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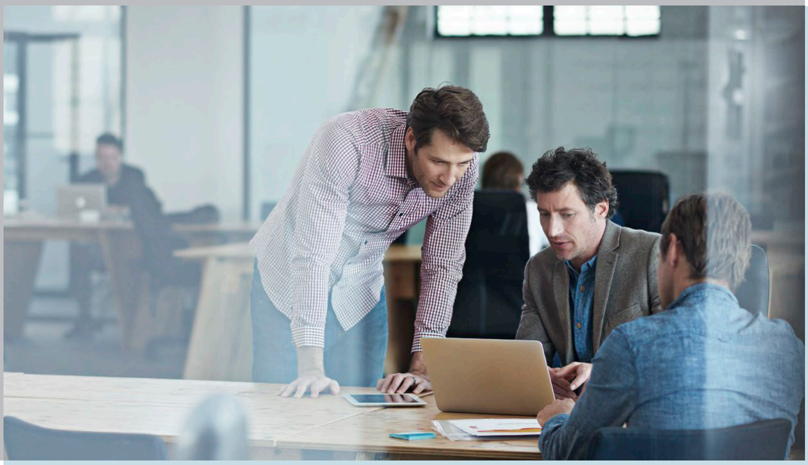

Egemen Eren
Bank for International Settlements (BIS)

Semyon Malamud
École Polytechnique Fédérale de Lausanne, Swiss Finance Institute, and CEPR

Dominant Currency Debt *

Egemen Eren† and Semyon Malamud‡

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Abstract

Why is the dollar the dominant currency for debt contracts and what are its macroeconomic implications? We develop an international general equilibrium model where firms optimally choose the currency composition of their debt. We show that there always exists a *dominant currency debt equilibrium*, in which all firms borrow in a single dominant currency. It is the currency of the country that effectively pursues aggressive expansionary monetary policy in global downturns, lowering real debt burdens of firms. We show that the dollar empirically fits this description, despite its short term safe haven properties. We provide further modern and historical empirical support for our mechanism across time and currencies. We use our model to study how the optimal monetary policy differs if the Federal Reserve reacts to global versus domestic conditions.

Keywords: dollar debt, dominant currency, exchange rates, inflation
JEL Classification Numbers: E44, E52, F33, F34, F41, F42, F44, G01, G15, G32

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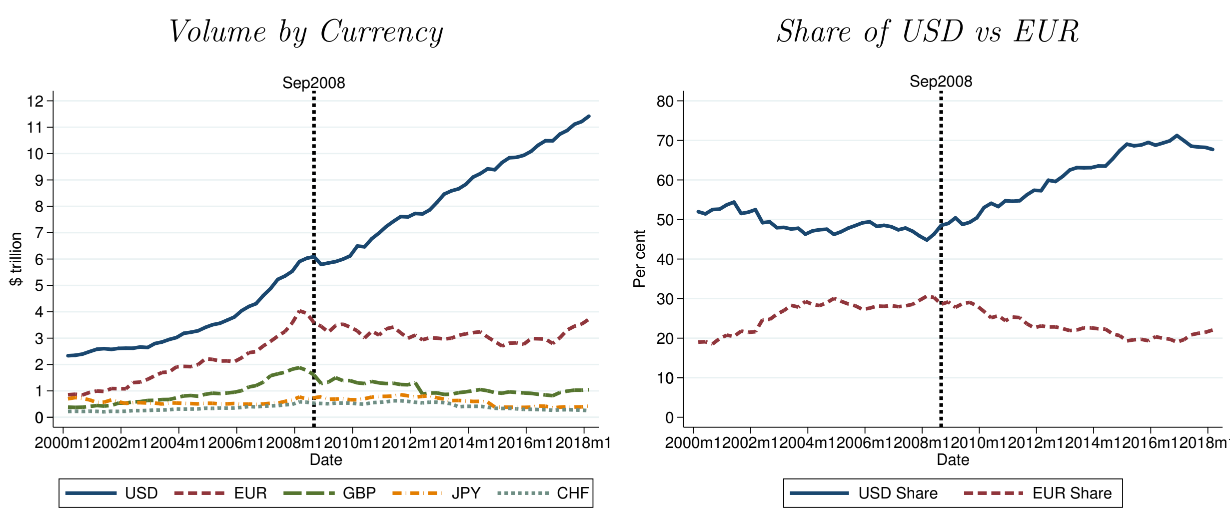
†Bank for International Settlements (BIS); Email: egemen.eren@bis.org
‡Swiss Finance Institute, EPF Lausanne, and CEPR; E-mail: semyon.malamud@epfl.ch

1 Introduction

The dollar is the dominant currency in debt contracts across the globe. According to the Bank for International Settlements, dollar-denominated credit to non-banks outside the United States amounts to around \$11.5 trillion. While the dominance of the dollar was in decline prior to 2008, it reinstated and strengthened its dominance since the Great Financial Crisis (Figure 1).[1]

In this paper, we show how such a dominant currency debt equilibrium may emerge, why the dominance of the dollar might have declined and recovered in the last two decades, and what the optimal monetary policy of the Federal Reserve should be in a global economy where global dollar debt creates spillovers between countries through trade linkages.

Figure 1: Currency Denomination of Foreign Currency Non-Bank Debt



Source: Bank for International Settlements (see BIS (2018) for details.)

We develop an international general equilibrium model with multiple countries where firms optimally choose the currency composition of their debt. All firms are exporters, prices are flexible and firms have fully diversified cash flows. This allows us to zero in on

[1]Similar patterns were previously documented for debt issuance (see, for example, ECB (2017), Maggiori, Neiman and Schreger (2018), Aldasoro and Ehlers (2018)), and for global cross-border bond holdings (Maggiori, Neiman and Schreger (2018)).

the capital structure. Firms issue equity and debt, potentially in any currency. Debt is nominal and defaultable. Firms receive productivity shocks that affect their profits. When firms have dollar debt, inflation in the US and the exchange rate movements affect their real debt burden. When debt servicing costs are high relative to profits, firms face debt overhang. They cut production and reduce demand for intermediate inputs imported from other countries. This demand channel spreads debt overhang costs along the global value chain and serves as a key mechanism for the international spillovers.

We model a central bank as a countercyclical monetary policy rule that eases financing conditions for firms in times when output gap is high, and vice versa. Central banks differ from each other in how strongly they are able to react to output gap, i.e. generate inflation in times when firms receive negative productivity shocks and are close to default. Relative inflations between two countries determine the exchange rates through a relative purchasing power parity condition.

Our first main result is that there always exists a dominant currency debt equilibrium. A single currency can be chosen as the dominant currency in denominating debt contracts, even though there are other currencies with almost identical characteristics. It is the currency of the country with the central bank that *aggressively* and *precisely* pursues expansionary monetary policy in global downturns, lowering real debt burdens of firms. Inflation in that country spikes the most in a recession, causing a depreciation of that currency. This alleviates the debt overhang problem of firms ex-post and makes borrowing in this currency attractive ex-ante.

Importantly, the safe haven status of the dollar does not overturn our main result. In the model, firms prefer to issue debt in the currencies that have a positive correlation with the stock market *at the horizons of their debt maturity*, i.e. depreciate when the stock market falls and vice versa. Cortina, Didier and Schmukler (2018) show that weighted average debt maturity for corporate firms globally is around seven years. While we abstract from exchange rate risk premia, hence the safe haven status of the dollar in the model, we empirically show that *the safe haven effect dies out after a quarter to a year.* In fact, for longer horizons, the dollar tends to depreciate against other major international currencies when stock markets fall.

We argue that firms choosing debt issuance currency learn from the actions of a central bank about its ability to stimulate the economy and generate inflation in bad times. We attribute the rise in the share of dollar-denominated debt after the crisis to an updating of beliefs by market participants regarding monetary policy effectiveness. In particular, following the Great Financial Crisis, the Federal Reserve was the first among its peers to cut interest rates and start quantitative easing. As a result, US inflation remained closer to the inflation target. These observations might have lead to an important shift in firms' expectations about the ability of the Federal Reserve to produce inflation in economic downturns, relative to that of the European Central Bank and other major central banks. [2]

While expectations about the monetary policy effectiveness are not directly observable, it is possible to extract information about these expectations from financial asset prices. For example, the inflation risk premium (IRP) is given by the covariance of inflation with investors' marginal utilities. Hence, *countries for which investors expect a more counter-cyclical inflation tend to have a higher inflation risk premium.* Thus, inflation risk premium is linked to the dominant currency status: In fact, in our model, controlling for other characteristics, the dominant currency country is always the one with the highest inflation risk premium. Interestingly enough, the estimates of Hördahl and Tristani (2014) show that inflation risk premium was higher in the Eurozone compared to the US prior to the crisis, consistent the rising share of euro-denomination during that period. However, inflation risk premium was higher in the US than in the Eurozone after the crisis, consistent with the post-crisis rise in the dollar share of debt denomination (Figure 1). Motivated by this evidence, we

[2] The importance of accommodative monetary policy in helping firms deleverage and the differences across central banks in accomplishing this is also acknowledged by the ECB. See, for example, Praet (2016).

formally test our theoretical prediction about the link between the share of dollar debt and inflation expectations. We find that the dynamics of IRP in the two countries explains about 80% of the variation in the share of dollar debt, with the signs of the regression coefficients consistent with our theory. Moreover, our results imply that the IRP is associated with the currency choice of debt issuance even at the quarterly level, controlling for year dummies. We interpret this fact as a strong evidence for a distinctive prediction of our theory that changes to the dominance of a currency can occur in high frequency.

A similar pattern to that in Figure 1 is documented by Maggiori, Neiman and Schreger (2018) for cross-border corporate bond holdings: They show that the share of dollar-denominated debt in cross-border corporate holdings has drastically increased in the post-crisis period compared to the euro. We argue that this pattern is to a large extent driven by the bond-supply channel of Figure 1, and bond investors hold what the firms issue to clear markets in general equilibrium. That said, while bond investors might generally dislike holding nominal bonds with a high inflation risk premium, there is an opposite force in our model that increases the attractiveness of dollar-denominated bonds for lenders. The default probability of these bonds is lower because it is easier for firms to repay dollar debt due to lower real debt burdens in bad times. In equilibrium, however, the latter is dominated by the former.

The inflation expectations channel can also be used to understand patterns of debt issuance in other major currencies. One puzzling observation is that, despite the fact that the relative share of Japan is larger than the United Kingdom in the world economy, the pound-denomination of foreign currency debt exceeds debt-denomination in yen. Through the lens of our model, this could be explained by the fact that inflation in the UK was often close to and above the inflation target of the Bank of England, with firms seeing the real value of their debt decline more often. On the other hand, firms borrowing in Japanese Yen have seen the real values of their nominal debt increase as inflation consistently undershot the target of the Bank of Japan. This negative inflation surprises made the yen an unattractive currency to borrow in, despite its low inflation and low interest rates.

Our model thus predicts that even in the absence of factors like network externalities and inertia, a dominant debt currency can switch within short periods of time due to differences in *inflation expectations* of the incumbent and the competitor currency. For example, during the interwar years, the British pound suffered deflation to a larger extent than the dollar at the beginning of 1920s during the 1920-21 recession. This corresponds to the rise of the dollar as the debt denomination currency. On the flip side, the US dollar faced greater deflation during the Great Depression, which corresponds to the subsequent rise of the pound, according to the evidence provided by Chiţu, Eichengreen and Mehl (2014).

A skeptical reader might argue that an emerging market currency, such as the Argentine peso, fits our description of the dominant currency more than the dollar. Two forces in the model ensure that this is not the case. First, even though firms prefer to issue debt in currencies that depreciate during global downturns, they also avoid currencies that are volatile for idiosyncratic reasons. We show that the Argentine peso is around four times more volatile than the dollar and this volatility is almost entirely due to idiosyncratic reasons. This makes borrowing in peso unattractive for firms in our model. Second, a more straightforward force is the issuance costs: due to their depth and liquidity, it is cheaper to issue in dollar and major currencies than in the Argentine peso.

The main focus of our paper is on debt denominated in dominant currency and why the dollar is the dominant currency as opposed to other advanced economy peers. Another important question is what determines the share of local currency versus the dominant currency. Guided by our main mechanism, we develop hypotheses regarding the local currency share of corporate borrowing in the cross-section emerging market economies. We find strong evidence that firms in countries in which domestic inflation correlates more with the US inflation tend to have a higher share of debt denominated in local currency, in line with our predictions. Firms in those countries taking on local currency debt benefit both from the insurance properties of the dominant currency in downturns, while still having a central bank that can react to domestic conditions in the face of idiosyncratic shocks.

Our model offers a "debt-centric" view of the dollar's dominance that is different from the "trade-centric" view in Gopinath and Stein (2018). In their model, dollar's dominance in debt is rooted in its dominance in trade invoicing. One prediction of their model would be that, all else equal, more dollar invoicing of trade should be associated with more mismatched dollar borrowing by firms outside the US. According to the World Bank, total trade as a share of world GDP has risen prior to the crisis and has fallen after the crisis. To reconcile this behaviour of trade with the opposite behaviour of the share of dollar-denominated corporate debt documented in Figure 1, a "trade-centric" view would thus require a significant increase in the share of dollar invoicing in the global trade. Furthermore, we find that the relationship between dollar debt outside the US and total international trade excluding the US is negative. While this negative relationship is difficult to reconcile with the trade-centric view, it is broadly consistent with the debt-centric view whereby high levels of dollar debt may increase debt overhang and thereby reduce trade.

Our general equilibrium framework also allows us to discuss the macroeconomic implications of a dominant currency debt equilibrium and the role of the Federal Reserve as the world's central bank and the difference it makes for the global welfare when it reacts to global versus domestic conditions. Indeed, in a dominant currency debt equilibrium, with all firms borrowing in dollars, local central banks are not anymore able to alleviate the debt burdens of firms. As a result, the monetary policy of the dominant currency country plays a key role in the functioning of the global economy: In equilibrium, unemployment and inflation in each country respond directly to inflation in the dominant currency country. We show that, under certain conditions, it might be optimal for the dominant currency central bank to target global instead of local output gap, shedding some light on the debate whether it is an "exorbitant duty" of the dominant currency country to react to global conditions and to maintain global economic stability. Our general equilibrium framework is designed to explicitly capture the channels for such spillovers.

We run the following thought experiment: In the dominant currency debt equilibrium, with firms in the entire world issuing dollar debt, how should the Federal Reserve assign weights to output gaps of each country in order to maximize global welfare? The main channel the optimal monetary policy operates is through its effects on leverage. In our model, leverage unambiguously reduces welfare. Therefore, the optimal weight that the Fed assigns to a given country must be decreasing in the welfare costs of providing insurance to firms in this country. We derive optimal weights analytically and show that they are indeed *lower* for countries with volatile TFP shocks, high debt restructuring costs and countries that are more important in world trade. Limiting insurance given to those countries reduces their firms' leverage, improving global (and domestic) welfare.

Roadmap. The remainder of the paper is structured as follows. Section 2 provides an overview of the relevant literature. Section 3 describes the model. Section 4 derives the macroeconomic equilibrium with fixed leverage. Section 5 studies the dominant currency debt equilibrium. Section 7 provides empirical support for our theory. Section 6 shows how optimal monetary policy differs if the Federal Reserve maximizes global welfare. Section 8 concludes.

2 Literature Review

The role of the dollar as a dominant global currency has received a lot of attention in recent academic research. Dollar is omnipresent in all parts of the global financial system, including international trade invoicing (see Goldberg and Tille (2008), Casas, Díez, Gopinath and Gourinchas (2017)); global banking (Shin (2012), Ivashina, Scharfstein and Stein (2015), Aldasoro, Ehlers and Eren (2018)); corporate borrowing (Avdjiev, Bruno, Koch and Shin (2018), Bruno, Kim and Shin (2018), Bruno and Shin (2017), Bräuning and Ivashina (2017)); central bank reserve holdings (Bocola and Lorenzoni (2018)); and global portfolios (Maggiori, Neiman and Schreger (2018)).

