Settlement agreements allow parties to avoid costly trials. Cohen and Hagist (2020) report that "the costs associated with litigating a patent infringement claim in the U.S. ranged from $1 million to $4 million on average," while settling a lawsuit "typically cost a small company roughly $50,000." However, trials have some benefits over settlements. Farmer and Pecorino (1998) argue that plaintiffs enjoy a reputation benefit from going to trial because their future lawsuits will be taken more seriously. For example, Oasis Research, a serial plaintiff in intellectual property lawsuits, was unwilling to bargain over settlement terms in its patent lawsuit against Carbonite for fear of "any precedential effect that might have" in Oasis's other lawsuits (Cohen et al., 2018). As a result, the lawsuit ended in a trial.

[14] Contingent fees also enable efficient risk sharing (Danzon, 1983) and mitigate attorney moral-hazard problems (Rubinfeld and Scotchmer, 1993; Dana Jr and Spier, 1993).

Defendants also enjoy reputational benefits from trials. Indeed, Miceli (1993) report that some insurance companies express policies of never settling lawsuits, discouraging future lawsuits. Similarly, when Rapid7 was sued by Finjan for patent infringement, Rapid7 was hesitant to settle for fear of " signaling to other patent asserters how best to follow" (Cohen and Hagist, 2020). Likewise, Cloudfare refused to settle when it was sued by Blackbird because of a desire to make "patent trolls think twice before attempting to take advantage of the system."[15]

While a settlement may encourage future plaintiffs to file nuisance lawsuits, a hefty trial award can be more damaging for defendants. Cutler and Summers (1988) show that when a jury ruled Texaco had to pay Pennzoil more than \$10 billion, each dollar of transfer from Texaco to Pennzoil resulted in 30 cents of deadweight loss. The authors interpret the deadweight loss associated with this large trial award as reflecting financial-distress costs incurred by Texaco. Similarly, in a sample of lawsuits involving publicly traded plaintiffs and defendants, Bhagat, Brickley, and Coles (1994) find that the combined value of the plaintiff and the defendant drops by \$21 million on average after a lawsuit is filed.[16] The paper also shows that part of this loss "is explained by the costs of increased financial distress imposed on the defendant." A large public trial award can also create a perception that the defendant engaged in serious misconduct, creating a deadweight loss associated with a loss of customers.

3 Illustrative model

In this section, we use a simplified illustrative setting — a two-period model with out-of-court settlement determined by Nash bargaining — to derive our main results and provide intuition. In Section 4, we extend this setting using a continuous-time stochastic bargaining framework to realistically model settlement. Our results continue to hold in this richer setting, which delivers further results.

Our model involves two agents: a plaintiff ("agent P") with a potential lawsuit, and the defendant ("agent D") in the potential lawsuit. All agents are risk neutral and share a

[15]See `https://blog.cloudflare.com/the-project-jengo-saga-how-cloudflare-stood-up-to-a-patent-troll-and-won/`.

[16]In a similar study, Raghu et al. (2008) estimate that the average combined cumulative abnormal return of plaintiff and defendant firms at the time of litigation announcements is equal to -1.6%.

common discount rate r. Agents share common information, represented by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a continuous-time filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t \geq 0}$.

There are two periods in our illustrative model. At time zero, agents choose legal strategies. At an exogenous deterministic time $\mathcal{T}_T$, agents decide through Nash bargaining whether to settle out of court or go to trial. Payoffs are then realized and the game ends. Figure 1 provides a timeline summarizing this illustrative model. This model takes as given that the plaintiff has filed the lawsuit. In Section 3.7, we use the solution of this model to consider the plaintiff's filing decision.

3.1 Time-zero legal-strategy choices

At time zero, agents P and D (the plaintiff and defendant) choose legal strategies. Each agent j chooses a real-valued strategy ϕ^j from a set $\Phi^j \subset \mathbb{R}$. A higher value of ϕ^j corresponds to a stronger legal strategy, increasing agent j's future payoff. We provide details in Section 3.2. The strategy choice ϕ^j requires a payment $C^j(\phi^j)$, where $C^j : \Phi^j \to \mathbb{R}$ is an increasing function. This payment represents both the monetary cost of lawyers and the nonmonetary costs of employees' attention.

The plaintiff is financially constrained. We assume that the plaintiff has access to internal funds $I > 0$ and external funds $F \geq 0$ that allow the plaintiff to choose a strategy from the set:

$$B_F \equiv \{\phi^P \in \Phi^P : C^P(\phi^P) \leq I + F\}. \tag{1}$$

We model the impact of litigation financing as the impact of an increase in the exogenous parameter F.

In exchange for the external funds F, the plaintiff promises its financiers a fraction $\xi \geq 0$ of any payment that the plaintiff receives as a result of the lawsuit. If the plaintiff internally funds the lawsuit, then $\xi = 0$. A value $\xi > 0$ might be interpreted as a contingent-fee arrangement with the plaintiff's lawyer. It might also be interpreted as a contract with a litigation financing firm. We assume that ξ, which refer to as the financing terms, is an exogenous parameter. For additional realism, in the model of Section 4, we assume instead that ξ is endogenously determined to make the financier break even.

3.2 The expected trial award

If a settlement is not reached by an exogenous deterministic trial time $\mathcal{T}_T$, then the game ends in a trial. We let the random variable $\mathbb{X}(\omega, \phi^P, \phi^D) \geq 0$ denote the trial award that the defendant pays the plaintiff. If the defendant wins, then $\mathbb{X} = 0$. For any time $t \leq \mathcal{T}_T$, we let $X_t \equiv \mathbb{E}[\mathbb{X}|\mathcal{F}_t]$ denote the time-t expectation of the future trial award amount. We model X_t as a geometric Brownian motion:[17]

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \tag{2}$$

where B_t is a Brownian motion, $\sigma > 0$ is a volatility parameter, and $\mu < r$ is a drift parameter. The starting value X_0 of the expected trial award is determined by the time-zero legal-strategy choices:[18]

$$X_0 = \chi + \phi^P - \phi^D. \tag{3}$$

In this equation, $\chi > 0$ is an exogenous parameter representing the plaintiff's trial prospects and ϕ^P, ϕ^D are the time-zero strategy choices (Section 3.1). The plaintiff can thus increase the expected trial award through stronger legal strategy choices. Likewise, the defendant can decrease the expected trial award through stronger strategy choices.

3.3 Trial payoffs

We assume that the defendant incurs a deadweight loss in trial. The loss is equal to $\alpha\mathbb{X}$ for an exogenous parameter α. The loss is thus proportional to the trial award. The loss could reflect the real effects of a trial loss on the defendant's reputation, costs of financial distress, or the possibility of incentivizing future litigation (Section 2). The cost may also reflect direct legal expenses.[19]

To provide a motive for trials, we assume that the defendant and the plaintiff enjoy fixed

[17]Empirical evidence suggests that expected trial awards vary stochastically prior to trial. Prescott, Spier, and Yoon (2014) empirically analyze "reserve amounts" associated with insurance company claims, which are the "best guess of the claim's litigation value." They find that "a claim's reserve amount can change dramatically month to month as newly relevant information comes to light."

[18]We assume that $\chi \geq \sup \Phi^D$ so that $X_0 \geq 0$.

[19]Hersch and Viscusi (2007) show empirically that defense expenses are larger in cases featuring high ex-ante expected trial awards.

reputation benefits R^D and R^P, respectively, from going to trial. Section 2 motivates this assumption. These benefits can be viewed as the reputation benefit net of any fixed costs associated with trials. The parameters R^P, R^D can thus be negative if fixed costs outweigh the reputation benefits associated with trial.

Given these assumptions, immediately prior to trial at time $\mathcal{T}_T$, the plaintiff's expected trial payoff is equal to $R^P + (1-\xi)X_{\mathcal{T}_T}$. The defendant's corresponding expected payoff is $R^D - (1+\alpha)X_{\mathcal{T}_T}$. The joint surplus in trial is defined as the sum of the agents' payoffs:

$$\mathbb{S}_{\mathcal{T}_T} \equiv R^D + R^P - (\alpha+\xi)X_{\mathcal{T}_T}. \tag{4}$$

3.4 Simplified Settlement Bargaining

In Section 4, we assume that settlement can occur at any time according to optimal bargaining strategies in the equilibrium of a continuous-time stochastic bargaining game. In this baseline model, we provide intuition through a simpler stylized settlement model. We assume that an out-of-court settlement can only be reached at time $\mathcal{T}_T$, immediately prior to trial. We assume that settlement occurs if and only if the expected joint surplus in trial (the sum of the agents' trial payoffs) is negative. If settlement occurs, we assume that the defendant receives the sum of (i) its outside option of going to trial and (ii) a fraction ψ of the surplus created by avoiding trial. The defendant's expected payoff at time $\mathcal{T}_T$ is thus:

$$\mathbb{V}^D_{\mathcal{T}_T} \equiv R^D - (1+\alpha)X_{\mathcal{T}_T} + \psi \max\left(0,\, -\mathbb{S}_{\mathcal{T}_T}\right). \tag{5}$$

The first two terms reflect the defendant's incentive to decline any settlement that results in a lower expected payoff than its outside option of trial. The rest of the terms reflect the assumed settlement bargaining outcome: if the joint surplus $\mathbb{S}_{\mathcal{T}_T}$ in trial is negative, the defendant extracts a fraction ψ of the surplus created by avoiding trial.

We assume that the plaintiff similarly extracts a fraction $1-\psi$ of any surplus created by settlement. The plaintiff's expected payoff at time $\mathcal{T}_T$ is thus the following:

$$\mathbb{V}^P_{\mathcal{T}_T} \equiv R^P + (1-\xi)X_{\mathcal{T}_T} + (1-\xi)(1-\psi)\max\left(0,\, -\mathbb{S}_{\mathcal{T}_T}\right). \tag{6}$$

Equations (5) and (6) reveal that the defendant's and plaintiff's time-$\mathcal{T}_T$ payoffs are nonlinear in $X_{\mathcal{T}_T}$. Specifically, the option to settle leads to time-$\mathcal{T}_T$ payoffs that are analogous to call-option payoffs. If the realized expected trial award $X_{\mathcal{T}_T}$ is large, the joint surplus $\mathbb{S}_{\mathcal{T}_T}$ in trial is negative and the settlement option is "in the money." If the realization of $X_{\mathcal{T}_T}$ is small relative to the fixed reputation benefits R^P, R^D, then $\mathbb{S}_{\mathcal{T}_T}$ is positive and the settlement option is not exercised.

3.5 Equilibrium

An equilibrium of our illustrative model consists of legal strategy choices $\phi^{*,D}, \phi^{*,P}$ solving:

$$\phi^{*,P} = \text{argmax}_{\phi^P \in B_F}\, F - C^P(\phi^P) + \mathbb{E}\left[e^{-r\mathcal{T}_T}\mathbb{V}^P_{\mathcal{T}_T}\,|X_0 = \chi + \phi^P - \phi^{*,D}\right] \tag{7}$$

$$\phi^{*,D} = \text{argmax}_{\phi^D \in \Phi^D} -C^D(\phi^D) + \mathbb{E}\left[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}\,|X_0 = \chi + \phi^{*,P} - \phi^D\right]. \tag{8}$$

Recall that the plaintiff receives external funds F to partially offset the cost $C^P(\phi^P)$ of its strategy choice ϕ^P. The plaintiff then receives the expected discounted value of its future payoff. The defendant does not need to rely on external financing and pays the full cost $C^D(\phi^D)$ of its strategy choice ϕ^D.

Before considering equilibrium ϕ^D, ϕ^P choices, we first examine the defendant's expected discounted payoff $\mathbb{E}[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}\,|X_0 = x]$. We calculate this expectation in closed form in Appendix A. Using parameter assumptions described in Table 1, we calculate this expectation over a grid of x values. Figure 2 shows the result. The expected discounted payoff is a declining function of X_0 because of the first two terms in equation (5), which reflect the defendant's trial payoff. The convexity arises because of the option to settle, which is reflected by the final term in equation (5). Because of the negative slope and convexity of the line in Figure 2, the defendant's value function is less responsive to changes in X_0 for large X_0 values.

Given our assumptions, we can calculate the objectives in equations (7) and (8) in closed form. Using these solutions, we solve for equilibrium strategy choices $\phi^{*,D}, \phi^{*,P}$ by searching numerically for values satisfying equations (7) and (8). Appendix A provides details.

3.6 Litigation financing and joint surplus

This section presents our first main result: litigation financing discourages defendants from overspending on defense, potentially improving the joint surplus of litigants. To show this result, we vary the exogenous parameter F over a grid of values and calculate equilibrium strategy choices corresponding to each F value.[20] Panel (a) of Figure 3 shows the result. The orange line plots the plaintiff's equilibrium strategy $\phi^{*,P}$, on the y axis, corresponding to each level of available financing F, on the x axis. The orange line increases in F, implying that the plaintiff is financially constrained — it optimally chooses the strongest feasible strategy $\phi^{*,P} = \sup B_F$ for each level of financing F.

For low values of $\phi^{*,P}$, the defendant optimally chooses the strongest available strategy $\phi^{*,D} = \sup \Phi^D$. However, Figure 3(a) shows that for sufficiently strong plaintiff strategies $\phi^{*,P}$, the defendant instead chooses the weakest strategy $\phi^{*,D} = \inf \Phi^D$. Litigation financing thus disincentives defense spending. This effect arises because the option to settle convexifies the defendant's value function (Figure 2). Intuitively, the defendant is less willing to spend money to lower X_0 if X_0 is high, because the defendant knows that a settlement is likely in such a case. The defendant's payoff is still sensitive to changes in $X_{\mathcal{T}_T}$ in a settlement, since such changes affect the bargaining outcome. However, the defendant is nonetheless less sensitive to changes in $X_{\mathcal{T}_T}$ when it expects that a trial is unlikely. Thus, because of the settlement option, the defendant is less willing to pay for stronger strategies when X_0 is high. Since litigation financing allows the plaintiff to choose a stronger legal strategy, increasing X_0, financing thus disincentives defense spending. Excluding unusual parameter values for which there are multiple equilibria, this result holds generically.

Proposition 1. *Let F_0, F_1 be distinct values of the parameter F and fix arbitrary values of all other parameters. Suppose that there is a unique equilibrium $\{\phi_j^{*,D}, \phi_j^{*,P}\}$ associated with each value F_j. If $F_0 < F_1$, then $\phi_1^{*,D} \leq \phi_0^{*,D}$.*

We next study the effect of litigation financing on the equilibrium joint surplus of plaintiffs and defendants, which we define as the sum of the two agents' payoffs:

$$\mathbb{S}_0 \equiv F + \sum_{j=P,D} \left(-C^j(\phi^{*,j}) + \mathbb{E}\left[e^{-r\mathcal{T}_T} \mathbb{V}^j_{\mathcal{T}_T} \,|X_0 = \chi + \phi^{*,P} - \phi^{*,D} \right] \right). \tag{9}$$

[20] We continue to use the parameter assumptions in Table 1.

The blue line in panel (b) of Figure 3 plots $\mathbb{S}_0$ as a function of the available financing F. Some increases in F lead to lower joint surplus. This is because litigation financing leads to stronger plaintiff strategies (Figure 3 (a)). These stronger plaintiff strategies increase trial awards, mechanically increasing the proportional deadweight loss suffered by defendants. However, stronger plaintiff strategies endogenously discourage defense spending (Figure 3(a)), which increases joint surplus. Thus, even though litigation financing has no direct social benefit in our model, it nonetheless can improve joint surplus in equilibrium.

3.7 Litigation financing and frivolous cases

This section presents our second main result: litigation financing does not necessarily encourage the filing of frivolous lawsuits. We define a frivolous lawsuit as one that has poor expected trial prospects (low χ) but high enough volatility (σ) to nonetheless create the possibility of a large trial award.

To model the plaintiff's filing decision, we fix an equilibrium $(\phi^{*,P}, \phi^{*,D})$ and define the plaintiff's and financier's time-zero expected payoffs:

$$\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D}) \equiv F - C^P(\phi^{*,P}) + \mathbb{E}\left[e^{-r\mathcal{T}_T}\mathbb{V}_{\mathcal{T}_T}^P \,\middle|\, X_0 = \chi + \phi^{*,P} - \phi^{*,D}\right] \tag{10}$$

$$\mathbb{V}_0^F(\phi^{*,P}, \phi^{*,D}) \equiv -F + \mathbb{E}\left[e^{-r\mathcal{T}_T}\left(\xi X_{\mathcal{T}_T} + \xi(1-\psi)\max\left[0, -\mathbb{S}_{\mathcal{T}_T}\right]\right) \,\middle|\, X_0 = \chi + \phi^{*,P} - \phi^{*,D}\right]. \tag{11}$$

The financier's payoff follows from equation (6) and the fact that the financier receives a fraction χ of the plaintiff's settlement payoff or trial award. We assume that the plaintiff chooses to file if there is an equilibrium $(\phi^{*,P}, \phi^{*,D})$ in which $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D}) \geq 0$ and $\mathbb{V}_0^F(\phi^{*,P}, \phi^{*,D}) \geq 0$. The plaintiff thus files if it expects a weakly positive equilibrium payoff and the financier is willing to provide funds.

