Capital Structure and Hedging Demand with Incomplete Markets*

Alberto Bisin† — NYU and NBER

Gian Luca Clementi‡ — NYU Stern and NBER

Piero Gottardi§ — Essex

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Abstract

In this paper we study the determinants of the capital structure of firms as an equilibrium phenomenon in financial markets. In particular, we focus on the role of *hedging demand*, in economies where different investors may develop an appetite for different types of securities depending on their risk properties. To this end we develop and study a general equilibrium model with production and incomplete markets where households differ in their risk–sharing needs. Value-maximizing firms take investors' hedging demand into consideration in their capital structure choices, possibly facing different investors for their bond and equity supply, which affects their choice of leverage as well as their choice of technology. We find that as the demand for hedging increases, corporates grow in size, to allow for greater precautionary saving, and issue more debt. How much more, depends on the availability of competing risk-sharing instruments, such as (government–issued) risk–free debt and derivatives. When the capital structure is jointly shaped by hedging demand and supply considerations, the latter, in the form of an asset–substitution problem, we find that (i) agency is relevant only when hedging demand is high and that (ii) larger investors' risk–sharing needs lead to equilibria featuring greater aggregate risk.

Key words: Capital structure, leverage, incomplete markets, hedging demand.

JEL Codes: D51, D52, D53, G32.

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†Department of Economics, New York University. Email: alberto.bisin@nyu.edu.

‡Department of Economics, Stern School of Business, New York University. Email:clem@nyu.edu.

§Department of Economics, University of Essex. Email: piero.gottardi@essex.ac.uk.

1 Introduction

In this paper we introduce a theoretical framework where non-financial firms issue liabilities with the intent of catering to investors' hedging needs. The extant literature has established that capital structure choices are likely shaped by tax considerations, costly default, agency, and asymmetric information. We refer to these as *supply considerations*, as they affect the relative costs of issuing one versus another type of securities. However, as documented in the empirical literature, firms' capital structure choices also depend on the prices of their liabilities – equity and bonds, in particular – which are in turn affected by *hedging demand considerations*. These are the factors leading investors to develop a differential appetite for different corporate securities, depending on their respective risk properties.[1]

While a large body of literature has investigated in detail the implications of supply considerations, much less attention has been paid to the role played by hedging demand. This may be due in part to the fact that capital structure responds to variations in hedging demand only in incomplete-market general equilibrium economies, whose analysis is often contrived.[2]

In this paper, we construct such an economy. To focus on the least studied hedging demand channel, we consider first a scenario free of any supply considerations. There is neither a tax advantage of debt, nor default costs, nor agency problems. It is only the cross–sectional variation in investors' hedging needs, via its effect on asset prices, that drives firms' capital structure.

Our model features a continuum of ex-ante identical firms, equipped with a decreasing-return-to-scale production function subject to an aggregate productivity shock – the only aggregate factor. Markets are incomplete, in the sense that consumers can only invest in firm-issued bonds and stocks. Investors differ in terms of the correlation of their endowment with the aggregate factor. Riskier investors – those with relatively higher correlation – display a higher willingness to pay for corporate bonds, which allows for better risk sharing. Although not default-free, corporate debt provides hedging services and therefore commands a safety premium over equity at equilibrium.

This safety premium depends negatively on the firm's default risk, hence on its lever-

[1]We are wary of a potential terminological confusion. Investors' demand for hedging instruments issued by firms can be also envisioned as their supply of capital to firms. Indeed, the latter convention is adopted by part of the literature. For example, see Baker (2009).

[2]Under complete markets, hedging demand can affect the capital structure only indirectly, by influencing the terms of the trade-offs that commonly determine financing in partial equilibrium. We discuss this literature in Section 1.1.

age. In our economy with incomplete markets, it is the trade-off between the quantity of bonds issued - the firm's leverage – and their unit price, reflecting the safety premium, that pins down the capital structure.

We show that, as the hedging demand by risky investors rises, firms find it value–maximizing to cater to their needs by increasing both their capital stock and their debt. The debt rises to allow investors to better allocate consumption across states of nature. The capital stock rises in order to attenuate the effect of higher debt on default risk.

Equilibria characterized by greater hedging demand are associated with higher debt, leverage, and price of debt, therefore rationalizing the finding that securities' issuance responds to market valuation. Ritter (1991), Baker and Wurgler (2000) and Ma (2019) among others, document that firms systematically react to hikes in security prices by increasing their issuance. Because of the well known Modigliani-Miller indeterminacy result, it is impossible to make sense of this evidence under complete markets.[3]

Market leverage increases when hedging demand increases, because debt issuance proceeds grow by more than investment. The difference is passed on to shareholders in the shape of greater dividends. This result further highlights that in our framework debt is issued by virtue of its role as hedging instrument, independently of investment-driven financing needs. This finding is consistent with Ma (2019) and Mota (2021), who document that when firms issue debt in response to higher bond prices, the proceeds tend to be rebated to shareholders rather than used to finance investment.[4]

In our framework, an increase in hedging demand may result from higher aggregate risk. This immediately implies that leverage is positively associated with aggregate risk. In spite of the large body of work on the time-variation of aggregate risk since Bloom (2009)'s seminal contribution, its relation with capital structure remains in large part to be studied. It appears safe to state, however, than since risk is found to be greater in recessions, our model rationalizes Halling et al. (2016)'s finding that leverage is countercyclical.

We are aware that corporate bonds are not the primary means of pursuing insurance against consumption risk. The literature has long recognized that so-called safe assets such as the sovereign bonds issued by a restricted set of developed economies play an outsize role in this arena. Our analysis identifies conditions under which low-risk corporate bonds are imperfect substitutes of such sovereigns, so that firms have the incentives of issuing them to cater to investors' hedging needs.

[3] We remind the reader to Section 1.1 for a discussion of alternative rationalization of this evidence.

[4] Kubitza (2021) also finds that when insurers' demand for corporate bonds raises bond prices, firms respond by issuing more debt. However, the proceeds are funnelled towards investment rather than equity payouts.

In an extension of our baseline model, we allow for the exogenous supply of a perfectly safe asset – a risk-free bond. In that framework, a rise in hedging needs that is not matched by a proportionate rise in the supply of perfectly safe asset leads firms to increase their debt issuance, and to destine a fraction of the greater proceeds to equity payout.

This prediction aligns well with the recent evidence on *safe-asset shortage*. Caballero et al. (2008), Gorton et al. (2012) and Caballero et al. (2017) among others have argued convincingly that the protracted decline in the level of interest rates that we witnessed over the last three decades finds its root cause in a sustained increase in worldwide precautionary saving in face of a stable supply of safe asset. Mota (2021) finds that in the period since the 2008 financial crisis, when the excess demand for safe asset likely intensified, non-financial corporations almost doubled their debt. At the same time, as documented by Gutiérrez and Philippon (2017) and Crouzet and Eberly (2019), aggregate investment was weak.

Modern financial markets can also accommodate a rise in demand for safety by increasing the supply of derivative assets. In order to understand the effect of such channel on firms' capital structure decisions, we relax the short-selling constraint and allow financial intermediaries to sell short positions on corporate debt. We show that a decline in transaction costs leading to an expansion of the derivative markets crowds out the underlying asset, leading to a drop in corporate debt and leverage.

We further show that when firms are allowed to choose among technologies of different risk, an increase in hedging demand is met by a rise in the fraction of firms opting for production plans yielding safer cash flows. The securities issued by relatively safer firms crowd out the debt of riskier firms, leading the latter to reduce their leverage.

Given the emphasis the capital structure literature has given to supply considerations in the past, we find it interesting to understand how they interact with hedging demand in shaping financing decisions in our incomplete-market model. In the spirit of the asset substitution literature, we assume that it is up to shareholders alone to choose the risk profile of the firm's cash flows and that such profile is unobservable to other investors. Compared to the symmetric-information benchmark, the risk chosen by shareholders is higher, as it is commonly the case in partial equilibrium. The effect of agency on the debt choice, however, is ambiguous. It is still the case that, as in the text-book asset substitution scenario, initial shareholders have the incentive to reduce leverage in order to nudge future shareholders into choosing less risk. In general equilibrium, however, the larger firms' cash-flow risk associated with agency leads to higher hedging demand by bondholders and calls for more debt.

Beyond uncovering a clear role for hedging demand as driver of capital structure choices, allowing for financial market incompleteness suggests that in general equilibrium the strength of supply considerations such as agency will depend on the risk-sharing opportunities available to investors.

The remainder of the paper is organized as follows. In Section 1.1 we relate our work to the extant literature. In Section 2 we introduce the model. In Section 3 we characterize the equilibrium relation between hedging demand, investment, and capital structure. There we also show how firms' financing choices respond to a larger supply of risk-free asset, a relaxation of short-selling constraints, and the availability of less risky technologies, respectively. In Section 4, we bring together demand and supply consideration and describe how asset substitution and hedging demand jointly shape capital structure. The detailed discussion of various aspects of the notion of equilibrium, as long as the proofs of existence, unanimity and welfare properties are in Section 5. Section 6 concludes.

1.1 Related Literature

Our main contribution is the analysis of the role played by investors' *hedging demand* in shaping firms' capital structure. Most of the theoretical and empirical literature centers instead on what we labeled *supply* determinants of the capital structure.

A large fraction of these studies are conducted in partial equilibrium. Typically, they assume a perfectly elastic provision of funds by investors at a given price, so that firms choose capital structure by trading off the relative benefits of issuing bonds vs. equity. A celebrated version of such trade-off is that between the tax advantage of debt and the cost of default, an advanced treatment of which can be found in Hennessy and Whited (2005). An alternative approach, pioneered by Jensen and Meckling (1976), relies on asymmetric information between claim-holders.

A limitation of this literature is that it has a hard time accounting for the time series of capital structure. Ritter (1991), Baker and Wurgler (2000) and Ma (2019) among others document that securities issuance respond strongly to market valuation.

This evidence prompted the development of what became to be known as capital-market driven corporate finance. Early contributions carved out a role for market conditions as capital structure driver in partial equilibrium models, by assuming a departure from rational expectations and/or frictions in financial intermediation. Baker (2009)'s survey shows how non-fundamental investor demand, due to investor inertia or overconfidence, as well as limited arbitrage and imperfectly competitive intermediaries may lead asset prices to deviate from their fundamental values and shape the capital structure of optimizing firms.

Other recent contributions, closer to ours in spirit, show how aggregate shocks may affect well-understood trade-offs and therefore impact capital structure decisions in dynamic partial-equilibrium environments. Hackbarth et al. (2006) show how aggregate productivity shocks impact capital structure via their effects on both the tax benefit of debt and bankruptcy costs. In Chen and Manso (2017)'s treatment of debt overhang, agency costs are higher in recession, since transfers from shareholders to bondholders are larger in downturns, when the former's stochastic discount factor is higher. Because of their partial equilibrium nature, there is no feedback from firms choices to investors' consumption processes and therefore state prices.

Bhamra et al. (2010), Chen (2010) and Gomes and Schmid (2021) go one step further by embedding the classical trade-off between tax advantage of debt and bankruptcy cost in general equilibrium economies with complete markets. Hedging demand contributes to shaping capital structure, since the equilibrium state prices affect the terms of the trade-off. However, because of the complete market assumption, the firms' debt choice has no effect on hedging opportunities available to investors. The pricing kernel, i.e. the intertemporal marginal rate of substitution of the representative investor, is fully determined by aggregate consumption.

In absence of the trade-off between tax advantage of debt and default costs, capital structure would be indeterminate – a direct implication of the Modigliani-Miller theorem. Absent that trade-off, for capital structure do be determined in a complete market economy, the literature has resorted to assuming mis-valuation, as described in Baker (2009), exogenous external debt demand as in Corbae and Quintin (2016), or safety services in the utility function, as in Krishnamurthy and Vissing-Jorgensen (2012) and Mota (2021).[5]

When analyzing G.E. economies with production and incomplete markets such as that introduced below, the identification of an appropriate objective function for the firm is not a trivial task. It is only in rather special environments that the complete market analysis can be directly extended to incomplete market economies.[6]

[5]In Mota (2021) the demand for safety services is interpreted as a consequence of regulation-driven portfolio requirements on institutional investors. A corporate bond's safety premium is measured by the excess-yield (with respect to Treasuries) on a position consisting of the bond and its associated maturity-matched credit default swap (CDS). In principle, such measure should reflect a purely idiosyncratic safety element. A residual aggregate component would be present if, due to counter-party risk and/or the risk of changes in the price and collateral requirements of CDS contracts, CDS did not fully insure against aggregate risk.

[6]This is the case when (i) firms' production and capital structure decisions do not affect investors' hedging opportunities, as in Diamond (1967) and Carceles-Poveda and Coen Pirani (2009), or (ii) firms operate with a backyard technology and are managed by households, so that equity is not traded, as in Heathcoate et al. (2009) and many other Bewley–type economies. In these environments, however,

Here we show instead that the analysis of general equilibrium production economies with incomplete markets and agency frictions can be grounded on solid theoretical foundations, providing the basis for the integrated study of macroeconomics and corporate finance. Key is the requirement that firm value is defined on the basis of *rational conjectures*, as in Makowski (1983a) and Makowski (1983b): Firms' beliefs about the prices of equity and debt in correspondence of any choice of investment and capital structure are given by the highest among the consumers' marginal valuations for each liability. Under this assumption, shareholders unanimously support value maximization and hence equilibrium firms' choices.[7]

On the one hand, our analysis re-formulates and extends the earlier findings by Makowski (1983a), Makowski (1983b) and Allen and Gale (1988) to economies with asymmetric information and agency frictions. On the other hand, it extends the analysis of general equilibrium with asymmetric information in Prescott and Townsend (1984) to economies with incomplete markets.

2 Benchmark model

There are two dates, indexed by $t = 0, 1$. The economy is perfectly competitive and all agents are price-takers. Only one good is available for both production and consumption. Markets are incomplete, as the only financial assets are firm-issued equity and debt.

There is a continuum of ex-ante identical firms, of unit mass, which produce according to the production function $y = e^{\varepsilon} f(k)$. Here $k > 0$ denotes investment at $t = 0$, y is output at $t = 1$, ε is a random variable, and f is a strictly increasing and strictly concave function. Each firm faces a production choice, given by its investment level k, and capital structure choice, given by the notional amount $B \geq 0$ of debt it issues.

When $e^{\varepsilon} f(k) < B$, the firm defaults on its debt obligations and output is divided pro–rata among all bondholders. It follows that the conditional payoffs to equity and

hedging demand has no effects on capital structure by construction.