Our paper belongs to the growing literature that tries to understand the dominant role of the dollar in a general equilibrium framework. For example, Matsuyama, Kiyotaki and Matsui (1993), Rey (2001), Devereux and Shi (2013) and Chahrour and Valchev (2017) investigate the emergence of a "vehicle" currency that serves as a medium of exchange; Mukhin (2017) studies dominant currency invoicing when prices are sticky; Doepke and Schneider (2017) show how a dominant unit of account equilibrium may arise in decentralized markets with bilateral contracts as a mechanism to avoid exchange rate risk and default risk; Farhi and Maggiori (2017) show how US government debt can emerge as the dominant safe asset due to the US monopoly power in the production of this asset (see also Caballero, Farhi and Gourinchas (2008) and Caballero and Krishnamurthy (2009)); He, Krishnamurthy and Milbradt (2016) show how safety of US government debt depends on the US fiscal capacity and investors' coordination; Bocola and Lorenzoni (2017) show that banks in emerging markets optimally issue dollar-denominated debt, which makes them more prone to runs and increases the probability of crises; and Wiriadinata (2018) shows how external dollar debt is linked to currency risk premia.

The most closely related to ours is the paper of Gopinath and Stein (2018), who show how the dollar can emerge as a single dominant currency in both trade invoicing and global banking, which in turn leads to emerging markets endogenously borrowing in dollars. The focus in Gopinath and Stein (2018) is on the interaction between the banking sector and invoicing decisions of exporters. By contrast, our goal is to characterize Fisherian debt deflation forces underlying the impact of nominal debt on the macroeconomy, as in Gomes, Jermann and Schmid (2016), but in an international setting. The dominant currency's special role arises from firms' demand for bonds with the optimal risk profile, linked to put-like policies pursued by the central bank (see Cieslak and Vissing-Jorgensen (2017)). It is this risk profile that makes dollar-denominated debt endogenously safe and hence also receive lower rates of return.[3] This perceived safety is determined by the ability of the dominant currency country to produce inflation in crisis states. For simplicity, we assume that prices are fully flexible and hence invoicing choices have no impact on equilibrium. Introducing sticky prices into our model (as in Mukhin (2017)) and understanding the interaction between invoicing, sticky prices, endogenous inflation dynamics, and corporate debt is an important direction for future research.

Similarly to Gopinath and Stein (2018), Jiang, Krishnamurthy and Lustig (2018) argue that the speciality of the dollar stems from the special demand for dollar safe assets. This demand creates a premium in dollar denominated assets and makes it optimal for firms to issue in dollars. By contrast, in our paper the decision to issue in dollars is driven mostly by the supply (debt issuers) side. Yet, demand side also plays a role because dollar debt is endogenously safer due to its lower default risk[4]

Drenik, Kirpalani and Perez (2018) develop a model in which agents choose the currency denomination of in contracts, and the government chooses the inflation rate. The problem of optimal choice of currency in Drenik, Kirpalani and Perez (2018) resembles that of firms choosing currency denomination of their debt in our model: Namely, it is all about the co-variance of price risk with the relative consumption needs of the different agents signing the contract (firms and creditors in our model). In particular, high domestic political risk makes it optimal to sign contracts denominated in the more stable foreign currency (US dollar). As a result, the model of Drenik, Kirpalani and Perez (2018) implies that a reduction in political risk may lead to de-dollarization of emerging markets. While our results also

[3] In particular, firms in our model issue dominant currency debt because they "reach for safety"; this leads to excessive leverage and can be destabilizing, as in Caballero and Krishnamurthy (2001, 2002); Caballero and Lorenzoni (2014); Caballero and Simsek (2018).

[4] In the real world, both demand and supply side seem to be important determinants of debt currency denomination. See, for example, Cohen (2005).

imply that policy uncertainty discourages issuance in local currency, we also argue that emerging markets may keep issuing in dollars if the US Federal reserve keeps convincing market participants in its superior ability to persue aggressive policy in crisis times.

A large literature shows that global credit conditions, and, in particular, the US dollar, serve as an important mechanism for the international transmission and amplification of credit supply shocks.[5] To the best of our knowledge, our paper is the first to develop a large open economy macroeconomic model with foreign firms optimally issuing dollar debt. In particular, we are able to explicitly characterize the financial channel outlined in Avdjiev, Bruno, Koch and Shin (2018) and Bruno, Kim and Shin (2018), whereby shocks to balance sheets of firms with dollar debt impact their investment and exporting activities. In our model, monetary policy of the the dominant currency country arises endogenously as an important driver of global credit conditions. Shocks to this monetary policy may thus endogenously lead to a "Global Financial Cycle", consistent with the findings in Miranda-Agrippino and Rey (2018). The mechanism through which firms endogenously choose to correlate their risk exposures to dollar is related to that in Farhi and Tirole (2012).

Numerous papers in international macroeconomics study the mechanisms underlying the exchange rate pass-through into prices of real goods. Most of these papers focus on the so-called trade channel whereby pass-through depends on price stickiness and the invoicing currency choice. See, for example, Engel (2006), Gopinath, Itskhoki and Rigobon (2010), Goldberg and Tille (2013), and Casas, Díez, Gopinath and Gourinchas (2017). We highlight a novel passthrough mechanism operating through the financial channel (see Avdjiev, Bruno, Koch and Shin (2018)). With dollar debt, a dollar appreciation shock puts leveraged firms in distress and increases their effective operational costs. Firms respond to this by raising prices, consistent with the mechanism highlighted in Gilchrist, Schoenle, Sim and Zakrajšek

[5] See Avdjiev, Bruno, Koch and Shin (2018), Bruno, Kim and Shin (2018), Gourinchas and Obstfeld (2012), Schularick and Taylor (2013), Baskaya, di Giovanni, Kalemli-Ozcan and Ulu (2017), Aguiar (2005), Miranda-Agrippino and Rey (2018).

(2017) and Malamud and Zucchi (2018). Importantly, this passthrough channel operates even when prices are fully flexible.

In our paper, in order to isolate the underlying mechanisms, we focus on a stylized equilibrium in which all firms in all countries borrow only in dollars. In the real world, as Maggiori, Neiman and Schreger (2018) and Salomao and Varela (2018) show, the vast majority of firms in developed markets borrow in their local currency, and only large and productive firms issue bonds in foreign currency. We show the results of our model regarding domestic currency vs dominant currency choice in subsection 7.3. The main mechanism remains the same also in this trade-off: Among currencies with similar issuance costs, firms prefer borrowing in the currencies of countries that are able to produce inflation in crisis times. [6]

Our focus in this paper is on corporate debt; as a result, we completely ignore another important pillars of the debt system: household debt, bank debt, and sovereign debt. In particular, currency composition of sovereign debt, especially for emerging markets, and its interaction with the currency composition of corporate debt is a key determinant of economic stability and shock propagation. See Du and Schreger (2016a) and Du and Schreger (2016b).

3 Model

3.1 Households and Exchange Rates

Time is discrete, indexed by $t = 0, 1, \cdots$ There are N countries, indexed by $i = 1, \cdots, N$. Households work and consume. They maximize

$$E\left[\sum_{t=0}^{\infty} e^{-\beta t} U(C_{i,t}, N_{i,t})\right]$$

[6] Of course, in reality, exchange rates also depend on factors other than inflation, for example the relative safe haven status of a currency. These considerations would also affect the debt currency choices of firms in a more complex model for exchange rate determination.

with

$$U(C_{i,t}, N_{i,t}) \;=\; \frac{C_{i,t}^{1-\gamma}}{1-\gamma} \;-\; \nu_i N_{i,t}$$

for some $\gamma \geq 2$,[7] where $N_{i,t}$ is the number of hours worked,[8] and

$$C_{i,t} \;=\; \left(\sum_j \theta(j) \int_0^1 (\tilde{C}_{i,t}(j,\omega))^{\frac{\eta-1}{\eta}} d\omega\right)^{\frac{\eta}{\eta-1}}$$

is the standard, constant elasticity of substitution (CES) consumption aggregator, with the elasticity of substitution η and with $\tilde{C}_{i,t}(j,\omega)$ denoting the consumption of type-ω good imported from country j into country i, with $\omega \in [0,1]$. Parameter $\theta(j)$ determines the global demand for all country j goods. Without loss of generality, we assume that these demand parameters are normalized, so that

$$\sum_{j=1}^{N} \theta(j) \;=\; 1\,,\; i = 1, \cdots, N\,.$$

Denote by $P_t^i(j,\omega)$ the price at which a country j firms sell type$-\omega$ goods in country i. The price is always in the domestic, country-i currency. We define the price index

$$\mathcal{P}_{i,t} \;\equiv\; \left(\sum_j \theta(j) \int_0^1 P_t^i(j,\omega)^{1-\eta} d\omega\right)^{1/(1-\eta)} \tag{1}$$

We will also always use the normalization $\mathcal{P}_{i,0} = 1$ for all i.

Households have access to a complete, frictionless financial market with a domestic, nominal pricing kernel $M_{i,t,\tau}$ *in the domestic currency*, for any $t < \tau$. The following lemma characterizes customers' optimal consumption choices.

[7]Condition $\gamma \geq 2$ is imposed for technical reasons and can be relaxed.

[8]The simplifying assumption that the inverse Frisch elasticity of labor is zero allows us to pin down equilibrium wage without the need keep track of equilibrium labor demand. It is made purely for technical reasons and can be removed at the cost of significant additional complexity of the calculations.

Lemma 1 *Optimal consumption demand is given by*

$$\tilde{C}_{i,t}(j,s) \;=\; (P^i_t(j))^{-\eta}(\mathcal{P}_{i,t})^{\eta} C_{i,t}\theta(j)\,,$$

consumption expenditures satisfy the inter-temporal Euler equation,

$$e^{-\beta}C^{-\gamma}_{i,t+1}/C^{-\gamma}_{i,t} \;=\; M_{i,t,t+1}\left(\mathcal{P}_{i,t+1}/\mathcal{P}_{i,t}\right) \tag{2}$$

and equilibrium wages are given by

$$w_{i,t} \;=\; \nu_i C^{\gamma}_{i,t}\mathcal{P}_{i,t}\,.$$

We will denote by $\mathcal{E}_{i,j,t}$ the value of a unit of currency i in the units of currency j. That is, when $\mathcal{E}_{i,j,t}$ goes up, currency i appreciates relative to currency j. We will select one reference country (the US), denoted by \$, and use $\mathcal{E}_{i,t} = \mathcal{E}_{i,\$,t}$ to denote the nominal exchange rate against the US Dollar.

Due to assumed market completeness, consumers in different countries attain perfect risk sharing and pricing kernels $M_{i,t,t+1}$ and $M_{j,t,t+1}$ of any two countries i, j are linked through the no-arbitrage identity:

$$\frac{M_{i,0,t}}{M_{j,0,t}} \;=\; \frac{\mathcal{E}_{i,j,t}}{\mathcal{E}_{i,j,0}}\,.$$

Substituting from the consumption Euler equation (2), we get the standard risk sharing identity:

$$\mathcal{E}_{i,j,t} \;=\; \frac{C^{\gamma}_{i,0}\mathcal{P}_{i,t}C^{-\gamma}_{i,t}}{C^{\gamma}_{j,0}\mathcal{P}_{j,t}C^{-\gamma}_{j,t}}\frac{\mathcal{E}_{i,0}}{\mathcal{E}_{j,0}}$$

Thus, defining

$$c_{i,0} \;\equiv\; C_{i,0}^{\gamma}\mathcal{E}_{i,0}\;,\; i = 1, \cdots, N\,,$$

and the *real exchange rates*

$$\widetilde{\mathcal{E}}_{i,j,t} \;\equiv\; C_{i,t}^{-\gamma}/C_{j,t}^{-\gamma}\frac{c_{i,0}}{c_{j,0}}, \tag{3}$$

we can rewrite nominal exchange rates as the product of real exchange rates and the inflation quotient:

$$\mathcal{E}_{i,j,t} \;=\; \widetilde{\mathcal{E}}_{i,j,t}\,\frac{\mathcal{P}_{i,t}^{-1}}{\mathcal{P}_{j,t}^{-1}}\,. \tag{4}$$

3.2 Firms' Choices with an Exogenous Debt Overhang

3.2.1 Production

Each country's productive sector is populated by a continuum of ex-ante identical firms, indexed by $\omega \in [0,1]$, with firm ω producing goods of type ω. We will often use (i,ω) to denote the firm ω in country i. Firms are taxed on profits at a country-specific tax rate τ_i. All firms use labor as well as goods[9] produces by other firms (domestic and foreign) as inputs in a standard Cobb-Douglas production technology: The output of an (i,ω) firm is given by

$$Y_{i,t}(\omega) \;=\; Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}}\, L_t(i,\omega)^{1-\alpha}\, X_t(i,\omega)^{\alpha}\,,$$

where, for each $i = 1, \cdots, N$,

- $Z_t(i,\omega)\; > 0$ is firm (i,ω) idiosyncratic production shock that is drawn from a country-

[9] For simplicity we assume that all goods are used both for production (as intermediate inputs) and consumption.

specific distribution with a density $\phi_i(z) = \ell_i z^{\ell_i - 1}$ on $[0,1]$ with a country specific parameter $\ell_i > 0$ and the cumulative distribution function $\Phi_i(z) = \int_0^z \phi_i(x)dx = z^{\ell_i}$. We assume that $Z_{i,t}$ are i.i.d. over time and across firms within a given country;

- $a_{i,t}$ is the country-i productivity shock;
- $L_t(i,\omega)$ is labour hired by the (i,ω) firm at time t;
- $X_t(i,\omega)$ is the CES aggregator of goods used by the firm as inputs:[10]

$$X_t(i,\omega) = \left(\sum_{j=1}^{N} \theta(j) \int_0^1 (\tilde{X}_{t,(i,\omega)}(j,s))^{\frac{\eta-1}{\eta}} ds \right)^{\frac{\eta}{\eta-1}} .$$

Here, $\tilde{X}_{t,(i,\omega)}(j,s)$ is the demand of a (i,ω) firm for goods of a (j,s)-firm in country j.

All firms in our model are exporters and sell goods both domestically and abroad. As above, we use $P_t^j(i,\omega)$ to denote the *nominal* price (in the units of country j currency) at which an (i,ω) firm sells its goods in country j at time t. For simplicity, we assume that prices are fully flexible and hence the aggregate nominal price level in any given country is indeterminate. We assume that the price level (inflation) in each country can be controlled by the monetary authority and follows a country-specific stochastic process $\mathcal{P}_{i,t}$.

Due to the assumed identical CES structure of consumption and production aggregators, for each time $t+1$, each firm (i,ω) faces the downward sloping demand for its goods sold in country j, given by

$$D_{t+1}^j(i) = \mathcal{D}_{t+1}^j(i)(P_{t+1}^j)^{-\eta}, \tag{5}$$

[10]For simplicity, we assume that the consumption aggregator coincides with the production aggregator. Without this assumption, we would need to separately consider consumer and producer price indices, which would complicate the analysis.

where the demand coefficients $\mathcal{D}^j_{t+1}(i)$ are determined in equilibrium. We will use

$$\bar{D}_{t+1}(i) \;=\; \sum_{j=1}^{N} D^j_{t+1}(i) \tag{6}$$

to denote the global demand for country i goods.