Before studying the filing decision, it is helpful to understand what types of lawsuits tend to have weakly positive plaintiff payoffs. Figure 4 shows that the plaintiff's equilibrium payoff $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D})$ is increasing in χ, σ, and F. High χ values correspond to high initial expected trial awards, which benefit plaintiffs through higher time-$\mathcal{T}_T$ payoffs. By the same logic, high F values benefit the plaintiff because they enable the plaintiff to increase X_0 through stronger strategies. Higher σ values increase the value of the option to settle, benefiting both the plaintiff and defendant.

To study the types of lawsuits that litigation financing encourages, we conduct the following exercise. Varying the parameters χ and σ, we calculate the smallest value $\hat{F}(\chi, \sigma)$ such that the plaintiff files in equilibrium. Figure 4(c) implies that $\hat{F}(\chi, \sigma)$ is well defined: if the plaintiff's equilibrium payoff is weakly positive for $\hat{F}(\chi, \sigma)$, it will be positive for $F > \hat{F}(\chi, \sigma)$. We confirm that the financier's equilibrium payoff $\mathbb{V}_0^F(\phi^{*,P}, \phi^{*,D})$ also increases in F. Figure 5 plots $\hat{F}(\chi, \sigma)$. Blue and yellow regions correspond to low and high values of $\hat{F}(\chi, \sigma)$, respectively.

By definition, if the quantity of available financing increases from F to F', it will lead to the filing of lawsuits in which $F < \hat{F}(\chi, \sigma) < F'$. Figure 5 shows that these tend to be lawsuits with low values of χ and σ. This is consistent with Figure 4: lawsuits with high values of χ, σ are more profitable for plaintiffs and thus do not require financing to be viable. Our second main result follows from comparing the lower right corner of Figure 5 to the lighter middle region. Litigation financing leads to some lawsuits (the lighter middle region) with lower σ and higher χ values than some lawsuits that are filed without financing (the lower right corner).

In this sense, litigation financing does not necessarily lead to frivolous lawsuits. Litigation financing actually encourages the filing of less risky lawsuits, because riskier lawsuits are viable without financing. Some of these less risky lawsuits will actually have better trial prospects (higher χ) than lawsuits that are filed without financing.

4 Stochastic bargaining model

In this section, we extend the model of Section 3 using a continuous-time stochastic bargaining model. The results of the previous section continue to hold in this more realistic framework. The stochastic bargaining framework also delivers two additional results. First, we show that litigation financing leads to faster settlement. Second, we show that greater volatility has a nonmonotonic effect on the cost of litigation financing.

4.1 Setup

Just as in Section 3, the risk neutral agents D and P choose time-zero strategies $\phi^D \in \Phi^D$ and $\phi^P \in B_F$ (equation (1)). Strategy choices entail costs $C^P(\phi^P)$ and $C^D(\phi^D)$. These strategies determine the initial expected trial award X_0 by equation (3). The expected trial award X_t then evolves as a geometric Brownian motion (equation (2)). The plaintiff's expected trial award at the trial time $\mathcal{T}_T$ is equal to $R^P + (1-\xi)X_{\mathcal{T}_T}$. The defendant's corresponding expected payoff is equal to $R^D - (1+\alpha)X_{\mathcal{T}_T}$. Thus, in this section, we maintain all of the assumptions of Section 3.1, Section 3.2, and Section 3.3.

However, we assume a settlement bargaining process that is very different from the one in Section 3.4. We assume that the plaintiff and defendant bargain in continuous time. A settlement can thus be reached in any moment prior to trial. We also assume that the settlement payoffs are determined by the endogenous bargaining outcome of a Markov perfect equilibrium (MPE). We provide details in Section 4.2. For tractability, we assume that the trial time $\mathcal{T}_T$ is an exponential random variable with mean $\mathbb{E}[\mathcal{T}_T] = 1/\lambda$ for a parameter λ. We assume that $\mathcal{T}_T$ is independent of all other random variables.

Finally, we make two further changes for additional realism. First, we assume that each agent i incurs a flow cost $\vartheta^i dt$ per unit time, where $\vartheta^D, \vartheta^P \geq 0$ are parameters. These flow costs represent the resources and attention that the plaintiff and defendant must dedicate to the lawsuit during settlement negotiations. Second, we assume that the financing terms, the fraction ξ of the plaintiff's payment that goes to the financier, are determined endogenously at time zero such that the financier breaks even. We discuss this further in Section 4.4. Figure 6 provides a timeline summarizing this model.

4.2 Stochastic bargaining assumptions

Prior to the trial, the plaintiff and the defendant may choose to end the lawsuit with a settlement. We assume that settlement negotiations occur after the financing terms ξ and strategy choices ϕ^D, ϕ^P are determined at time zero. We model the settlement negotiation as a stochastic-bargaining game.[21] At any time $t < \mathcal{T}_T$, exactly one agent is the proposer. The proposer in any instant is given exogenously by a time-homogeneous two-state Markov chain s_t. When $s_t = P$, only the plaintiff may propose a settlement. When $s_t = D$, only

[21]See Merlo and Wilson (1995).

the defendant may propose a settlement. We assume that $s_t, \mathcal{T}_T$, and $\{X_t\}_{t\geq 0}$ are mutually independent. We assume that s_t transitions from state j to state $i \neq j$ with constant intensity ρ^j, $j = P, D$. The stochastic-proposer bargaining protocol is standard in the literature (see Antill and Grenadier (2019) for a review). The rates of transitions are a tractable representation of bargaining power. In this setting, agent i has a strong bargaining position if ρ^i, the rate of transition away from state i, is low, and if the corresponding rate of transition ρ^j into state i is high.

At each time $t < \mathcal{T}_T$, the proposer may make a settlement offer to the other agent, the receiver. If the receiver accepts the offer, then the game ends immediately. We model a settlement offer at time t as a real number Y_t, which has the following interpretation. If the settlement offer Y_t is accepted, then: (i) the plaintiff receives a payoff equal to $(1-\xi)Y_t$; (ii) the financier receives a payoff equal to ξY_t; and (iii) the defendant's payoff is equal to $-Y_t$. We assume that settlement does not entail any costs.[22] We let $\mathcal{T}_S$ denote the equilibrium settlement time.

We will show that settlement does not always occur immediately in equilibrium. Delaying settlement allows parties to observe whether a trial will be jointly beneficial relative to settlement. A trial is beneficial if the reputational benefits R^P, R^D outweigh the proportional costs described in Section 3.3. In equilibrium, the parties settle when the expected trial award, and the associated proportional costs, are large relative to the reputational benefits.

After the time-zero legal-strategy choices, the plaintiff and the defendant form strategies in the dynamic-bargaining game.

Definition 1. *A strategy for agent i consists of*

1. *A real-valued process A_t^i representing the lowest payoff that agent i will accept at time t if it is the receiver. Thus, the plaintiff will only accept a settlement offer with $(1-\xi)Y_t \geq A_t^P$, and the defendant will only accept a settlement offer with $-Y_t \geq A_t^D$.*

2. *A stopping time τ^i with $s_{\tau^i} = i$ such that agent i makes a settlement offer at time τ^i, offering $A^j_{\tau^i}$ to agent j.*

Our definition of a strategy rules out some obviously suboptimal behavior. For example, it rules out the possibility that agent i offers more than A_t^j to agent j. Likewise, it rules out

[22]The costs and benefits of trial discussed in Section 3.3 may be interpreted as the costs and benefits relative to settlement.

the possibility that agent i is willing to accept an offer x but unwilling to accept an offer $y > x$ in the same instant.[23]

For any fixed strategies $\{A_t^i, \tau^i\}_{i\in\{P,D\}}$ for the plaintiff and the defendant, we define the corresponding settlement payoff for each agent as follows:

$$J_t^P \equiv \mathbf{1}\ \big(\ s_t = P\ \big)\ (1-\xi)\ \bigg(\ -A_t^D\ \bigg)\ +\mathbf{1}\ \big(\ s_t = D\ \big)\ A_t^P \tag{12}$$

$$J_t^D \equiv \mathbf{1}\ \big(\ s_t = D\ \big)\bigg(\ -\frac{1}{1-\xi}A_t^P\ \bigg)\ +\mathbf{1}\ \big(\ s_t = P\ \big)\ A_t^D. \tag{13}$$

4.3 Stochastic bargaining equilibrium

We solve the model working backward in time. In this section, we calculate equilibrium bargaining strategies $\{A_t^i, \tau^i\}_{i=P,D}$ in the post-strategy-choice bargaining game. In Section 4.4, we solve for the equilibrium ξ value and optimal time-zero strategy choices, given the subsequent bargaining equilibrium.

In the continuous-time bargaining game, agent i takes the strategy of the other agent as given and chooses an admissible strategy $\{A_t^i, \tau^i\}$ to solve:

$$\begin{aligned} V^i(X,s) = \sup_{A^i,\tau^i} \mathbb{E}^{(X,s)}\big[-\int_0^{\tau^j\wedge\tau^i\wedge\mathcal{T}_T} e^{-rs}\vartheta^i ds + \mathbf{1}(\tau^j\wedge\tau^i \le \mathcal{T}_T)e^{-r\tau^j\wedge\tau^i} J^i_{\tau^j\wedge\tau^i} \\ + \mathbf{1}(\tau^j\wedge\tau^i > \mathcal{T}_T)e^{-r\mathcal{T}_T}\bigg(\ R^i + X_{\mathcal{T}_T}\big[(1-\xi)\mathbf{1}(i=P) - (1+\alpha)\mathbf{1}(i=D)\big]\ \bigg)\big], \end{aligned} \tag{14}$$

subject to equations (12)-(13) and the constraint that $s_{\tau^i} = i$. We define an admissible strategy as one for which the expectation in (14) exists. Following Antill and Grenadier (2019), we define an equilibrium of the dynamic-bargaining game as follows:

Definition 2. *An equilibrium (MPE) is a collection of strategies $\{A^i, \tau^i\}_{i\in\{P,D\}}$ such that*

1. *For each i, taking the strategy $\{A^j, \tau^j\}$ of agent j as given, the strategy (A^i, τ^i) solves problem (14).*

2. *For each i, letting $V^i(X,s)$ denote the value function for problem (14), $A_t^i = V^i(X_t, s_t)$.*

[23] Additionally, the fact that A_t^i is real valued rules out the possibility that in some instant t and state ω agent i is unwilling to accept any offer. This turns out to be without loss of generality.

Condition 1 ensures that the equilibrium strategies correspond to a Nash equilibrium in stationary strategies for any starting values. Condition 2 is our notion of subgame perfection in continuous time: agents must optimally accept offers if and only if the offer exceeds their continuation value in the equilibrium.

We characterize a MPE in which the proposer makes an offer, which the receiver accepts, the first time that X_t exceeds a threshold $\overline{X}$. To solve for this MPE, we conjecture that each agent's value function satisfies particular Hamilton-Jacobi-Bellman (HJB) equations. We calculate such value functions in closed form, then verify that these value functions indeed correspond to an MPE. The following proposition, which is proved in Appendix B, characterizes the resulting MPE.

Proposition 2. *Let ζ, κ denote the positive and negative roots of*

$$r + \lambda + \rho^D + \rho^P - \mu z - \frac{\sigma^2}{2} z(z-1) = 0.$$

Let β, γ denote the positive and negative roots of the quadratic

$$r + \lambda - \mu z - \frac{\sigma^2}{2} z(z-1).$$

Define

$$\begin{aligned}
\bar{\vartheta} &\equiv \vartheta^P/(1-\xi) + \vartheta^D \\
\bar{R} &\equiv R^P/(1-\xi) + R^D \\
\overline{X} &\equiv \left(\frac{\lambda \bar{R} - \bar{\vartheta}}{r+\lambda} \right) \frac{\beta}{\beta - 1} \frac{r + \lambda - \mu}{\lambda \alpha} \\
\nu_1 &\equiv \frac{\rho^D \left(\lambda R^P - \vartheta^P + (1-\xi)(\lambda R^D - \vartheta^D) \right)}{(\rho^P + \rho^D)(r + \lambda + \rho^P + \rho^D)} \\
\nu_2 &\equiv \frac{\rho^P(\lambda R^P - \vartheta^P) - (1-\xi)\rho^D(\lambda R^D - \vartheta^D)}{(\rho^P + \rho^D)(r+\lambda)}
\end{aligned}$$

$$\begin{aligned}
\eta_1 &\equiv \frac{-\rho^D \lambda (1-\xi)\alpha}{(\rho^P + \rho^D)(r + \lambda + \rho^P + \rho^D - \mu)} \\
\eta_2 &\equiv \lambda \frac{\rho^P(1-\xi) + (1-\xi)\rho^D(1+\alpha)}{(\rho^P + \rho^D)(r + \lambda - \mu)},
\end{aligned}$$

and

$$\begin{bmatrix} H_1^P \\ H_2^P \\ K_1 \\ K_2 \end{bmatrix} \equiv \begin{bmatrix} -\frac{\rho^P}{\rho^D}\overline{X}^{\zeta} & \overline{X}^{\beta} & \frac{\rho^P}{\rho^D}\overline{X}^{\kappa} & -\overline{X}^{\gamma} \\ -\frac{\rho^P}{\rho^D}\zeta\overline{X}^{\zeta-1} & \beta\overline{X}^{\beta-1} & \frac{\rho^P}{\rho^D}\kappa\overline{X}^{\kappa-1} & -\gamma\overline{X}^{\gamma-1} \\ \overline{X}^{\zeta} & \overline{X}^{\beta} & -\overline{X}^{\kappa} & -\overline{X}^{\gamma} \\ \zeta\overline{X}^{\zeta-1} & \beta\overline{X}^{\beta-1} & -\kappa\overline{X}^{\kappa-1} & -\gamma\overline{X}^{\gamma-1} \end{bmatrix}^{-1}$$
$$\times \begin{bmatrix} -\frac{\lambda R^P-\vartheta^P}{r+\lambda} - \frac{\lambda(1-\xi)\overline{X}}{r+\lambda-\mu} - \frac{\rho^P}{\rho^D}(\nu_1+\eta_1\overline{X}) + \nu_2 + \eta_2\overline{X} \\ -\frac{\lambda(1-\xi)}{r+\lambda-\mu} - \frac{\rho^P}{\rho^D}\eta_1 + \eta_2 \\ -\frac{\lambda R^P-\vartheta^P}{r+\lambda} - \frac{\lambda(1-\xi)\overline{X}}{r+\lambda-\mu} + \nu_1 + \eta_1\overline{X} + \nu_2 + \eta_2\overline{X} \\ -\frac{\lambda(1-\xi)}{r+\lambda-\mu} + \eta_1 + \eta_2 \end{bmatrix}.$$

Then there exists a MPE with value functions

$$V^P(X,P) \equiv \begin{cases} -\frac{\rho^P}{\rho^D}H_1^P X^{\zeta} + H_2^P X^{\beta} + \frac{\lambda R^P-\vartheta^P}{r+\lambda} + \frac{\lambda(1-\xi)X}{r+\lambda-\mu} & X < \overline{X} \\ -\left(\frac{\rho^P}{\rho^D}K_1 X^{\kappa} - K_2 X^{\gamma} + \frac{\rho^P}{\rho^D}(\nu_1+\eta_1 X) - \nu_2 - \eta_2 X \right) & X \geq \overline{X}, \end{cases} \tag{15}$$

$$V^P(X,D) \equiv \begin{cases} H_1^P X^{\zeta} + H_2^P X^{\beta} + \frac{\lambda R^P-\vartheta^P}{r+\lambda} + \frac{\lambda(1-\xi)X}{r+\lambda-\mu} & X < \overline{X} \\ K_1 X^{\kappa} + K_2 X^{\gamma} + \nu_1 + \nu_2 + (\eta_1+\eta_2)X & X \geq \overline{X}, \end{cases} \tag{16}$$

$$V^D(X,s) \equiv \begin{cases} \frac{-V^P(X,s)}{1-\xi} + \left[\overline{X}^{-\beta} \left(\frac{\lambda\alpha\overline{X}}{r+\lambda-\mu} - \frac{-\bar{\vartheta}+\lambda\bar{R}}{r+\lambda} \right) X^{\beta} + \frac{-\bar{\vartheta}+\lambda\bar{R}}{r+\lambda} - \frac{\lambda\alpha X}{r+\lambda-\mu} \right] & X < \overline{X} \\ \frac{-V^P(X,s)}{1-\xi} & X \geq \overline{X}, \end{cases} \tag{17}$$

and strategies

$$A_t^i \equiv V^i(X_t, s_t) \tag{18}$$
$$\tau^i = \inf\{t : X_t \geq \overline{X}, s_t = i, A_t^j \leq V^j(X_t, s_t)\}. \tag{19}$$