[7]Rational conjectures are central to Allen and Gale (1988)'s path-breaking study of optimal security design. However, they have been somewhat overlooked in the literature on the objective function of the firm with incomplete markets. Makowski' work is either not cited or not prominently cited in the main later contributions in the field, among which Dreze (1985), Duffie and Shafer (1986), Demarzo (1993), Kelsey and Milne (1996), Dierker and Dierker (2002), Bonnisseau and Lachiri (2004), Dreze et al. (2007), Carceles-Poveda and Coen Pirani (2009) and not even in surveys such as Grossman and Hart (1990) and Magill and Shafer (1991).

debt at date 1 associated to any choice (k, B) are

$$d^e(k, B; \varepsilon) = \max\left\{e^\varepsilon f(k) - B, 0\right\},$$
$$d^b(k, B; \varepsilon) = \min\left\{1, e^\varepsilon f(k)/B\right\},$$

respectively.

There is a continuum of households of I different types, each of unit mass. A type-i household's endowment consists of $w_0^i \geq 0$ units of commodity at $t = 0$ and a non-negative random allotment $w_1^i(\varepsilon)$ at $t = 1$. Her initial equity stake in the firm is $\theta_0^i \geq 0$.

Households' preferences are described by a common strictly increasing, strictly quasi-concave, Von Neumann-Morgerstern utility function over random consumption sequences $\left\{c_0^i, c_1^i(\varepsilon)\right\}$:

$$U(c_0^i, c_1^i) \equiv u(c_0^i) + \beta\mathbb{E}\left[u(c_1^i)\right], \quad \beta > 0.$$

Trade in financial assets occurs at $t = 0$. Let $d^b(\varepsilon)$, $d^e(\varepsilon)$ denote the payoff of bonds and equity resulting from the values of k, B chosen by firms.[8] Consumers can purchase corporate bonds at the price p and rearrange their stock-holdings by either buying or selling equity at price q.

Letting V denote the cum-dividend market value of the firm and using θ^i and b^i to denote type i's post-trade holdings of equity and bonds, respectively, the household optimization problem writes as

$$\max_{c_0^i, \theta^i, b^i, c_1^i(\varepsilon)} u(c_0^i) + \beta\mathbb{E}\left[u(c_1^i)\right] \tag{1}$$
$$\text{s.t.} \quad c_0^i = w_0^i + \theta_0^i V - q\theta^i - pb^i, \tag{2}$$
$$c_1^i(\varepsilon) = w_1^i(\varepsilon) + \theta^i d^e(\varepsilon) + b^i d^b(\varepsilon) \;\; \forall\, \varepsilon,$$
$$\theta^i \geq 0, \;\; b^i \geq 0.$$

The two inequality constraints rule out short-selling.

Notice that since $V \equiv q - k + pB$, condition (2) can be rewritten as $c_0^i = w_0^i + \theta_0^i(-k + pB) + q(\theta_0^i - \theta^i) - pb^i$. Whenever $k < pB$, firms pay out dividends to initial shareholders as of $t = 0$. Conversely, when $k > pB$, initial shareholders are asked to fund investment.

The firm's problem consists in choosing the pair $\{k, B\}$ that maximizes its market value, i.e.

$$\max_{k,B} -k + q(k, B) + p(k, B)B, \tag{3}$$

[8] For simplicity, we illustrate the model in the case in which all firms make the same production and financial choices. However, as illustrated in Section 3.5, this is not true in general, as identical firms may end up making different choices in equilibrium.

where $q(k, B)$ and $p(k, B)$ indicate the price conjectures regarding the values of equity and debt associated to any possible plan $\{k, B\}$. We shall require such conjectures to be *rational*. For any $\{k, B\}$,

$$q(k, B) = \max_i \; \mathbb{E}\left[m^i d^e(k, B)\right], \tag{4}$$

$$p(k, B) = \max_i \; \mathbb{E}\left[m^i d^b(k, B)\right], \tag{5}$$

where m^i denotes agent's i's inter-temporal marginal rate of substitution evaluated at equilibrium consumption, or $m^i = \beta \frac{u'(c_1^i)}{u'(c_0^i)}$.

This notion of rationality is the Makowski's criterion introduced in Makowski (1980) and Makowski (1983a). Since markets are incomplete there is not a single stochastic discount factor that allows to determine the price conjectures. The rationality condition requires the conjectured prices of equity and debt associated with any pair $\{k, B\}$ to equal the highest marginal valuation across all consumers of the securities' payoffs associated with that pair. In other words, conditional on any choice of investment and capital structure, each security is valued by the investor that values its payoff the most. The price conjectures are determined taking the households' marginal rates of substitution as given, i.e. without internalizing their dependence on the firm's decisions.

When conjectures are rational, they are also *consistent*, i.e. they are correct in equilibrium. This follows immediately from the observation that conditions (4)-(5), when evaluated at the equilibrium choices, coincide with the agents' Euler equations. Rationality requires that those conditions hold true for all feasible pairs (k, B), including all out-of-equilibrium ones!

With complete financial markets, marginal rates of substitutions are equalized across households and hence a unique stochastic discount factor prices all possible payoffs of existing assets. This is no longer true when markets are incomplete. *Rationality* implies that the stochastic discount factor pricing equity may be different from the one pricing bonds, and both may vary for different plans $\{k, B\}$.

Adapting arguments due to Louis Makowski, in Section 5 we show that, when firms operate on the basis of rational conjectures, their objective is well defined. That is, initial shareholders unanimously support firms' choices. We close this section by stating our definition of competitive equilibrium.

Definition 1 ***Competitive Equilibrium.*** *A competitive equilibrium consists of firms' choices k, B, cum-dividend value V, asset prices q, p, price conjectures $p(k, B)$, $q(k, B)$, as well as consumption choices $\left(c_0^i, c_1^i(\varepsilon)\right)$ and portfolio choices $\left(\theta^i, b^i\right)$ for each agent*

$i = 1, \ldots, I$, such that (i) k, B attain the maximum in problem (3), (ii) V is the value of problem (3), (iii) $\left(c_0^i, c_1^i(\varepsilon)\right)$ and $\left(\theta^i, b^i\right)$ solve problem (1) for each agent $i = 1, \ldots, I$, (iv) $p(k, B)$ and $q(k, B)$ are rational, i.e. satisfy (4)-(5), (v) price conjectures and asset payoffs at the equilibrium choices k, B are consistent, i.e.

$$q = q(k, B), \ \ p = p(k, B), \ \ d^e = d^e(k, B), \ \ d^b = d^b(k, B), \tag{6}$$

and (vi) markets clear:

$$\sum_i b^i \leq B, \ \sum_i \theta^i \leq 1. \tag{7}$$

3 Hedging and capital structure

Due to financial market incompleteness, in our model firms' production and financing decisions affect households' hedging opportunities. For this reason, the Modigliani-Miller theorem does not hold. We now discuss in detail how firms' investment and capital structure choices are shaped by their incentives to cater to households' insurance needs.

We specialize our economy by assuming that $I = 2$ and $y = e^{\varepsilon} A k^{\alpha}$, $\alpha \in (0, 1)$, with $\varepsilon \sim N(\mu, \sigma^2)$, $\sigma > 0$. Initial equity ownership is uniformly distributed across households, i.e. $\theta_0^i = 1/2$ for $i = 1, 2$.

Households' temporary utility function is $u(c) = \frac{c^{1-\psi}}{1-\psi}$, with $\psi > 0$. The endowment is the same across types at $t = 0$, i.e. $w_0^i = w_0 \ \ \forall\, i$, but not at $t = 1$. We assume that

$$w_1^i = e^{-\chi_i \mu - \frac{1}{2}\chi_i^2 \sigma^2 + \chi_i \varepsilon},$$

with $\chi_i \geq 0 \ \ \forall\, i$ and $\chi_1 \neq \chi_2$. That is, w_1^i is log-normally distributed with mean parameter $-\frac{1}{2}\chi_i^2 \sigma^2$ and variance parameter $\chi_i^2 \sigma^2$.

The exogenous variation in χ_i disciplines the heterogeneity in endowment risk across groups. A higher value of χ_i is associated with strictly greater variance and skewness of w_1^i, as well as greater covariance with the risk factor, but no change in expected value, as $\mathbb{E}\left(w_1^i\right) = 1$ for all i.[9] Hence households differ in the exposure of their future income to aggregate shocks. Their endowment growth exhibits different levels of correlation with the risk factor and so with equity returns.[10]

[9] It is immediate to verify that $var(w_1^i) = e^{\chi_i^2 \sigma^2} - 1$ and $cov(w_1^i, e^{\varepsilon}) = e^{\mu + \frac{1}{2}\sigma^2}(e^{\chi_i \sigma^2} - 1)$.

[10] Whether households' endowments have a positive loading on the aggregate factor is the subject of debate in the empirical literature. See for example Storesletten et al. (2004), Cocco et al. (2005), Benzoni et al. (2007), Lynch and Tan (2011), Catherine (2020). However, the outcome of this dispute does not impinge on our analysis. We have assumed non-negative loading for convenience, but our results extend to economies with idiosyncratic income shocks, uncorrelated with firms' cash-flows.

We will focus on situations where the degree of consumers' heterogeneity is sufficiently large and the equilibrium features complete market segmentation: All equity is held by agents of one type, while all debt is purchased by agents of the other type.

In the remainder, we posit that type-1 households' endowment is riskless, while type-2's is risky. That is, we set $\chi_1 = 0$ and consider a range of strictly positive values for χ_2. Refer to Figure 1 for an illustration of how the shocks affect production and consumers' income. The remaining parameters are chosen with the only objective of facilitating the illustration of our results.[11]

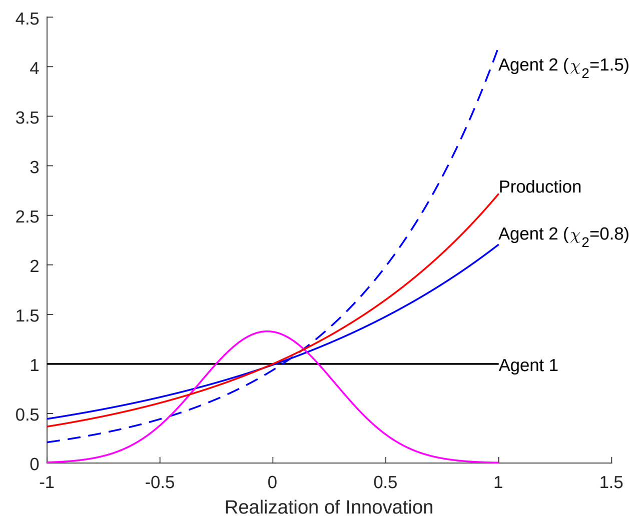

Figure 1: Households' Endowments and Firms' Output.

Since the future income of type-2 households is exposed to shocks, their portfolio choices will be motivated in part by hedging needs. Due to risk aversion, they will be interested in shifting consumption from high- to low-realization states. Since the income of these consumers is positively correlated with firms' cash-flows, the assets issued by firms are not ideal hedging instruments. This is particularly true for equity, which pays a larger share of the firms' cashflow in the higher states, when consumers' income is also higher. In contrast debt pays the largest share of the firms' cash-flow for lower realizations of ε. This explains why, in equilibrium, type-2 households shun equity and turn to debt instead. Because of convexity in marginal utility, they will also respond to increases in risk by increasing mean consumption growth – the classical precautionary saving's motive.

[11] The relative risk aversion coefficient is $\psi = 3$, discount factor $\beta = 0.96$, and initial endowment $w_0 = 1$. The span of control parameter is $\alpha = 0.6$, while the scale factor is $A = 2.5$. The distribution of the productivity shock is fully characterized by $\sigma = 0.3$ and $\mu = -0.025$.

Equity is held by type-1 households, whose endowment is not exposed to shocks.

As intuitive as these features of the equilibrium allocation may be, they could never arise in the familiar complete market environment. In that scenario, all investors have the same valuations for all financial assets and, most important, firms cannot improve upon with their hedging opportunities.

In the equilibrium configuration described above, the households' Euler equations holding with equality are

$$q = \int_{\varepsilon^*(k,B)}^{+\infty} m^1(\varepsilon)\left(Ae^{\varepsilon}k^{\alpha} - B\right)g(\varepsilon)d\varepsilon \tag{8}$$

and

$$p = \int_{\varepsilon^*(k,B)}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon + \frac{Ak^{\alpha}}{B}\int_{-\infty}^{\varepsilon^*(k,B)} m^2(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon, \tag{9}$$

where g denotes the density of the normal distribution and $\varepsilon^*(k,B) \equiv \log\left(\frac{B}{Ak^{\alpha}}\right)$ is the lowest among the realizations of ε consistent with solvency of the firm.

Differently from the complete markets scenario, the pricing kernels for equity and debt do not coincide. Type-1 agents have a strictly higher marginal valuation for equity, while type-2 agents have a strictly higher valuation for debt.

By the consistency requirement, the asset prices in (8)-(9) equal the firm's price conjectures evaluated at the equilibrium choices of capital and debt. The first-order conditions of the firm value maximization problem with respect to capital and debt are then, respectively,[12]

$$\alpha Ak^{\alpha-1}\left[\int_{\varepsilon^*(k,B)}^{+\infty} m^1(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon + \int_{-\infty}^{\varepsilon^*(k,B)} m^2(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon\right] = 1 \tag{10}$$

and

$$\int_{\varepsilon^*(k,B)}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon = \int_{\varepsilon^*(k,B)}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon. \tag{11}$$

Equation (10) requires that the marginal benefit of adding capital equals its marginal cost. The marginal unit of capital benefits shareholders (type-1 households) in the solvency states and bondholders (type-2 households) over the default region.

At the margin, raising debt shifts resources from shareholders to bondholders over the solvency states. Condition (11) pins down the optimal level of debt chosen by firms by equating bondholders' marginal benefit to shareholders' marginal loss. If markets were complete, condition (11) would hold as an identity, leaving debt undetermined – the Modigliani-Miller result.

[12]The firm's first order conditions are only necessary, since with incomplete markets and rational conjectures, firms' optimization problem (3) is not necessarily convex.

3.1 Comparative statics: Hedging demand

We have argued that, because of incomplete markets, firms' production and financing decisions respond to investors' hedging needs. To better understand how this actually unfolds, we carry out a comparative statics analysis of the effects of an increase in χ_2 on the equilibrium values of capital, leverage, consumption, and financial assets' returns. We study the equilibria that obtain for $\chi_2 \in [0.8, 5]$.