With flexible prices and CES demand, it is always optimal for each firm to set prices in different countries using the law of one price: $P^j_{t+1}(i) \;=\; P^i_{t+1}(i)/\mathcal{E}_{j,i,t+1}$. Therefore, global demand (6) can be rewritten as $\bar{D}_{t+1}(i) \;=\; (P^i_{t+1}(i))^{-\eta}\tilde{\mathcal{D}}_t(i)$, with

$$\tilde{\mathcal{D}}_t(i) \;\equiv\; \sum_j \mathcal{D}^j_t(i)\,\mathcal{E}^{\eta}_{j,i,t}\,. \tag{7}$$

The following is true.[11]

Lemma 2 *Total after tax profits of country i firms are given by*

$$\Pi_{i,t} \;=\; \Omega_{i,t} Z_{i,t} \tag{8}$$

with

$$\Omega_{i,t} \;=\; \tilde{\mathcal{D}}_t(i)\mathcal{P}^{1-\eta}_{i,t}(1-\tau_i)\bar{\eta}\left((\nu_i C^{\gamma}_{i,t})^{1-\alpha} e^{-a_{i,t}}\right)^{1-\eta}\,. \tag{9}$$

for some constant $\bar{\eta}$ given in the Appendix.

3.2.2 Debt

In order to highlight the mechanisms through which debt affects real outcomes in our model, we first introduce debt exogenously.[12] We assume that when firms with nominal debt receive

[11] See Lemma 14 in the Appendix for firms' production decisions.

[12] In section 5, we endogenize the choice between debt and equity, as well as the choice of the compositions of currency denomination of debt.

bad productivity draws, they become distressed and produce less efficiently. This form of *debt overhang* is the key mechanism through which debt is linked to real outcomes in our model.

Assumption 1 (Debt Overhang) *Firms are short-lived and enter period t with short term nominal debt with a face value of $\mathcal{B}_{i,t}$ in domestic currency.*

- *Having observed the idiosyncratic shock realization, the firm computes its optimal profits:*
  - *If the after-tax profits are sufficient to cover the debt servicing cost, the firm hires labor, buys intermediate inputs and makes optimal production decisions.*
  - *If the after-tax profits are insufficient to cover the debt servicing cost, the firm enters a financial distress state, and is only able to produce at a fraction $\zeta_i \in (0,1)$ of its capacity $Z_{i,t}$.*

The simple nature of debt overhang in Assumption 1 implies that production decisions in distress are identical to those in Lemma 14, but with $Z_{i,t}^{(\eta-1)^{-1}}$ replaced by $\zeta_i Z_{i,t}^{(\eta-1)^{-1}}$. Furthermore, Assumption 1 also implies that the firm enters a financial distress when $Z_{i,t}$ falls below the distress threshold

$$\Psi_{i,t} \;\equiv\; \frac{\mathcal{B}_{i,t}}{\Omega_{i,t}}\,. \tag{10}$$

Thus, Assumption 1 allows us to capture two key features of the behaviour of financially constrained firms: In distress, effective marginal costs surge, forcing the firms to raise prices and cut production.[13] Both features are important for our results: In equilibrium, high prices hit global demand; firms respond by cutting their production and raising prices even further, potentially leading to a debt crisis.

[13] In Section 5 we micro-found these costs by assuming that, in distress, debt-holders take over the firm, and are less efficient in running production. In Section G.1 in the Appendix, we introduce investment decisions and into the firm problem.

4 General Equilibrium

In our model, all goods are used both for consumption and for production. Total demand $D_t^i(j)$ of country i for country j goods is thus given by the sum of consumers' and firms' demand:

$$D_t^i(j) \;=\; \tilde{C}_{i,t}(j) \;+\; \int_0^1 \tilde{X}_{t,(i,\omega)}(j)d\omega\,. \tag{11}$$

By Lemma 1, consumers' demand is given by

$$\tilde{C}_{i,t}(j) \;=\; \theta(j)(P_t^i(j))^{-\eta}\mathcal{P}_{i,t}^{\eta}C_{i,t}\,.$$

At the same time, country i firms' demand can be decomposed into the demand of distressed and non-distressed firms. By the law of large numbers, formula (38) and Assumption 1 imply that total country i firms' demand for country j goods can be rewritten as

$$\begin{aligned}&\int_0^1 \tilde{X}_{t,(i,\omega)}(j)d\omega\\ &=\; \theta(j)(P_t^i(j))^{-\eta}\mathcal{P}_{i,t}^{\eta-1}\frac{\bar{\chi}}{\bar{\eta}(1-\tau_i)}\Omega_{i,t}\;\int_0^1(Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)>\Psi_{i,t}}+\zeta_i Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)<\Psi_{i,t}})d\omega\,,\end{aligned}$$

where $\Psi_{i,t}$ is the distress threshold (10). Define

$$G_i(\Psi_{i,t}) \;=\; \ell_i(\ell_i+1)^{-1}\left((\zeta_i-1)\Psi_{i,t}^{\ell_i+1}+1\right)\,.$$

Then, by direct calculation

$$\int_0^1(Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)>\Psi_{i,t}}+\zeta_i Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)<\Psi_{i,t}})d\omega \;=\; G_i(\Psi_{i,t})$$

and therefore, using (11), we can rewrite the coefficient $\mathcal{D}_t^i(j)$ in the demand schedule $D_t^i(j) = \mathcal{D}_t^i(j)(P_t^i(j))^{-\eta}$ (see (5)) as

$$\mathcal{D}_t^i(j) \;=\; (\mathcal{P}_{i,t})^{\eta}\theta(j)\hat{\mathcal{D}}_{i,t}\,, \tag{12}$$

with

$$\hat{\mathcal{D}}_{i,t} \;=\; \left(C_{i,t} + G_i(\Psi_{i,t})\,\frac{\bar{\chi}}{\bar{\eta}(1-\tau_i)}\mathcal{P}_{i,t}^{-1}\Omega_{i,t}\right),\; i = 1,\cdots,N\,. \tag{13}$$

Equation (13) defines the equilibrium system for global demands: demand of country i firms depends on the global demand $\tilde{\mathcal{D}}_t(i)$ (see (7)) for country i goods, as reflected in formula (40) for $\Omega_{i,t}$. Substituting (12) into (7), we get, after some algebra, that $\mathcal{P}_{j,t}^{-\eta}\tilde{\mathcal{D}}_t(j) = \theta(j)\sum_i \hat{\mathcal{D}}_{i,t}\,\widetilde{\mathcal{E}}_{i,j,t}^{\eta}$. Substituting formula (4) for real exchange rates, we get

$$\mathcal{P}_{j,t}^{-\eta}\tilde{\mathcal{D}}_t(j) \;=\; \theta(j)(c_{j,0}^{-1}C_{j,t}^{\gamma})^{\eta}\bar{\mathcal{D}}_t\,, \tag{14}$$

where we have defined the *global demand factor*

$$\bar{\mathcal{D}}_t \;\equiv\; \sum_i \hat{\mathcal{D}}_{i,t}\,c_{i,0}^{\eta}C_{i,t}^{-\gamma\eta}\,. \tag{15}$$

Now, in order to derive the equilibrium system for global consumption, we need to compute price indices (1) and their response to debt overhang. By (37), the Law of One Price, and formula (4) for real exchange rates, we get that the total contribution of country j firms to country i price index is given by

$$\begin{aligned}\int_0^1 (P_t^i(j,\omega))^{1-\eta}d\omega \;&=\; \int_0^1 (\mathcal{E}_{j,i,t}P_t^j(j,\omega))^{1-\eta}d\omega \\ &=\; (\mathcal{P}_{i,t}^{-1}\tilde{\mathcal{E}}_{j,i,t})^{1-\eta}\,C_{j,t}^{\gamma(1-\alpha)(1-\eta)}\left(\frac{\eta}{\eta-1}\nu_j^{1-\alpha}\bar{\alpha}e^{-a_{j,t}}\right)^{1-\eta}G_j(\Psi_{j,t})\,.\end{aligned} \tag{16}$$

Therefore, debt overhang directly affects price level: Consistent with the evidence in Gilchrist, Schoenle, Sim and Zakrajšek (2017), financially constrained firms raise prices because their effective marginal cost of production is higher in distress.[14] Substituting (16) into formula (1) for the price index and using the explicit expression (3) for real exchange rates, we get

$$1 \;=\; \sum_j \theta(j)\left(C_{j,t}^{-\gamma}/C_{i,t}^{-\gamma}\frac{c_{j,0}}{c_{i,0}}\right)^{1-\eta} C_{j,t}^{\gamma(1-\alpha)(1-\eta)}\left(\frac{\eta}{\eta-1}\nu_j^{1-\alpha}\bar{\alpha}e^{-a_{j,t}}\right)^{1-\eta} G_j(\Psi_{j,t})\,. \qquad (17)$$

By assumption, all agents in all countries have identical preferences, markets are complete, and hence consumption is perfectly aligned across countries. As a result, *real exchange rates equal one for all country pairs* i, j and hence nominal exchange rates move one-to-one with relative inflation. This is important: Absent financial frictions, our model would be at odds with the exchange rate disconnect puzzle (see, e.g., Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017)) and generate counter-factual behaviour of currency risk premia. Define $\bar{C}_t \;=\; (C_{i,t}^{\gamma}c_{i,0}^{-1})^{\eta-1}$. Then, (17) takes the form

$$\bar{C}_t^{1-\alpha} \;\equiv\; \sum_{j=1}^{N}\theta(j)(c_{j,0}^{1-\eta})^{(1-\alpha)}\left(\frac{\eta}{\eta-1}\nu_j^{1-\alpha}\bar{\alpha}e^{-a_{j,t}}\right)^{1-\eta} G_j(\Psi_{j,t})\,. \qquad (18)$$

Equation (18) characterizes equilibrium consumption in the presence of debt overhang. In the frictionless case, we have $G_j(\Psi_{j,t}) = \ell_j(\ell_j+1)^{-1}$ and hence the frictionless aggregate consumption index, which we denote by $\bar{C}_{t,*}$, is a (non-linear) aggregate of total factor productivities. To solve for the equilibrium with debt overhang, we first need to derive the relationship between global consumption and production demands. To this end, we note that global production demand is proportional to the total exchange-rate weighted sum of domestic production demands, (15). Each domestic demand (13) is a sum of consumption demand $C_{i,t} \;=\; (c_{i,0}\bar{C}_t^{(\eta-1)^{-1}})^{\gamma^{-1}}$, and production demand. The latter is proportional to

[14]See also Malamud and Zucchi (2018).

profits, $\Omega_{i,t}$, which are in turn proportional to the global demand factor (15). This leads to an equilibrium fixed point system, whose solution is reported in the following proposition.

Proposition 3 *We have* $\bar{\mathcal{D}}_t = d_* \bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}}$ *for some* $d_* > 0$.

We will restrict our attention to equilibria in which a strictly positive fraction of firms in each country is not in distress. That is, $\Psi_{i,t} < 1$ for all $i = 1, \cdots, N$. Define

$$\xi_j \equiv \theta(j) d_* (1-\tau_j) \bar{\eta} (c_{j,0}^{1-\eta})^{(1-\alpha)} \left(\nu_j^{1-\alpha}\right)^{1-\eta} .$$

By direct calculation, we have

$$\Omega_{i,t} = \xi_i e^{a_{i,t}(\eta-1)} \bar{C}_t^{\hat{\eta}} , \ \hat{\eta} \equiv \frac{\gamma^{-1}}{\eta-1} + \alpha - 1 . \tag{19}$$

The following is true.

Theorem 4 *There exists a unique equilibrium solution* $\bar{C}_t$ *to the equation*

$$\bar{C}_t^{1-\alpha} = A \sum_{j=1}^{N} \frac{\xi_j}{1-\tau_j} e^{a_{j,t}(\eta-1)} G_j \left(\frac{\mathcal{B}_{j,t} \mathcal{P}_{j,t}^{-1}}{\xi_j e^{a_{j,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}} \right) \tag{20}$$

satisfying $\max_i \Psi_{i,t} < 1$ *and such that* $\bar{C}_t$ *that is monotonically increasing in* $a_{j,t}$, $j = 1, \cdots, N$.[15]

We complete this section with two results that are crucial for understanding the real effects of debt overhang. First, since financial distress lowers production, debt overhang leads to and output gap and unemployment. Second, due to the input-output linkages, rising debt burdens in one country always transmit to other countries; we show that, under

[15] In the case of $\hat{\eta} > 0$, there might exist a "non-economic equilibrium" with an unreasonable feature that consumption is decreasing in productivity. For the rest of the paper, we neglect this equilibrium and only focus on the one that we call the "normal equilibrium;" that is, the equilibrium in which $\bar{C}_t$ is monotonically increasing in $a_{j,t}$ for all j.

natural conditions, a rising debt burden in one country always leads to higher debt overhang costs in all other countries. The following two corollaries formalize this intuition.

Corollary 5 *Denote by $\bar{L}_t(i)$ and $\bar{O}_t(i)$ the country i equilibrium labour demand (employment) and output, respectively. Let also $\bar{L}^*_t(i)$ and $\bar{O}^*_t(i)$ denote the corresponding frictionless benchmarks absent debt overhang. Then, both the output gap and the employment gap are given by*

$$\frac{\bar{L}_t(i)}{\bar{L}^*_t(i)} = \frac{\bar{O}_t(i)}{\bar{O}^*_t(i)} = \frac{G_i(\Psi_{i,t})}{\ell_i(\ell_i+1)^{-1}} < 1\,. \tag{21}$$

Corollary 6 (Default transmission) *Suppose that $\hat{\eta} > 0$. Then, a shock to debt burden $\mathcal{B}_{j,t}$ or the debt overhang cost $1-\zeta_j$ of a country j always leads to an increase in the fraction of distressed firms in all other countries $i \neq j$.*

5 Dominant Currency Debt

In this section, we study the firms' choice of leverage and the composition of currency denomination of their debt in general equilibrium.

We assume that firms finance themselves by issuing both equity and defaultable short-term nominal bonds in *any* of the N currencies.[16] Each bond has a nominal face value of one currency unit, and the firm is required to pay a coupon of c currency units per unit of outstanding debt.[18] We denote by $B_{j,t}(i)$ the stock of outstanding nominal debt at time t of country i firms, denominated in the currency of country j. We also denote

[16]For the sake of analytical tractability, we assume that firms only issue short-term debt. However, our arguments about the special risk properties of the dollar are based on the observation that dollar tends to depreciate over long-term. At the same time, dollar safe haven properties imply that it tends to appreciate over short term during crises (see, for example, Maggiori (2013) and Farhi and Maggiori (2017)), making it unattractive for short-term borrowing. In Section G.2, we show that our main results still hold true for currencies with such risk profiles.[17]

[18]Apart from the multiple currencies assumption, in modelling the financing side we closely follow Gomes, Jermann and Schmid (2016).

by $B_t = (B_{j,t}(i))_{j=1}^N$ the vector of debt stocks in different currencies. Firms also have a possibility of hedging foreign exchange risk by acquiring h_t units of a financial derivative contract with a payoff of $X_{t+1} \geq 0$ and a price of $E_t[M_{i,t,t+1}X_{t+1}]$ to be paid at time t. As in Gomes, Jermann and Schmid (2016), we assume that coupon payments are shielded from taxes, so that

$$\mathcal{B}_{i,t+1}(B_t) \;=\; ((1-\tau_i)c+1)\sum_{j=1}^{N}\mathcal{E}_{j,i,t+1}B_{j,t}$$

is the total debt servicing cost in local currency, net of tax shields. The choice of firm leverage therefore depends on the trade-off between tax advantages and the distress costs.[19] Then, absent default, nominal distribution to shareholders at time $t+1$ is given by

$$\Pi_{t+1}(i,\omega) \;+\; h_t(1-\tau_i)X_{t+1} \;-\; \mathcal{B}_{i,t+1}(B_t)\,. \tag{22}$$

The first term captures the firm's after tax nominal operating profits (39); the second term is the payoff from hedging using $h_t \geq 0$ units of the derivative; and the third one is the debt repayment net of tax shields, denominated in country i currency. If the cash flows (22) are non-positive, shareholders default on firm's debt. Upon default, debt-holders take over the firm and shareholders get zero. Our first important result is that hedging is always suboptimal.