4.4 Completing the equilibrium

A complete equilibrium of the model consists of (i) bargaining strategies $\{A^i_t, \tau^i\}_{i=P,D}$ that comprise an equilibrium of the stochastic bargaining game; (ii) financing terms ξ; and (iii) legal strategies choices $\phi^{*,D}, \phi^{*,P}$ such that:

$$\phi^{*,P} = \text{argmax}_{\phi^P \in B_F}\, F - C^P(\phi^P) + V^P(\chi + \phi^P - \phi^{*,D}\,,\, s_0) \tag{20}$$

$$\phi^{*,D} = \text{argmax}_{\phi^D \in \Phi^D}\, -C^D(\phi^D) + V^D(\chi + \phi^{*,P} - \phi^D\,,\, s_0) \tag{21}$$

$$F = V^F(\chi + \phi^{*,P} - \phi^{*,D}\,,\, s_0), \tag{22}$$

where $V^F(X,s)$ is the financier's expected payoff given the bargaining strategies:

$$V^F(X,s) = \mathbb{E}^{(X,s)}\big[\mathbf{1}(\tau^P \wedge \tau^D \leq \mathcal{T}_T)e^{-r\tau^P\wedge\tau^D}\frac{\xi}{1-\xi}J^P_{\tau^P\wedge\tau^D} + \mathbf{1}(\tau^P \wedge \tau^D > \mathcal{T}_T)e^{-r\mathcal{T}_T}\xi X_{\mathcal{T}_T}\big]. \tag{23}$$

This equilibrium definition is analogous to the one in Section 3.5. Each strategy choice ϕ^i must be optimal given the choice ϕ^j of the other agent and given the equilibrium of the subsequent settlement bargaining game (Proposition 2). Equation (22) is a novel condition implying that the financier must break even. The value F of the financing provided must equal the expected value of the financier's stake in the plaintiff's lawsuit. We calculate $V^F(X,s)$ in closed form in Appendix B.5. Given the closed-form solutions for $V^P(X,s)$ and $V^D(X,s)$ (Proposition 2), we solve numerically for values $\xi, \phi^{*,D}, \phi^{*,P}$ that satisfy equations (20)-(22).

In Appendix C, we show that our main results (Sections 3.6 and 3.7) continue to hold in this more realistic model. We now show that this stochastic-bargaining model provides two additional results.

4.5 Litigation financing expedites lawsuits

In this section, we show that litigation financing leads to faster settlement, expediting lawsuits. To show this result, we vary the exogenous parameter F over a grid of values and calculate an equilibrium (Section 4.4) corresponding to each F value. For each equilibrium, we simulate the expected time until resolution by settlement or trial:[24]

$$\text{Time to resolution} \equiv \mathbb{E}\big[\mathcal{T}_S \wedge \mathcal{T}_T \mid X_0 = \chi - \phi^{*,D} + \phi^{*,P}\ ,\ s_0\big]. \tag{24}$$

Panel (a) of Figure 7 shows the result. When the available financing F increases, the plaintiff uses the financing to choose a costlier equilibrium strategy: $\phi^{*,P}$ increases. This increases the initial expected trial award X_0. Since X_0 is higher, the process X_t reaches the endogenous settlement threshold $\bar{X}$ faster in expectation. As a result, settlement occurs more quickly when F is larger. Because there is no offsetting delay in trials, Figure 7(a) shows that the time to resolution declines as F increases. Panel (b) of Figure 7 shows an obvious corollary: equilibrium expected flow costs decline when F increases.

4.6 The role of volatility in stochastic bargaining

In this section, we show that the volatility of a lawsuit has a nonmonotonic effect on the plaintiff's payoff and the financing terms ξ. In our illustrative model, we find that the plaintiff's payoff is a monotonically increasing function of the lawsuit volatility (Figure 4 (b)). The plaintiff and the defendant share an option to settle, which becomes more valuable with increases in volatility. This same effect exists in our continuous-time model of litigation.

However, volatility plays an additional role in our continuous-time stochastic bargaining model: it determines the equilibrium settlement time. As volatility increases, the option to wait — to see how the expected trial award evolves — becomes more valuable. As a result, increases in volatility delay settlement in equilibrium. To show this formally, we vary the exogenous parameter σ over a grid of values and calculate an equilibrium (Section 4.4) corresponding to each σ value. For each equilibrium, we simulate the expected time to resolution (equation (24)).[25] Figure 8(a) shows the result, confirming that increases in volatility delay settlement and time to resolution.

By delaying settlement, increases in volatility can harm the plaintiff. If the plaintiff begins the bargaining as the proposer ($s_0 = P$), then an early settlement offer is more likely to be proposed by the plaintiff. The plaintiff fares better in settlement offers that it proposes

[24]We use the parameter values listed in Table D.1. Appendix D describes the simulation methodology.

[25]We use the parameter values listed in Table D.2.

because the defendant's outside option is weaker in such settlements: the defendant must wait to become the proposer to counteroffer. This implies that, for some parameter values, the plaintiff benefits from early settlement. Since increases in volatility delay settlement (Figure 8(a)), such increases can harm the plaintiff. We show this formally in Figure 8(b). As volatility increases on the x axis, the plaintiff's value function first declines because of the delay in settlement. As volatility further increases, settlement is delayed so long that the plaintiff's initial bargaining position becomes irrelevant. At this point, the plaintiff begins to benefit from increases in volatility by the same intuition described above (Figure 4(b)). The plaintiff's equilibrium payoff is thus a nonmonotonic function of the lawsuit riskiness σ. This novel result can only be demonstrated in a stochastic bargaining model like ours. A simpler model like the one in Section 3 cannot capture the dynamic fluctuations in the plaintiff's bargaining strength and incentive to settle that drive the result.

Finally, we note that the financier's payoff is positively correlated with the plaintiff's payoff. The financier's stake in the plaintiff's future payoff is more valuable when that payoff is large. We formalize this in Appendix B.5. If an increase in volatility causes the plaintiff's equilibrium payoff to decline (Figure 8(b)), then ξ must increase to make the financier break even. Likewise, ξ must decrease if the plaintiff's payoff increases. Figure 8(c) shows that the equilibrium value of ξ, shown on the y axis, can increase or decrease when volatility increases on the x axis. This implies that the equilibrium cost ξ of litigation financing is a nonmonotonic function of lawsuit risk. This novel prediction implies that it might be difficult to empirically document a link between lawsuit riskiness and the use of litigation financing.[26]

5 Conclusion

We build a theoretical model of third-party litigation financing. In our litigation model, a plaintiff imposes a negative externality on a defendant, who loses more than the plaintiff gains in trial. By strengthening plaintiffs, litigation financing directly exacerbates this externality. One would thus expect that litigation financing reduces the joint surplus of the plaintiff and defendant. However, we show that litigation financing changes the defendant's equilibrium

[26] Because some increases in volatility benefit the plaintiff, we continue to find that some lawsuits requiring litigation financing are less risky than lawsuits that do not require financing. We show this in Figure C.3.

behavior, leading to surprising consequences. First, we show that litigation financing deters defendants from engaging in wasteful bullying strategies. Because of this effect, litigation financing can actually improve the joint surplus of plaintiffs and defendants. Second, we show that litigation financing does not lead to the filing of risky frivolous lawsuits. In fact, litigation financing encourages lawsuits that are less risky than lawsuits that are filed without financing.

We extend this setting using a novel continuous-time model of litigation. In this richer model, we find that litigation financing expedites the resolution of lawsuits. We also show that volatility plays a surprising role in dynamic settlement bargaining. Higher volatility can be beneficial or harmful for plaintiffs and can lead to higher or lower costs of financing. More generally, our model provides a framework for studying the arrival of new information during settlement negotiation.

In a study of civil lawsuits in Arizona, 70% of lawsuits are settled out of court within 30 days of the trial date (Williams, 1983). The same study finds that 13% of lawsuits are settled on the day of trial. This phenomenon, sometimes called "settling on the court steps," is wasteful to the extent that legal fees associated with lengthy settlement negotiations could be avoided by faster resolution. Our model implies that costly delays in settlement can be prevented by strengthening plaintiffs and leveling the playing field. The potential for litigation financing to expedite lawsuits and deter defendant spending could be relevant for future regulation.

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Table 1: Parameter values for illustrative model

This table shows our baseline parameter values for the model of Section 3.

Parameter	Description	Value
r	Discount rate	0.05
μ	Drift of X_t	0.00
σ	Volatility of X_t	0.25
$\mathcal{T}_T$	Time until trial	5
α	Defendant's proportional trial cost	1.5
R^P	Plaintiff's net benefit of trial	0
R^D	Defendant's net benefit of trial	2.66
ψ	Defendant's bargaining strength	0.9
ξ	Financier's share of plaintiff's payoff	0.05
χ	Baseline value of X_0	1
I	Plaintiff's internal funds	0.0001
$C^D(\phi^D)$	Defendant's cost function	$1.35\ \phi^D$
$C^P(\phi^P)$	Plaintiff's cost function	$0.01\ \phi^P$
Φ^P	Plaintiff's strategy set	$[0, 0.5]$
Φ^D	Defendant's strategy set	$[0, 0.5]$
F	Baseline level of available financing	0

Figure 1: Illustrative model timeline

This figure shows the timeline for our illustrative model.

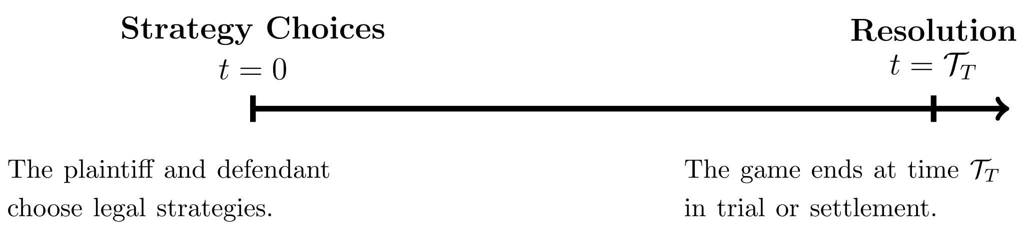


Figure 2: The defendant's value function

The blue line shows the defendant's expected discounted future payoff $\mathbb{E}[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}|X_0]$ on the y axis as a function of X_0 on the x axis. We assume the parameter values listed in Table 1.

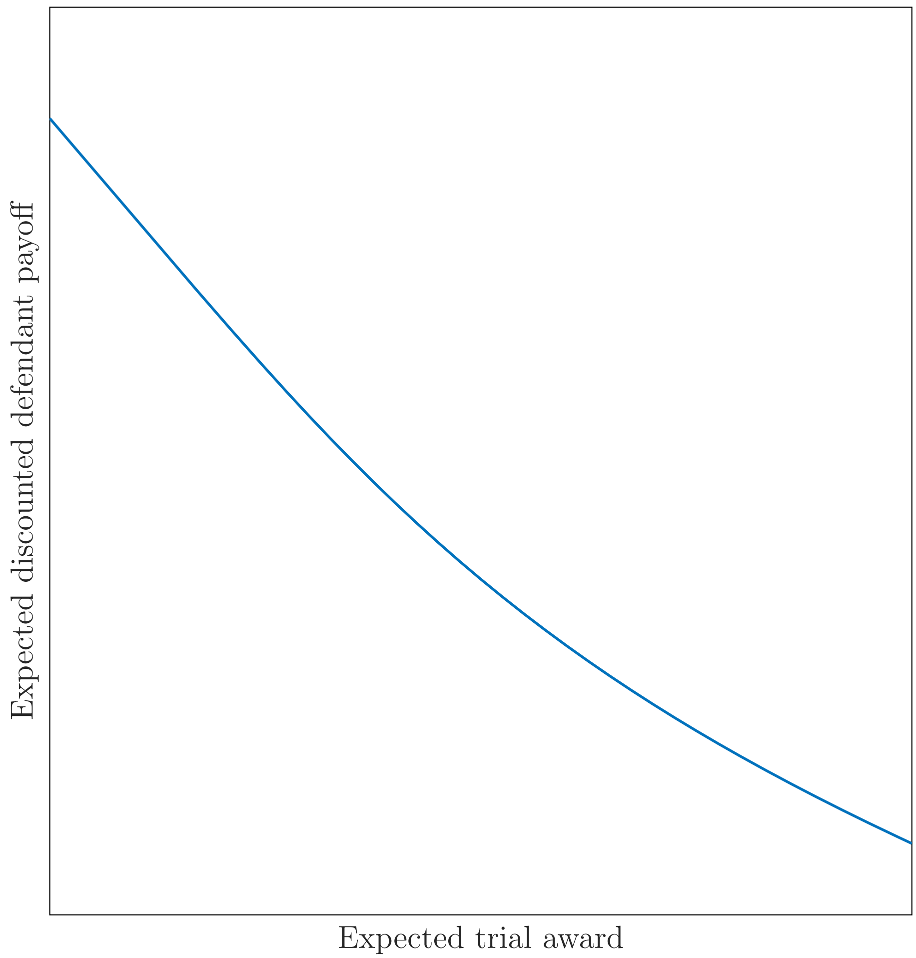
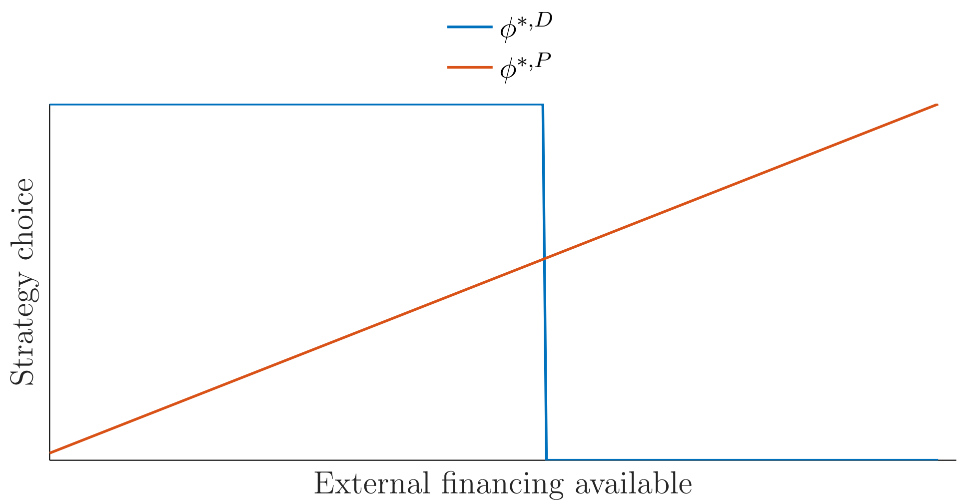
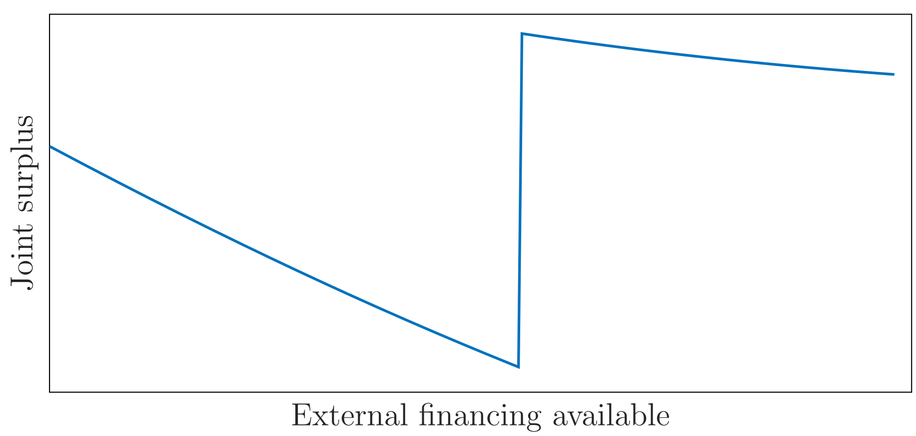


Figure 3: Financing and joint surplus

Starting with the parameter values listed in Table 1, we increase the parameter F that represents external financing available and numerically solve for equilibrium strategies $(\phi^{*,D}, \phi^{*,P})$ satisfying equations (7) and (8). In panel (a), the orange and blue lines show $\phi^{*,P}$ and $\phi^{*,D}$, respectively, on the y axis as a function of F on the x axis. In panel (b), the blue line shows the joint surplus $\mathbb{S}_0$ on the y axis as a function of F on the x axis.