3.1.1 Investment and leverage

Recall that raising χ_2 induces a mean–preserving spread of type-2's endowment. Hence the greater is χ_2, the larger are those consumers' hedging needs. Because of convexity in marginal utility, all else equal this implies a rise in the right-hand-side term of condition (11) – the firm's marginal benefit from issuing debt. Firms respond to the increase in consumers' hedging demand by issuing more bonds so as to cater to those needs. This happens in spite of the fact that a higher χ_2 also increases the covariance between type 2's consumption growth and bond returns over the default region, thus making corporate bonds a worse hedging instrument.

A higher χ_2 also induces firms to increase k. To see why, rewrite the terms between square brackets in condition (10) as

$$\begin{aligned}
&[1 - G(\varepsilon^*)]\; cov\left[m^1(\varepsilon), e^{\varepsilon}|\varepsilon > \varepsilon^*\right] + G(\varepsilon^*)\; cov\left[m^2(\varepsilon), e^{\varepsilon}|\varepsilon < \varepsilon^*\right] + \\
&+ [1 - G(\varepsilon^*)]\; \mathbb{E}\left[m^1(\varepsilon)|\varepsilon > \varepsilon^*\right] \mathbb{E}\left(e^{\varepsilon} \mid \varepsilon > \varepsilon^*\right) + \\
&+ G(\varepsilon^*)\; \mathbb{E}\left[m^2(\varepsilon)|\varepsilon < \varepsilon^*\right] \mathbb{E}\left(e^{\varepsilon}|\varepsilon < \varepsilon^*\right),
\end{aligned} \tag{12}$$

where G is the CDF of the normal distribution. Expression (12) is the generalization to the incomplete-market scenario of terms familiar in complete-markets asset pricing models. The marginal unit of capital adds to equity-holders' payoff in solvency states and to bondholders' payoff in default states, respectively.

In the first line are conditional covariances between the investors' marginal rates of substitution and the productivity innovation. Both terms are negative, reflecting the adverse impact of the marginal unit of investment on investors' utility deriving from the positive conditional correlation between assets' payoffs and investors' consumption growth. An increase in χ_2 is associated with a rise in the absolute value of these terms, therefore discouraging investment.

The remaining terms are positive and reflect the precautionary saving motive. Consider the last addendum. As noted above, due to convexity of marginal utility, a higher

χ_2 leads to an increase in debt-holders' conditional expected marginal rate of substitution. In other words, it leads to a higher demand for consumption in default states. This encourages investment. The latter effect prevails for all values of χ_2 we have considered. Hence an increase in the demand for hedging induces an increase in investment.

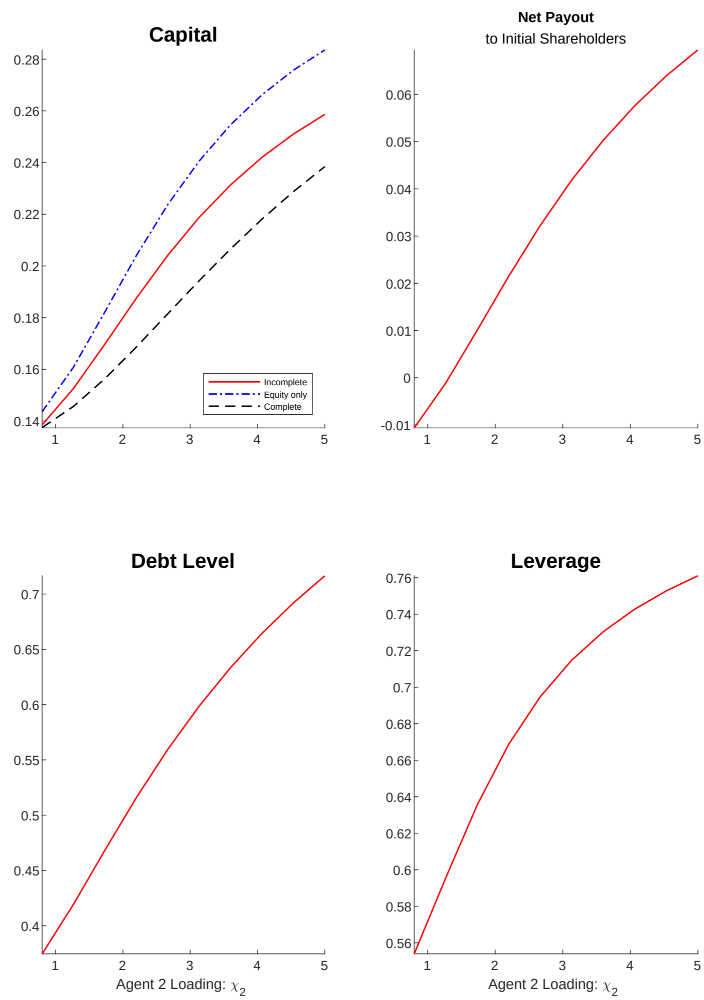

Figure 2: Firms' Choices.

We illustrate our comparatives statics in Figure 2. The red solid curves describe the values obtained at the equilibrium allocations of the incomplete market economy under consideration. We benchmark these values against those arising under two alternative financial market arrangements. In one (dashed, black curves), markets are complete. Equilibria feature the equalization of marginal rates of substitution. In the other (dash-dot, blue lines), equity is the only asset available to investors.

A higher χ_2 is associated with greater values for both B and k, i.e. a greater supply of hedging instruments for the benefit of type-2 agents. At the margin, more debt is appealing to them because it allows for a better distribution of consumption over the solvency region. More capital gives them the opportunity to improve upon their consumption allocation over default states.

Our theory also features a non-trivial complementarity between the two firm's choices. For given k, increasing B has a negative impact on type-1 agents, since they only hold equity. A higher k has the effect of reducing the magnitude of such impact, thereby allowing for an even greater increase in debt. To see this, simply substitute (11) into (12).

The increase in debt issuance is associated with an increase in market leverage $pB/(pB+q)$. To a large extent, this is due to the relative increase in B and k. The top-right panel of Figure 2 illustrates $pB-k$, i.e. the dividend paid to initial shareholders at $t=0$. For low values of χ_2, dividends are negative, indicating that initial shareholders are called to fund investment. However, as χ_2 rises, dividends eventually become positive and progressively larger. Not only firms respond to the increase in hedging demand by issuing more debt. They increase debt at a higher rate than investment, transferring a larger fraction of debt issuance proceeds to shareholders. Firms determine their capital structure not just based on their investment financing needs. Rather, they generate value for their shareholders by catering to investors' hedging needs. This is a novel factor driving capital structure choice, which can only arise in an incomplete market economy.

Our theory rationalizes recent evidence that other approaches to capital structure choice have a hard time accounting for. Using cross-sectional data for US non-financial firms, Mota (2021) constructs estimates of corporate bonds' safety premia. It then documents that while an high premium forecasts debt issuance, it has little or no predictive power for investment or acquisitions. In other words, companies with relatively high safety premia issue more debt and distribute most of the issuance proceeds to their shareholders.

Neither dividends, nor debt nor leverage are reported for the alternative market arrangements, since debt is not available in one scenario and is indeterminate in the other. Investment and firm value are uniformly higher when equity is the only asset, since the more limited insurance possibilities afforded by agents when firms cannot issue debt imply a stronger type-2's precautionary motive.

With complete markets, a full set of contingent claims allow the agents to efficiently share risk. The portion that is left, monotonically increasing in χ_2, is aggregate in nature. As χ_2 rises, firms cater to the larger precautionary motive by providing investors with more of the only means to move resources intertemporally.

3.1.2 Consumption and asset returns

An increase in the demand for hedging also affects equilibrium asset prices and households' consumption. In Figure 3, we plot the mean and variance of consumption growth for both agents. The variance of agent-2's consumption growth increases monotonically with χ_2.

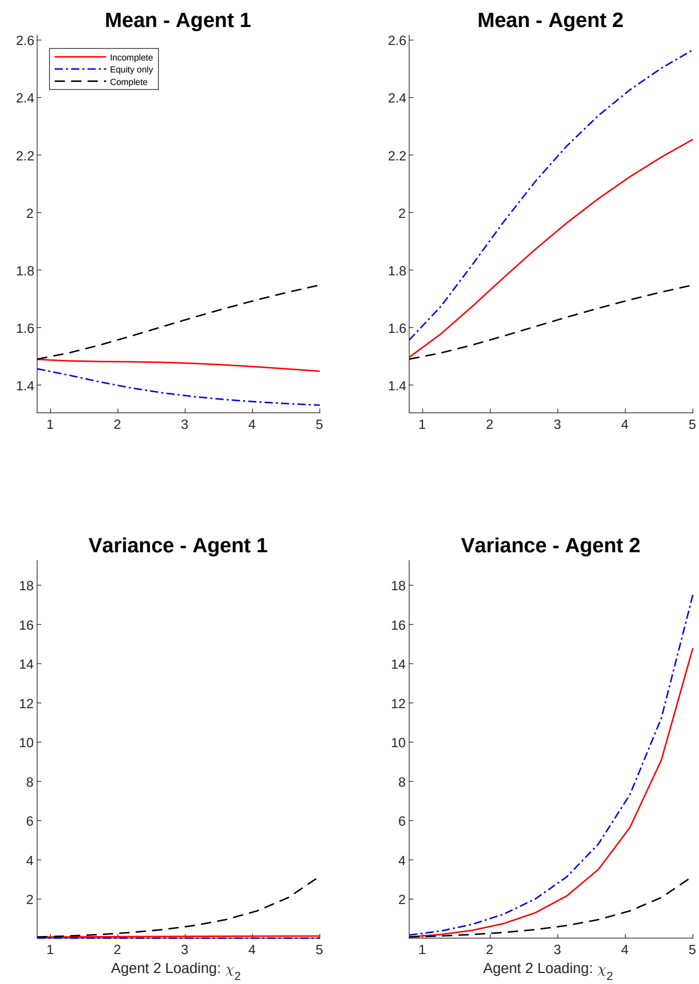

Figure 3: Consumption growth.

However, this is also the case with complete markets, since the aggregate endowment's risk is monotonically increasing in χ_2.

What is most relevant is that the difference between the variances that obtain in the incomplete and complete markets setups also increases with χ_2. The reason is that with incomplete markets type-2 agents only rely on bonds, an imperfect hedging instrument, to satisfy their larger hedging needs.

Firms cater to the greater hedging demand by issuing more debt, which allows to limit the pass-through of endowment risk on consumption risk. As a by-product, type-2 agents' mean consumption growth also increases. Since firms split the larger receipts from bond sales between investment and dividends, this results in a lower payout to equity-holders at $t = 1$ and hence a lower mean consumption growth for type-1 agents.

We turn next to asset returns. Since a risk-free bond is not available for trade, the risk-free rate displayed in Figure 4 is the inverse of the shadow price of an asset with riskless unit payoff. Since type-1 agents would always value such asset strictly less than type-2 agents, the price coincides with the latter's marginal valuation for this asset and is an indicator of their hedging needs.

The monotonicity of the risk-free rate in χ_2 reflects type-2 agents' consumption growth process. On the one hand, higher mean growth is associated with a higher risk-free rate. On the other hand, higher variance of growth tends to reduce it. The latter effect dominates.

The corporate bond spread is the difference between the expected return on the bond – its expected payoff divided by the equilibrium price – and the risk-free rate. One can easily show that

$$\mathbb{E}(R^b) - R^f = -cor(m^2, R^b)\frac{\sigma\left(m^2\right)}{\mathbb{E}\left(m^2\right)}\sigma\left(R^b\right).$$

As χ_2 increases, both the standard deviation of returns $\sigma\left(R^b\right)$ and the price of risk – measured as $\sigma\left(m^2\right)/\mathbb{E}\left(m^2\right)$ – rise. Since $cor(m^2, R^b) < 0$, the corporate bond spread also rises with χ_2.

The excess return on equity is also monotonically increasing in χ_2. To investigate why, let's express it as

$$\mathbb{E}(R^e) - R^f = \left[\frac{1}{\mathbb{E}\left(m^1\right)} - \frac{1}{\mathbb{E}\left(m^2\right)}\right] - cor(m^1, R^e)\frac{\sigma\left(m^1\right)}{\mathbb{E}\left(m^1\right)}\sigma\left(R^e\right). \tag{13}$$

The term in square brackets, which is specific to the incomplete market environment, reflects the fact that the pricing kernels of equity and the risk–free bond are different. Since type-1 agents value the risk-free asset strictly less than type-2s, that term is strictly positive.

The pattern of the excess equity return is mainly the result of two forces. The rise in χ_2 triggers an increase in leverage and in the risk of type-2 agents' consumption growth. The former drives the variance of equity returns $\sigma\left(R^e\right)$ higher. The latter leads to a decline in the risk–free $1/\mathbb{E}(m^2)$.

The risk-free rate lies between the values that obtain with complete markets and with

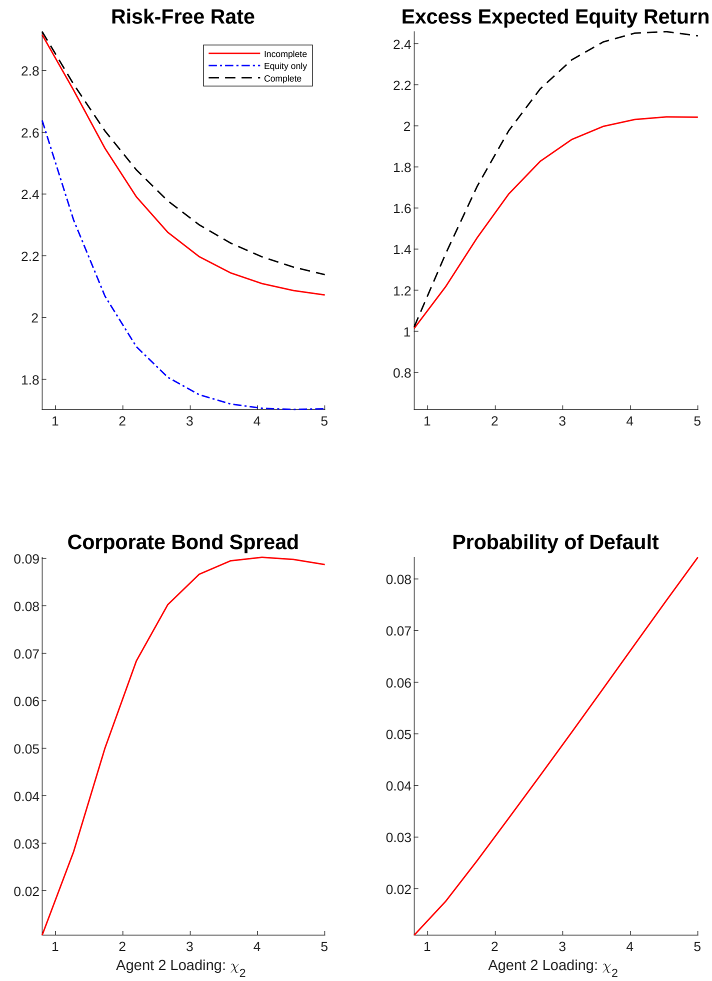

Figure 4: Asset returns

equity only, respectively. The pattern is driven by the variance of consumption growth of type-2 agents, who price the bonds in all three scenarios. As shown in Figure 3, the variance is largest with equity only and smallest with complete markets.