Proposition 7 *The firm always chooses $h_t = 0$.*

The intuition behind this result is straightforward. Hedging effectively plays a role of investment, and the firm only gets the payoff X_{t+1} from this investment in good (survival) states, while paying the market price at time t to get the payoff in all states. Thus, hedging

[19]For simplicity, as in Gomes, Jermann and Schmid (2016), we assume that tax shields are the only motivation for issuing debt. However, one could also interpret τ_i as reduced form of gains from debt issuance, such as alleviation of adverse selection costs.

is just a transfer of funds from shareholders to debt-holders, and firms optimally decide to minimize this transfer.[20]

5.1 Optimal Leverage and Dominant Currency Debt

As we explain above, shareholders default whenever cash flows (22) are non-positive; that is, when $\Pi_{t+1}(i,\omega) \leq \mathcal{B}_{i,t+1}(B_t)$. By (39), $\Pi_{i,t+1} = \Omega_{i,t+1}Z_{i,t+1}$ and hence default occurs whenever $Z_{i,t+1}$ falls below the default threshold

$$\Psi_{i,t+1}(B_t) \equiv \frac{\mathcal{B}_{i,t+1}(B_t)}{\Omega_{i,t+1}}.$$

We assume that, upon default, debt-holders recover a fraction ρ_i of their promised value, $1+c$.[21] Thus, by direct calculation, the nominal price in country i currency of one unit of debt denominated in currency j is given by

$$\delta_i^j(B_t) = E_t\left[M_{i,t,t+1}\left(1-(1-\rho_i)\Phi(\Psi_{i,t+1}(B_t))\right)(1+c)\mathcal{E}_{j,i,t+1}\right].$$

As is common in the literature, we assume that firms face a proportional cost $q_i(j)$ of issuing in country j currency[22] and are maximizing equity value plus the proceeds from debt issuance

[20] There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (2018): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." See also Niepmann and Schmidt-Eisenlohr (2017), Bruno and Shin (2017). While it is known that costly external financing makes hedging optimal (see, for example, Froot, Scharfstein and Stein (1993) and Hugonnier, Malamud and Morellec (2015)), Rampini, Sufi and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less.

[21] We assume that ρ_i is sufficiently small relative to productivity in default parameter, ζ_i, so that debt holders can recover at most what they get from (inefficiently) running the firm net of (unmodelled) default costs paid to lawyers, etc. We assume that these costs go directly to the representative consumer and hence have no impact on equilibrium outcomes. There are big differences in these default costs across countries. See, Favara, Morellec, Schroth and Valta (2017).

[22] While we do not micro-found these costs, it is not difficult to do so. These costs may originate from underwriting costs, limited risk bearing capacity of intermediaries (in case of bank loans), or the actual debt placement costs incurred by the investment banks (such as locating bond investors). These costs differ

net of issuance costs:

$$\max_{B_t} \left\{ \sum_{j=1}^{N} \delta_i^j(B_t) B_{j,t}(1 - q_i(j)) + E_t[M_{i,t,t+1} \max\{\Pi_{t+1}(i,\omega) \; - \; \mathcal{B}_{i,t+1}(B_t), 0\}] \right\}.$$

In the case when $\ell_i = 1$, this problem can be solved explicitly, and we report the solution in Proposition 19 in the Appendix. However, in the main text we only characterize the case of a corner solution when issuing all debt in dollars is optimal. Everywhere in the sequel, we will use $E_t^{\$}$ and $\mathrm{Cov}_t^{\$}$ to denote conditional expectation and covariance under the US Dollar risk neutral measure with the conditional density $E_t[M_{\$,t,t+1}]^{-1} M_{\$,t,t+1}$. Furthermore, for each stochastic process X_t, we will consistently use the notation

$$X_{t,t+1} \;\equiv\; \frac{X_{t+1}}{X_t}\,.$$

We will need the following assumption ensuring that the leverage choice problem has a non-trivial solution.

Assumption 2 *We have*

$$\begin{aligned} &(1 - q_i(j))(1+c) \;>\; (1 + c(1-\tau_i)) \qquad \textit{and} \\ &\bar{q}_i(j,\$) \;\equiv\; \frac{((1 - q_i(j))(1+c) - (1 + c(1-\tau_i)))}{(1-\rho_i)(1+c)[(1-q_i(j)) + \ell_i(1 - q_i(\$))] - (1 + c(1-\tau_i))} \;>\; 0 \end{aligned}$$

for all i, $j = 1, \cdots, N$. Let also $\bar{q}_i(\$) \equiv \bar{q}_i(\$,\$)$.

The first condition ensures that the cost $q_i(j)$ of issuing debt is less than the gains, measured by the value of tax shields, so that there is positive debt issuance. The second condition ensures that the recovery rate ρ_i is sufficiently small: Otherwise, funding becomes

drastically depending on the currency in which debt is issued. For example, according to Velandia and Cabral (2017), "... in the case of Mexico, the average bid- ask spread of the yield to maturity on outstanding USD-denominated international bonds is 7 basis points, compared to 10 basis points for outstanding EUR-denominated bonds; and Mexico is an example with very liquid benchmarks on both currencies."

so cheap for the firm that it may want issuing infinite amounts of debt. The following is true.

Theorem 8 *Issuing only in US Dollars is optimal if and only if*

$$\frac{\bar{q}_i(j,\$)}{\bar{q}_i(\$)} - 1 \;\leq\; \frac{\mathrm{Cov}_t^{\$}\left(\left(\mathcal{E}_{i,t,t+1}\Omega_{i,t+1}\right)^{-\ell_i}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[(\Omega_{i,t+1}\mathcal{E}_{i,t,t+1})^{-\ell_i}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]} \tag{23}$$

for all $j = 1, \cdots, N$. In this case, optimal dollar debt satisfies

$$b_{\$,t}(i) \;=\; \mathcal{E}_{i,t}^{-1} B_{\$,t} \;=\; (1 + c(1-\tau_i))^{-1} \left(\frac{\bar{q}_i(\$)}{E_t^{\$}\left[(\Omega_{i,t+1}\mathcal{E}_{i,t,t+1})^{-\ell_i}\right]}\right)^{\ell_i^{-1}} . \tag{24}$$

Condition (45) shows that the incentives for issuing in dollars are determined by two forces: The effective cost of issuance, $\bar{q}_i(\$)$, and the risk profile of the dollar. The low effective cost of issuance, $\bar{q}_i(\$)$, is an obvious factor favoring the dollar as the dominant currency of choice for debt contracts. Dollar capital and derivative markets are deep and liquid. Furthermore, it is the vehicle currency in FX transactions (see Moore, Sushko and Schrimpf (2016)). However, our main result does not rely on the dollar having low issuance costs: Theorem 8 implies that the dollar can arise as the dominant debt-denomination currency purely due to its risk profile. The underlying mechanism works as follows: Absent heterogeneity in effective issuance costs (that is, when $q_i(j)$ is independent of j), (45) takes the form

$$\mathrm{Cov}_t^{\$}\left(\left(\mathcal{E}_{i,t,t+1}\Omega_{i,t+1}\right)^{-\ell_i}, \mathcal{E}_{j,t,t+1}\right) \;\geq\; 0\,. \tag{25}$$

Here, $\mathcal{E}_{i,t,t+1}\Omega_{i,t+1}$ is the value of country i firms in US dollars,[23] while $(\mathcal{E}_{i,t,t+1}\Omega_{i,t+1})^{-\ell_i}$ is the *effective marginal utility of profits.* Naturally, firms are attracted by assets that co-move positively with their effective marginal utility; as a result, they like issuing dollar debt if

[23] While firms are short-lived in our model, their decision to default is determined by the present value of their cash flows, which is exactly their stock market value.

dollar tends to depreciate at times when their marginal utility is high. The strength of this effect increases in the parameter ℓ_i, that captures the sensitivity of company default risk to shocks.[24]

5.2 Dominant Currency Debt in General Equilibrium

In this section, we combine the equilibrium characterization in Theorem 4 with the dominant currency debt condition of Theorem 8 to answer the question: When does dominant currency debt arise in general equilibrium?

We will make the following simplifying assumption.

Assumption 3 *We have*

- *issuing costs are independent of currency denomination:* $q_i(\$) = q_i(j)$ *for all* $i, j = 1, \cdots, N$.
- *TFP shocks satisfy* $a_{j,t} = a_t + \varepsilon^a_{j,t}$ *for some common shock* a_t *and idiosyncratic TFP shocks* $\varepsilon^a_{j,t}$ *with small variance that are independent across countries and are also independent of* a_t.

As we explain above, in our model consumption is perfectly aligned across countries, real exchange rates equal one, and nominal exchange rates changes are determined purely by the relative inflation rates:

$$\mathcal{E}_{i,t,t+1} = \mathcal{P}^{-1}_{i,t,t+1}\mathcal{P}_{\$,t,t+1} . \tag{26}$$

[24]Condition (25) corresponds to the problem of a firm choosing between dollar debt and debt denominated in other key currencies such as, e.g., the euro, the yen, the franc and the pound. For an emerging markets firm choosing between local currency debt and dollar debt, heterogeneity in issuance costs may be as (if not more) important as the currency risk profile.

Thus, substituting the profits $\Omega_{i,t}$ from (19) and using Assumption 3, we get from (25) that dominant currency debt is an equilibrium if and only if

$$\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}} \mathcal{P}_{\$,t,t+1}\right)^{-\ell_i}, \mathcal{P}_{j,t,t+1}^{-1} \mathcal{P}_{\$,t,t+1}\right) \;\geq\; 0 \tag{27}$$

for all $i, j = 1, \cdots, N$.

Consistent with the standard asset pricing logic, the key contributions to this covariance are coming from the states with high firm's effective marginal utility; that is, states with low nominal Dollar profits $\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}} \mathcal{P}_{\$,t,t+1}$. Thus, firms want to issue debt in the currency of the country that is able to produce inflation in crisis states. Furthermore, the higher ℓ_i is, the stronger is this effect.

In our model, inflation is completely exogenous and hence can be directly controlled by the monetary authority. Expansionary monetary policy leads to an immediate currency depreciation in (26) and, hence, we interpret an increase in $\mathcal{P}_{i,t}$ as a "monetary easing shock."[25] To proceed further, we need to specify how the central bank responds to domestic economic conditions. We assume a standard, counter-cyclical monetary rule whereby the monetary authority eases (respectively, tightens) when employment or output falls (respectively, rises) relative to the frictionless benchmark (see (21)):

Assumption 4 *Country i central bank sets inflation rate according to*

$$\mathcal{P}_{i,t,t+1} \;=\; \left(1 - \bar{L}_{t+1}(i)/\bar{L}_{t+1}^*(i)\right)^{\phi_i} \; e^{\varepsilon_{i,t+1}}\,. \tag{28}$$

Here, $\phi_i > 0,\; i = 1, \cdots, N$ is a country specific parameter that measures the aggressiveness of domestic monetary policy, and $\varepsilon_{i,t+1}$ is a country-specific monetary policy shock, $\varepsilon_{i,t+1} \sim N(\bar{\varepsilon}_i, \sigma_{i,\varepsilon}^2)$.

[25] There is ample evidence that monetary easing leads to a simultaneous currency depreciation. See, for example, Ferrari, Kearns and Schrimpf (2017).

In our model, debt overhang is the only source of frictions. When firms are in trouble, the central bank tries to stimulate the domestic economy by reducing debt burden of domestic firms by effectively reducing their borrowing costs. Under such a policy, standard Phillips curve holds (though the underlying mechanism seems to be new): high inflation reduces firms' debt burden, and hence stimulates production and reduces unemployment and the output gap.

The ability of this debt inflation channel to stimulate domestic economy is limited in the presence of foreign currency debt. In the extreme case of the dominant currency debt equilibrium, this ability vanishes completely. Substituting the debt servicing costs $\mathcal{B}_{\$,t}(B_{\$,t-1}) = ((1-\tau_\$)c+1)b_{\$,t-1}(j)\mathcal{P}_{\$,t-1,t}^{-1}\mathcal{P}_{j,t-1,t}$ into equation (21) for unemployment, and then using the assumed policy equation (28), we get that the US monetary policy satisfies the fixed point equation

$$\mathcal{P}_{\$,t-1,t} = \left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_{\$,t}}\left(\xi_\$^{-1}((1-\tau_\$)c+1)b_{\$,t-1}(\$)\,\mathcal{P}_{\$,t-1,t}^{-1}\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$} e^{\varepsilon_{\$,t+1}} \tag{29}$$

Indeed, the inflation policy is assumed to respond to unemployment, while the unemployment in turn responds to the inflation policy. The counter-cyclical policy assumption (that is, $\phi_\$ > 0$) implies that there is a unique solution to (29), given by

$$\mathcal{P}_{\$,t-1,t} = \left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_{\$,t}}\left(\xi_\$^{-1}((1-\tau_\$)c+1)b_{\$,t-1}(\$)\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$} e^{\varepsilon_{\$,t+1}}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}. \tag{30}$$

We can now characterize the conditions when a dominant currency debt equilibrium emerges. The following theorem is the main result of this paper.

Theorem 9 *Suppose that monetary policy uncertanty $\sigma_{i,\varepsilon}$, $i = 1, \cdots, N$ are sufficiently* *small and that the indices $(\ell_j + 1)\phi_j$ are all pairwise different. Then, there always exists a unique Dominant Currency Debt equilibrium. The dominant debt currency is always the currency of the country with the highest index $(\ell_j + 1)\phi_j$.*

Theorem 9 has an interesting relationship with the results in Du, Pflueger and Schreger (2016). Namely, Du, Pflueger and Schreger (2016) show that, surprisingly, sovereigns of countries with more countercyclical inflation issue more foreign-currency debt. This result is counter-intuitive: If we extrapolate the logic of Theorem 9 from private firms to sovereigns, one would expect to see more domestic currency issuance in countries with more countercyclical inflation. As Du, Pflueger and Schreger (2016) argue, their finding can be explained by monetary policy credibility. Namely, it is precisely low credibility government governments that tend to inflate their debt during recessions. In our model, one could interpret low credibility as a form of monetary policy uncertainty. Naturally, firms view this uncertainty as an additional and undesirable form of risk. The following is true.

Proposition 10 *Absent heterogeneity in the indices $(\ell_i + 1)\phi_i$, firms always issue in the currency of the country with the lowest degree of idiosyncratic policy uncertainty, $\sigma_{i,\varepsilon}$.*

Proposition 10 suggests that, in addition to insufficient market liquidity (modelled by high issuance costs), the significant idiosyncratic volatility of emerging market currencies may serve as an additional important mechanism explaining why firms do not want to issue in these currencies, despite the fact that such currencies do tend to significantly depreciate during crises. As an illustration, consider a typical emerging market currency, the Argentinian Peso (ARS). During the period of November 1995-September 2018, the standard deviation of the monthly returns on the dollar index was 1.9%, while the standard deviation of monthly returns on the ARS/USD exchange rate was 7.1%. Furthermore, this volatility was almost entirely due to idiosyncratic shocks: Indeed, the R^2 of a regression of the monthly ARS/USD returns on the returns on the dollar index is only 0.0033.

6 The Federal Reserve as the World's Central Bank

Our general equilibrium framework also allows us to discuss the macroeconomic implications of a dominant currency debt equilibrium and the role of the Federal Reserve as the world's central bank and the difference it makes for the global welfare when it reacts to global versus domestic conditions.