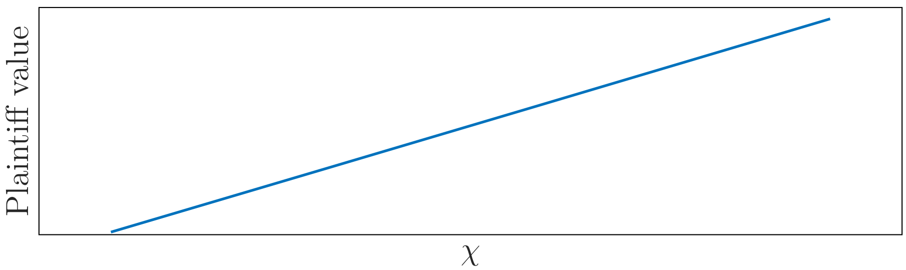
Panel (a) Equilibrium strategy choices as a function of F



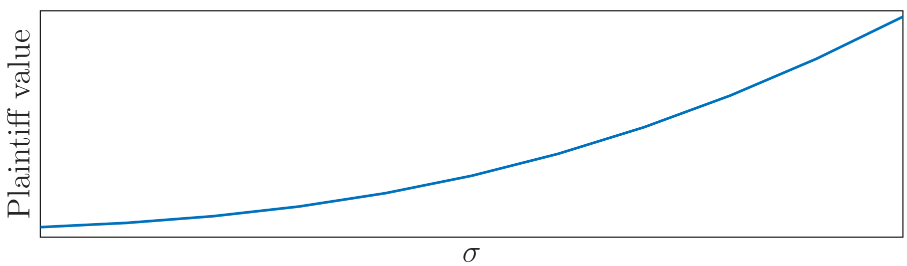
Panel (b) Equilibrium joint surplus as a function of F

Figure 4: Plaintiff value

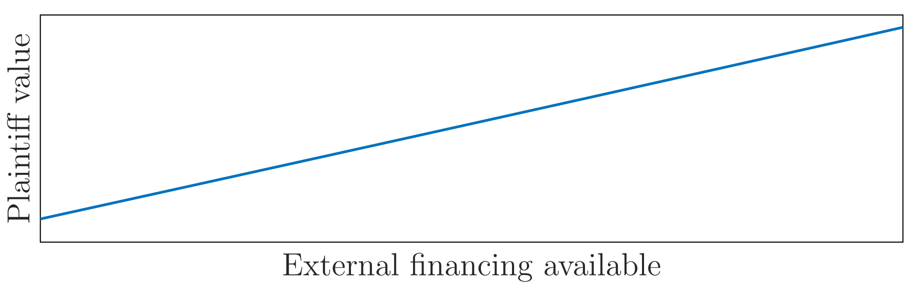
Starting with the parameter values listed in Table 1 and $F = 0$, we vary the parameters χ, σ, and F and numerically solve for equilibrium strategies $(\phi^{*,D}, \phi^{*,P})$ satisfying equations (7) and (8). In panel (a), the blue line shows $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D})$ on the y axis as a function of χ on the x axis. Panels (b) and (c) show $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D})$ as a function of σ and F, respectively.



Panel (a) $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D})$ as a function of χ



Panel (b) $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D})$ as a function of σ



Panel (c) $\mathbb{V}_0^P(\phi^{*,P}, \phi^{*,D})$ as a function of F.

Figure 5: Financing and frivolous cases

Starting with the parameter values listed in Table 1 (except $C^P(x) = .4 + .01x$ and $I = .40001$), we vary the parameters χ, σ and F and numerically solve for equilibrium strategies $(\phi^{*,D}, \phi^{*,P})$ satisfying equations (7) and (8). For each (χ, σ) pair, we calculate the smallest F value such that the plaintiff files (Section 3.7). This figure shows a heat map in which, for each χ on the y axis and σ on the x axis, the color indicates the smallest F value such that the plaintiff files. Blue values correspond to small F values while yellow values correspond to larger F values.

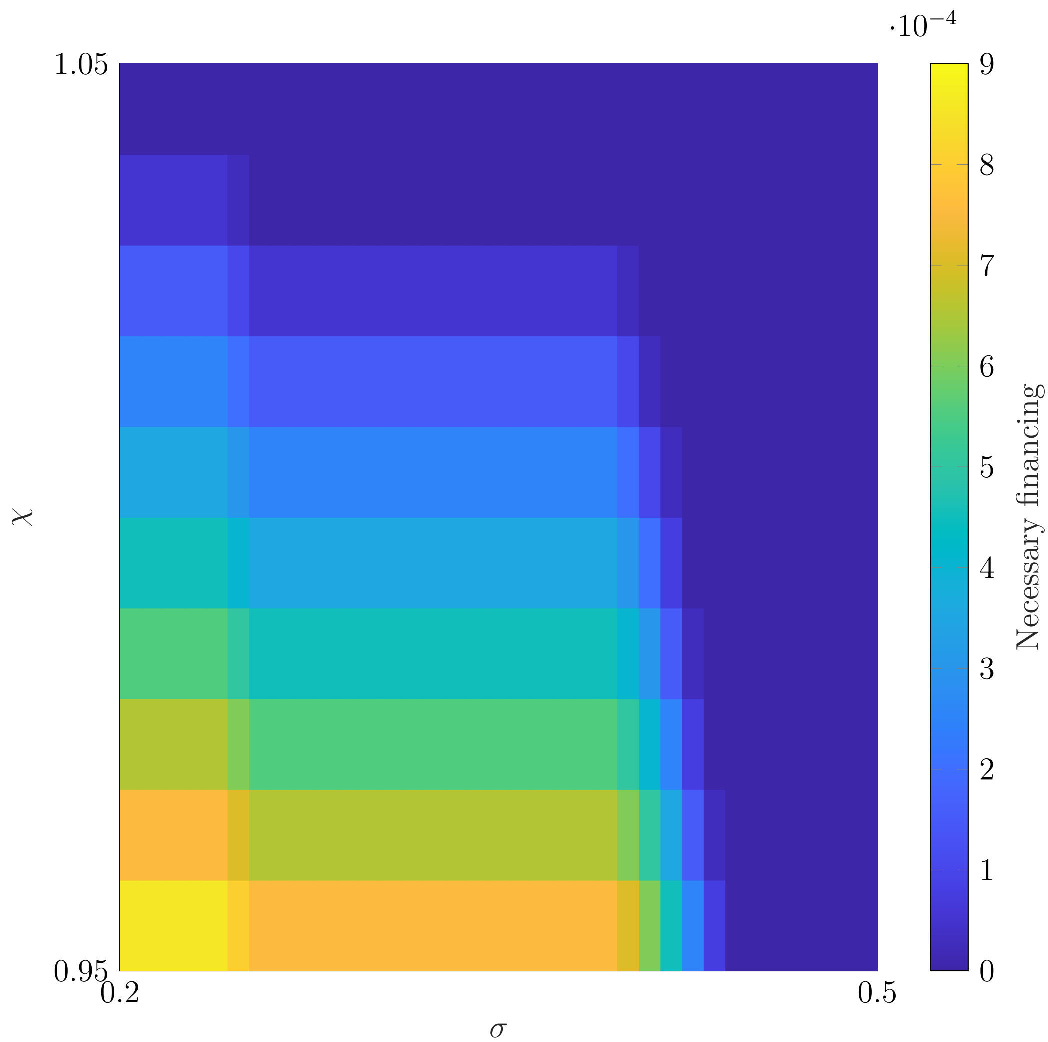


Figure 6: Stochastic bargaining model timeline

This figure shows the timeline for the stochastic bargaining model in Section 4.

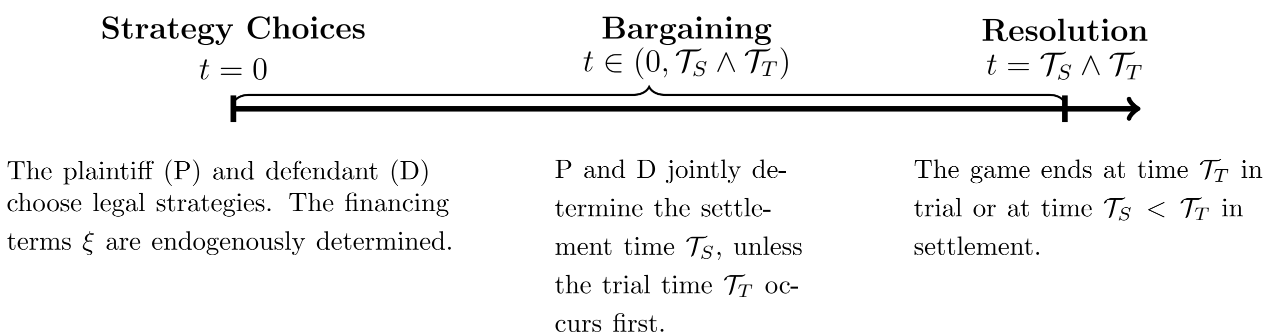
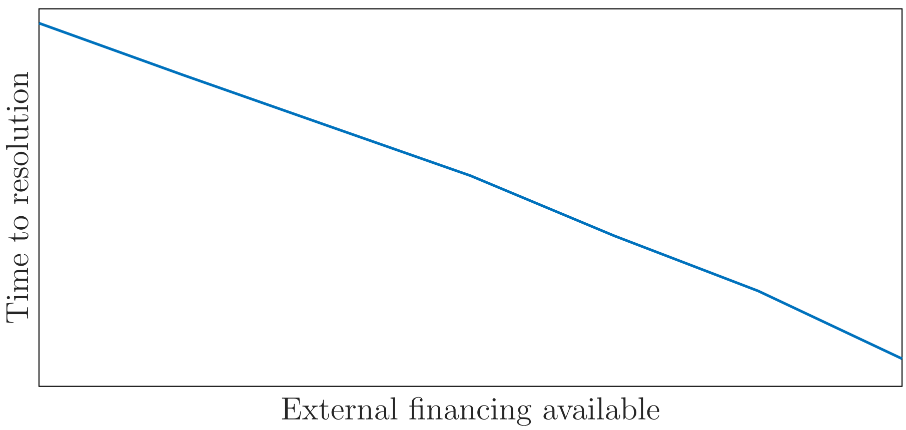
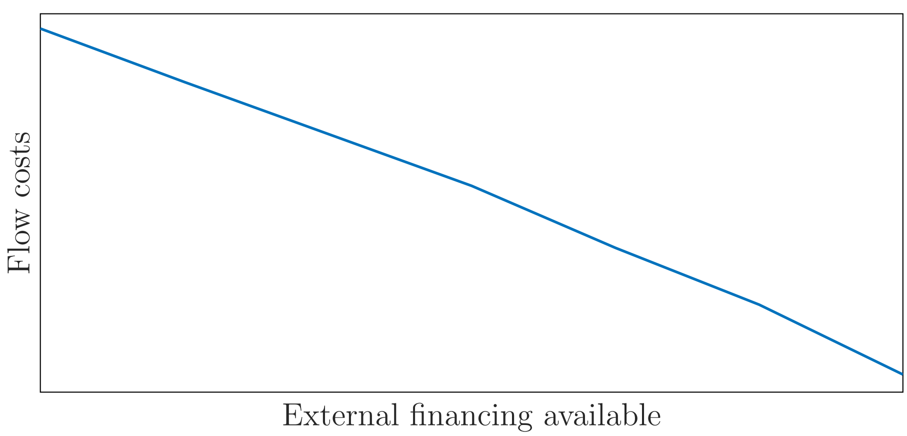


Figure 7: Financing and time to resolution

Starting with the parameter values listed in Table D.1, we increase the parameter F that represents external financing available and numerically solve for equilibrium values of $\xi, \phi^{*,D}$, and $\phi^{*,P}$ that satisfy equations (20)-(22). For each equilibrium, we calculate the expected time to resolution $\mathbb{E}[\mathcal{T}_T \wedge \mathcal{T}_S]$ using a methodology described in Appendix D. Panel (a) shows the expected time to resolution on the y axis as a function of F on the x axis. Panel (b) shows the expected flow costs $(\vartheta^P + \vartheta^D)\mathbb{E}[\mathcal{T}_T \wedge \mathcal{T}_S]$ on the y axis as a function of F on the x axis.



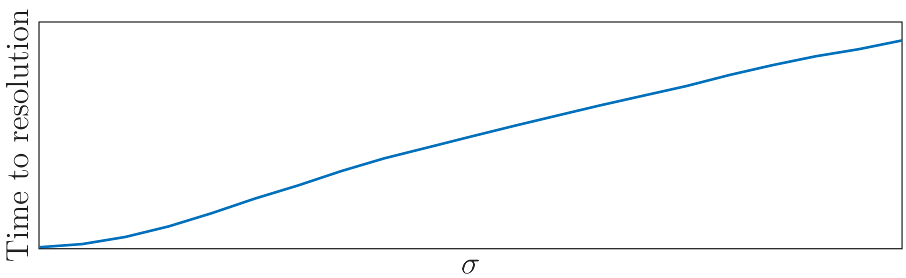
Panel (a) Equilibrium time to resolution as a function of F



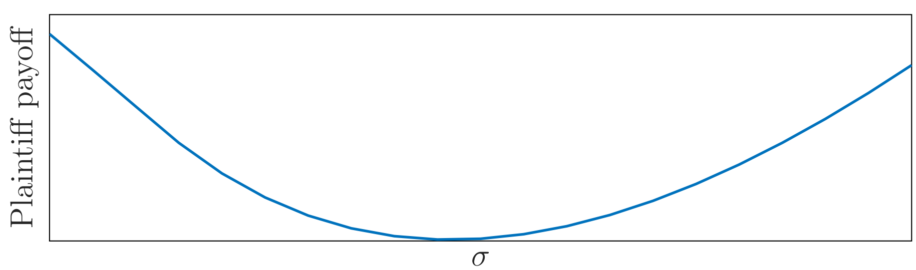
Panel (b) Equilibrium flow costs as a function of F

Figure 8: The role of volatility

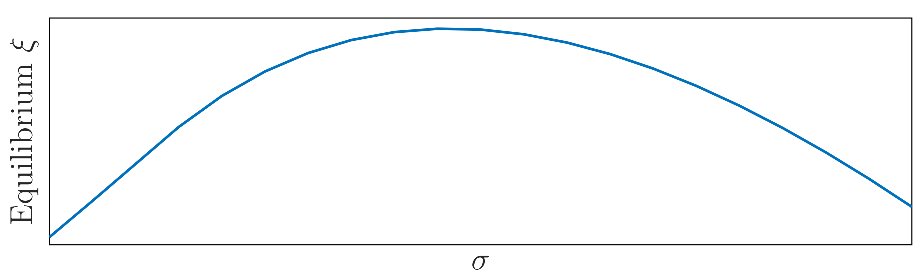
Starting with the parameter values listed in Table D.2, we increase the volatility parameter σ and numerically solve for equilibrium values of $\xi, \phi^{*,D}$, and $\phi^{*,P}$ that satisfy equations (20)-(22). For each equilibrium, we calculate the expected time to resolution $\mathbb{E}[\mathcal{T}_T \wedge \mathcal{T}_S]$ using a methodology described in Appendix D. Panel (a) shows the expected time to resolution on the y axis as a function of σ on the x axis. Panel (b) shows the plaintiff's equilibrium payoff $F - C^P(\phi^{*,P}) + V^P(\chi + \phi^{*,P} - \phi^{*,D}, s_0)$ on the y axis as a function of σ on the x axis. Panel (c) shows the equilibrium value of ξ on the y axis as a function of σ on the x axis.