In the right-top panel of Figure 4, debt in the complete-market scenario was chosen so that the default probability is the same as in the incomplete-market case. This choice eliminates any effect of leverage on the difference in expected excess equity returns via its impact on $\sigma(R^e)$. Refer to equation (13). When markets are complete, the term in square brackets is identically zero. However, the market price of risk is higher, because the complete-market variance of consumption growth is higher than type-1's variance under incomplete markets. In our example, this second consideration dominates, leading to a higher excess equity return under complete markets.

3.2 Comparative statics: Aggregate risk

We now turn to the study of how key features of equilibrium allocations vary with respect to changes in the variance of the aggregate shocks. The goal is to gain some insight into the properties of more general versions of our setup, featuring time-varying aggregate risk. In Figures 5 and 6 we illustrate how the firms' equilibrium choices and asset returns vary with σ for $\chi_2 = 0.8$ and $\chi_2 = 3.0$, respectively.

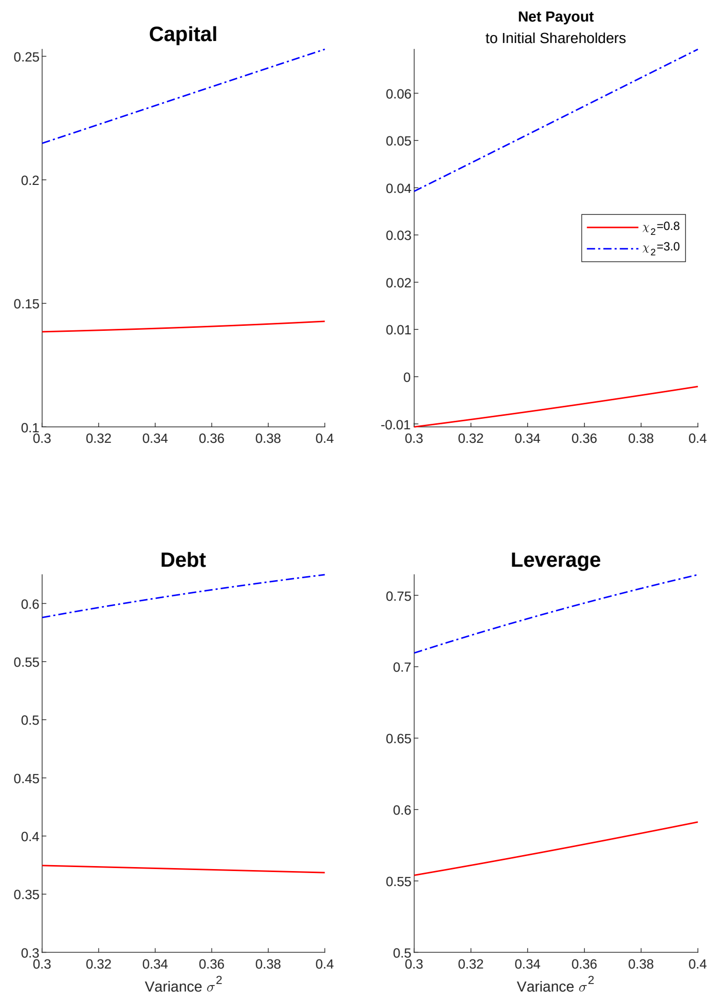

Figure 5: Capital Structure and Aggregate Risk

An increase in σ has countervailing effects on firms' incentives to issue debt. Refer once again to condition (11). On the one hand, the variance of type–2 households' endowment rises, leading to greater hedging demand. On the other hand, the variance of firms' cash-flows also rises, leading to a higher variance of consumption growth for type-1 agents. In

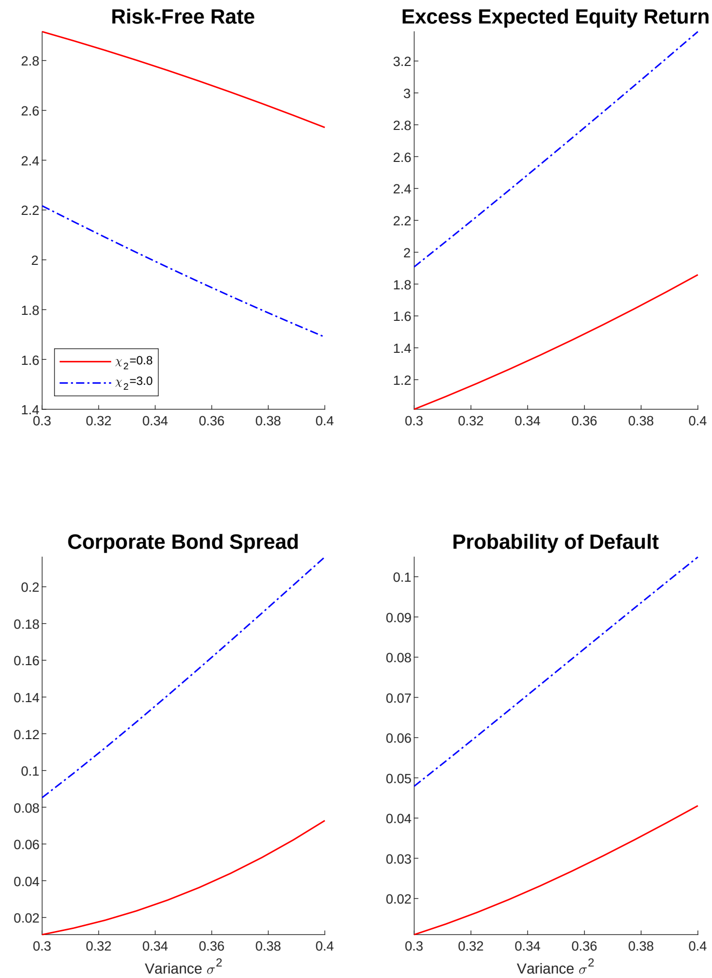

Figure 6: Asset returns and Aggregate Risk

turn, this means a higher firm's marginal loss of increasing debt.

Figure 5 suggests that the net effect on debt depends on other parameters, among which the loading of type-2 agents on the aggregate shock. When χ_2 is relatively low ($\chi_2 = 0.8$), corporate debt declines with σ, while the opposite occurs for higher χ_2.

Leverage, however, is monotone increasing in σ. Since empirical proxies for aggregate risk covary negatively with output, our finding provides a rationalization for the recent finding – see Halling et al. (2016) – that US firms' leverage is countercyclical.

Beyond the obvious impact of debt, the comparative statics of market leverage $pB/(pB+q)$ depends on the effects of σ on the price of debt and on investment. The top-right panel in Figure 5 shows that dividends $pB - k$ increase with σ. This is in part due to the fact that in equilibrium a larger demand for bonds leads to a higher price p.

The above discussion clarifies that the forces shaping the response of firms' capital structure to aggregate shocks are different from those at work in the partial equilibrium model of Hackbarth et al. (2006) and in the complete market model of Bhamra et al. (2010). In those frameworks, fluctuations in the economy's stochastic discount factor induced by time-varying aggregate risk affect the trade-off between the tax advantage of debt and the cost of bankruptcy.

3.3 Comparative statics: Supply of public debt

We have so far illustrated the hedging services provided by corporate debt in a scenario where equity is the only alternative asset. In reality, investors with insurance needs also rely on other financial instruments, among which sovereign bonds and derivatives.

We now turn to investigating how the equilibria described above change when households can also purchase risk-free debt, available in fixed and exogenous supply. We will refer to such asset as public debt. In the next section we will explicitly model derivatives.

Figures 7 and 8 illustrate equilibria in scenarios that only differ in the provision of public debt. The entire supply of this new asset is purchased by type–2 agents, reducing the demand for firms' hedging services and leading to lower capital and corporate debt. Notice however that the crowding out of privately-provided hedging instruments is only partial – both aggregate savings and average consumption growth increase.

As a result of the improved risk–sharing opportunities afforded by type-2 agents, the risk–free rate increases. The excess return on equity declines, mostly because of the lower volatility of equity's payoff implied by lower leverage.

In their study of US non-financial firms, Graham et al. (2014) find that government debt is negatively correlated with corporate debt and investment, and strongly so for relatively safer issuers. Demirci et al. (2019) find similar results in a large cross-section of countries. Our finding provides a firm theoretical grounding for Graham et al. (2014)'s suggestion that "*financially healthy corporations act as liquidity providers by supplying relatively safe securities to investors when alternatives are in short supply, and that this financial strategy influences firms' capital structures and investment policies.*"

3.4 Comparative statics: Short-selling costs

In all the equilibria considered above, the short–sale constraint on debt is binding for type-1 agents. We now relax this constraint, by allowing for intermediated short-sales.[13]

[13]The general analysis of this extension is important from a theoretical standpoint. See Section the Appendix for a discussion of the more general properties of the equilibria of this economy.

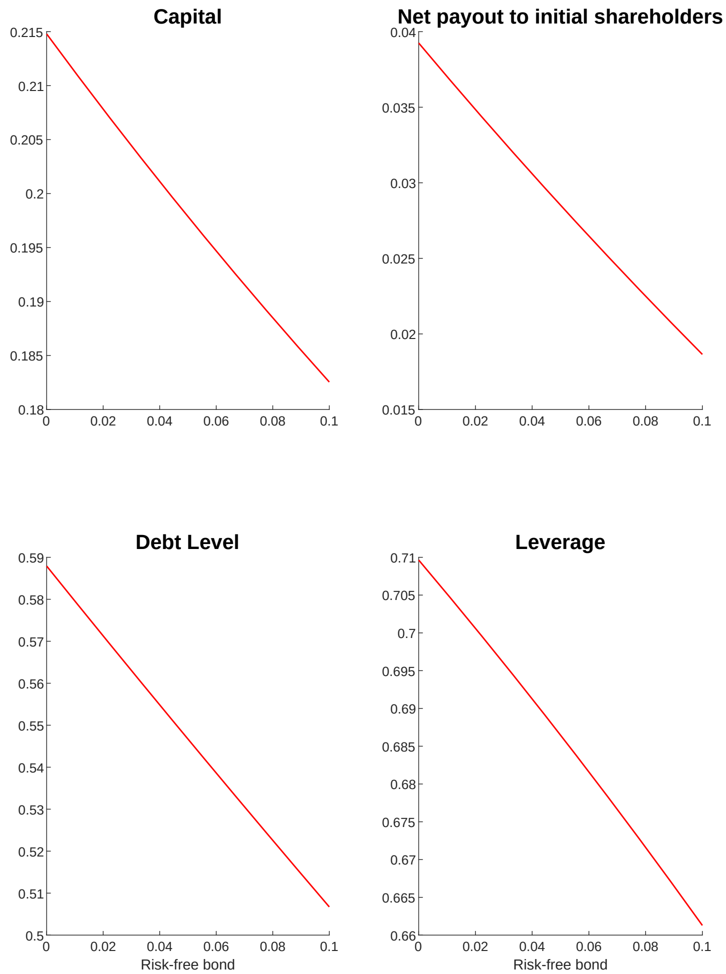

Figure 7: Capital structure choices with risk–free public debt

We introduce financial intermediaries who issue derivatives corresponding to short positions on firms' debt for the benefit of type-1 agents and long positions on the same underlying security, for the benefit of type-2 investors.

This exercise has clear analogies to the addition of public debt considered above. In common with that scenario, the supply of hedging instruments available to type-2 consumers increases. It differs however in two key dimensions. To start with, the supply of derivatives is endogenous, i.e. it depends upon type–1 households' appetite for short positions and on intermediation costs. Secondly, the net supply of assets still equals the sum of equity and debt issued by firms.

We assume that consumers taking a short position on firm's debt partly default. Default is costly, in the sense that a portion of the repayment from short-positions' holders

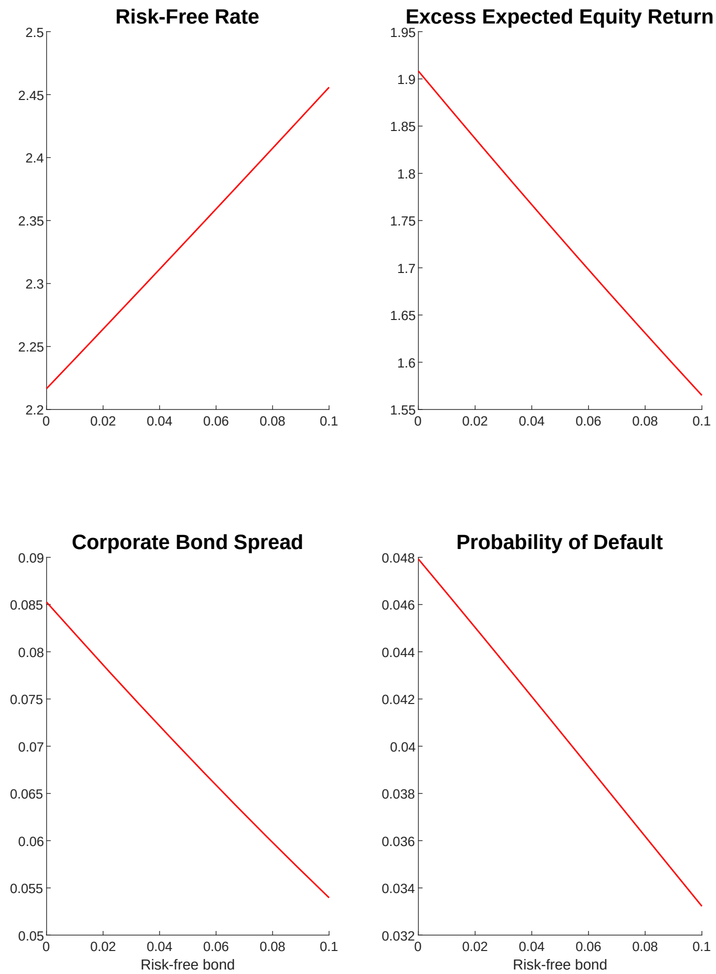

Figure 8: Asset returns with risk–free public debt

does not reach the intermediaries. For simplicity, we posit that the deadweight loss is a fraction $\delta \in (0,1)$ of the amount due.