An aggressive monetary policy of the dominant currency country lowers *ex-post* real debt burdens of firms through higher inflation and exchange rate depreciation. Therefore, it reduces the cost of issuing debt in that currency, increasing leverage *ex-ante.* However, higher leverage means higher distress costs in the face of more severe shocks. Even though aggressive monetary policy in crises is optimal ex-post when a crisis state is realized, it is never optimal ex-ante. Namely, the welfare gains from reducing distress costs of firms are more than offset by the welfare costs of higher leverage. Central banks would prefer not to provide this insurance to firms ex-ante, but cannot credibly do so.

Given this policy trade-off, in what follows we characterize the optimal monetary policy of a hypothetical central bank of the world in our dominant debt currency equilibrium, where firms in the entire globe issue dollar debt. The global central bank optimally assigns weights on the output gaps of different countries to maximize global welfare, taking into account all spillovers arising from the interconnectedness of different countries due to global value chains. We make the following assumption:

Assumption 5 *There exist $\chi_i \geq 0,\ i = 1, \cdots, N$, such that*

$$\mathcal{P}_{\$,t,t+1} \;=\; \prod_i \left(1 - \bar{L}_{t+1}(i)/\bar{L}^*_{t+1}(i)\right)^{\chi_i} \,.$$

Furthermore, all pairwise correlations are identical: $Corr_{t-1}(a_{i,t}, a_{j,t}) \;=\; \rho_{t-1}$ is independent of $i \neq j$.

The following is true.

Proposition 11 *The welfare maximizing policy is to only react to output gap in countries with:*

- *low TFP variance of* $a_{i,t}$
- *low default sensitivity* ℓ_i
- *low restructuring cost* $1-\zeta_i$
- *low importance in global trade,* ξ_j

This result arises from the trade-off that firms face in issuing debt. Debt is cheaper to issue than equity for firms due to tax shields it provides. Aggressive monetary policy in downturns reduces the probability that the firms default and hence makes debt even cheaper. This incentivizes firms to exploit this and issue even more debt. However, higher indebtedness intensifies the debt burdens in the bad states of the world, reducing expected welfare ex-ante.

The global central bank chooses weights to precisely limit the leverage of firms in countries where the adverse effects of leverage are the highest as shown in Proposition 11.[26] That in turn reduces leverage ex-ante and improves global welfare. Our results have interesting implications for the recent academic literature about the Global Financial Cycle (see, for example, Gourinchas and Rey (2007), Gourinchas, Govillot and Rey (2010), Rey (2013), Cerutti, Claessens and Rose (2017), Miranda-Agrippino and Rey (2018)). First, the exorbitant duty of the Federal Reserve to respond to global conditions might in fact be optimal for the US welfare; and second, the dynamics of global expectations about the US monetary policy might be as important as the monetary policy itself.

[26]There are big differences in restructuring costs across countries. See, Favara, Morellec, Schroth and Valta (2017).

7 Empirical Evidence

In this section, we provide evidence on the viability of the dollar as a dominant currency through the channels described in our model, as well as other evidence that is consistent with the predictions of our theory. We also compare predictions of the debt-centric view with those of the trade-centric view of dollar dominance, in light of the evidence provided in Figure 1.

7.1 Why is the dollar dominant?

The condition (25) suggests that firms prefer to issue in dollars if the dollar co-moves positively with their stock market value, that is the dollar depreciates when the stock market falls. In order to test the empirical relevance of this condition, we use the trade-weighted dollar index against major currencies, including the Eurozone, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden, obtained from the FRED database. Note that we are abstracting from the stochastic discount factor. In effect, we look at the correlation between the dollar and stock markets under the physical measure, while the dominant currency debt condition (25) is formulated under the risk neutral measure.

Our first hypothesis is that the returns on the dollar index are positively correlated with the returns on the stock market indices at horizons corresponding to the weighted average corporate debt maturity, that is around 6-7 years (Choi, Hackbarth and Zechner (2018), Cortina, Didier and Schmukler (2018)).

We ask two questions: First, are US firms better off issuing debt in dollars? Second, are firms in the rest of the world better off issuing debt in dollars? To answer the first, we regress the returns on the dollar index on the returns on the S&P 500 index at different horizons. To answer the second, we repeat the same procedure, using the MSCI World Index instead.

In order to calculate different horizons, we roll the returns and create quarter-on-quarter, year-on-year, 2-year-on-2-year returns and so on. We run the following regressions for each

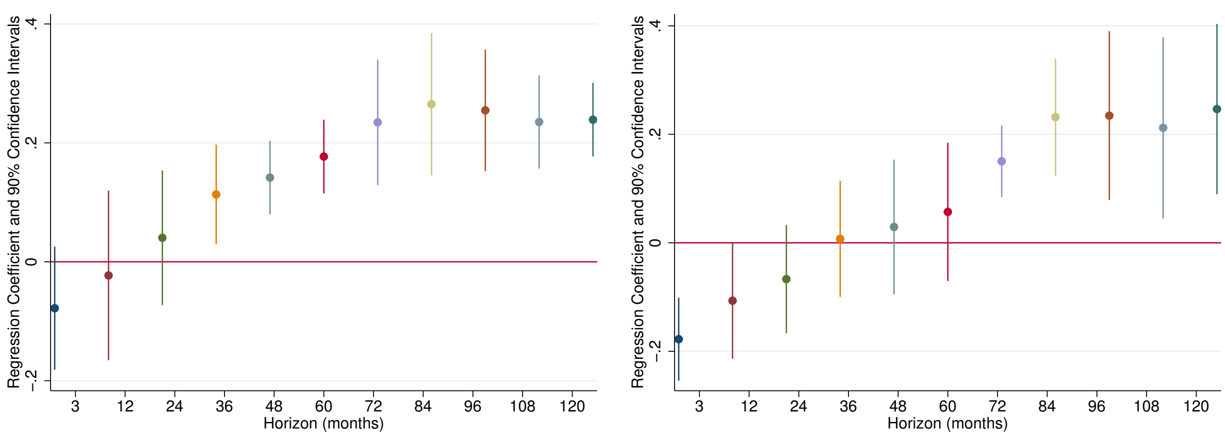
$h \in \{3, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120\}$ months:[27]

$$Return\_USD_t^h = \alpha^h + \beta_h Return\_SP500_t^h + \epsilon_t^h \,. \tag{31}$$

The left-hand panel of Figure 2 reports the results for the regression coefficient β_h at different horizons together with the 95% confidence intervals for the sample period between December 1987 and September 2018.

Figure 2: Correlation of the USD index with Stock Market Indices

S&P 500 Index *MSCI World Index*



Notes: The left-hand side graph reports the regression coefficients β_h from the regressions (31). The right-hand side graph reports the regression coefficients from the regressions (32). X_t^h refers to the returns for the variable X at horizon h at a given month t. For example, $h = 3$ refers to quarter-on-quarter returns, $h = 12$ refers to year-on-year returns and so on. The dots are the corresponding β_h and the lines are the 95% confidence intervals. Standard errors are corrected using the Newey-West procedure, with h lags in each regression.

The results show a pattern of negative correlation at short horizons and positive and mostly increasing correlation at longer horizons. These findings suggest that US firms are

[27]We control for autocorrelation at the respective horizons by using the Newey-West correction with the respective lag.

better off borrowing in dollars than other major international currencies if their debt maturity exceeds roughly two years, which is the case.

Next, we turn to the rest of the world and compute the correlation of the returns on the dollar index with the returns on the MSCI World Index. The question in mind is whether the average firm in the rest of the world is better off borrowing in dollars than other major international currencies. We follow the same procedure as before and run the following regressions, for each $h \in \{3, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120\}$ months:

$$Return\_USD_t^h = \alpha^h + \beta_h Return\_MSCIWorld_t^h + \epsilon_t^h \,. \tag{32}$$

The results from these regressions align similarly with the results for the S&P 500. The safe haven effect (negative correlation with the stock market) dies out after a year and firms with debt maturity of longer than five years are statistically significantly better off borrowing in dollars than in other major international currencies, providing support for our hypothesis.

7.2 The share of the dollar in the last two decades

The last two decades have witnessed the fall of the share of the dollar in denominating debt contracts in early 2000s and a rise after the Global Financial crisis. In this section, we argue that our model offers an explanation for the experience of the last two decades. The mechanism we highlight operates through expectations of countercyclicality of inflation. Namely, firms choose to issue debt in currencies for which they expect the central bank to be able to generate inflation in bad times. While expectations about the monetary policy effectiveness are not directly observable, one might infer some information about these expectations from financial asset prices; for example, from the inflation risk premium.

First, we provide a link between inflation risk premium and the dominant currency status of a currency. Then, we provide empirical evidence that ties our theory to the experience of the last two decades. Finally, we combine data on trade, inflation risk premium and debt to compare and test the predictions of our "debt-centric" view of dollar dominance and the "trade-centric" view of dollar dominance highlighted in Gopinath and Stein (2018). We find strong evidence in favor of our predictions. Furthermore, although not conclusive due to several data limitations, we find no evidence for the predictions of Gopinath and Stein (2018) in the time-series data of the last two decades.

7.2.1 The debt view: Inflation risk premium

In our model, the preference of firms to issue dollar denominated debt linked to inflation risk premium: Effectively, dollar debt is a claim on $\mathcal{P}_{\$,t}^{-1}$. If the market puts large weight on the states with US deflation (or low US inflation), dollar nominal bonds will be expensive relative to their real counterparts. In this case, inflation risk premium is negative. On the contrary, if the market puts weight on the states with high US inflation, the dollar nominal bonds will be cheaper and the inflation risk premium will be positive. Formally, the inflation risk premium can be defined as

$$IRP_{i,t} \;=\; \frac{BEIR_{i,t}}{E_t[\mathcal{P}_{i,t,t+1}]} \;-\; 1 \;=\; \frac{e^{r_{i,t}}\mathrm{Cov}_t(M_{i,t,t+1},\mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t,t+1}]}\,,$$

where the breakeven inflation rate (BEIR) is given by

$$BEIR_{i,t} \;=\; \frac{E_t[M_{i,t,t+1}\mathcal{P}_{i,t,t+1}]}{E_t[M_{i,t,t+1}]} \;=\; E_t^i[\mathcal{P}_{i,t,t+1}]\,.$$

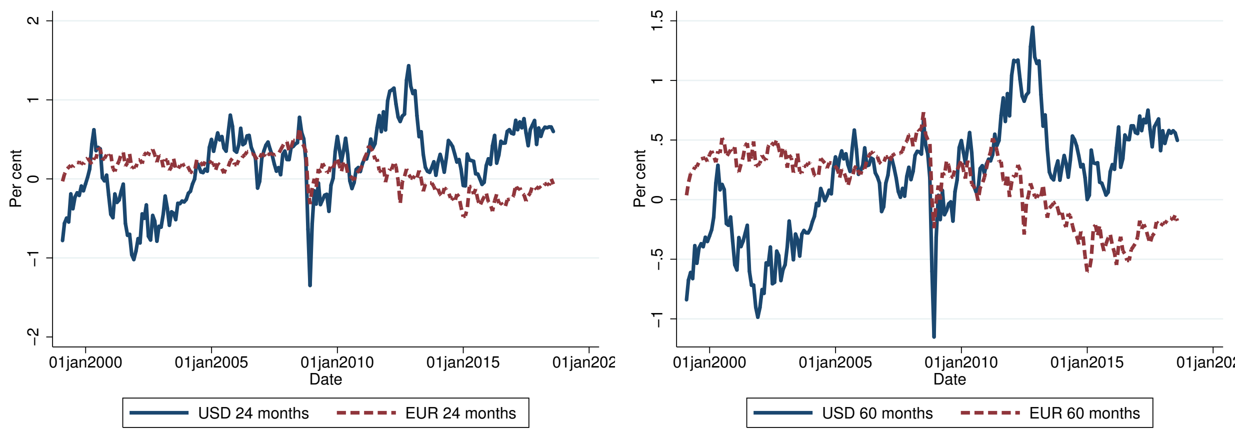
The following proposition ties the dominant currency to the inflation risk premium:

Proposition 12 *The inflation risk premium, $IRP_{i,t}$, has the largest value for the dominant currency country.*

The intuition behind Proposition 12 is straightforward: More aggressive counter-cyclical policy leads to more countercyclical inflation, making nominal bonds unattractive. In particular, in our model, *the inflation risk premium reflects investor expectations about inflation cyclicality.* Suppose that the pre- and post-crisis trends in the euro and dollar shares of debt denomination (Figure 1) are indeed driven by a shift in the dominant currency equilibrium due to inflation expectations. Then, our model would also predict a higher inflation risk premium in the Eurozone prior to the crisis and higher inflation risk premium in the US after the crisis.

Figure 3: Inflation Risk Premia in the US and the Eurozone

2-year IRP *5-year IRP*



Source: Hördahl and Tristani (2014), authors' calculations.

We use the estimates from Hördahl and Tristani (2014) for the inflation risk premia in the US and the Eurozone. They use a joint macroeconomic and term structure model, also using survey data on inflation and interest expectations, together with data from nominal and inflation index-linked bonds. To the best of our knowledge, Hördahl and Tristani (2014) are the only authors that apply the same rigorous methodology to recover IRP for the US and the Eurozone, making them comparable for our purposes.

We report their estimates for the 2-year and 5-year horizons in Figure 3, though the results are qualitatively similar and more pronounced for other horizons as well. Interestingly enough, the estimates of Hördahl and Tristani (2014) show that inflation risk premium was indeed higher in the Eurozone compared to the US prior to the crisis, consistent with the rising share of euro-denomination during that period (Figure 1). At the same time, the US inflation risk premium has been higher than the Euro since the crisis, consistent to the post-crisis rise in the dollar share of debt denomination.

Consistent with Figure 1, Maggiori, Neiman and Schreger (2018) show that the share of dollar-denominated debt in cross-border corporate holdings has drastically increased in the post-crisis period compared to the euro. We argue that this pattern is to a large extent driven by the bond-supply channel of Figure 1, and bond investors hold what the firms issue to clear markets in general equilibrum. That said, while bond investors might generally dislike holding nominal bonds with a high inflation risk premium, there is an opposite force in our model that increases the attractiveness of dollar-denominated bonds for lenders. The default probability of these bonds is lower because it is easier for firms to repay dollar debt due to lower real debt burdens in bad times. This is nevertheless dominated by the former channel.

7.2.2 The trade view: Dollar invoicing and dollar debt

Our model offers a debt-centric view of the dollar's dominance that is different from the trade-centric view of Gopinath and Stein (2018). While our papers are complimentary to each other in some aspects, they also differ in certain predictions and have different implications for the dynamics of the dominance of the dollar, as well as for the implications of this dominance for the global economy. In this regard, the last two decades of the variation in the share of the dollar's dominance provide an interesting testing ground.

In the trade-centric view, dollar's dominance in debt is rooted in its dominance in trade invoicing. Since trade is invoiced primarily in dollars, importers keep deposits in dollars due to a constraint resembling cash-in-advance. In their model, local currency projects outside the US are financed by issuing dollar "safe assets" because the interest rates on those are lower than home currency safe assets, which is in turn tied to trade invoicing in dollars. Therefore, a prediction of their model would be that, all else equal, more dollar invoicing of trade should be associated with more (mismatched) dollar borrowing by firms outside the US.