Panel (a) Equilibrium time to resolution as a function of σ



Panel (b) Equilibrium plaintiff payoff as a function of σ



Panel (c) Equilibrium ξ as a function of σ

A Proofs and Calculations for Section 3

A.1 Equilibrium calculation

Consider the expectation $\mathbb{E}[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}|X_0 = x]$. Applying the definition of a geometric Brownian motion and the formula for the mean of a lognormal random variable,

$$\mathbb{E}[e^{-r\mathcal{T}_T}X_{\mathcal{T}_T}|X_0 = x] = e^{(\mu-r)\mathcal{T}_T}x. \tag{A.1}$$

Defining $K \equiv (R^D + R^P)/(\alpha+\xi)$ and applying the Black-Scholes Formula (Duffie, 2010),

$$\mathbb{E}[e^{-r\mathcal{T}_T}\max(0, X_{\mathcal{T}_T} - K)|X_0 = x] = x\Phi_N\left(d_1(x)\right) - Ke^{-r\mathcal{T}_T}\Phi_N\left(d_1(x) - \sigma\sqrt{\mathcal{T}_T}\right), \tag{A.2}$$

where Φ_N is the cdf of a standard normal random variable and

$$d_1(x) \equiv \frac{\log(x/K) + (r + \sigma^2/2)\mathcal{T}_T}{\sigma\sqrt{\mathcal{T}_T}}. \tag{A.3}$$

Putting this together,

$$\begin{aligned}\mathbb{E}[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}|X_0 = x] &= e^{-r\mathcal{T}_T}R^D - (1+\alpha)e^{(\mu-r)\mathcal{T}_T}x &\text{(A.4)}\\ &+ \psi(\alpha+\xi)\left(x\Phi_N\left(d_1(x)\right) - Ke^{-r\mathcal{T}_T}\Phi_N\left(d_1(x) - \sigma\sqrt{\mathcal{T}_T}\right)\right). &\text{(A.5)}\end{aligned}$$

It follows that the objective in equation (8) is equal to

$$\begin{aligned}&- C^D(\phi^D) + e^{-r\mathcal{T}_T}R^D - (1+\alpha)e^{(\mu-r)\mathcal{T}_T}x^{*,D} &\text{(A.6)}\\ &+ \psi(\alpha+\xi)\left(x^{*,D}\Phi_N\left(d_1(x^{*,D})\right) - Ke^{-r\mathcal{T}_T}\Phi_N\left(d_1^*(x^{*,D}) - \sigma\sqrt{\mathcal{T}_T}\right)\right), &\text{(A.7)}\end{aligned}$$

where

$$x^{*,D} \equiv \chi + \phi^{*,P} - \phi^D. \tag{A.8}$$

A nearly identical calculation shows that the objective in equation (7) is equal to

$$- C^P(\phi^P) + e^{-r\mathcal{T}_T} R^P + e^{(\mu-r)\mathcal{T}_T}(1-\xi)x^{*,P} \tag{A.9}$$

$$+ (1-\xi)(1-\psi)(\alpha+\xi)\left(x^{*,P}\Phi_N\left(d_1(x^{*,P}) \right) - Ke^{-r\mathcal{T}_T}\Phi_N\left(d_1^*(x^{*,P}) - \sigma\sqrt{\mathcal{T}_T} \right)\right), \tag{A.10}$$

where

$$x^{*,P} \equiv \chi + \phi^P - \phi^{*,D}. \tag{A.11}$$

We solve our model by searching numerically over potential $\phi^{*,D}, \phi^{*,P}$ values for mutual best responses in the sense of equations (7) and (8).

A.2 Proof of Proposition 1

Applying the well known result that the delta of a call option is equal to $\Phi_N(d_1(x))$,

$$\frac{\partial}{\partial x}\mathbb{E}[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}|X_0 = x] = -(1+\alpha)e^{(\mu-r)\mathcal{T}_T} + \psi(\alpha+\xi)\Phi_N(d_1(x)). \tag{A.12}$$

It follows immediately that

$$\frac{\partial}{\partial \phi^D}\mathbb{E}[e^{-r\mathcal{T}_T}\mathbb{V}^D_{\mathcal{T}_T}|X_0 = x^{*,D}] = (1+\alpha)e^{(\mu-r)\mathcal{T}_T} - \psi(\alpha+\xi)\Phi_N(d_1(x^{*,D})). \tag{A.13}$$

Noting that

$$\frac{\partial d_1(x^{*,D})}{\partial \phi^P} = \frac{\partial d_1(x^{*,D})}{\partial x^{*,D}} > 0, \tag{A.14}$$

we have

$$\frac{\partial^2}{\partial \phi^D \partial \phi^P} \left(-C^D(\phi^D) + \mathbb{E}[e^{-r\mathcal{T}_T} \mathbb{V}^D_{\mathcal{T}_T} | X_0 = \chi - \phi^D + \phi^P] \right) < 0. \tag{A.15}$$

It then follows from Topkis's theorem (Milgrom and Shannon, 1994) that $\phi_1^{*,D} \leq \phi_0^{*,D}$ as long as $\phi_1^{*,P} \geq \phi_0^{*,P}$. We now show that $\phi_1^{*,P} \geq \phi_0^{*,P}$ if $F_1 > F_0$. Suppose by contradiction this were not true: $\phi_1^{*,P} < \phi_0^{*,P}$. Then $C^P(\phi_1^{*,P}) \leq C^P(\phi_0^{*,P}) \leq I + F_0$, so $\phi_1^{*,P}$ is feasible with financing F_1. Combining this with the fact that $(\phi_1^{*,P}, \phi_1^{*,D})$ are mutual best responses by assumption, it follows that $(\phi_1^{*,P}, \phi_1^{*,D})$ are a distinct equilibrium associated with financing F_0, a contradiction.

B Proof of Proposition 2

The proof proceeds in steps.

We call a function $f(X)$ "smooth" if it is continuously differentiable everywhere and the second derivative $f''(X)$ exists almost everywhere. We call a function $f(X,s)$ smooth if $f(\cdot,s)$ is a smooth function of X for any fixed s.

To simplify expressions, we define $\pi \equiv 1-\xi$. We let ϑ^P, ϑ^D denote the flow costs paid by the plaintiff and the defendant, respectively. We let R^P, R^D denote fixed net benefits of trial and we let α denote the proportional costs of trial. We define

$$\bar{\vartheta} = \vartheta^P/\pi + \vartheta^D$$
$$\bar{R} = R^P/\pi + R^D.$$

First, we construct the value function $V(X)$ of a social planner that chooses the time of settlement $\mathcal{T}_S$ to maximize the sum of all agents' value functions. We show that, by construction, this value function V is smooth and satisfies the variational inequality

$$0 = \max\left(-V(X), -rV(X) + \mathcal{D}V(X) - \bar{\vartheta} + \lambda\left(\bar{R} - \alpha X - V(X) \right)\right), \tag{B.1}$$

where, for any smooth f, the operator $\mathcal{D}$ is defined by

$$\mathcal{D}f(X,s) \equiv \mu X f_X(X,s) + \frac{\sigma^2}{2} X^2 f_{XX}(X,s). \tag{B.2}$$

We also calculate a threshold $\overline{X}$ such that $V(X)=0$ if and only if $X \geq \overline{X}$.

Second, we calculate smooth candidate equilibrium value functions $V^P(X,s), V^D(X,s)$ that satisfy

$$V(X) = V^D(X,s) + \frac{1}{\pi} V^P(X,s) \tag{B.3}$$

everywhere, and that satisfy

$$rV^j(X,s) = \mathcal{D}V^j(X,s) - \vartheta^j + \rho^s\left(V^j(X,s') - V^j(X,s) \right) + \lambda\left(R^j + M^j X - V^j(X,s) \right) \tag{B.4}$$

on the set $\mathcal{Q} \equiv \{(X,s,j) : X \leq \overline{X} \,\text{or}\, s \neq j\}$, where $M^P = \pi$, $M^D = -(1+\alpha)$.

Next, we put together equations (B.1), (B.3), and (B.4) to show that each candidate value function $V^j(X,s)$, $j = P, D$ satisfies the variational inequality

$$\begin{aligned} 0 = \max \Bigg(& J^j(X,s) - V^j(X,s), -rV^j(X,s) + \mathcal{D}V^j(X,s) + \rho^s \left(V^j(X,s') - V^j(X,s) \right) \\ & - \vartheta^j + \lambda \left(R^j + M^j X - V^j(X,s) \right) \Bigg), \end{aligned} \tag{B.5}$$

where

$$\begin{aligned} J^P(X,s) &\equiv \pi[-V^D(X,s)] - V^P(X,s) \\ J^D(X,s) &\equiv -\frac{1}{\pi} V^P(X,s) - V^D(X,s). \end{aligned}$$

In the final step, we use standard technical arguments to show that equation (B.5) implies the results of the Proposition 2.

B.1 The auxiliary problem of a social planner

In this section, we consider the auxiliary problem of a social planner who chooses a settlement stopping time $\mathcal{T}_S$ to maximize the sum of all agents' value functions:

$$V(X) \equiv \sup_{\mathcal{T}_S} -\mathbb{E}\left[\int_0^{\mathcal{T}_S \wedge \mathcal{T}_T} e^{-rs} \bar{\vartheta} ds + e^{-r\mathcal{T}_T} \mathbf{1}\{\mathcal{T}_S \geq \mathcal{T}_T\} \left(-\bar{R} + \alpha X_{\mathcal{T}_T} \right) \right]. \tag{B.6}$$

We now prove that the optimal strategy solving (B.6) is given by a barrier strategy $\mathcal{T}_S = \inf\{t : X_t \geq \overline{X}\}$ for a threshold $\overline{X}$ that we will calculate. We also calculate V in closed form and show that it satisfies equation (B.1)

For any $X \leq \overline{X}$, we conjecture the value function $V(X) = V(X,s)$, which does not depend on s, satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rV(X) = \mathcal{D}V(X) - \bar{\vartheta} + \lambda \left(\bar{R} - \alpha X - V(X) \right).$$

The general solution to this ODE is the usual one: let β, γ be the positive and negative roots of

$$r + \lambda - \mu z - \frac{\sigma^2}{2} z(z-1) = 0.$$

Then the general solutions take the form

$$A_1 X^\beta + A_2 X^\gamma$$

for constants A_1, A_2. The particular solution can then be written as

$$\frac{-\bar{\vartheta} + \lambda \bar{R}}{r+\lambda} - \frac{\lambda \alpha X}{r+\lambda-\mu},$$

so the value function is

$$V(X) = A_1 X^\beta + A_2 X^\gamma + \frac{-\bar{\vartheta} + \lambda \bar{R}}{r+\lambda} - \frac{\lambda \alpha X}{r+\lambda-\mu}.$$

We conjecture an upper threshold $\overline{X}$ such that the social planner chooses to settle when $X_t \geq \overline{X}$ (the trial is costly). The value V should stay finite as $X \to 0$, so that $A_2 = 0$. Since V must be smooth, it must be continuously differentiable at $\overline{X}$. The value matching and smooth pasting conditions are then

$$\begin{aligned} 0 &= A_1 \overline{X}^\beta + \frac{-\bar{\vartheta} + \lambda \bar{R}}{r+\lambda} - \frac{\lambda \alpha \overline{X}}{r+\lambda-\mu} \\ 0 &= A_1 \beta \overline{X}^\beta - \frac{\lambda \alpha \overline{X}}{r+\lambda-\mu}. \end{aligned}$$

Rearranging the first equation gives

$$A_1 \overline{X}^\beta = \frac{\lambda \alpha \overline{X}}{r+\lambda-\mu} - \frac{-\bar{\vartheta} + \lambda \bar{R}}{r+\lambda}. \tag{B.7}$$

Plugging this into the second equation,

$$(\beta - 1)\frac{\lambda \alpha \overline{X}}{r+\lambda-\mu} + \beta \left(-\frac{-\bar{\vartheta} + \lambda \bar{R}}{r+\lambda} \right) = 0,$$

or

$$\overline{X} = \left(\frac{\lambda \bar{R} - \bar{\vartheta}}{r+\lambda} \right) \frac{\beta}{\beta - 1} \frac{r+\lambda-\mu}{\lambda\alpha}. \tag{B.8}$$

Equation (B.7) then immediately implies the closed form solution

$$A_1 = \overline{X}^{-\beta} \left(\frac{\lambda\alpha\overline{X}}{r+\lambda-\mu} - \frac{-\bar{\vartheta}+\lambda\bar{R}}{r+\lambda} \right). \tag{B.9}$$

We thus conjecture that the value function of a social planner that solves (B.6) is equal to $V(X) = 0$ for $X \geq \overline{X}$ and is equal to

$$V(X) = \overline{X}^{-\beta} \left(\frac{\lambda\alpha\overline{X}}{r+\lambda-\mu} - \frac{-\bar{\vartheta}+\lambda\bar{R}}{r+\lambda} \right) X^{\beta} + \frac{-\bar{\vartheta}+\lambda\bar{R}}{r+\lambda} - \frac{\lambda\alpha X}{r+\lambda-\mu}$$

for $X \leq \overline{X}$.

To show this, note that for $X \geq \overline{X}$, we have $\mathcal{D}V(X) = 0$, so

$$-rV(X) + \mathcal{D}V(X) - \bar{\vartheta} + \lambda \left(\bar{R} - \alpha X - V(X) \right) = -\bar{\vartheta} + \lambda(\bar{R} - \alpha X). \tag{B.10}$$

Note that $\mu < r$ combined with the definition of β,

$$r + \lambda - \mu\beta - \frac{\sigma^2}{2}\beta(\beta-1) = 0,$$

implies that $\beta > 1$ and

$$\begin{aligned} 0 &= \frac{r+\lambda}{\beta-1} - \mu\frac{\beta}{\beta-1} - \frac{\sigma^2}{2}\beta \\ &= -(r+\lambda) + \frac{\beta}{\beta-1}\left(r+\lambda-\mu \right) - \frac{\sigma^2}{2}\beta \\ &\leq -(r+\lambda) + \frac{\beta}{\beta-1}\left(r+\lambda-\mu \right). \end{aligned}$$

This implies that

$$r+\lambda \le \frac{\beta}{\beta-1}\left(r+\lambda-\mu \right),$$

so

$$(r+\lambda)(\lambda\bar{R}-\bar{\vartheta}) \le (\lambda\bar{R}-\bar{\vartheta})\frac{\beta}{\beta-1}\left(r+\lambda-\mu \right) = (r+\lambda)\lambda\alpha\overline{X}.$$

We thus have that $(\lambda\bar{R}-\bar{\vartheta}) \le \lambda\alpha\overline{X}$. Combining this with equation (B.10), it follows that $X \ge \overline{X}$ implies that

$$-rV(X)+\mathcal{D}V(X)-\bar{\vartheta}+\lambda\left(\bar{R}-\alpha X-V(X) \right) \le 0.$$

We now show that $V(X) \ge 0$. First, note that if $\lambda\bar{R} \le \bar{\vartheta}$ then $\bar{X} \le 0$, so $V(0)=0$ and the statement holds trivially. Suppose instead that $\lambda\bar{R} > \bar{\vartheta}$. By inspection of V, we see that V' can equal zero for at most one X on the interval $[0,\overline{X}]$. But $V'(\overline{X})=0$ by construction. Thus either V' is weakly positive on $[0,\overline{X}]$, or it is weakly negative on that interval. By inspection, $V(0)=\frac{-\bar{\vartheta}+\lambda\bar{R}}{r+\lambda}$ and $V(\overline{X})=0$, so it must be that V' is weakly negative on $[0,\overline{X}]$. It follows that if there were an $X \le \overline{X}$ such that $V(X)<0$, then $V(\overline{X})<0$, a contradiction. Since $V(X)=0$ for $X \ge \overline{X}$ by construction, we have that $V(X) \ge 0$.

It follows that V is a bounded[27] continuously differentiable function satisfying

$$0=\max\left(-V(X), -rV(X)+\mathcal{D}V(X)-\bar{\vartheta}+\lambda\left(\bar{R}-\alpha X-V(X) \right) \right). \tag{B.11}$$

A standard verification argument similar to the one we use later shows that V is indeed the value function in equation (B.6). For our purposes, all that matters is that V is continuously differentiable and satisfies (B.1).

B.2 Calculation of the candidate value functions

Recall that $\pi = 1-\xi$. In this section we calculate candidate equilibrium value functions $V^P(X,s), V^D(X,s)$ that satisfy

$$V(X)=V^D(X,s)+\frac{1}{\pi}V^P(X,s)$$

[27] V is continuous on the compact interval $[0,\overline{X}]$ and is constant for $(\overline{X},\infty)$.

everywhere, and that satisfy

$$rV^j(X,s) = \mathcal{D}V^j(X,s) - \vartheta^j + \rho^s \left(V^j(X,s') - V^j(X,s) \right) + \lambda \left(R^j + M^j X - V^j(X,s) \right)$$

on the set $\mathcal{Q} \equiv \{(X,s,j) : X \leq \overline{X} \,\text{or}\, s \neq j\}$. We verify that these value functions comprise an equilibrium in a later section.