To ensure its ability to meet its future obligations in the presence of default risk, an intermediary who issues H long and short positions, respectively, will hold corporate debt in an amount γ such that

$$H \leq H(1-\delta)+\gamma. \tag{14}$$

Hence originating short positions involves a linear cost, given by the face value of the debt – δH – needed to fully cover the shortfall in proceeds due to investors' default. To cover such cost, a spread will arise between the price of long positions p^+ and the price of short positions p^-.

Let p still denote the market price of debt. The intermediary chooses the number of positions H and the quantity γ of debt held (collateral) so as to maximize its profits at $t = 0$, given by $(p^+ - p^-)H - p\gamma$, subject to the solvency constraint (14).

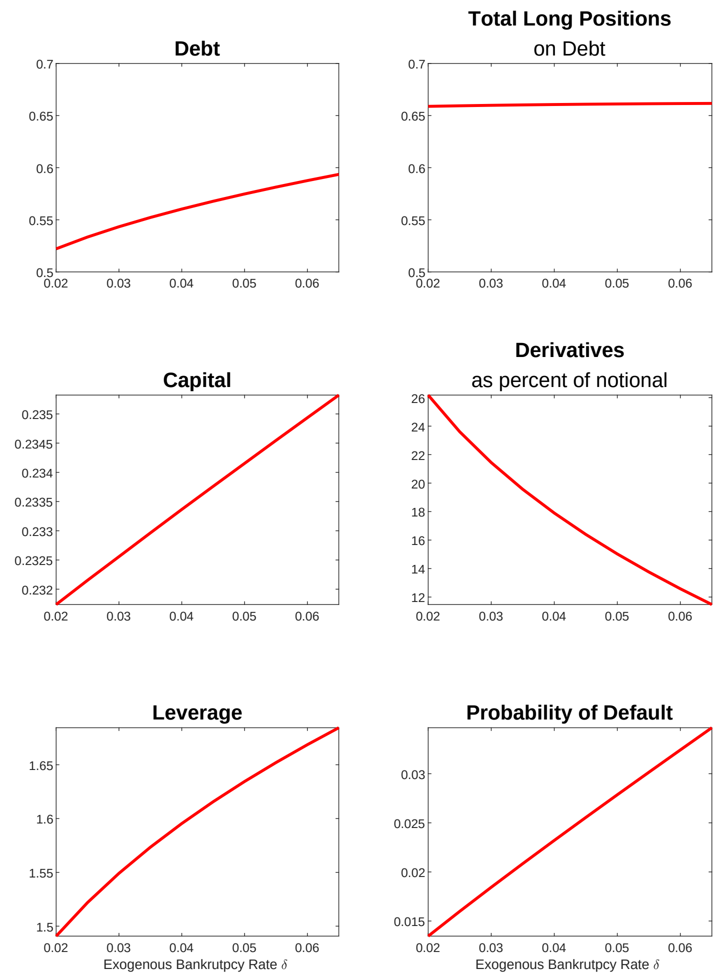

Figure 9: Short Sales and Capital Structure.

A solution to the intermediary's problem exists and features a strictly positive level of intermediation, provided that the spread between the price of long and short positions satisfies the no-arbitrage condition $p^+ - p^- = \delta p$. In such case, the spread allows to fully recoup the default cost of intermediation. Intermediaries make zero profits and purchase an amount δ of corporate debt per unit of derivative issued – just enough to cover the shortfall due to default. It follows that overall intermediation activity is limited by the amount of the firm's outstanding debt, B.

When the volume of intermediation is not constrained, i.e. $\delta H < B$, a portion of the outstanding debt is directly held by consumers. In this case, debt and long positions trade at the same price, i.e. $p = p^+$. If instead $\delta H = B$, the firm's debt is entirely held by intermediaries. In such scenario, debt sells at a premium over the long positions, due to its additional role as collateral. That is, $p > p^+$.

In Figure 9 we compare equilibria that differ only in the default rate δ, which is also the unit cost of intermediation. For our parameter values, intermediation is never constrained. Type–1 investors acquire the short positions on debt, while type–2 households purchase all long positions. When intermediation costs are relatively high, the volume of intermediation is negligible. As the cost declines, intermediation becomes rises and the availability of derivatives increases the supply of hedging opportunities available to type-2 households. As a consequence, firms optimally choose to lower their investment and leverage. These features are reminiscent of those arising when an inelastic supply of risk-free debt is introduced in the economy.

Differently from the scenario of Section 3.3, however, the net supply of assets only changes due to the firms' equilibrium response. The improved hedging services available to type-2 agents are provided by type–1 investors, who purchase the short positions. In turn, this means that te implications for households' consumption processes are different. In particular, as intermediation increases, mean consumption growth declines for both types. Furthermore, since the long positions are perfect substitutes for corporate bonds in the eyes of type-2 agents, the crowding out of corporate bonds is essentially complete. The top-right panel of Figure 9, which reproduces the total of long positions on debt, reveals that the increase in the supply of derivatives is accompanied by an equivalent decline in corporate debt.

3.5 Comparative statics: Technology choice

In this section we generalize our model to show how the supply of hedging instruments may depend on firms' technology choices. We expand the production possibilities allowing firms to choose between the risky technology $e^{\varepsilon} A k^{\alpha}$ and an alternative, safer technology. For simplicity, let the latter be entirely deterministic, i.e. $A_w k^{\alpha}$, with $A_w < A\mathbb{E}(e^{\varepsilon})$.[14] The production function becomes

$$F(k, \phi; \varepsilon) = \phi e^{\varepsilon} A k^{\alpha} + (1 - \phi) A_w k^{\alpha}, \;\; \phi \in \{0, 1\}.$$

We look for conditions under which ex-ante identical firms specialize, with a positive measure of them opting for the safe technology, i.e. setting $\phi = 0$, and the remainder

[14] In the numerical example, $A_w = 1.75$ and $A\mathbb{E}(e^{\varepsilon}) = 2.5$.

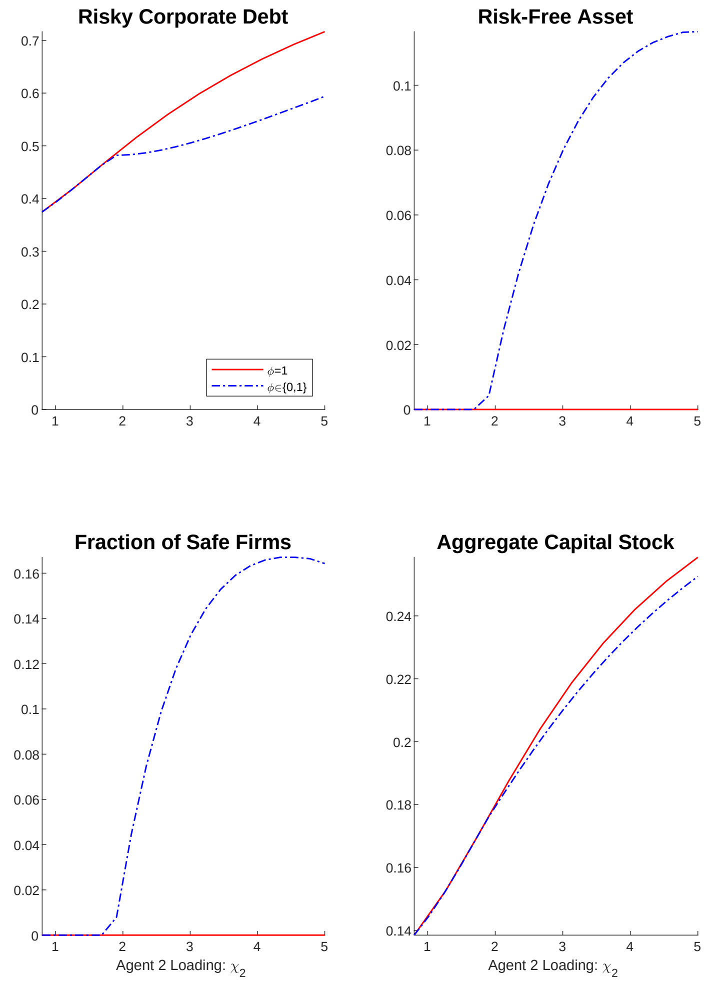

Figure 10: Specialization

still operating the risky technology ($\phi = 1$).[15] Figures 10, 11 and 12 illustrate equilibria indexed by $\chi_2 \in [0.8, 5]$. Red solid lines describe equilibrium values arising in the scenario above, when $\phi = 1$ by assumption for all firms. The blue (dash-dot) lines illustrate instead equilibria when firms are free to randomize over the feasible choices of $\phi = 0, 1$.

The lower-left panel in Figure 10 shows that when χ_2 is relatively low, all firms choose the risky, more productive technology. As the endowment of type–2 households becomes riskier, specialization arises. Equilibria feature a non-zero fraction of firms choosing the

[15]The non-convexity in the choice of technology is actually not needed to generate specialization. When markets are incomplete, rationality of the price conjectures already implies that the firm's choice problem is not convex. Equilibria where ex-ante identical firms specialize in their production or financing choices may arise in the absence of any further assumption. We make this further assumption with the only purpose of sharpening our exposition.

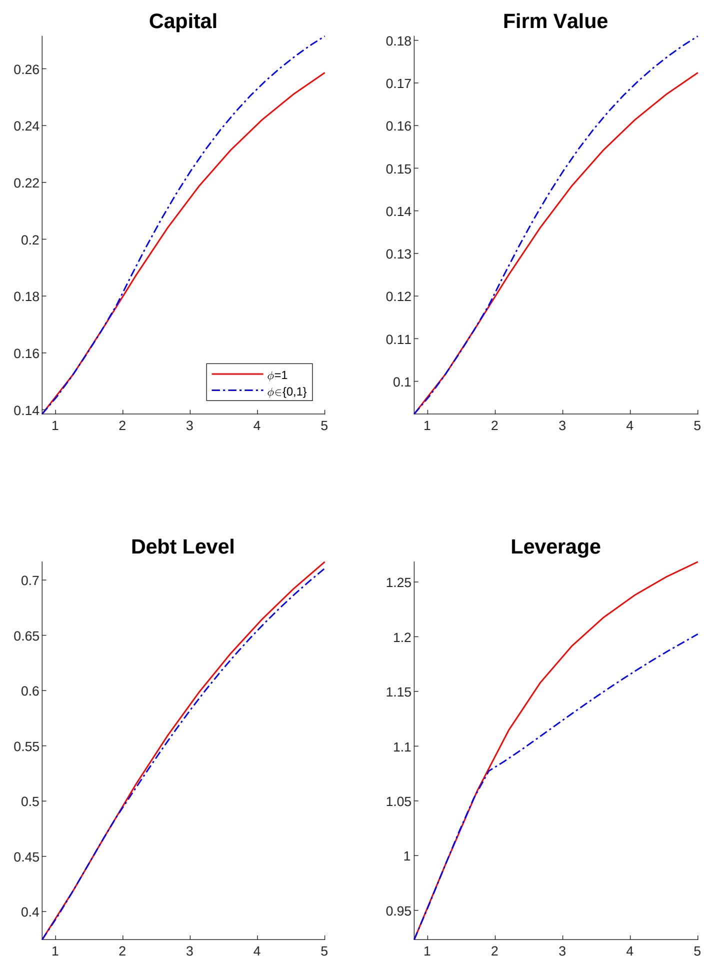

Figure 11: Choices of risky firms with specialization

safe technology.

Since the output of firms selecting $\phi = 0$ is risk-free, type-2 agents always value their equity strictly more than type-1's. Thus, the conjectured equilibrium value of operating the risk-free technology is

$$\max_{k} \; -k + A_w k^{\alpha} \int_{-\infty}^{+\infty} m^2(\varepsilon) g(\varepsilon) d\varepsilon = \max_{k} \; -k + \frac{A_w k^{\alpha}}{R^f}. \tag{15}$$

For relatively low χ_2, the risk-free rate – which proxies for the hedging value of the riskless asset – is not small enough to compensate for the lower productivity of the safe technology. The value of problem (15) is strictly lower than that guaranteed by the risky technology. As a result, the lottery over ϕ is degenerate. All firms choose risk.

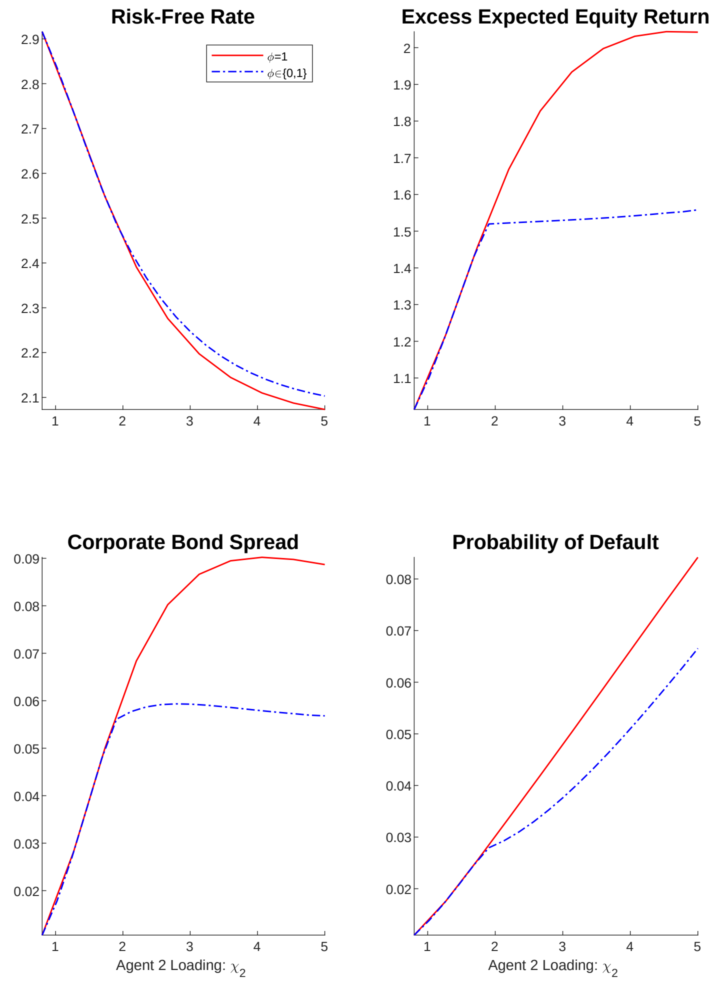

Figure 12: Specialization and asset returns.

