Homotopic insights into topological band invariants

Homotopy theory is a branch of mathematics that classifies continuous maps between a pair of spaces. In particular, two maps are described as homotopically equivalent if one can be continuously deformed into the other. Such a definition of equivalence bears resemblance to topological invariants of energy bands in crystalline solids, which capture features unaffected by continuous deformations of the underlying Hamiltonian. As topological invariants often indicate the presence of robust metallic boundary states inside the bulk energy gap, one anticipates homotopic ideas to play an important role in the design and discovery of materials with novel spectral and transport properties.

In this colloquium, I will first review the homotopic tools within the familiar context of topological defects in ordered media, such as magnets and liquid crystals. We will then use a mathematical analogy to transfer the technique to the context of energy bands in crystalline systems. The presented applications of homotopy theory will include the characterization of band degeneracies in semimetals and superconductors, including their non-commutative braiding in momentum space, and the theoretical concepts will be illustrated with concrete materials predictions.