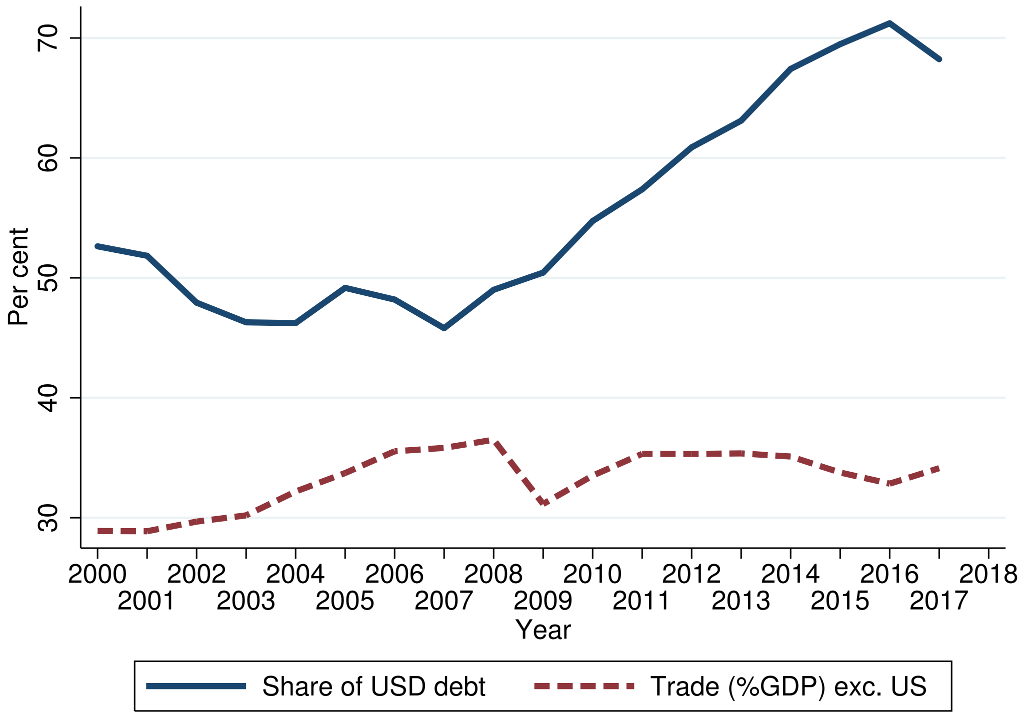
There are three reasons why our tests regarding the trade-centric view are not conclusive: First is due to the fact that it is hard to find a counterpart of their definition of a safe asset in the data. Second, there is no available aggregate time-series data for dollar invoiced trade. However, evidence presented for the countries in Gopinath (2015) suggests that dollar invoicing has remained roughly stable. That said, an ECB study shows that the share of euro invoicing in extra-euro area trade declined from 63.6% to 57.1% in 2017. (ECB (2018)). However, it is hard to gauge the quantitative importance of this for dollar trade invoicing in the world. Third, in a short time series, it is hard to keep all else equal.[28]

With the caveats above in mind and assuming that the dollar invoicing share of trade has remained roughly stable since 2000, to explain the rise in the dollar's dominance in debt issuance after the crisis, and its decline in the early 2000s, the trade-centric model in Gopinath and Stein (2018) would potentially require both a rise in world trade after the crisis and a fall in trade prior to the crisis.[29] This seems to be at odds with the data: According to the World Bank data, trade has increased not only in value, but also as a share of GDP prior to the crisis, while the trade share in GDP has fallen since the crisis (Figure 4). Furthermore, according to the evidence in Gopinath (2015), the share of trade invoiced in dollars does not seem to have increased significantly to overcompensate for the fall in trade after the crisis; similarly, it does not seem to have decreased significantly prior to the crisis.

[28]We partially deal with this concern by using a dummy variable for the post-crisis period. We do not report the results in the paper, but they are available upon request.

[29]Alternatively, a rise in the share of trade invoiced in dollars after the crisis and a fall prior to the crisis would have the same effect.

Figure 4: Trade versus Debt



Source: BIS, World Bank, authors' calculations

7.2.3 The last two decades: The debt view versus the trade view

In this subsection, we look for evidence for the debt view and the trade view of dollar's dominance using time-series information since 2000. In particular, we test the following hypotheses:

Hypothesis - Debt view: *The difference between the inflation risk premium of the dollar and the euro is positively associated with the share of the dollar in debt contracts outside the US. Alternatively put, the inflation risk premium in the dollar is positively associated with the share of dollar in debt contracts, while this association is negative for the inflation risk premium in the euro.*

Hypothesis - Trade view: Assuming that the trade invoicing share of the dollar is constant, more trade is associated with more (mismatched) dollar borrowing by non-US firms.

In order to test these hypotheses, we create the following variables: USD_t^{shr} refers to the share of dollar debt including both bank loans and debt securities outside the US. $USD_t^{shr,BL}$ refers to the share of dollar debt outside the US for only bank loans (using only bank loans gets us closer to the setup in Gopinath and Stein (2018)). $IRP_{\$,t}^{5Y} - IRP_{€,t}^{5Y}$ refers to the difference of the 5-year inflation risk premium for the dollar versus the euro as measured by Hördahl and Tristani (2014). $TotalTrade(\%GDP)_t^{exUS}$ is the total trade to world GDP, excluding the US. In some specifications, where the analysis is conducted using quarterly data, we also include year dummies.

The hypotheses above could then be restated as follows: To constitute evidence for the debt view, we would expect to get a positive association between USD_t^{shr} (or $USD_t^{shr,BL}$) and $IRP_{\$,t}^{5Y} - IRP_{€,t}^{5Y}$. Similarly, for the trade view, we would expect a positive association between USD_t^{shr} (or $USD_t^{shr,BL}$) and $TotalTrade(\%GDP)_t^{exUS}$. We test these in a linear regression.

Table 1 presents the results: Our main finding is that while there is strong evidence for the debt view in the time series data for the last two decades, we fail to find any evidence for the trade view.

Testing for the hypothesis of the debt view of a positive regression coefficient on $IRP_{\$,t}^{5Y} - IRP_{€,t}^{5Y}$, we find a large, positive, statistically significant coefficient and an R^2 of 52.8% unconditionally (Column (1)). Even after including year dummies, the positive sign remains positive and statistically significant (Column (3)). The estimates are also economically large: unconditionally, a 1 percentage point increase in the inflation risk premia difference between the dollar and the euro predicts an increase in the share of dollar debt of around 10 percentage points.

These results also corroborate a main distinction of our theory compared to others, that

Table 1: The debt view versus the trade view

	(1)	(2)	(3)	(4)	(5)	(6)
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	$USD_t^{shr,BL}$
$IRP_{\$,t}^{5Y} - IRP_{€,t}^{5Y}$	9.636***		1.553**		11.45***	13.55***
	(1.129)		(0.718)		(2.288)	(2.637)
$IRP_{\$,t}^{5Y}$		2.079*		0.841		
		(1.053)		(0.688)		
$IRP_{€,t}^{5Y}$		-22.71***		-4.863***		
		(1.495)		(0.967)		
$TotalTrade(\%GDP)_t^{exUS}$					-1.036*	-1.224*
					(0.589)	(0.686)
Year dummy			✓	✓		
Freq.	Q	Q	Q	Q	Y	Y
Observations	72	72	72	72	18	18
R-squared	0.528	0.786	0.986	0.989	0.567	0.582

Notes: Robust standard errors in parantheses. *, **, *** denote significance at the 10, 5 and 1% level respectively. USD_t^{shr} refers to the share of dollar debt including both bank loans and debt securities. $USD_t^{shr,BL}$ refers to the share of dollar debt for only bank loans. $IRP_{\$,t}^{5Y}$ and $IRP_{€,t}^{5Y}$ refer to the 5-year inflation risk premium for the dollar and the euro as measured by Hördahl and Tristani (2014), respectively. $TotalTrade(\%GDP)_t^{exUS}$ is the total trade to world GDP, excluding the US. Q and Y refer quarterly and yearly frequency since 2000.

is changes in the dominance of a currency can be high-frequency events. Even controlling for yearly patterns with the year dummies, we take the fact that within year changes in the IRP are significantly associated with currency choice in debt issuance to be a strong evidence for our theory.

So far, we have interpreted our results as the Federal Reserve implementing countercyclical monetary policy effectively. An alternative interpretation could be that its peers have lost the effectiveness of countercyclical monetary policy, at least in the expectations of the market participants. We find strong evidence for this alternative interpretation in the last two decades.

Instead of including $IRP^{5Y}_{\$,t} - IRP^{5Y}_{€,t}$ in the regression, we include the inflation risk premia in the US and the Euro area separately. Columns (2) and (4) of Table 1 presents the results. First observation is that the R^2 of the unconditional specification in column (2) rise significantly compared to column (1). Second, the coefficient on the $IRP^{5Y}_{€,t}$ is economically large: a 1 percentage point increase in the inflation risk premia in the euro predicts a decrease in the share of dollar debt of around 22 percentage points. These results imply that post-crisis share in the dollar denomination of debt is primarily due to the declining inflation risk premium in the Euro area. The negative coefficient survives and remains economically significant even after the inclusion of year dummies in quarterly regressions in column (4).

We fail to find any evidence for the trade view. When $IRP^{5Y}_{\$,t} - IRP^{5Y}_{€,t}$ and $TotalTrade(\%GDP)^{exUS}_t$ are included in the same regression in column (5), the sign of the coefficient on the $TotalTrade(\%GDP)^{exUS}_t$ variable is the opposite of the hypothesis of the trade view. In column (6), we include only bank loans instead of total debt to become closer to the "safe asset" definition of the trade view, but the results remain similar.

7.3 Cross-sectional evidence: Local currency shares

Next, we test the predictions of our model in a cross-section of emerging market economies for which data on corporate debt in different currencies are available.[30] To this end, we use

[30] Data are obtained from Institute for International Finance (IIF) since 2005. The countries are: Argentina, Brazil, Chile, China, Colombia, Czechia, Hong Kong, Hungary, India, Indonesia, Israel, Republic of Korea, Malaysia, Mexico, Poland, Russian Federation, Saudi Arabia, Singapore, South Africa, Thailand and Turkey.

an extension of Theorem 8 for the case when firms issue a mixture of local currency (LC) and Dollar debt to derive the following result.[31]

Proposition 13 *Suppose that (1) $q_i(i) = q_i(\$)$ (that is, issuing in LC costs the same as issuing in Dollars); (2) the variance of all shocks is sufficiently small; (3) all countries follow the monetary policy rules (28); and (4) issuing in both LC and Dollars is optimal.*[32] *Then,*

(a) *the fraction $\frac{B_{i,t}(i)}{B_{i,t}(\$)\mathcal{E}_{\$,i,t}}$ is monotone increasing in the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ if and only if $B_{i,t}(i) \geq B_{i,t}(\$)\mathcal{E}_{\$,i,t}$;*

(b) *the fraction $\frac{B_{i,t}(i)}{B_{i,t}(\$)\mathcal{E}_{\$,i,t}}$ is always monotone decreasing in $\sigma_{i,\varepsilon}$.*

(c) *the fraction $\frac{B_{i,t}(i)}{B_{i,t}(\$)\mathcal{E}_{\$,i,t}}$ is always monotone increasing in ϕ_i.*

Items (a)-(c) of Proposition 13 directly translate into the following three empirical hypotheses.

Hypothesis CS-1: The local currency share of corporate debt is *higher* for countries in which domestic inflation correlates more with the US inflation controlling for relevant factors.

The intuition for this hypothesis is as follows: In the face of non-negligible idiosyncratic shocks, as well as global shocks, for firms in countries whose inflation correlates more with the US inflation, taking on local currency debt is advantageous. Indeed, this is because local currency debt partly replicates the insurance properties of the dominant currency in downturns, while still having a central bank that can react to domestic conditions in the face of idiosyncratic shocks.

[31] See Theorem 18 in the Appendix.

[32] See Theorem 18 in the appendix for the necessary and sufficient conditions for the optimality of such a mixed issuance policy.

In order to test this hypothesis as close as possible to the theory, we proceed as follows. For each country i in our sample, we estimate the following time series regression:

$$\pi_t^i = \gamma_0 + \gamma_1 Return\_MSCIWorld_t + \gamma_2 Return\_DomesticStockIndex_t^i + \pi_t^{res,i}\,, \quad (33)$$

where π_t^i is the domestic monthly inflation rate in country i and $Return\_MSCIWorld_t$ is the monthly return on the MSCI World Index. $Return\_DomesticStockIndex_t^i$ is the monthly return on the domestic stock market index. $\pi_t^{res,i},\ t \geq 0$ is the sequence of residuals from this regression. We also run the following regression for the analogous variables in the US:

$$\pi_t^{US} = \mu_0 + \mu_1 Return\_MSCIWorld_t + \pi_t^{res,US}\,, \quad (34)$$

We then run the following regression in order to compute a proxy for the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ between the residual domestic inflation and residual US inflation (see item (a) of Proposition 13),

$$\pi_t^{res,i} = \alpha + \beta \pi_t^{res,US} + \epsilon_t\,,$$

where $\pi_t^{res,i}$ is the residual domestic monthly inflation rate in country i from (33) and $\pi_t^{res,US}$ is the residual monthly inflation rate in the US from (34). We denote the estimated slope coefficient by $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$.

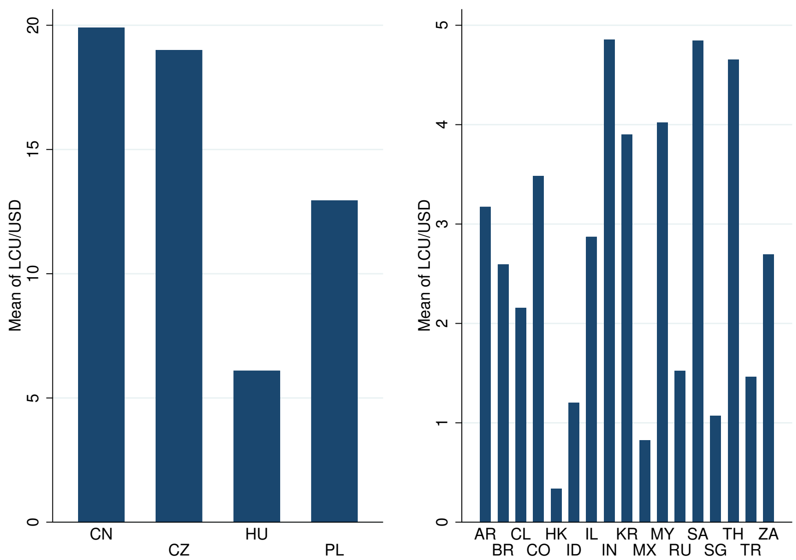
We then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_1 + \beta_1 \hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}} + X_i + \eta_i\,.$$

Here, $\frac{L\bar{C}U}{USD}_i$ is the average local currency to US debt ratio for corporates in the countries in the dataset, and X_i denote other control variables.

Figure 5 shows the mean local currency to USD debt ratio by country. The left-hand panel

Figure 5: Mean of the local currency to USD debt ratio by country



Source: IIF, authors' calculations

shows the outliers, namely China and the EU countries in the sample (Czechia, Hungary and Poland) and the right-hand panel shows the rest. We exclude the outliers to test our hypotheses and focus only on the sample countries on the right-hand panel.

Item (a) of Proposition 13 sugests that $\beta_1 > 0$. The first three columns of Table 2 show that this is indeed the case. In column (1), we show the results of a univariate regression. In column (2), we add an additional control variable $\bar{kaopen}_i$, which is a financial openness index obtained from Chinn and Ito (2006). In column (3), we take the predictions of the model literally as appear in item (a) of Proposition 13: $\beta_1 > 0$ for countries where $\frac{L\bar{C}U}{USD}_i > 1$ and exclude the two countries where $\frac{L\bar{C}U}{USD}_i < 1$, namely Hong Kong and Mexico. In all three columns, regressions corroborate *Hypothesis CS-1*. [33]

[33] All our results are qualitatively and quantitatively similar if we use raw domestic and US inflation rates, instead of residuals. Moreover, all results go through if we use the share of local currency debt in total debt instead of the ratio of local currency debt to USD debt.