As χ_2 grows larger, so do type-2 agents' hedging needs and their valuation of the risk-free asset. For χ_2 greater than a certain threshold, a non–zero measure of firms respond to the higher hedging demand and associated lower value of R^f by selecting the safe technology.

Because of firms' specialization, a new asset becomes available to households, making markets endogenously more complete. In addition to the corporate debt issued by firms selecting the risky technology, type–2 agents purchase the riskless asset issued by safe firms.

This outcome bears obvious analogies with the one reached in the previous section. There, the increase in hedging instruments was due to financial intermediaries that found it optimal to increase their production of derivatives on corporate debt. Here, the novel asset is directly produced by non-financial firms who alter their technology choice and hence the risk of their liabilities.

The positive association between hedging demand and the supply of assets by safer firms is in line with Mota (2021), who finds that debt issuance by safer firms is larger and more responsive to increases in the aggregate safety premium.

The lower right panel of Figure 10 shows that equilibria with specialization feature a lower capital stock. This is the case because the safe technology is less productive. In fact, Figure 11 suggests that in response to specialization, risky firms increase their capital stock in order to make up for the negative effect of the extensive margin on the volume of risky equity. A larger choice of k by risky firms is needed to cater to type-1 agents, in a world where fewer firms supply risky equity. Their debt issuance, instead, barely changes, leading to lower leverage.

Figure 12 illustrates the impact of specialization on returns. The supply of risk-free assets by firms choosing the safe technology reduces the variance of type–2 agents' consumption growth. As a result, the risk–free rate increases. Lower leverage by risky firms leads then to lower excess equity return, default probability, and corporate bond spreads. These changes mirror those observed in Sections 3.3 and 3.4. In all such scenarios, their root cause lies in the greater supply of hedging instruments.

4 Agency

We generalize our environment by introducing an agency friction akin to the standard asset substitution problem pioneered by Jensen and Meckling (1976). The production function is the same as in Section 3.5 except for the assumption that firms can now choose any combination of the safe and the risky technology. The production possibility frontier is

$$F(k, \phi; \varepsilon) = \phi e^{\varepsilon} A k^{\alpha} + (1 - \phi) A_w k^{\alpha}, \quad \phi \in [0, 1].$$

For given k, a larger ϕ is associated with greater output volatility and higher expected output. Figure 13 provides a graphical rendition of how – everything else equal – changes in the loading ϕ affect the payoffs of equity and debt. As ϕ rises, the yield of equity tends to increase, while the yield of debt tends to decrease.

The choices of k and B are taken by initial shareholders to maximize firm value. Both are observable by outside investors and even contractible upon. The choice of ϕ, instead, is not observable by outside investors and is taken by end-of-period shareholders to maximize their benefits from holding equity alone, hence the agency friction.

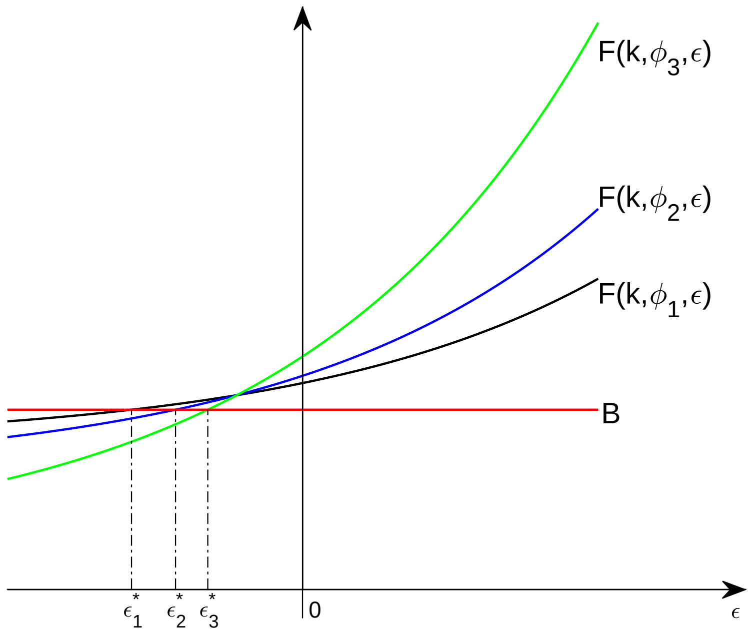

Figure 13: The effect of ϕ on the payoffs of debt and equity.

The conflict between bondholders and end-of-period shareholders arises because, for given k and B, the shareholders' valuation of equity will be maximal for a loading ϕ greater than the level that maximizes firm value. This follows from the effect of ϕ on the payoffs of equity and debt described above.

We posit that increasing the expected value of the firm's production as well as its risk to levels implied by ϕ requires end-of-period shareholders to incur a cost $C(\phi)$, where C is a twice continuously differentiable, strictly convex, and strictly increasing function of ϕ. Since the cost is borne by end-of-period shareholders, acquiring one unit of equity at $t = 0$ requires from them an outlay $q + C$.

We introduce the function C with the only purpose of avoiding the uninteresting scenario in which shareholders always opt for the corner solution $\phi = 1$. In the numerical exercises to follow, we will assume $C(\phi) = c/(1 - \phi)$, for $c > 0$.

The specification of the *rational* price conjectures associated to any triplet $\{k, B, \phi\}$ is the natural extension of that in (4)-(5) to the scenario where the payoffs of equity and debt are also a function of ϕ. For any choice $\{k, B\}$, the anticipated risk loading is the value of ϕ that maximizes the value of equity

$$q(k,\phi,B) = k^\alpha \int_{\varepsilon^*(k,B,\phi)}^{+\infty} m^1(\varepsilon)[\phi A e^\varepsilon + (1-\phi)A_w - B]g(\varepsilon)d\varepsilon - C(\phi), \tag{16}$$

where $\varepsilon^*(k,B,\phi) = \log\left[\frac{Bk^{-\alpha}-(1-\phi)A_w}{\phi A}\right]$ is the default threshold. Equation (16) results from observing that in equilibrium type-1 households have the highest valuation for equity.

The firm's optimization problem then writes as

$$\max_{k,B} \; -k + q(k,B,\phi) + p(k,B,\phi)B \tag{17}$$

$$\text{s.t.} \;\; \phi \in \arg\max \; q(k,B,\phi), \tag{18}$$

where (18) is the end-of-period shareholders' incentive compatibility constraint.

Consisting of a simple modification of that in Section 2, the definition of equilibrium is omitted for brevity. However, a general analysis of the properties of equilibria with incomplete markets and agency is of independent interest. We refer the interested reader to Section 5.

Provided that the second derivative of the cost function is large enough, ϕ obtains as the unique solution to the first-order condition

$$k^\alpha \left[\int_{\varepsilon^*(k,B,\phi)}^{+\infty} m^1(\varepsilon)[Ae^\varepsilon - A_w]g(\varepsilon)d\varepsilon\right] - C'(\phi) = 0. \tag{19}$$

Recall now that in the benchmark scenario without agency, the choice of ϕ is observable and is taken by equityholders so as to maximize firm's value. In that scenario, the first order condition is

$$k^\alpha \left[\int_{-\infty}^{\varepsilon^*(k,B,\phi)} m^2(\varepsilon)[Ae^\varepsilon - A_w]g(\varepsilon)d\varepsilon + \int_{\varepsilon^*(k,B,\phi)}^{+\infty} m^1(\varepsilon)[Ae^\varepsilon - A_w]g(\varepsilon)d\varepsilon\right] - C'(\phi) = 0. \tag{20}$$

The additional term appearing in (20) reflects the fact that without agency the shareholders internalize the effect of ϕ on bondholders' debt value. Provided that $B < A_w k^\alpha$, i.e. B is not too large, such term is going to be negative. In turn, this implies that – everything else equal – agency induces shareholders to choose an inefficiently high risk loading.

We turn next to the effects of agency on the initial shareholders' choice problem and on the equilibrium values of k, B. Let $\phi(k,B)$ denote the map defined by (19) for any given k, B. Such relation describes the risk loading ϕ that is anticipated by outside investors for any k, B. Initial shareholders will take such mapping as given when choosing capital and debt. In other words, requiring $\phi = \phi(k, B)$ implements the incentive constraint (18). Necessary conditions for a solution of the initial shareholders' problem are

$$\phi \alpha A k^{\alpha-1} \left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon) e^{\varepsilon} g(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon^*} m^2(\varepsilon) e^{\varepsilon} g(\varepsilon) d\varepsilon \right] +$$
$$+(1-\phi)\alpha A_w k^{\alpha-1} \left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon) g(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon^*} m^2(\varepsilon) g(\varepsilon) d\varepsilon \right] +$$
$$+ \frac{\partial \phi}{\partial k} k^{\alpha} \int_{-\infty}^{\varepsilon^*} m^2(\varepsilon) [A e^{\varepsilon} - A_w] g(\varepsilon) d\varepsilon = 1 \tag{21}$$

and

$$-\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon) g(\varepsilon) d\varepsilon + \int_{\varepsilon^*}^{+\infty} m^2(\varepsilon) g(\varepsilon) d\varepsilon + \frac{\partial \phi}{\partial B} k^{\alpha} \int_{-\infty}^{\varepsilon^*} m^2(\varepsilon) [A e^{\varepsilon} - A_w] g(\varepsilon) d\varepsilon = 0, \tag{22}$$

where ε^* is a shorthand for $\varepsilon^*(k, \phi, B)$.

The first two terms in both (21) and (22) are in common with the necessary conditions of the no-agency case. In fact, they constitute the immediate generalization of conditions (10) and (11). The last term on the left-hand-side of either equation is instead special to the environment with agency, reflecting the effects of k and B, respectively, on the end-of-period shareholders' choice of ϕ.

Consider the last term on the left-hand-side of (22). It can be established that, in equilibria where $B < A_w k^{\alpha}$, $\partial\phi/\partial B > 0$. Everything else equal, a lower debt induces end-of-period shareholders to select a lower risk loading ϕ. This is the defining feature of the asset substitution problem. Since the term that multiplies $\partial\phi/\partial B$ is negative, coeteris paribus initial shareholders will want to lower their debt choice to prevent end-of-period shareholders from taking inefficiently high risks.

In our incomplete-market environment, however, this consideration does not guarantee that agency leads to a lower equilibrium's corporate debt. This is the case because the higher ϕ induced by agency also impacts the investors' consumption processes, differentially so for shareholders and bondholders. In turn, this modifies the firms' incentives to issue debt when catering to investors hedging needs.

We saw in Section 3 that, absent the choice of ϕ, it is optimal for firms to increase debt as long as the marginal gain from bondholders is greater than the marginal loss of shareholders. The first two terms in (22) highlight that such trade-off is still present when ϕ is a choice variable. Differentiating those terms with respect to ϕ informs us on how equilibrium changes in ϕ affect that trade-off:

$$\frac{\partial \varepsilon^*}{\partial \phi}\left[m^1(\varepsilon^*) - m^2(\varepsilon^*)\right] g(\varepsilon^*) + \int_{\varepsilon^*(\phi,k,B)}^{+\infty} \left[\frac{\partial m^2(\varepsilon)}{\partial \phi} - \frac{\partial m^1(\varepsilon)}{\partial \phi}\right] g(\varepsilon) d\varepsilon. \qquad (23)$$

The first addendum in (23) reflects the effect resulting from the marginal increase in the probability of default. The second illustrates the differential effect of greater aggregate output risk on the state prices of agents 1 and 2, respectively, over the solvency region.

Since the bondholders' equilibrium consumption for $\varepsilon > \varepsilon^*$ does not depend on ϕ, $\partial m^2(\varepsilon)/\partial \phi = 0$ for all $\varepsilon \in (\varepsilon^*, +\infty)$. Turning to equityholders, notice that $\partial m^1(\varepsilon)/\partial \phi < 0$ for $\varepsilon > \log(A_w/A)$ and $\partial m^1(\varepsilon)/\partial \phi > 0$ for $\varepsilon \in (\varepsilon^*, \log(A_w/A))$. While in general it is not possible to sign the second addendum, the property $A_w < A$ all but guarantees that it is positive.

Similar considerations lead us to argue the first addendum is also positive.

It follows that the larger ϕ induced by agency raises the bondholders' marginal benefit with respect to the shareholders' marginal loss. Hedging demand increases. Everything else equal, this second effect of agency on firms' debt calls for a higher debt issuance.

In summary, with incomplete markets agency affects optimal capital structure via two channels. The first is the textbook asset substitution mechanism that leads to lower debt. The second is the general equilibrium effect of the higher risk loading on consumers' marginal rate of substitution, which calls for higher debt.

In Figure 14, we illustrate how equilibria with agency (blue, dash-dot lines) and without agency (red, solid lines) vary with χ_2.

Consider the scenario without agency first. As it was the case in Section 3, firms cater to larger type-2 agents' hedging needs by issuing more debt. Since a larger debt level increases agent 1's state prices over the solvency region, however, it is now optimal for firms to accompany a higher debt with higher levels of both ϕ and k.

With agency, firms always opt for a higher ϕ. The rationale can be found in the asset substitution effect. Interestingly, the difference between the loadings that obtain in the two scenarios is increasing in χ_2. When debt is essentially riskless, there is no asset substitution to speak of. Since shareholders also suffer the negative consequences of higher ϕ, i.e. lower firm cash-flows for low realizations of ε, they have no interest in increasing risk. Higher values of χ_2, commanding higher and riskier debt, generate the incentives for shareholders to select higher ϕ. Agency costs, as substantiated in excess risk, are higher when hedging demand is higher.