B.2.1 Solution in the Settlement Region

Let $Y(X,s)$ denote the endogenous settlement payment that the defendant makes to the plaintiff and its financier. Then in the settlement region,

$$V^D(X,s) = -Y(X,s).$$

Since the plaintiff only receives a fraction π of the payment,

$$V^P(X,s) = \pi Y(X,s).$$

Putting this together, in the settlement region $X \geq \overline{X}$, the candidate value functions should satisfy

$$V^D(X,s) = -\frac{1}{\pi} V^P(X,s) \tag{B.12}$$

$$V^P(X,s) = -\pi V^D(X,s). \tag{B.13}$$

We conjecture that in the settlement region, the value functions are characterized by equation (B.12), (B.13), and the following HJB equations:

$$
\begin{aligned}
rV^P(X,D) &= \mathcal{D}V^P(X,D) + \rho^D \left(\pi(-V^D(X,P)) - V^P(X,D) \right) && \text{(B.14)}\\
&\quad - \vartheta^P + \lambda \left(R^P + \pi X - V^P(X,D) \right) && \text{(B.15)}\\
rV^D(X,P) &= \mathcal{D}V^D(X,P) + \rho^P \left(-(\frac{1}{\pi})V^P(X,D) - V^D(X,P) \right) && \text{(B.16)}\\
&\quad - \vartheta^D + \lambda \left(R^D - (1+\alpha)X - V^D(X,P) \right) . && \text{(B.17)}
\end{aligned}
$$

Define

$$
B \equiv \begin{bmatrix} r+\lambda+\rho^D & \pi\rho^D \\ \frac{\rho^P}{\pi} & r+\lambda+\rho^P \end{bmatrix},
$$

so that

$$
B \begin{bmatrix} V^P(X,D) \\ V^D(X,P) \end{bmatrix} = \mathcal{D} \begin{bmatrix} V^P(X,D) \\ V^D(X,P) \end{bmatrix} + \begin{bmatrix} \lambda R^P - \vartheta^P \\ \lambda R^D - \vartheta^D \end{bmatrix} + \lambda \begin{bmatrix} \pi X \\ -(1+\alpha)X \end{bmatrix}. \tag{B.18}
$$

Defining

$$
\begin{aligned}
E &\equiv \begin{bmatrix} r+\lambda+\rho^D+\rho^P & 0 \\ 0 & r+\lambda \end{bmatrix} \\
T &\equiv \begin{bmatrix} 1 & 1 \\ \frac{\rho^P}{\pi\rho^D} & -\frac{1}{\pi} \end{bmatrix},
\end{aligned}
$$

direct calculation shows that

$$
T^{-1} = \frac{\pi\rho^D}{\rho^P+\rho^D} \begin{bmatrix} \frac{1}{\pi} & 1 \\ \frac{\rho^P}{\pi\rho^D} & -1 \end{bmatrix}
$$

and

$$
TE(T)^{-1} = B.
$$

We can thus use the transformation

$$\begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix} \equiv T^{-1} \begin{bmatrix} V^P(X,D) \\ V^D(X,P) \end{bmatrix}$$

to write

$$B \begin{bmatrix} V^P(X,D) \\ V^D(X,P) \end{bmatrix} = TET^{-1} \begin{bmatrix} V^P(X,D) \\ V^D(X,P) \end{bmatrix} = TE \begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix}.$$

Multiplying equation (B.18) by T^{-1}, by the linearity of the operator $\mathcal{D}$, we arrive at the separate ODEs

$$E \begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix} = \mathcal{D} \begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix} + T^{-1} \begin{bmatrix} \lambda R^P - \vartheta^P \\ \lambda R^D - \vartheta^D \end{bmatrix} + \lambda T^{-1} \begin{bmatrix} \pi X \\ -(1+\alpha)X \end{bmatrix}. \tag{B.19}$$

The ODEs

$$E \begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix} = \mathcal{D} \begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix}$$

have general solutions

$$\begin{aligned} \hat{V}^P(X,D) &= K_1 X^\kappa + H_1 X^\zeta \\ \hat{V}^D(X,P) &= K_2 X^\gamma + H_2 X^\beta, \end{aligned}$$

where ζ, κ are the positive and negative roots of the quadratic

$$r + \lambda + \rho^D + \rho^P - \mu z - \frac{\sigma^2}{2} z(z-1)$$

and β, γ are the positive and negative roots of the quadratic

$$r + \lambda - \mu z - \frac{\sigma^2}{2} z(z-1).$$

From equation (B.19), the particular solutions are

$$\nu + \eta X \equiv \begin{bmatrix} \frac{\rho^D\left(\lambda R^P - \vartheta^P + \pi(\lambda R^D - \vartheta^D)\right)}{(\rho^P+\rho^D)(r+\lambda+\rho^P+\rho^D)} \\ \frac{\rho^P(\lambda R^P - \vartheta^P) - \pi\rho^D(\lambda R^D - \vartheta^D)}{(\rho^P+\rho^D)(r+\lambda)} \end{bmatrix} + \lambda X \begin{bmatrix} \frac{-\rho^D\pi\alpha}{(\rho^P+\rho^D)(r+\lambda+\rho^P+\rho^D-\mu)} \\ \frac{\rho^P\pi+\pi\rho^D(1+\alpha)}{(\rho^P+\rho^D)(r+\lambda-\mu)} \end{bmatrix}.$$

We conjecture that solutions (which we later verify) are bounded by an affine function of X as $X \to \infty$, so it must be that $H_2 = H_1 = 0$, and thus the solutions are

$$\begin{aligned} \begin{bmatrix} V^P(X,D) \\ V^D(X,P) \end{bmatrix} &= T \begin{bmatrix} \hat{V}^P(X,D) \\ \hat{V}^D(X,P) \end{bmatrix} \\ &= T\left(\begin{bmatrix} K_1 X^\kappa \\ K_2 X^\gamma \end{bmatrix} + \nu + \eta X \right) \\ &= \begin{bmatrix} K_1 X^\kappa + K_2 X^\gamma \\ \frac{1}{\pi}\left(\frac{\rho^P}{\rho^D} K_1 X^\kappa - K_2 X^\gamma \right) \end{bmatrix} + \begin{bmatrix} \nu_1 + \nu_2 + (\eta_1+\eta_2)X \\ \frac{1}{\pi}\left(\frac{\rho^P}{\rho^D}(\nu_1 + \eta_1 X) - \nu_2 - \eta_2 X \right) \end{bmatrix}. \end{aligned}$$

B.2.2 Solution in the Waiting Region

Now, consider the region $X \leq \bar{X}$. We conjecture and later verify that prior to settling, the agent j's value function V^j should satisfy the following HJB equation in each state s:

$$rV^j(X,s) = \mathcal{D}V^j(X,s) - \vartheta^j + \rho^s\left(V^j(X,s') - V^j(X,s) \right) + \lambda\left(R^j + M^j X - V^j(X,s) \right), \tag{B.20}$$

where

$$\begin{aligned} M^P &= \pi \\ M^D &= -(1+\alpha). \end{aligned}$$

Define

$$B_2 \equiv \begin{bmatrix} r+\lambda+\rho^D & -\rho^D \\ -\rho^P & r+\lambda+\rho^P \end{bmatrix}.$$

Then for $j = P, D$, we can write the HJB for the value function $V^j(X,s)$ as

$$B_2 \begin{bmatrix} V^j(X,D) \\ V^j(X,P) \end{bmatrix} = \mathcal{D} \begin{bmatrix} V^j(X,D) \\ V^j(X,P) \end{bmatrix} + (-\vartheta^j + \lambda R^j + \lambda M^j X) \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Defining

$$E_2 \equiv \begin{bmatrix} r+\lambda+\rho^D+\rho^P & 0 \\ 0 & r+\lambda \end{bmatrix}$$
$$T_2 \equiv \begin{bmatrix} 1 & 1 \\ \frac{-\rho^P}{\rho^D} & 1 \end{bmatrix},$$

direct calculation shows that

$$T_2^{-1} = \frac{\rho^D}{\rho^P+\rho^D} \begin{bmatrix} 1 & -1 \\ \frac{\rho^P}{\rho^D} & 1 \end{bmatrix}$$

and

$$T_2 E_2 (T_2)^{-1} = \begin{bmatrix} r+\lambda+\rho^P+\rho^D & r+\lambda \\ \frac{-\rho^P}{\rho^D}(r+\lambda+\rho^P+\rho^D) & r+\lambda \end{bmatrix} (T_2)^{-1} = \begin{bmatrix} r+\lambda+\rho^D & -\rho^D \\ -\rho^P & r+\lambda+\rho^P \end{bmatrix} = B_2$$

We can thus use the transformation

$$\begin{bmatrix} \tilde{V}^j(X,D) \\ \tilde{V}^j(X,P) \end{bmatrix} \equiv T_2^{-1} \begin{bmatrix} V^j(X,D) \\ V^j(X,P) \end{bmatrix}$$

to write

$$B_2 \begin{bmatrix} V^j(X,D) \\ V^j(X,P) \end{bmatrix} = T_2 E_2 T_2^{-1} \begin{bmatrix} V^j(X,D) \\ V^j(X,P) \end{bmatrix} = T_2 E_2 \begin{bmatrix} \tilde{V}^j(X,D) \\ \tilde{V}^j(X,P) \end{bmatrix}.$$

Noting that $E_2 = E$, the HJB above can thus be multiplied on the left by T_2^{-1} to obtain

$$
\begin{aligned}
E\begin{bmatrix}\tilde{V}^j(X,D)\\ \tilde{V}^j(X,P)\end{bmatrix} &= \mathcal{D}\begin{bmatrix}\tilde{V}^j(X,D)\\ \tilde{V}^j(X,P)\end{bmatrix} + (-\vartheta^j + \lambda R^j + \lambda M^j X)T_2^{-1}\begin{bmatrix}1\\ 1\end{bmatrix}\\
&= \mathcal{D}\begin{bmatrix}\tilde{V}^j(X,D)\\ \tilde{V}^j(X,P)\end{bmatrix} + \begin{bmatrix}0\\ -\vartheta^j + \lambda R^j + \lambda M^j X\end{bmatrix}.
\end{aligned}
$$

As above, each $\tilde{V}^j(X,k)$ has general solutions of the form $L_k^j X^{a_k} + H_k^j X^{b_k}$, where the constants $a_k > 0, b_k < 0$ are the same as before since $E_2 = E$. We conjecture that solutions (which we later verify) remain bounded as $X \to 0$ so the coefficients on X^{b_k} must equal zero. The general solutions are thus

$$
\begin{bmatrix}\tilde{V}^j(X,D)\\ \tilde{V}^j(X,P)\end{bmatrix} = \begin{bmatrix}H_1^j X^{\zeta}\\ H_2^j X^{\beta}\end{bmatrix}.
$$

Following the usual approach, the particular solutions are

$$
\begin{bmatrix}0\\ \frac{\lambda R^j - \vartheta^j}{r+\lambda} + \frac{\lambda M^j X}{r+\lambda-\mu}\end{bmatrix},
$$

so the solutions are

$$
\begin{aligned}
\begin{bmatrix}V^j(X,D)\\ V^j(X,P)\end{bmatrix} &= T_2\begin{bmatrix}\tilde{V}^j(X,D)\\ \tilde{V}^j(X,P)\end{bmatrix}\\
&= T_2\left(\begin{bmatrix}H_1^j X^{\zeta}\\ H_2^j X^{\beta}\end{bmatrix} + \begin{bmatrix}0\\ \frac{\lambda R^j - \vartheta^j}{r+\lambda} + \frac{\lambda M^j X}{r+\lambda-\mu}\end{bmatrix}\right)\\
&= \begin{bmatrix}H_1^j X^{\zeta} + H_2^j X^{\beta}\\ -\frac{\rho^P}{\rho^D} H_1^j X^{\zeta} + H_2^j X^{\beta}\end{bmatrix} + \begin{bmatrix}\frac{\lambda R^j - \vartheta^j}{r+\lambda} + \frac{\lambda M^j X}{r+\lambda-\mu}\\ \frac{\lambda R^j - \vartheta^j}{r+\lambda} + \frac{\lambda M^j X}{r+\lambda-\mu}\end{bmatrix}.
\end{aligned}
$$

Defining

$$
\begin{aligned}
\psi_j &\equiv \frac{\lambda M^j}{r+\lambda-\mu}\\
\Psi_j &\equiv \frac{\lambda R^j - \vartheta^j}{r+\lambda}
\end{aligned}
$$

this is

$$\begin{bmatrix} V^j(X,D) \\ V^j(X,P) \end{bmatrix} = \begin{bmatrix} H_1^j X^\zeta + H_2^j X^\beta \\ -\frac{\rho^P}{\rho^D} H_1^j X^\zeta + H_2^j X^\beta \end{bmatrix} + (\Psi_j + \psi_j X) \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

B.2.3 The Solution

We conjecture that $V^P(X,s)$ is smooth, so we require that $V^P(X,s)$ value matches and smooth pastes at $\overline{X}$ for $s = P, D$. Value matching and smooth pasting for $V^P(X,s)$ give the following four equations:

$$\begin{aligned} \lim_{X\uparrow\overline{X}} V^P(X,P) &= \lim_{X\downarrow\overline{X}} V^P(X,P) \\ \lim_{X\uparrow\overline{X}} \frac{\partial}{\partial X} V^P(X,P) &= \lim_{X\downarrow\overline{X}} \frac{\partial}{\partial X} V^P(X,P) \\ \lim_{X\uparrow\overline{X}} V^P(X,D) &= \lim_{X\downarrow\overline{X}} V^P(X,D) \\ \lim_{X\uparrow\overline{X}} \frac{\partial}{\partial X} V^P(X,D) &= \lim_{X\downarrow\overline{X}} \frac{\partial}{\partial X} V^P(X,D). \end{aligned}$$

Plugging in solutions and inferring $V^P(X,P)$ from $V^D(X,D)$ and equation (B.13),

$$\begin{aligned} -\frac{\rho^P}{\rho^D} H_1^P \overline{X}^\zeta + H_2^P \overline{X}^\beta + \Psi_P + \psi_P \overline{X} &= -\left(\frac{\rho^P}{\rho^D} K_1 \overline{X}^\kappa - K_2 \overline{X}^\gamma + \frac{\rho^P}{\rho^D}(\nu_1 + \eta_1 \overline{X}) - \nu_2 - \eta_2 \overline{X} \right) \\ -\frac{\rho^P}{\rho^D} H_1^P \zeta \overline{X}^{\zeta-1} + H_2^P \beta \overline{X}^{\beta-1} + \psi_P &= -\left(\frac{\rho^P}{\rho^D} K_1 \kappa \overline{X}^{\kappa-1} - K_2 \gamma \overline{X}^{\gamma-1} + \frac{\rho^P}{\rho^D} \eta_1 - \eta_2 \right) \\ H_1^P \overline{X}^\zeta + H_2^P \overline{X}^\beta + \Psi_P + \psi_P \overline{X} &= K_1 \overline{X}^\kappa + K_2 \overline{X}^\gamma + \nu_1 + \nu_2 + (\eta_1 + \eta_2)\overline{X} \\ H_1^P \zeta \overline{X}^{\zeta-1} + H_2^P \beta \overline{X}^{\beta-1} + \psi_P &= K_1 \kappa \overline{X}^{\kappa-1} + K_2 \gamma \overline{X}^{\gamma-1} + (\eta_1 + \eta_2). \end{aligned}$$

This can be rearranged to write the closed form solution

$$
\begin{bmatrix} H_1^P \\ H_2^P \\ K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -\frac{\rho^P}{\rho^D}\overline{X}^{\zeta} & \overline{X}^{\beta} & \frac{\rho^P}{\rho^D}\overline{X}^{\kappa} & -\overline{X}^{\gamma} \\ -\frac{\rho^P}{\rho^D}\zeta\overline{X}^{\zeta-1} & \beta\overline{X}^{\beta-1} & \frac{\rho^P}{\rho^D}\kappa\overline{X}^{\kappa-1} & -\gamma\overline{X}^{\gamma-1} \\ \overline{X}^{\zeta} & \overline{X}^{\beta} & -\overline{X}^{\kappa} & -\overline{X}^{\gamma} \\ \zeta\overline{X}^{\zeta-1} & \beta\overline{X}^{\beta-1} & -\kappa\overline{X}^{\kappa-1} & -\gamma\overline{X}^{\gamma-1} \end{bmatrix}^{-1}
$$

$$
\times \begin{bmatrix} -\Psi_P - \psi_P\overline{X} - \frac{\rho^P}{\rho^D}(\nu_1 + \eta_1\overline{X}) + \nu_2 + \eta_2\overline{X} \\ -\psi_P - \frac{\rho^P}{\rho^D}\eta_1 + \eta_2 \\ -\Psi_P - \psi_P\overline{X} + \nu_1 + \eta_1\overline{X} + \nu_2 + \eta_2\overline{X} \\ -\psi_P + \eta_1 + \eta_2 \end{bmatrix}
$$

For any conjectured $\overline{X}$, these coefficients define an explicit solution for $V^P(X,s)$. Now, suppose that we define

$$
V^D(X,s) = V(X,s) - \frac{1}{\pi}V^P(X,s),
$$

where $V(X,s) = V(X)$ is the social planner's value function. Suppose also that we take $\overline{X} = \inf\{X : V(X) = 0\}$ to be the social planner's exercise threshold. Since $V^P(X,s)$ and $V(X)$ value match and smooth paste at $\overline{X}$ by construction, it follows immediately that $V^D(X,s)$ does as well. In what follows, we show that $V^D(X,s)$ satisfies the appropriate HJB equations.