Table 2: The cross section of the local currency to USD debt ratio

	(1)	(2)	(3)	(4)	(5)
	$\bar{\frac{LCU}{USD}}_i$	$\bar{\frac{LCU}{USD}}_i$	$\bar{\frac{LCU}{USD}}_i$	$\bar{\frac{LCU}{USD}}_i$	$\bar{\frac{LCU}{USD}}_i$
$\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$	3.843***	3.694***	3.396***		
	(0.684)	(0.629)	(0.783)		
$\bar{kaopen}_i$		-0.0748	0.0403	-0.404	-0.376
		(0.327)	(0.409)	(0.356)	(0.346)
$\sigma_i^{\pi_t^{res,i}}$				-1.440	
				(1.476)	
$\hat{\beta}_i^{\pi_t^i,Stock_t^i}$					-31.44*
					(16.73)
Observations	17	17	15	17	17
R-squared	0.513	0.516	0.366	0.184	0.245

Notes: Robust standard errors in parantheses. *, **, *** denote significance at the 10, 5 and 1% level respectively. $\bar{\frac{LCU}{USD}}_i$ is the mean share of local currency debt obtained from the IIF for each of the 17 emerging market economies since 2005. $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$ is the estimated regression coefficient of a linear regression of residuals of monthly domestic inflation rate from (33) on the residuals of the US inflation rate from (34). $\bar{kaopen}_i$ is the mean of Chinn-Ito financial openness index for each country. $\sigma_i^{\pi_t^{res,i}}$ is the standard deviation of the residuals of the monthly domestic inflation rate obtained from (33). $\hat{\beta}_i^{\pi_t^i,Stock_t^i}$ is the estimated regression coefficient of a linear regression of monthly domestic inflation rate on the domestic stock market returns also controlling for the returns on the world stock market index in (33).

Next, we test two other related hypotheses:

Hypothesis CS-2: Firms in countries with more volatile domestic inflation tend to have less debt denominated in local currency.

To test this hypothesis, we calculate the standard deviation of $\pi_t^{res,i}$ as a proxy for $\sigma_{\varepsilon,i}$ then and run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_2 + \beta_2 \sigma_i^{\pi_t^{res,i}} + X_i + \eta_i\,. \tag{35}$$

Proposition 13, item (b) predicts that $\beta_2 < 0$. Column (4) shows the results of regression (35). Although the result lacking statistical significance, the sign of the coefficient is consistent with our theoretical prediction.

Hypothesis CS-3: Firms in countries in which domestic inflation is more procyclical (i.e. correlates positively with the stock returns) tend to have less debt denominated in local currency.

In order to test this hypothesis we recover the coefficient γ_2 from the regression of domestic inflation on the domestic stock market and the MSCI World Index, (33), and denote it as $\hat{\beta}_i^{\pi_t^i, Stock_t^i}$, and run the following cross-sectional regression:

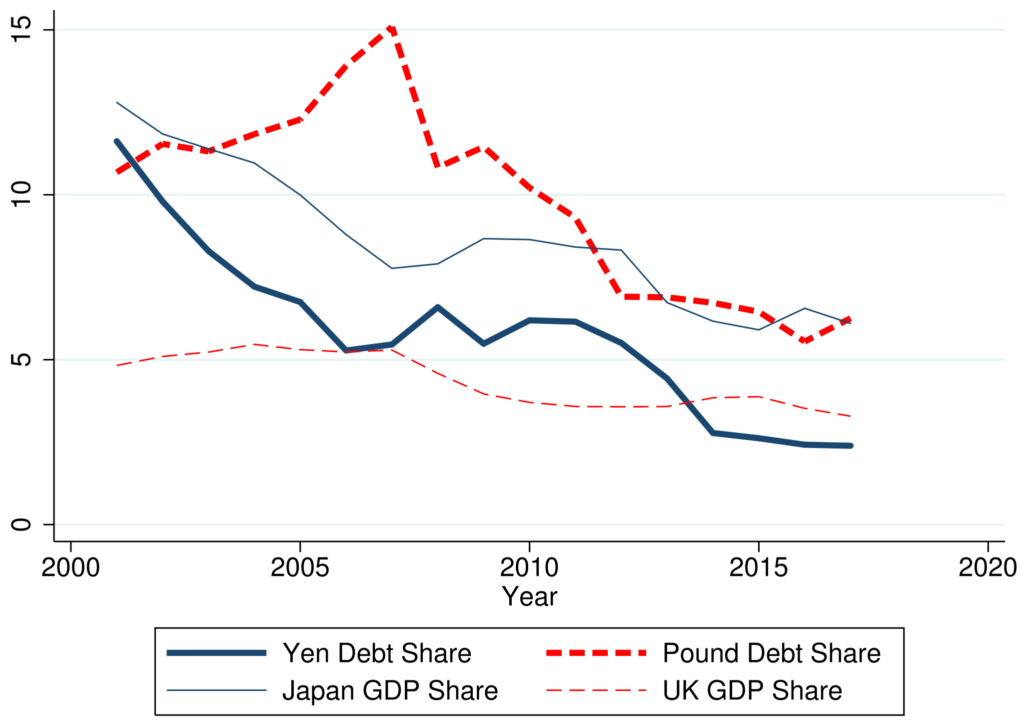
$$\frac{L\bar{C}U}{USD}_i = \alpha_3 + \beta_3 \hat{\beta}_i^{\pi_t^i, Stock_t^i} + X_i + \eta_i\,. \tag{36}$$

Proposition 13, item (c) predicts that $\beta_3 < 0$. Column (5) presents the results of regression (36). The coefficient has the predicted sign and is significant at 10% level.

7.4 Evidence from the yen versus the pound

The inflation expectations channel discussed above can also be used to understand patterns of debt issuance in other major currencies. Figure 6 is a case in point. It shows the shares of Japan and the United Kingdom in the world economy and contrasts it with the share of their currencies in denominating debt by foreigners. Despite the fact that Japan has a larger size than the United Kingdom, and the fact that inflation is lower in Japan, pound-denominated debt punches above the weight of the United Kingdom economy, while it is the opposite case for yen-denominated debt.

Figure 6: Japan versus the United Kingdom



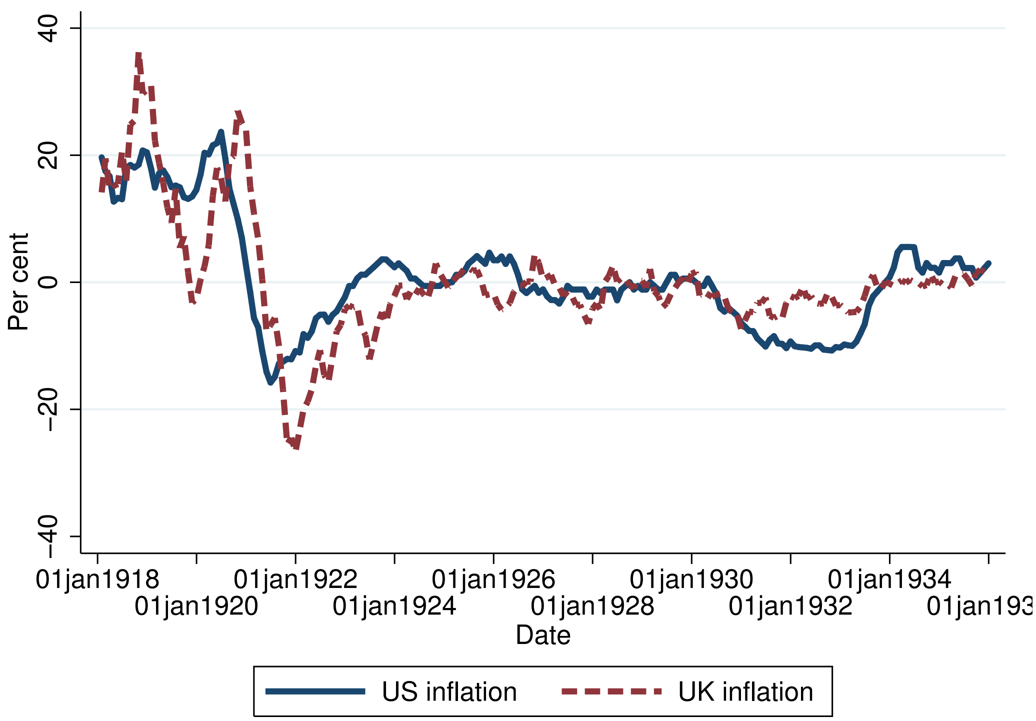
Source: BIS, IMF WEO, authors' calculations

Through the lens of our model, this could be explained by the fact that inflation in the UK was often close to and above the inflation target of the Bank of England, with firms seeing the real value of their debt decline more often. On the other hand, firms borrowing in yen have seen the real values of their nominal debt increase as inflation undershot the target of the Bank of Japan consistently. This made the yen an unattractive currency to borrow in, despite its low inflation.

7.5 Evidence from the interwar years

New historical accounts of the switch from the pound to the dollar as the main reserve currency and the currency choice of debt denomination suggest that, contrary to previous beliefs, it is not the case that once a currency loses dominance it cannot get it back. Chiţu, Eichengreen and Mehl (2014) show that this is the case during the interwar years for the pound and the dollar.

Figure 7: Historical Inflation Rates



Source: Global Financial Data

Our model predicts that even in the absence of factors like network externalities and inertia, a dominant debt currency can switch within short periods of time due to differences in *inflation expectations* of the incumbent and the competitor currency. This was indeed the case for interwar years. Pound suffered deflation to a larger extent than the dollar at the beginning of 1920s during the 1920-21 recession (Figure 7). This corresponds to the rise of the dollar as the debt denomination currency. On the flip side, the dollar faced greater deflation during the Great Depression, which corresponds to the subsequent rise of the pound, according to the evidence provided by Chiţu, Eichengreen and Mehl (2014).

8 Conclusion

Motivated by the omnipresence of the dollar in the denomination of debt contracts globally, we develop a simple international general equilibrium model. Our main result offers an explanation of why the dollar is the dominant debt currency, despite the presence of rival currencies with deep and liquid debt markets. This happens if firms believe that the Federal Reserve is both able and willing to effectively and precisely pursue aggressive monetary policies in global downturns, generating more inflation than its peers for horizons of the debt maturity of corporates. This feeds into dollar depreciation, lowering the real debt burdens of firms, pushing them away from default and hence stimulating the global economy. The empirically observed behaviour of the dollar vis-a-vis other major currencies since 2000 is supportive of this mechanism. Our mechanism can also account for a host of other empirical observations. Our key modelling contribution is the introduction of the main mechanisms of the Fisher debt-deflation theory into a standard international general equilibrium model.

What do our results imply for the future of the dollar? Many explanations of the dominant role of the dollar in the international monetary system feature arguments like inertia, size, network externalities, and market liquidity. All of these imply that changes of dominant currencies occur only slowly. Our results suggest that if the Federal Reserve loses its status of being able to effectively pursue aggressive monetary policy during global crises, the dollar might lose its status as the dominant debt currency. This status thus rests fully on the beliefs of market participants, and hence may change abruptly. This is in line with the evidence provided by Chiţu, Eichengreen and Mehl (2014) suggesting multiple switches between the pound and the dollar during the interwar years.

In our model, we have abstracted from many realistic features of the world to effectively highlight our main mechanism. Our model can be extended in multiple avenues. One potential extension could address the interactions between the role of the dollar in trade and finance by introducing sticky prices and the choice of currency denomination in trade invoicing. Second, modelling the demand for safety of investors more realistically, we can address the interactions between household investment demand and firm borrowing demand in different currencies. We leave these important questions for future research.

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A Production Decisions

Lemma 2 is a direct consequence of the following result.

Lemma 14 *Let*

$$\bar{\alpha} \;=\; \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \;+\; \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}.$$

A country i firm optimally sets the price

$$P_t^i(i) \;=\; \mathcal{P}_{i,t}\frac{\eta}{\eta-1}\bar{\alpha}(\nu_i C_{i,t}^{\gamma})^{1-\alpha} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \tag{37}$$

in the domestic market (in domestic currency) and sets prices in other countries using the law of one price; it hires labour

$$L_t(i) \;=\; \bar{\mathcal{L}}_i(\nu_i C_{i,t}^{\gamma})^{-(\alpha+\eta(1-\alpha))}\mathcal{P}_{i,t}^{-\eta}\tilde{\mathcal{D}}_t(i) Z_{i,t} e^{a_{i,t}(\eta-1)},$$

where we have defined

$$\bar{\mathcal{L}}_i \;=\; \left(\frac{\eta}{\eta-1}\bar{\alpha}\right)^{-\eta}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha},$$

and spends

$$X_t(i) \;=\; \bar{\chi}(\nu_i C_{i,t}^{\gamma})^{(1-\alpha)(1-\eta)}\mathcal{P}_{i,t}^{-\eta}\,\tilde{\mathcal{D}}_t(i)Z_{i,t}e^{a_{i,t}(\eta-1)}$$

on intermediate goods, where we have defined

$$\bar{\chi} \;=\; \left(\frac{\eta}{\eta-1}\bar{\alpha}\right)^{-\eta}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}.$$

The demand of country i firms for country-j goods is given by

$$\tilde{X}_{i,t}(j) \;=\; \theta(j)(P_t^i(j))^{-\eta}\bar{\chi}(\nu_i C_{i,t}^{\gamma})^{(1-\alpha)(1-\eta)}\tilde{\mathcal{D}}_t(i)e^{a_{i,t}(\eta-1)}Z_{i,t} \tag{38}$$

Total after tax profits of country i firms are given by

$$\Pi_{i,t} \;=\; \Omega_{i,t}Z_{i,t} \tag{39}$$

with

$$\Omega_{i,t} \;=\; \tilde{\mathcal{D}}_t(i)\mathcal{P}_{i,t}^{1-\eta}(1-\tau_i)\bar{\eta}\left((\nu_i C_{i,t}^{\gamma})^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta}. \tag{40}$$

Proof of Lemma 14. For a flexible price firm, the global demand for its goods is inversely proportional to the prices vector $P_t^j(i,\omega)$, and the firm will be choosing that price vector. Given that vector, the firm will face a vector of demands,

$$D_t^j((i,\omega),P) \;=\; \mathcal{D}_t^j(i)\,P_t^j(i,\omega)^{-\eta},$$

and hence the nominal income in the domestic currency will be given by

$$\mathcal{I}((P_t^j)_{j=1}^N) \;=\; \sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{1-\eta}\mathcal{E}_{j,i,t}\,.$$

Thus, first, the objective of the firm is to maximize its income given the fixed demand:

$$\max\{\mathcal{I}((P_t^j)_{j=1}^N):\; \sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta} \;=\; \bar{D}\}\,.$$

The Lagrangian of this problem is

$$(1-\eta)\mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta}\mathcal{E}_{j,i,t} \;+\; \lambda\eta\mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta-1} \;=\; 0\,,$$

which gives

$$P_t^j(i,\omega)\mathcal{E}_{j,i,t} \;=\; \lambda\frac{\eta}{\eta-1}$$

implying that a flexible price monopolist always sets the prices satisfying the law of one price. Thus, total demand satisfies

$$\begin{aligned}\bar{D} \;&=\; \sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta} \;=\; \sum_j \mathcal{D}_t^j(i)\, (P_t^i(i,\omega)/\mathcal{E}_{j,i,t})^{-\eta} \;=\; P_t^i(i,\omega)^{-\eta}\sum_j \mathcal{D}_t^j(i)\, \mathcal{E}_{j,i,t}^{\eta} \\ &=\; P_t^i(i,\omega)^{-\eta}\tilde{\mathcal{D}}_t\,,\end{aligned}$$