Chen and Manso (2017) reach a similar conclusion in their partial-equilibrium anal-

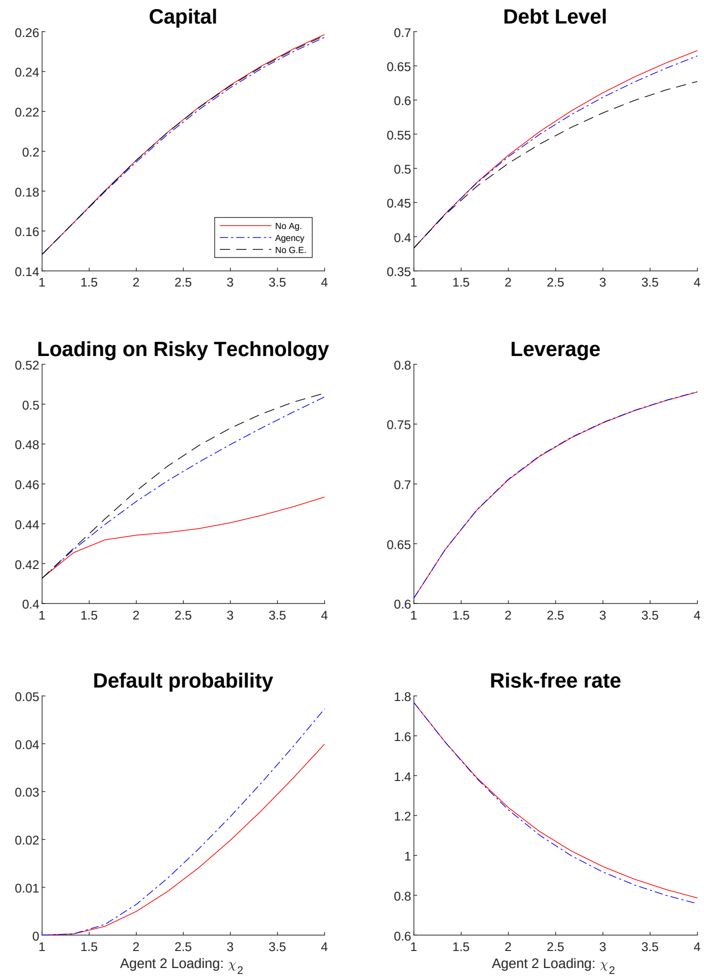

Figure 14: Agency

ysis of debt overhang, but for a very different reason. In their framework agency costs are higher in recession, because transfers from shareholders to bondholders are larger in recession, when the former's stochastic discount factor is higher.

Figure 14 shows that in our example agency leads to a small decrease in debt issuance. The asset substitution effect prevails over the hedging demand effect: Initial shareholders reduce debt to restrain end-of-period shareholders from introducing too much risk. To detect the hedging effect, we report in Figure 14 (black dashed lines) the firms' optimal choices with agency, when investors' state prices are set equal to those arising in the equilibrium without agency. When we suppress the general equilibrium effect of higher ϕ on consumption, the debt chosen by firms is considerably lower. The difference is due to the hedging effect.

Additional evidence on the interaction between agency and hedging demand is provided by the comparative statics of the interest rate, illustrated in the bottom-right panel of Figure 14. The risk-free rate is lower with agency, because the larger risk loading ϕ leads the consumption of type-2 investors to decrease in bad states of nature and hence their discount factor to increase.

For the same reason, the price of debt is also higher with agency. Since investment is barely impacted by agency, it follows that market leverage is roughly the same with and without agency.

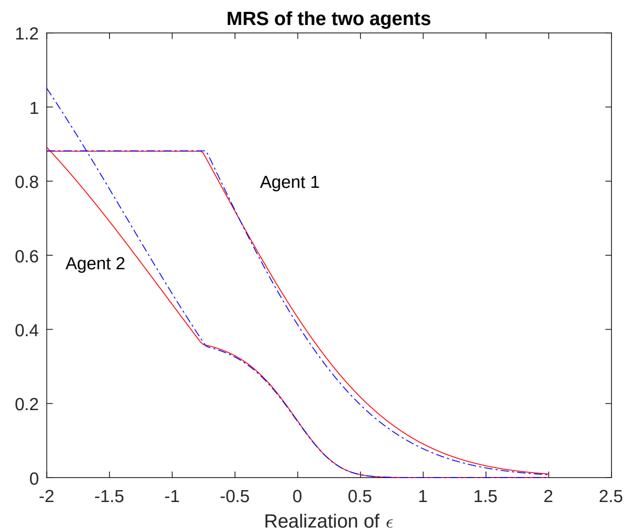

Figure 15: Agency

5 The objective function of the firm

The analysis of equilibrium in economies with production when markets are incomplete - and, possibly, with agency frictions - requires addressing the issue of what is the appropriate objective function of firms. When financial markets are complete, the market value of firms is uniquely determined. In this case, in fact, the value of any possible production plan is evaluated by the stochastic discount factor, which is unique, as intertemporal marginal utilities are equalized across agents. In turn, therefore, firms' conjectures about equity and bond prices are uniquely determined. When markets are incomplete, on the other hand, the stochastic discount factor is not unique. Each security is valued by (the stochastic discount factors of) the investors that in equilibrium buy the security. When evaluating out of equilibrium production and capital structure plans, however, different investors might have different marginal valuation of equity and bonds (hence of firms values). In this case we say that shareholders' unanimity may break down and the objective function of firms may not be uniquely determined.[16]

Many different objective functions have been proposed to address this issue, e.g., firms' market value withe respect to (some weighted average of) the stochastic discount factors of the equilibrium shareholders (Dreze, 1974), or of the initial, time 0, shareholders (Grossman and Hart, 1979), or even of a winning coalition of equilibrium shareholders (Demarzo, 1993, Boyarchenko, 2004, Cres and Tvede, 2005).[17] It has also been argued that the breakdown of shareholders' unanimity with incomplete markets may open up a normative role for some form of stakeholder governance objectives (Hart and Zingales, 2017).[18]

In this paper, we have postulated that firms' conjectures satisfy a *rationality condition* introduced by Makowski (1983a), Makowski (1983b).[19] Rational conjectures about the prices of equity and debt equal the highest marginal valuation, across all consumers (not just initial or equilibrium shareholders), of these securities' payoffs. This rationality condition on conjectures can be justified conceptually and intuitively.[20] It allows shareholders (either equilibrium shareholders or shareholders at the initial time 0), in their calculus of the value of equity for out of equilibrium production and financial plans, to contemplate the possibility of selling the firm in the market, to allow the buyers of equity to operate the plan they instead prefer. Indeed, under rational conjectures each firm evaluates different production and financial plans using possibly different marginal valuations, those which value these plans maximally.[21]

[16]A large theoretical literature has dealt with this issue, starting with the contributions of Dreze (1974), Grossman and Hart (1979) and Duffie and Shafer (1986).

[17]A large theoretical literature has dealt with this issue; see Duffie and Shafer (1986) and Magill et al. (2015).

[18]See also the forceful position along these lines taken e.g., by Bebchuk and Tallarita (2020), Bebchuk et al. (2021), Bebchuk and Tallarita (2021).

[19]See also Allen and Gale (1988).

[20]Note that *rational* price conjectures are consistent with competitive markets: the consumers' marginal rate of substitution used to determine the conjectures over the market valuation of debt and equity are taken as given, evaluated at the equilibrium consumption values and unaffected by the firm's choice of k, ϕ, B. In this sense each firm is price taker, is "small" relative to the market, and we can think of each consumer as holding a negligible amount of shares of any given firm. Some restrictions on short-sales also play an important role on this dimension. With unrestricted short sales, e.g., of equity, a *small* firm can in fact have a *large* effect on the economy by choosing a production plan with cash flows which, when traded as equity, change the asset span and hence the admissible trades of all consumers, allocations and equilibrium prices; see Allen and Gale (1991). While the no-short-sales assumption we have imposed in Sections (2- 3) is unnecessarily restrictive in general, some frictions in the intermediation as in Section (3.4) are necessary.

[21]We discuss these issues, in relation to the literature, carefully in Appendix A, where we also argue that our equilibrium notion with agency frictions is equivalent to the notion proposed by Prescott and Townsend (1984), once extended to production economies with incomplete markets.

Most importantly, we show in the remainder of the next section that rational conjectures imply shareholders' unanimity. In other words, shareholders unanimously support firms' decisions which maximize market value evaluated by rational conjectures, without any role to stakeholders. Furthermore, unanimity is linked to the welfare properties of equilibrium allocations. We show then that, in production economies with incomplete markets, provided the economy is not plagued by additional pecuniary (or other form of) externalities and agents' and firms' conjectures about prices are rational, equilibria are constrained efficient. Inefficiencies arise however when agency frictions are introduced, besides market incompleteness.

5.1 Unanimity and welfare

In this section we introduce a systematic analysis of the unanimity and welfare properties of competitive equilibria of the economy with incomplete markets and heterogeneous consumers introduced in Section 2, extended to account for agency frictions modeled as in Section 4. The definition of a competitive equilibrium is the natural extension of Definition 1, accounting for agency in the problem of the firm.

5.1.1 Unanimity

In both the economies with and without the agency friction shareholders unanimously agree on the firm's production and financing decisions, that is on the choice of k, B, ϕ which maximizes the firm's market value, determined on the basis of rational price conjectures (subject, when ϕ is unobservable, to the agency constraint (18).

Proposition 1 *Let (k, B, ϕ) be the firms' choice at a competitive equilibrium and $\left(c_0^i, c_1^i(\varepsilon)\right)_{i=1}^I$ be the consumption allocation. Then every agent i holding a positive initial amount θ_0^i of equity of a firm will be made - weakly - worse off by any other possible incentive compatible choice of the firm (k', B', ϕ').*

Proof. Note that we can always consider a situation where, in equilibrium, each consumer holds at most a negligible amount of equity of any individual firm and so the effects on a consumer's utility of alternative choices by a firm can then be evaluated using the consumer's marginal utility. Let $c(\varepsilon) = \left(c_0^i, c_1^i(\varepsilon)\right)_{i=1}^I$ be the equilibrium consumption allocation. For any possible choice k', ϕ', B' by a firm, with $\phi' \in \phi(k', B'; c(\varepsilon))$, the (marginal) utility of a type i consumer if he/she holds the firm's equity and debt is

$$-k' + \mathbb{E}\left[m^i(c^i(\varepsilon))d^e(k', \phi', B')\right] + \\ \mathbb{E}\left[m^i(c^i(\varepsilon))d^b(k', \phi', B')\right] B'.$$

But this is always lower or equal than the agent's utility if instead he sells the firm's equity and bonds at the market price, evaluated on the basis of price conjectures satisfying M),

$$-k' + \max_j \mathbb{E}\left[m^j(c^j(\varepsilon))d^e(k', \phi', B')\right] \\ + \max_j \mathbb{E}\left[m^j(c^j(\varepsilon))d^b(k', \phi', B')\right] B';$$

and the latter is in turn lower than the corresponding expression if the firm adopts the equilibrium choice k, ϕ, B, since this choice solves problem (17)–(18). ■

5.1.2 Welfare

In an economy with incomplete markets and possibly agency frictions, when the objective function of the firm does not imply unanimity, as e.g., when firm is evaluated with respect to the shareholders (at equilibrium or at time 0) marginal valuations, the competitive equilibrium allocations maybe constrained inefficient. This is because - the shareholders are not allowed to contemplate the possibility of selling the firm in the market to investors with a higher value, when such higher value is obtained under out of equilibrium production and financial plans. This will not be the case under rational conjectures as, in this case, (we have just shown) unanimity holds.

The appropriate efficiency notion for our economy is constrained efficiency: attainable allocations are restricted not only by the limited set of financial assets that are available but also by the presence of agency frictions. More formally, a consumption allocation $c^i(\varepsilon))\left(c_0^i, c_1^i(\varepsilon)\right)_{i=1}^I$ is *admissible* if:[22]

1. it is *feasible*: there exists a production plan k, ϕ such that

$$\begin{aligned} \sum_i c_0^i + k + C(\phi) &\leq \sum_i w_0^i \\ \sum_i c_1^i(\varepsilon) &\leq \sum_i w_1^i(\varepsilon) + F(k, \phi; \varepsilon); \end{aligned} \tag{24}$$

2. it is *attainable with the existing asset structure*: that is, in addition to k, ϕ as specified in 1. there exists B and, for each consumer's type i, a pair $\theta^i, b^i \geq 0$

[22] Again, we restrict notation for simplicity to symmetric allocations.

such that

$$c_1^i(\varepsilon) = w_1^i(\varepsilon) + d^e\,(k,\phi,B;\varepsilon)\,\theta^i \; + d^b(k,\phi,B;\varepsilon)b^i; \tag{25}$$

3. it is *incentive compatible*: the risk loading ϕ satisfies (18).

We then say that a competitive equilibrium allocation is *constrained Pareto efficient* if we cannot find another admissible allocation which is Pareto improving.

Proposition 2 *In the absence of agency frictions competitive equilibria are constrained Pareto efficient. With agency frictions constrained efficiency may fail.*

The case without agency frictions is the one where ϕ is observable and hence firms maximize their market value without the constraint imposed by (18). In this case, which encompasses the economy studied in Sections 2- 3, constrained efficiency always holds.On the other hand competitive equilibria with agency frictions may be constrained inefficient. The reason is that the incentive constraint, given by condition (18), generates what is essentially a pecuniary externality. The values of the risk loading ϕ chosen by shareholders does not only depend on the level of investment k and debt B chosen by the firm, but also on the equilibrium stochastic discount factors, that is the the consumers' marginal rate of substitution, which are used to determine the market value of equity for all possible values of ϕ. These marginal rates of substitution are taken as given by the firm, but depend on consumption allocation, so that a change in this allocation may relax the constraint. In other words, the externality affecting firms' incentive constraints may turn out to be more severe when some types of agents are shareholders than when others are. Therefore, when equilibrium allocations are constrained inefficient, a Pareto improvement might be achieved by modifying the types of agents owning equity with respect to those who do so in equilibrium.

It is important to note that the pecuniary externality through the incentive constraint (18) we have identified is the only source of inefficiency in our economy. In all other respects, firms' decisions are efficient and unanimously supported by shareholders. In particular, since the unanimity result in Proposition 1 always holds, even when equilibrium allocations are not constrained efficient, this misallocation of equity ownership is not a consequence of the lack of unanimity of shareholders.

6 Conclusions

In this paper we have provided an equilibrium foundation to the study of corporate finance by showing how a consistent definition of competitive equilibria can be provided in environments with production and incomplete financial markets. We have shown that, once firms are postulated to operate under rational conjectures, along the lines of Makowski (1983a) and Makowski (1983b), equilibria exist, ensure unanimity, and display appealing welfare properties.

We have shown that when households differ in their risk–sharing needs, ex–ante identical value–maximizing firms issue different securities, in order to cater to different groups of investors. As the demand for hedging increases, corporates grow in size – to allow for greater precautionary saving – and issue more debt. How much more, depends on the availability of competing risk-sharing instruments, such as (government–issued) risk–free debt and derivatives.