B.2.4 Verifying HJBs

By the linearity of the HJB, for $X \leq \bar{X}$,

$$
\begin{aligned}
&rV^D(X,s)\\
&= r\left(V(X,s) - \frac{1}{\pi}V^P(X,s) \right)\\
&= \mathcal{D}V(X,s) + \rho_s \left(V(X,s') - V(X,s) \right) - \bar{\vartheta} + \lambda \left(\bar{R} - \alpha X - V(X,s) \right)\\
&- \left(\mathcal{D}\frac{1}{\pi}V^P(X,s) + \rho_s \frac{1}{\pi} \left(V^P(X,s') - V(X^P,s) \right) - \frac{\vartheta^P}{\pi} + \lambda \left(\frac{R^P}{\pi} + X - \frac{1}{\pi}V^P(X,s) \right)\right)\\
&= \mathcal{D} \left(V(X,s) - \frac{1}{\pi}V^P(X,s) \right) + \lambda \left(\bar{R} - \frac{R^P}{\pi} - (1+\alpha)X - \{V(X,s) - \frac{1}{\pi}V^P(X,s)\} \right)\\
&- (\bar{\vartheta} - \frac{\vartheta^P}{\pi}) + \rho_s \left(\{V(X,s') - \frac{1}{\pi}V^P(X,s')\} - \{V(X,s) - \frac{1}{\pi}V^P(X,s)\} \right)\\
&= \mathcal{D}V^D(X,s) - \vartheta^D + \rho_s \left(V^D(X,s') - V^D(X,s) \right) + \lambda(R^D - (1+\alpha)X - V^D(X,s)).
\end{aligned}
$$

It follows that V^D solves the HJB for $X \leq \bar{X}$. Since $V(X,s)$ and $V^P(X,s)$ value match and smooth paste at $\overline{X}$ (at the $\bar{X}$ defined by equation (B.8)), so too does $V^D(X,s)$. It follows from the construction of $V^P(X,s)$ that $-\frac{1}{\pi}V^P(X,s)$ defines a function that satisfies the same HJB for $X \geq \bar{X}$, $s = P$.

B.3 The variational inequalities

Recall that for $X \geq \bar{X}$, by the definition of V,

$$
rV(X) \geq \mathcal{D}V(X) - \bar{\vartheta} + \lambda(\bar{R} - \alpha X - V(X)).
$$

Define $V(X,s)$ by $V(X,P) = V(X,D) = V(X)$. It follows that for $X \geq \bar{X}$,

$$
\begin{aligned}
&rV^D(X,s)\\
&= r\left(V(X,s) - \frac{1}{\pi}V^P(X,s) \right)\\
&\geq \mathcal{D}V(X,s) + \rho_s\left(V(X,s') - V(X,s) \right) - \bar{\vartheta} + \lambda\left(\bar{R} - \alpha X - V(X,s) \right)\\
&- \left(\mathcal{D}\frac{1}{\pi}V^P(X,s) + \rho_s\frac{1}{\pi}\left(V^P(X,s') - V(X^P,s) \right) - \frac{\vartheta^P}{\pi} + \lambda\left(\frac{R^P}{\pi} + X - \frac{1}{\pi}V^P(X,s) \right)\right)\\
&= \mathcal{D}\left(V(X,s) - \frac{1}{\pi}V^P(X,s) \right) + \lambda\left(\bar{R} - \frac{R^P}{\pi} - (1+\alpha)X - \{V(X,s) - \frac{1}{\pi}V^P(X,s)\} \right)\\
&- (\bar{\vartheta} - \frac{\vartheta^P}{\pi}) + \rho_s\left(\{V(X,s') - \frac{1}{\pi}V^P(X,s')\} - \{V(X,s) - \frac{1}{\pi}V^P(X,s)\} \right)\\
&= \mathcal{D}V^D(X,s) - \vartheta^D + \rho_s\left(V^D(X,s') - V^D(X,s) \right) + \lambda(R^D - (1+\alpha)X - V^D(X,s)).
\end{aligned}
$$

Note that the inequality holds with equality for $X \geq \overline{X}$, $s = P$ by construction. An identical calculation shows that

$$
rV^P(X,s) \geq \mathcal{D}V^P(X,s) - \vartheta^P + \rho_s\left(V^P(X,s') - V^P(X,s) \right) + \lambda(R^P + \pi X - V^P(X,s))
$$

for $X \geq \overline{X}$, with equality for $s = D$ by construction. Putting these together with the HJB equations that V^P, V^D satisfy, and the fact that $V^D(X,s) = V(X,s) - \frac{1}{\pi}V^P(X,s)$ everywhere by construction, we have shown that V^P, V^D satisfy the variational inequalities.

B.4 Martingale verification

This final section of the proof concludes with a standard verification argument. Fix a player i. Suppose player $j \neq i$ uses the conjectured equilibrium strategy

$$
\begin{aligned}
A_t^j &= V^j(X_t, s_t)\\
\tau^j &= \inf\{t : X_t \geq \overline{X}, s_t = j, A_t^i \leq V^i(X_t, s_t)\}.
\end{aligned}
$$

for the functions V^j, V^i given in the proposition. First, at each time t, it is without loss of generality to assume that player i either sets $A_t^i = V^i(X_t, s_t)$, or $A_t^i = \infty$. There is no point in setting $A_t^i < V^i(X_t, s_t)$ since changing the inequality to equality weakly improves player i's payoff. Likewise, setting A_t^i to any value higher than $V^i(X_t, s_t)$ has the same effect as setting $A_t^i = \infty$. It follows that $A_t^i = V^i(X_t, s_t)$ for all i in any moment t at which settlement could possibly occur, so defining

$$J^P(X,s) \equiv \mathbf{1}\left(\, s = P\,\right)(1-\xi)\left(\, -V^D(X,s)\,\right) + \mathbf{1}\left(\, s = D\,\right) V^P(X,s) \tag{B.21}$$

$$J^D(X,s) \equiv \mathbf{1}\left(\, s = D\,\right)\left(\, -\frac{1}{1-\xi}V^P(X,s)\,\right) + \mathbf{1}\left(\, s = P\,\right) V^D(X,s), \tag{B.22}$$

we may rewrite $J_t^i = J^i(X_t, s_t)$ without loss of generality. We may rewrite player i's problem as

$$\begin{aligned}\tilde{V}^i(X,s) = \sup_{\tau: s_\tau = i} \mathbb{E}^{(X,s)}\big[&\mathbf{1}(\tau^j \wedge \tau \leq \mathcal{T}_T)e^{-r\tau^j\wedge\tau} J^i\left(\, X_{\tau^j\wedge\tau}, s_{\tau^j\wedge\tau}\,\right) \\ &- \int_0^{\tau^j\wedge\tau\wedge\tau_T} e^{-rs}\vartheta^i ds + \mathbf{1}(\tau^j \wedge \tau > \mathcal{T}_T)e^{-r\mathcal{T}_T}(R^i + M^i X_{\mathcal{T}_T})\big].\end{aligned} \tag{B.23}$$

It is clear that the value $\tilde{V}^i(X,s)$ is weakly lower than the value $\hat{V}^i(X,s)$ in the equivalent problem for which player i is free to choose $\tau^j \wedge \tau$:

$$\hat{V}^i(X,s) = \sup_{\tau} \mathbb{E}^{(X,s)}\big[\mathbf{1}(\tau \leq \mathcal{T}_T)e^{-r\tau} J^i\left(\, X_\tau, s_\tau\,\right) - \int_0^{\tau\wedge\tau_T} e^{-rs}\vartheta^i ds \tag{B.24}$$

$$+ \mathbf{1}(\tau > \mathcal{T}_T)e^{-r\mathcal{T}_T}(R^i + M^i X_{\mathcal{T}_T})\big]. \tag{B.25}$$

We now show that by following the conjectured equilibrium strategy τ^i, player i may achieve the value $\hat{V}^i(X,s)$, and $\hat{V}^i(X,s) = V^i(X,s)$. It follows that player i's equilibrium strategy is optimal, since $\hat{V}^i(X,s) \geq \tilde{V}^i(X,s)$.

Define $Z_t = \mathbf{1}(s = D)$ and $C_t = \mathbf{1}(t > \mathcal{T}_T)$. Define $Y_t = [X_t, Z_t, C_t, t]$ and define

$$U(Y) = e^{-rt}\left([1-C]\Big[V^i(X,P) + Z\{V^i(X,D) - V^i(X,P)\}\Big] + C[R^i + M^i X] \right)$$

Let $m = 1, ...4$ index the 4 components of Y and let $\Delta Y_u^m = Y_u^m - Y_{u-}^m$. Since Z,t, and C are finite-variation processes, we can apply Lemma 3 of Appendix H of Duffie (2010):

$$\begin{aligned} U(Y_t) = U(Y_0) &+ \sum_m \int_{0+}^t \frac{\partial}{\partial Y^m} U(Y_{u-}) dY_u^m + \frac{\sigma^2}{2}\int_0^t X^2 \frac{\partial^2}{\partial^2 X} U(Y_u) du \\ &+ \sum_{0\le u\le t}\Big[U(Y_u) - U(Y_{u-}) - \sum_m \frac{\partial}{\partial Y^m} U(Y_{u-})\Delta Y_u^m\Big]. \end{aligned}$$

For the rest of the proof, fix some time $t < \mathcal{T}_T$. Note that $C_u = \Delta C_u = 0$ for $u \le t$. Also, note that $V^i(X_u, P) + Z_u\{V^i(X_u, D) - V^i(X_u, P)\} = V^i(X_u, s_u)$ by definition of Z. We can plug in the definitions of Y, U to write

$$\begin{aligned} U(Y_t) = U(Y_0) &+ \int_{0+}^t e^{-ru}\ \big(\ V^i(X_u, D) - V^i(X_u, P)\ \big)\ dZ_u \\ &+ \int_{0+}^t e^{-ru}\ \big(\ R^i + M^i X_u - V^i(X_u, s_u)\ \big)\ dC_u \\ &+ \int_{0+}^t e^{-ru}\ \Big(\ -rV^i(X_u, s_u) + \mu X_u V_X^i(X_u, s_u) + \frac{\sigma^2}{2} X_u^2 V_{XX}^i(X_u, s_u)\ \Big)\ du \\ &+ \int_{0+}^t e^{-ru}\sigma X_u V_X^i(X_u, s_u) dB_u, \end{aligned}$$

where we have used the fact that

$$\Big[V^i(X_u, s_u) - V^i(X_{u-}, s_{u-})\Big] = \frac{\partial}{\partial Z} U(Y_{u-})\Delta Z.$$

Define $\rho(Z) \equiv \rho^P + Z\ \big(\ -\rho^P - \rho^D\ \big)$. It follows by the definitions of Z, C that we can define

$$\Theta_u \equiv -rV^i(X_u, s_u) + \mathcal{D}V^i(X_u, s_u) + \lambda \left(R^i + M^i X_u - V^i(X_u, s_u) \right)$$
$$+ \rho(Z_u) \left(V^i(X_u, D) - V^i(X_u, P) \right)$$

to write

$$U(Y_t) = U(Y_0) + \int_{0+}^{t} e^{-ru} \Theta_u du + \mathcal{M}_t$$

for a local martingale $\mathcal{M}_t$ with $\mathcal{M}_0 = 0$. Since V^i satisfies the variational inequality, $\Theta_u \leq \vartheta^i$ and $\Theta_u = \vartheta^i$ for $u < \inf\{t : X_t \geq \overline{X}\}$.

Now, fix some arbitrary stopping time τ_A and starting state (X_0, s_0). Note that since $C_0 = \mathcal{M}_0 = 0$, we have $U(Y_0) = V^i(X_0, s_0)$.

Let $\tau_n, n = 1, 2, ...$ denote the sequence of localizing stopping times for the local martingale M_t, let $\gamma_n = \inf\{t : |\mathcal{M}_t| > n\}$ and define the stopping time

$$q_n \equiv \max(0, -\frac{1}{n} + n \wedge \tau_n \wedge \tau_A \wedge \mathcal{T}_T).$$

Since q_n is bounded by n, for any n we have $\mathbb{E}[\mathcal{M}_{q_n}] = \mathcal{M}_0 = 0$ by the Optional Sampling Theorem (i.e., Protter (2005) theorem I.17) and the definition of τ_n. Since $q_n < \mathcal{T}_T$ for any n, we can apply the above calculations to write

$$\mathbb{E}[U(Y_{q_n})] = V^i(X_0, s_0) + \mathbb{E}[\int_{0+}^{q_n} e^{-ru} \Theta_u du].$$

By definition of q_n, we have $\lim_{n\to\infty} q_n = \tau_A \wedge \mathcal{T}_T$. Since $\mu < r$ and U is bounded by an affine function of X, we can apply the bounding argument of Antill and Grenadier (2019) to apply the Dominated Convergence Theorem to write

$$\mathbb{E}[U(Y_{\tau_A \wedge \mathcal{T}_T})] = V^i(X_0, s_0) + \mathbb{E}[\int_{0+}^{\tau_A \wedge \mathcal{T}_T} e^{-ru} \Theta_u du].$$

Since V^i satisfies the variational inequality, this implies that

$$
\begin{aligned}
&\mathbb{E}[\int_0^{\mathcal{T}_T \wedge \tau_A} -e^{-ru}\vartheta^i du + e^{-r\mathcal{T}_T}\mathbf{1}(\tau_A > \mathcal{T}_T)(R^i + M^i X_{\mathcal{T}_T}) + e^{-r\tau_A}\mathbf{1}(\tau_A \le \mathcal{T}_T) J^i(X_{\tau_A}, s_{\tau_A})] \\
&\le \mathbb{E}[\int_0^{\mathcal{T}_T \wedge \tau_A} -e^{-ru}\vartheta^i du + e^{-r\mathcal{T}_T}\mathbf{1}(\tau_A > \mathcal{T}_T)(R^i + M^i X_{\mathcal{T}_T}) + e^{-r\tau_A}\mathbf{1}(\tau_A \le \mathcal{T}_T) V^i(X_{\tau_A}, s_{\tau_A})] \\
&= \mathbb{E}[-\int_0^{\mathcal{T}_T \wedge \tau_A} e^{-rs}\vartheta^i ds + U(Y_{\tau_A \wedge \mathcal{T}_T})] \\
&= V^i(X_0, s_0) + \mathbb{E}[\int_{0+}^{\tau_A \wedge \mathcal{T}_T} e^{-ru}(\Theta_u - \vartheta^i) du] \\
&\le V^i(X_0, s_0).
\end{aligned}
$$

Note that if $\tau_A = \tau^i \wedge \tau^j$, then $X_u \le \overline{X}$ for all $u \le \tau_A$ so $\Theta_u = \vartheta^i$ for $u \le \tau_A$ and $J^i(X_{\tau_A}, s_{\tau_A}) = V^i(X_{\tau_A}, s_{\tau_A})$, so the inequalities holds with equality. This implies that $\tau^i \wedge \tau^j$ solves the problem (B.25) with value function $V^i(X_0, s_0)$. By the logic above, this implies that τ^i solves the problem (B.23) with value function $V^i(X_0, s_0)$. Finally, note that setting $A_t^i = V^i(X_t, s_t)$ everywhere (i.e., never setting $A_t^i = \infty$) achieves the same value: even if this induces settlement in a state where it otherwise would not occur, it could not change agent i's value function by definition. This completes the proof.

B.5 The financier's value function

Finally, we calculate the financiers value function $V^F(X, s)$. The financier's value function can be written as

$$V^F(X, s) \equiv \mathbb{E}\left[e^{-r\mathcal{T}_S}\mathbf{1}\{\mathcal{T}_S < \mathcal{T}_T\}\frac{\xi}{1-\xi}V^P(X_{\mathcal{T}_S}, s_{\mathcal{T}_S}) + e^{-r\mathcal{T}_T}\mathbf{1}\{\mathcal{T}_S \ge \mathcal{T}_T\}\xi X_{\mathcal{T}_T}\right].$$

We note that this is equal to

$$V^F(X, s) = \frac{\xi}{1-\xi}V^P(X, s) + \frac{\xi}{1-\xi}\mathbb{E}\left[\int_0^{\mathcal{T}_S \wedge \mathcal{T}_T} e^{-rs}\vartheta^P ds - \mathbf{1}\left(\mathcal{T}_T < \mathcal{T}_S\right) e^{-r\mathcal{T}_T} R^P\right],$$

where the second term reflects the term $\mathbb{E}\left[\int_0^{\mathcal{T}_S \wedge \mathcal{T}_T} -e^{-rs}\vartheta^P ds\right]$ in $V^P(X, s)$ and the third term reflects the term

$$\mathbb{E}\bigg[\mathbf{1}\left(\mathcal{T}_T < \mathcal{T}_S\right) e^{-r\mathcal{T}_T} R^P\bigg]$$

in $V^P(X,s)$. We note that standard arguments (i.e., those used in Appendix B.1) imply that the sum of the second term and third term is equal to

$$A_1 X^\beta + \frac{\xi}{1-\xi}\frac{\vartheta^P - \lambda R^P}{r+\lambda}$$

for $X \leq \overline{X}$, where

$$A_1 = \overline{X}^{-\beta}\left[-\frac{\xi}{1-\xi}\frac{\vartheta^P - \lambda R^P}{r+\lambda}\right].$$

Thus,

$$V^F(X,s) = \frac{\xi}{1-\xi}V^P(X,s) + \frac{\xi}{1-\xi}\frac{\vartheta^P - \lambda R^P}{r+\lambda}\left(1 - \overline{X}^{-\beta}X^\beta\right).$$

C Main Results in the Model of Section 4

Figure C.1: The defendant's value function

The blue line shows the defendant's bargaining value function $V^D(X_0, s_0)$ on the y axis as a function of X_0 on the x axis. We assume the parameter values listed in Table D.1.

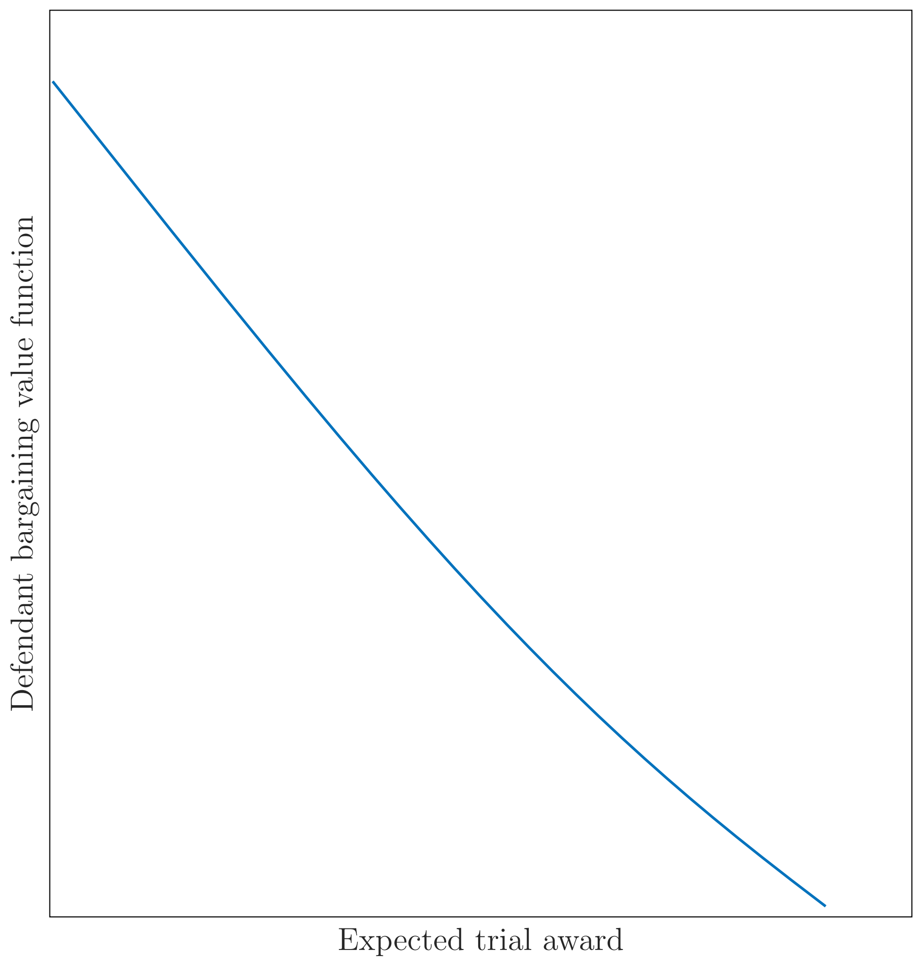
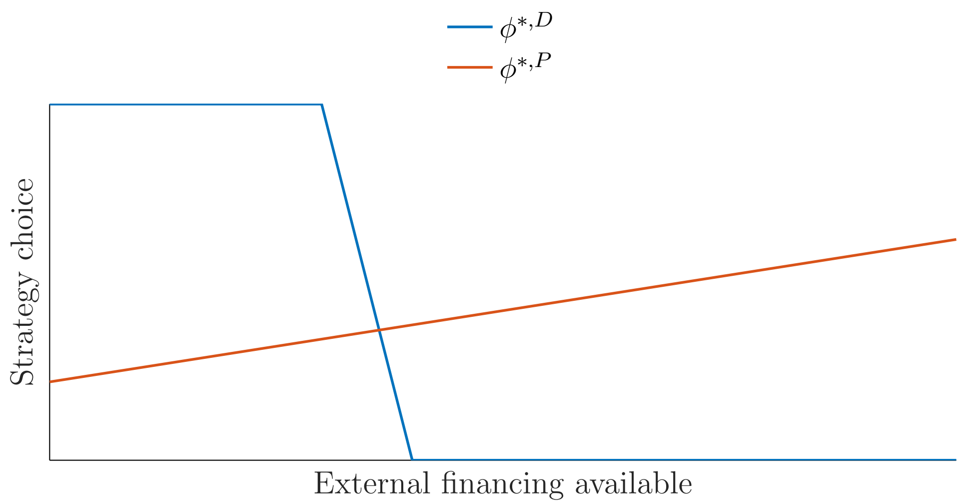
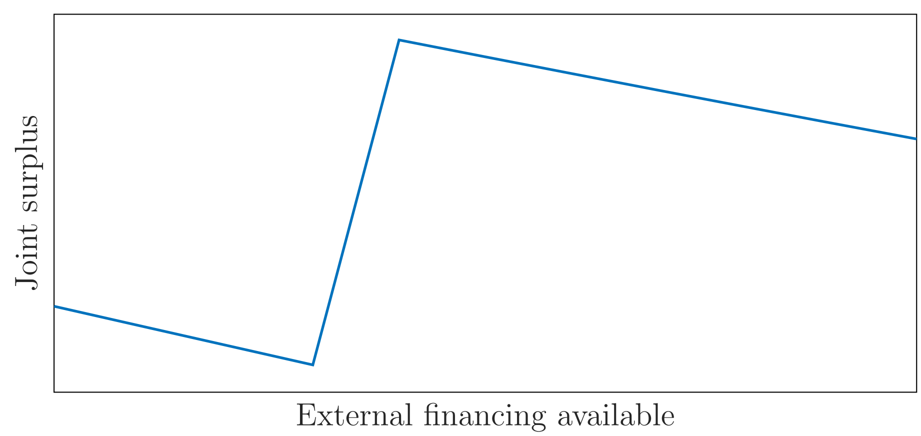


Figure C.2: Financing and joint surplus

Starting with the parameter values listed in Table D.1, we increase the parameter F that represents external financing available and numerically solve for equilibrium values of $\xi, \phi^{*,D}$, and $\phi^{*,P}$ that satisfy equations (20)-(22). In panel (a), the orange and blue lines show $\phi^{*,P}$ and $\phi^{*,D}$, respectively, on the y axis as a function of F on the x axis. In panel (b), the blue line shows the joint surplus, defined analogously to $\mathbb{S}_0$, on the y axis as a function of F on the x axis.



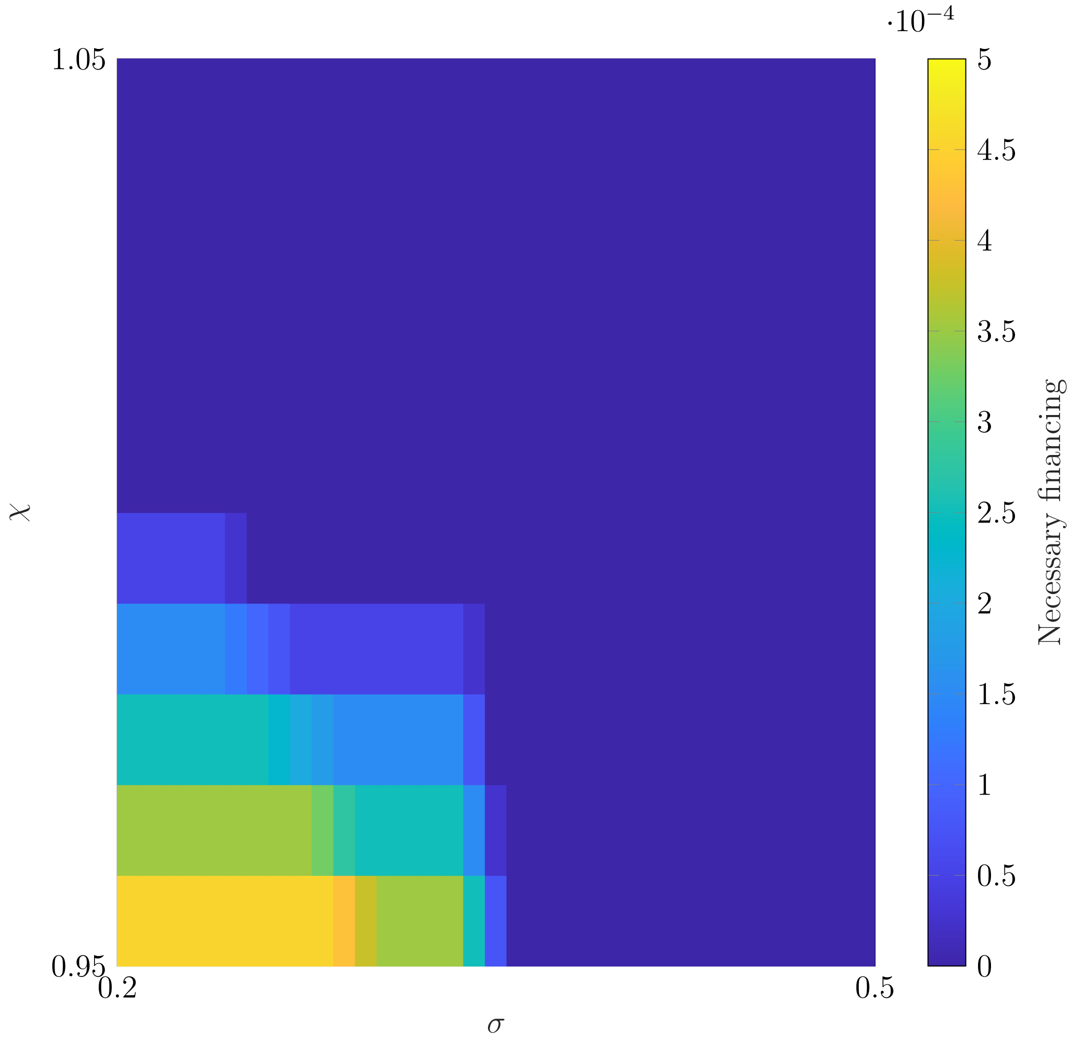
Panel (a) Equilibrium strategy choices as a function of F



Panel (b) Equilibrium joint surplus as a function of F

Figure C.3: Financing and frivolous cases

Starting with the parameter values listed in Table D.1 (except $C^P(x) = .4 + .01x$ and $I = .40001$), we vary the parameters χ, σ and F and numerically solve for equilibrium values $\xi, \phi^{*,D}$, and $\phi^{*,P}$ that satisfy equations (20)-(22). For each (χ, σ) pair, we calculate the smallest F value such that the plaintiff files: $F - C^P(\phi^{*,P}) + V^P(\chi + \phi^{*,P} - \phi^{*,D}, s_0) \geq 0$. This figure shows a heat map in which, for each χ on the y axis and σ on the x axis, the color indicates the smallest F value such that the plaintiff files. Blue values correspond to small F values while yellow values correspond to larger F values.



D Numerical Methodology in Section 4

For a given equilibrium, we simulate $10,000$ values of the trial time $\mathcal{T}_T$. We then simulate $10,000$ values of the settlement time $\mathcal{T}_S$ by the following algorithm. For each $i = 1, ..., 10,000$, we simulate $50,000$ i.i.d standard normal random variables $\{N_j^i\}_{j=1}^{50,000}$. Letting $dt = .01$, we simulate a Brownian motion B_t^i as

$$B_{k\times dt}^i = \sqrt{dt}\sum_{j=1}^{k} N_j^i. \tag{D.1}$$

We define

$$X_{k\times dt}^i \equiv \left(\chi + \phi^{*,P} - \phi^{*,D} \right) \exp\left((\mu - \sigma^2/2)kdt + \sigma B_{k\times dt} \right). \tag{D.2}$$

We define the ith simulated value $\mathcal{T}_S^i$ as the first time $t = kdt$ that $X_{k\times dt} \geq \bar{X}$. If none of the 50,000 simulated values $X_{k\times dt}^i$ are larger than $\bar{X}$, then we say $\mathcal{T}_S^i = \infty$. The ith time to resolution is calculated as the minimum of $\mathcal{T}_S^i$ and $\mathcal{T}_T^i$, the ith simulated trial time. The expected time to resolution is calculated as

$$\frac{1}{10,000}\sum_{i=1}^{10,000} \mathcal{T}_T^i \wedge \mathcal{T}_S^i. \tag{D.3}$$

Table D.1: Parameter values for the model of Section 4

This table shows our baseline parameter values for the model of Section 4.

Parameter	Description	Value
r	Discount rate	0.05
μ	Drift of X_t	0.00
σ	Volatility of X_t	0.25
λ	Trial arrival rate	0.2
α	Defendant's proportional trial cost	1.5
R^P	Plaintiff's net benefit of trial	0
R^D	Defendant's net benefit of trial	2.66
ρ^P	Transition rate from $s = P$ (defendant's bargaining strength)	0.9
ρ^D	Transition rate from $s = D$ (plaintiff's bargaining strength)	0.1
ϑ^P	Plaintiff's flow costs	0.00001
ϑ^D	Defendant's flow costs	0.00001
χ	Baseline value of X_0	1
I	Plaintiff's internal funds	0.0001
$C^D(\phi^D)$	Defendant's cost function	$1.9\ \phi^D$
$C^P(\phi^P)$	Plaintiff's cost function	$0.01\ \phi^P$
Φ^P	Plaintiff's strategy set	$[0, 0.5]$
Φ^D	Defendant's strategy set	$[0, 0.5]$
s_0	Starting value of s_t	$s_0 = P$

Table D.2: Parameter values for Figure 8

This table shows our baseline parameter values for Figure 8.

Parameter	Description	Value
r	Discount rate	0.05
μ	Drift of X_t	0.00
σ	Volatility of X_t	0.25
λ	Trial arrival rate	0.03
α	Defendant's proportional trial cost	1.5
R^P	Plaintiff's net benefit of trial	0
R^D	Defendant's net benefit of trial	2.66
ρ^P	Transition rate from $s = P$ (defendant's bargaining strength)	0.05
ρ^D	Transition rate from $s = D$ (plaintiff's bargaining strength)	0.03
ϑ^P	Plaintiff's flow costs	0.00001
ϑ^D	Defendant's flow costs	0.00001
χ	Baseline value of X_0	2.5
I	Plaintiff's internal funds	0.0001
$C^D(\phi^D)$	Defendant's cost function	$1.9\ \phi^D$
$C^P(\phi^P)$	Plaintiff's cost function	$0.4 + 0.0002\ \phi^P$
Φ^P	Plaintiff's strategy set	$[0, 0.5]$
Φ^D	Defendant's strategy set	$\{0\}$
s_0	Starting value of s_t	$s_0 = P$
F	Baseline level of available financing	0.4