When capital structure is jointly shaped by demand and supply considerations – the latter, in the form of an asset–substitution problem – we find that (i) agency is relevant only when hedging demand is high and that (ii) larger investors' risk–sharing needs lead to equilibria featuring greater aggregate risk.

The next step, which we leave for future work, consists in adapting the equilibrium concept and extending the analysis to the dynamic economies typically considered in macroeconomics and finance.

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Appendix A: More on the objective function of the firm

The literature on incomplete markets with production has emphasized the problems concerning the specification of the firms' objective function. These problems do not arise for the equilibrium notion we propose: as shown in Section (5), in the set-up typically considered in this literature (that is, with no agency frictions) both unanimity and constrained efficiency hold. The key difference lies in the specification of the firms' price conjectures. It is useful then to compare the (*Makoswki criterion* for) *rational conjectures* we consider to the two main alternative specifications in the literature in the literature, the *Dreze* and the *Grossman-Hart criteria*, in the context of an economy without agency frictions.

Applied to our environment, the criterion proposed by Dreze (1974) for equity price conjectures is as follows:

$$q(k, B, \phi) = \mathbb{E}\left[\sum_i \theta^i \frac{\beta u'(c_1^i)}{u'(c_0^i)} d^e(k, \phi, B)\right], \ \forall k, B, \phi. \tag{26}$$

It requires the conjectured price of equity for any plan k, B, ϕ to equal - pro rata - the marginal valuation of the agents who in equilibrium are shareholders of the firm (that is, the agents who value the most the plan chosen by the firm in equilibrium and hence choose to buy equity). It does not however require that the firm's shareholders are those who value the most any possible plan of the firm. Intuitively, the choice of a plan which maximizes the firm's value with $q(k, B, \phi)$ as in (26) corresponds to a situation in which the firm's shareholders choose the plan which is optimal for them without contemplating the possibility of selling the firm in the market, to allow the buyers of equity to operate the plan they instead prefer. Equivalently, the value of equity for out of equilibrium production and financial plans is determined using the - possibly incorrect - conjecture that the agents who in equilibrium own the equity of a firm remain the firm's shareholders also for any alternative production and financial plan.[23]

Grossman and Hart (1979) propose an alternative criterion for price conjectures which, when applied to the price of equity in our environment, requires:

$$q(k, B, \phi) = \mathbb{E}\left[\sum_i \theta_0^i \frac{\beta u'(c_1^i)}{u'(c_0^i)} d^e(k, \phi, B)\right], \ \forall k, B, \phi \tag{27}$$

[23]It is then easy to see that any allocation constituting an equilibrium with rational conjectures is also an equilibrium under the Dreze criterion: all shareholders of a firm have in fact the same valuation for the firm's production and financial plan and their marginal utility for any other possible plan is lower, hence a fortiori the chosen plan maximizes the weighted average of the shareholders' valuations. But the reverse implication is not true, i.e., an equilibrium under the Dreze criterion is not always an equilibrium under rational conjectures.

We can interpret this specification as describing a situation where the firm's plan is chosen by the initial shareholders (i.e., those with some predetermined endowment of equity at the beginning of date 0) so as to maximize their welfare, again without contemplating the possibility of selling the equity to other consumers who value it more. According to this criterion, the value of equity for all production and financial plans is derived on the basis of the conjecture that the firm's initial shareholders stay in control of the firm whatever is the plan.

In contrast, according to the Makowski criterion for rational conjectures each firm evaluates different production and financial plans using possibly different marginal valuations (that is, possibly different pricing kernels, but all still consistent with the consumers' marginal rate of substitution at the equilibrium allocation). This is essential to ensure the unanimity of shareholders' decisions and is a key difference with respect to Dreze (1974) and Grossman and Hart (1979), both of whom rely on the use of a single pricing kernel.

Finally, we note that the equilibrium concept with rational conjectures we have adopted is equivalent to one where we allow for *all markets to be open at market clearing prices.* Indeed, consider a specification where markets for all possible 'types' of equity and bonds are open: that is, equity and bonds corresponding to any possible value of k', ϕ', B' (consistent wit the agency constraint) are available for trade to consumers at the prices $q(k', \phi', B')$, $p(k', \phi', B')$. It is immediate to see that all such markets - except the one corresponding to the firms' equilibrium choice k', ϕ', B' - clear at zero trades. As a consequence, $q(k', \phi', B')$ and $p(k', \phi', B')$ correspond to the equilibrium prices of equity and bonds of a firm who were to "deviate" from the equilibrium choice and choose k', ϕ', B' instead. In this sense, we can say that rational conjectures impose a consistency condition on the out of equilibrium values of the equity and bonds price conjectures, that corresponds to a "refinement" somewhat analogous to subgame perfection.[24]

Turning then to asymmetric information and agency frictions, most of the competitive equilibrium concepts which have been proposed for production economies build on the one proposed by Prescott and Townsend (1984) for exchange economies, therefore exhibiting no traded equity.[25] While Prescott and Townsend's approach, rooted in mechanism design, is rather different from ours, which instead relies on the extension of *rational conjectures* to economies with asymmetric information, our equilibrium notion is indeed equivalent to the one of Prescott and Townsend once this is extended to economies with incomplete

[24]This is in line with what already observed in Section 2: under the consistency conditions (v), rationality is always satisfied for the prices associated with the firms' equilibrium choices. Rationality of conjectures requires that the same property holds also for all out of equilibrium choices.

[25]See, e.g., Magill and Quinzii (2002), Prescott and Townsend (2006), and Zame (2007).

markets where firms rather than consumers face agency frictions.[26] Indeed, consider the equilibrium concept adopted by Prescott and Townsend (1984) for exchange economies with asymmetric information. In this concept prices depend both on observable and unobservable choices (or states) and this is sustained, drawing a parallel with mechanism design formulations of related problems relying on the Revelation Principle, by restricting admissible choices to those which are incentive compatible. This is analogous to what we do in the firm's problem (17)-(18) where price conjectures also depend on the choice of the risk loading ϕ, though this choice is not observable by outside investors, but the values of ϕ are restricted by the agency constraint (18). Via this constraint, the level of ϕ is determined by the observable choices of the firm, k, B. Hence price conjectures reflect the correct anticipation of the firm's unobservable choices.

Nonetheless, interesting and important conceptual differences emerge between the properties of equilibria in the environments studied by Prescott and Townsend and in our. While competitive equilibria are always constrained efficient in the exchange economies with moral hazard considered by Prescott and Townsend, this is not the case in production economies, where agency frictions enter the firms' choice problem, as we have shown in Proposition 2. The nature of the equilibrium concept considered plays no role in this, given the equivalence recalled above. Rather, the incentive constraint in the firm's choice problem features a pecuniary externality, due to the presence of price conjectures needed to determine the market value of equity for any risk loading choice.[27]

Short sales

We extend here the environment described in Section 5 by introducing intermediated short-sales, along the lines of Section 3.4. Now intermediaries can issue derivatives both on corporate debt and an equity. In both cases the origination of a derivative entails a cost, due to the fact that consumers taking a short position repay only a fraction $(1 - \delta)$ of the amount due.

The intermediary's problem consists then in the choice of the amount H^b, H^e issued of long and short positions in the derivatives on debt and equity and the amounts γ^b, γ^e

[26] We do not discuss economies with adverse selection in this paper. We conjecture that the equilibrium concepts studied by Bisin and Gottardi (2006) have an equivalent reformulation in terms of equilibria with *rational conjectures* in economies with production along similar lines to those considered in the present paper.

[27] Prescott and Townsend also assume that markets are complete, while we do not. But whether markets are complete or not, and hence whether marginal rates of substitution are equalized or not across consumers, is not crucial for the welfare result. What is crucial is that these marginal rates of substitution enter the incentive constraint.

of debt and equity held as reserve, to maximize its total revenue at $t = 0$,

$$\max_{H,\gamma\in R_+^2} \left[(p^+ - p^-)H^b - p\gamma^b + (q^+ - q^-)H^e - q\gamma^e\right], \tag{28}$$

subject to the solvency constraints

$$\begin{aligned} H^b &\leq H^b(1-\delta) + \gamma^b, \\ H^e &\leq H^e(1-\delta) + \gamma^e. \end{aligned} \tag{29}$$

The latter ensure the reserves held suffice to allow the intermediary to cover all shortfalls in future revenue due to consumers' default and hence to meet all its future obligations. The presence of a bid ask spread on the long and short positions issued allows the intermediary to cover the cost of the debt and equity held as reserve.

A solution to the intermediary's choice problem exists provided that

$$p \geq \frac{p^+ - p^-}{\delta}, \;\; q \geq \frac{q^+ - q^-}{\delta} \tag{30}$$

and is characterized by $H^j > 0$ and $\gamma^j = \delta H^j$, $j = b, e$, only if the inequalities in (30) hold as equalities

Let $h_+^{i,j} \in R_+$ denote consumer i's holdings of long positions in the derivative $j = b, e$ issued by intermediaries, and $h_-^{i,j} \in R_+$ his holdings of short positions. The consumer's choice problem consists in maximizing his expected utility subject to the budget constraints

$$c_0^i = w_0^i + \theta_0^i V - q\theta^i - q^+ h_+^{i,e} + q^- h_-^{i,e} - pb^i - p^+ h_+^{i,b} + p^- h_-^{i,b} \tag{31}$$

$$c_1^i(\varepsilon) = w_1^i(s) + R^b(\varepsilon)(b^i + h_+^{i,b} - (1-\delta)h_-^{i,b}) + R^e(\varepsilon)(\theta^i + h_+^{i,e} - (1-\delta)h_-^{i,e}) \tag{32}$$

and $\left(\theta^i, b^i, h_+^{i,b}, h_-^{i,b}, h_+^{i,e}, h_-^{i,e}\right) \geq 0$. Note that a fraction $1-\delta$ of each agent's short position is defaulted on.[28]

The asset market clearing condition for debt and equity become

$$\begin{aligned} \gamma^b + \sum_{i\in I} b^i &\leq B, \\ \gamma^e + \sum_{i\in I} \theta^i &\leq 1 \end{aligned}$$

[28]Default rate is modeled exogenously for simplicity. This could be justified, for instance, by setting $1-\delta$ as the cutoff above which the intermediary would gain from enforcing a court ruling again the agent defaulting. Also, we show in the Online Appendix how the analysis and results extend to situations where default rates are endowgeneously chosen by consumers.

and for the derivative securities

$$\sum_{i\in I} h_{+}^{i,j} = \sum_{i\in I} h_{-}^{i,j} = H^j,\ j = b, e$$

The firm's choice problem is the same as in Section 2. The most significant change concerns the conditions specifying the *rationality* of the price conjectures for debt and equity, which need to be adjusted to reflect the fact that now intermediaries also demand debt and equity in the market:

$$p(k,\phi,B) \quad = \quad \max \left\{ \begin{array}{c} \max_i \ \mathbb{E}\left[\frac{\beta u'(c_1^i)}{u'(c_0^i)} d^b(k,\phi,B)\right], \\ \frac{\max_i \ \mathbb{E}\left[\frac{\beta u'(c_1^i)}{u'(c_0^i)} d^b(k,\phi,B)\right] - \min_i \max_i \ \mathbb{E}\left[\frac{\beta u'(c_1^i)}{u'(c_0^i)} d^b(k,\phi,B)\right]}{\delta} \end{array} \right\}, \qquad (33)$$

$$q(k,\phi,B) \quad = \quad \max \left\{ \begin{array}{c} \max_i \ \mathbb{E}\left[\frac{\beta u'(c_1^i)}{u'(c_0^i)} d^e(k,\phi,B)\right], \\ \frac{\max_i \ \mathbb{E}\left[\frac{\beta u'(c_1^i)}{u'(c_0^i)} d^e(k,\phi,B)\right] - \min_i \max_i \ \mathbb{E}\left[\frac{\beta u'(c_1^i)}{u'(c_0^i)} d^e(k,\phi,B)\right]}{\delta} \end{array} \right\} \qquad (34)$$

for all k, B. The above expressions state that the conjecture of a firm over the prices of its debt and equity when it chooses the plan k, ϕ, B equals the maximal marginal valuation of the corresponding payoffs, *among both intermediaries and consumers*. The second term on the right hand–side of the above expressions is in fact the intermediaries' marginal valuation for debt and equity and can be interpreted as the *value of intermediation*.

Since an appropriate amount of debt and equity are needed, as reserves, to ensure the intermediary can operate and fulfil its obligations, the intermediary's willingness to pay for these assets is determined by the consumers' marginal valuation for the corresponding derivative claims which can be issued.[29] Hence the above specification of the debt price conjectures allows firms to take into account the effects of consumers' willingness to pay for the derivatives issued on the value of intermediation.

In all other respects, a competitive equilibrium of the economy with intermediation and short sales is defined along similar lines to Section 2. The existence, constrained efficiency and shareholders' unanimity can then be established by similar arguments to Propositions **??**, 2, 1.

The model of intermediation proposed in this section is admittedly quite stylized. We believe however it allows to capture in a simple way the relationship between the financial claims issued by firms and the intermediation process. The key feature is that

[29] More precisely, the first term on the numerator of the second expression in (33) equals the consumers' marginal valuation for long positions in the derivative on debt, the second one their valuation for short positions; dividing by δ yields the revenue from intermediation, per unit of debt purchased. Similarly for equity, in (34).

the derivatives issues by intermediaries are backed by the claims issued by firms in two ways. First, the yields of these derivatives are pegged to the yield of the claims issued by firms; second, the intermediaries must hold some amount of these claims to back the derivatives issued. Hence part of the demand for the firms' claims now also comes from intermediaries (as such claims enter as a sort of input in the intermediation technology).

Finally, we can provide the following simple characterization of the intermediation levels at equilibrium, which follows from (30):

Proposition 3 *In the economy with financial intermediation and short sales, at an equilibrium, either (i)* $p = (p^+ - p^-)/\delta > p^+$ *and intermediation for debt derivatives is full (the whole amount of outstanding debt is purchased by intermediaries) or (ii)* $p = p^+$ *and intermediation is partial (some if not all the amount of outstanding debt is held by consumers). Similarly for equity derivatives.*

At an equilibrium where intermediation is full, debt sells at a premium over the long positions on the derivative claim issued by the intermediary, due to its additional value as input in the intermediation technology. Intermediaries in turn recoup the higher cost of debt through a sufficiently high spread $p^+ - p^-$ between the price of long and short positions on the derivative. When intermediation is partial, debt and long positions in the derivative trade at the same price, intermediaries may not be active in equilibrium and the bid ask spread $p^+ - p^-$ is low (in particular, less or equal than δp).