At the same time, the income is given by

$$\sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{1-\eta}\mathcal{E}_{j,i,t} \;=\; \sum_j \mathcal{D}_t^j(i)\, (P_t^i(i,\omega)/\mathcal{E}_{j,i,t})^{1-\eta}\mathcal{E}_{j,i,t} \;=\; P_t^i(i,\omega)^{1-\eta}\tilde{\mathcal{D}}_t\,.$$

Hence, the maximization problem becomes to maximize

$$-\left(w_{i,t}L_{i,t}(\omega)+\sum_j\int_0^1 P_t^i(j)\tilde{X}_{t,(i,\omega)}(j,s)ds\right)\;+\;\tilde{\mathcal{D}}_t(i)\,P_t^i(i,\omega)^{1-\eta}$$

Keeping the demand fixed, we are maximizing

$$\begin{aligned}
&-\left(w_{i,t}L_{i,t}(\omega)+X_t(i,\omega)\mathcal{P}_{i,t}\right)\;+\;\tilde{\mathcal{D}}_t(i)\,P_t^i(i,\omega)^{1-\eta}\\
&=\;-\left(w_{i,t}\left(\frac{\mathcal{P}_{i,t}}{w_{i,t}}\right)^{\alpha}\bar{D}_{i,t}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}+\left(\frac{w_{i,t}}{\mathcal{P}_{i,t}}\right)^{1-\alpha}\bar{D}_t(i,\omega)Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\mathcal{P}_{i,t}\right)\\
&+\;\mathcal{D}_t(i)\,P_t^i(i,\omega)^{1-\eta}\\
&=\;-\bar{\alpha}w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}\bar{D}_t(i,\omega)Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\;+\;\mathcal{D}_t(i)\,P_t^i(i,\omega)^{1-\eta}\\
&=\;-\bar{\alpha}w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}\mathcal{D}_t(i)\,P_t^i(i,\omega)^{-\eta}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\;+\;\mathcal{D}_t(i)\,P_t^i(i,\omega)^{1-\eta}\,,
\end{aligned}$$

where we have defined

$$\bar{\alpha}\;=\;\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\;+\;\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}.$$

Thus, the optimal price set by a flexible firm is given by

$$P_t^i\;=\;\frac{\eta}{\eta-1}\bar{\alpha}w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\,,$$

and the total revenue is given by

$$\begin{aligned}
&-\bar{\alpha}w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}\tilde{\mathcal{D}}_t(i)\left(\frac{\eta}{\eta-1}\bar{\alpha}w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\right)^{-\eta}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\\
&+\;\tilde{\mathcal{D}}_t(i)\left(\frac{\eta}{\eta-1}\bar{\alpha}w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\right)^{1-\eta}\\
&=\;\bar{\eta}\left(w_{i,t}^{1-\alpha}(\mathcal{P}_{i,t})^{\alpha}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\right)^{1-\eta}\tilde{\mathcal{D}}_t(i)
\end{aligned}$$

where we have defined

$$\bar{\eta} \;\equiv\; \frac{(\eta-1)^{\eta-1}\bar{\alpha}^{1-\eta}}{\eta^{\eta}}\,.$$

Q.E.D.

Proof of Proposition 3. By (14) and (40), we have

$$\Omega_{i,t} \;=\; \theta(i)(c_{i,0}^{-1}C_{i,t}^{\gamma})^{\eta}\bar{\mathcal{D}}_t\mathcal{P}_{i,t}(1-\tau_i)\bar{\eta}\left((\nu_i C_{i,t}^{\gamma})^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta},$$

so that

$$\begin{aligned}
\bar{\mathcal{D}}_t \;&=\; \sum_i \hat{\mathcal{D}}_{i,t}\, c_{i,0}^{\eta}C_{i,t}^{-\gamma\eta} \;=\; \sum_i \left(C_{i,t} + G_i(\Psi_{i,t})\,\frac{\bar{\chi}}{\bar{\eta}(1-\tau_i)}\mathcal{P}_{i,t}^{-1}\Omega_{i,t}\right)c_{i,0}^{\eta}C_{i,t}^{-\gamma\eta}\\
&=\; \sum_i \Bigg((c_{i,0}(\bar{C}_t)^{(\eta-1)^{-1}})^{\gamma^{-1}}\bar{C}_t^{-\eta/(\eta-1)}\\
&+\theta(i)G_i(\Psi_{i,t})\bar{\mathcal{D}}_t\bar{C}_t^{\alpha-1}(c_{i,0}^{1-\eta})^{(1-\alpha)}\left((\nu_i)^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta}\Bigg)\\
&=\; \bar{c}_0\bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}} \;+\; \bar{\mathcal{D}}_t\bar{C}_t^{\alpha-1}\bar{\chi}\sum_i \theta(i)G_i(\Psi_{i,t})(c_{i,0}^{1-\eta})^{(1-\alpha)}\left((\nu_i)^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta}.
\end{aligned}$$

Substituting from (18), we get

$$\bar{\mathcal{D}}_t \;=\; \bar{c}_0\bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}} \;+\; \bar{\mathcal{D}}_t\bar{C}_t^{\alpha-1}\bar{\chi}\,\bar{C}_t^{1-\alpha}(\bar{\alpha}^{1-\eta}\left(\frac{\eta}{\eta-1}\right)^{1-\eta})^{-1}$$

implying that

$$\bar{\mathcal{D}}_t \;=\; d_*\bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}}$$

and hence

$$\Omega_{i,t} = \theta(i)(c_{i,0}^{-1}C_{i,t}^{\gamma})^{\eta} d_* \bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}} \mathcal{P}_{i,t}(1-\tau_i)\bar{\eta} \left((\nu_i C_{i,t}^{\gamma})^{1-\alpha} e^{-a_{i,t}}\right)^{1-\eta}$$
$$= \theta(i)(c_{i,0}^{-1})^{\eta} d_* \mathcal{P}_{i,t}(1-\tau_i)\bar{\eta} \left((\nu_i)^{1-\alpha}\right)^{1-\eta} e^{a_{i,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}\,.$$

Q.E.D.

Proof of Theorem 4. We have

$$\Psi_{i,t} = \frac{\mathcal{B}_{i,t}\mathcal{P}_{i,t}^{-1}}{k_i \bar{C}_t^{\eta/(\eta-1)} \bar{\mathcal{D}}_t (1-\tau_i)\bar{\eta}\bar{C}_t^{\alpha-1}(c_{i,0}^{1-\eta})^{(1-\alpha)} \left((\nu_i)^{1-\alpha} e^{-a_{i,t}}\right)^{1-\eta}}$$
$$= \frac{\mathcal{B}_{i,t}\mathcal{P}_{i,t}^{-1}}{k_i \bar{C}_t^{\eta/(\eta-1)} d_* \bar{C}_t^{-1} (1-\tau_i)\bar{\eta}\bar{C}_t^{\alpha-1}(c_{i,0}^{1-\eta})^{(1-\alpha)} \left((\nu_i)^{1-\alpha} e^{-a_{i,t}}\right)^{1-\eta}}$$

and substituting this into equation (41), we get

$$\bar{C}_t^{1-\alpha} = \bar{\alpha}^{1-\eta} \left(\frac{\eta}{\eta-1}\right)^{1-\eta} \sum_{j=1}^{N} \theta(j)(c_{j,0}^{1-\eta})^{(1-\alpha)} \left(\nu_j^{1-\alpha} e^{-a_{j,t}}\right)^{1-\eta} G_i(\Psi_{i,t}) \tag{41}$$

Q.E.D.

B Leverage

Proof of Proposition 7. The maximization problem is

$$\max_{h_t} \Bigg\{ - E_t[M_{t,t+1}X_{t+1}]h_t$$
$$+ E_t \left[M_{t,t+1} \int_{\Omega_{i,t+1}Z_{i,t+1} > \mathcal{B}_{t+1}(B_t) - h_t(1-\tau_i)X_{t+1}} (\Omega_{i,t+1}Z_{i,t+1} - \mathcal{B}_{t+1}(B_t) + h_t(1-\tau_i)X_{t+1})\phi(Z_{i,t+1})dZ_{i,t+1} \right] \Bigg\}.$$

The derivative of this objective function with respect to h_t is given by

$$-E_t[M_{t,t+1}X_{t+1}] + (1-\tau_i)E_t\left[M_{t,t+1}X_{t+1}\left(1-\Phi_i\left(\frac{\mathcal{B}_{t+1}(B_t)-h_t(1-\tau_i)X_{t+1}}{\Omega_{i,t+1}}\right)\right)\right] < 0,$$

and hence $h_t = 0$ is optimal. Q.E.D.

Proof of Theorem 8. Firm's problem is to maximize

$$\sum_j E_t\left[M_{i,t,t+1}\left[\left(1-(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right]B_{j,t}(1-q_i(j))$$
$$+ \; E_t\left[M_{i,t,t+1}\left[-\mathcal{B}_{t+1}(B_t)\left(1-\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\right)+\Omega_{i,t+1}\ell_i(\ell_i+1)^{-1}\left(1-\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i+1}\right)\right]\right]$$

Differentiating, we get that from the standard Kuhn-Tucker conditions that borrowing only in dollars is optimal if and only if

$$E_t\left[M_{i,t,t+1}\left[\left(1-(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right](1-q_i(j))$$
$$+ E_t\left[M_{i,t,t+1}\left[\left(-\ell_i(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i-1}\Omega_{i,t+1}^{-1}\right)(1+c)\mathcal{E}_{\$,i,t+1}(1+c(1-\tau_i))\mathcal{E}_{j,i,t+1}\right]\right]B_{\$,t}(1-q_i(\$))$$
$$-(1+c(1-\tau_i))E_t\left[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}\right]$$
$$+ E_t\left[M_{i,t,t+1}(\ell_i+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}(1+c(1-\tau_i))\mathcal{E}_{j,i,t+1}\right.$$
$$\left.-\ell_i\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}(1+c(1-\tau_i))\mathcal{E}_{j,i,t+1}\right] \leq 0$$

for all j with the identity for $j = \$$. This inequality can be rewritten as

$$
\begin{aligned}
& E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}]((1-q_i(j))(1+c)-(1+c(1-\tau_i))) \\
\leq\; & E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\mathcal{E}_{j,i,t+1}\right]((1-\rho_i)(1+c)[(1-q_i(j))+\ell_i(1-q_i(\$))]-(1+c(1-\tau_i)))
\end{aligned}
$$

At the same time, for the dollar debt we get

$$
\begin{aligned}
& E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]((1-q_i(\$))(1+c)-(1+c(1-\tau_i))) \\
=\; & E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\mathcal{E}_{\$,i,t+1}\right]((1+\ell_i)(1-\rho_i)(1+c)(1-q_i(\$))-(1+c(1-\tau_i)))
\end{aligned}
$$

implying that

$$
B_{\$,t}(1+c(1-\tau_i)) \;=\; \left(c_{i,0}(\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\Omega_{i,t+1}^{-\ell_i}\mathcal{E}_{\$,i,t+1}^{1+\ell_i}\right]}\right)^{\ell_i^{-1}}
$$

and we get the Kuhn-Tucker conditions

$$
\frac{\bar{q}_i(j,\$)}{\bar{q}_i(\$)}\frac{E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}]}{E_t\left[M_{i,t,t+1}\Omega_{i,t+1}^{-\ell_i}\mathcal{E}_{j,i,t+1}\mathcal{E}_{\$,i,t+1}^{\ell_i}\right]} \;\leq\; \frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\Omega_{i,t+1}^{-\ell_i}\mathcal{E}_{\$,i,t+1}^{1+\ell_i}\right]}
$$

In the case when $q_i(j) = q_i(\$)$, this condition takes the form

$$
\mathrm{Cov}_t^{\$}\left((\mathcal{E}_{i,t+1}\Omega_{i,t+1})^{-\ell_i},\mathcal{E}_{j,t+1}\right) \;\geq\; 0\,.
$$

Q.E.D.

C Proof of Theorem 9

Using (30), we arrive at the following result.

Lemma 15 *Country j firms' default threshold is given by*

$$\Psi_{j,t}(b_{t-1}) \;=\; \tilde{\psi}_j(b_{t-1}) e^{-(\eta-1)a_{j,t} + \frac{-[(\eta-1)[-(\ell_\$+1)\phi_\$ a_{\$,t}]+\varepsilon_{\$,t}]}{1+(\ell_\$+1)\phi_\$}} \bar{C}_t^{-\frac{\hat{\eta}}{1+(\ell_\$+1)\phi_\$}},$$

where we have defined

$$\tilde{\psi}_j(b_{t-1}) \;=\; \frac{((1-\tau_j)c+1)b_{\$,t-1}(j)}{\xi_j((1-\zeta_\$)(\xi_\$^{-1}((1-\tau_\$)c+1)b_{\$,t-1}(\$))^{\ell_\$+1})^{\frac{\phi_\$}{1+(\ell_\$+1)\phi_\$}}}\,. \tag{42}$$

As above, we are only interested in equilibria in which $\Psi_{j,t} < 1$ for all j, which is equivalent to $\bar{C}_t^{\hat{\eta}} > (\bar{C}_t^M)^{\hat{\eta}}$, where

$$\bar{C}_t^M \;\equiv\; \max_j \left(\tilde{\psi}_j(b_{t-1}) e^{\frac{-1}{1+(\ell_\$+1)\phi_\$}[(\eta-1)a_t+\varepsilon_{\$,t}]}\right)^{\frac{1+(\ell_\$+1)\phi_\$}{\hat{\eta}}}.$$

The equilibrium condition of Theorem 4 implies the following result.

Proposition 16 *Let $\bar{C}_{t,*}(a_t)$ be the frictionless consumption, solving (20) for the case with no debt overhang and exogenous monetary policy. All equilibria with active monetary policy (28) are then characterized by solutions $\bar{C}_t(b_{t-1}, a_t)$ to the equation*

$$\bar{C}_t^{1-\alpha} \;+\; A\sum_{j=1}^N \frac{\xi_j}{1-\tau_j} e^{a_{j,t}(\eta-1)} \frac{\ell_j(1-\zeta_j)}{\ell_j+1}\left(\Psi_{j,t}(b_{t-1})\right)^{\ell_j+1} \;=\; \bar{C}_{t,*}^{1-\alpha}\,,$$

There is at most one economic equilibrium that is monotonically increasing in the common TFP shock a_t.

We can now prove the characterization of the dominant currency debt equilibrium.

Proof of Theorem 9. By (30)

$$
\begin{aligned}
&\mathcal{P}_{j,t-1,t}^{-1}\mathcal{P}_{\$,t-1,t}\\
&= \left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(b_{\$,t-1}(\$)\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\\
&\times \Bigg((1-\zeta_j)e^{-(\ell_j+1)(\eta-1)a_t}\\
&\times \Bigg(b_{j,t-1}(\$)\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(b_{\$,t-1}(\$)\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{-(1+(\ell_\$+1)\phi_\$)^{-1}}\\
&\times \bar{C}_t^{-\hat{\eta}}\Bigg)^{\ell_j+1}\Bigg)^{-\phi_j}\\
&= \widetilde{b}_{t-1}\left(e^{-(\eta-1)a_t}\bar{C}_t^{-\hat{\eta}}\right)^{\frac{(\ell_\$+1)\phi_\$(1+\phi_j(\ell_j+1))}{1+(\ell_\$+1)\phi_\$}}\left(e^{(\eta-1)a_t}\bar{C}_t^{\hat{\eta}}\right)^{(\ell_j+1)\phi_j}\ .
\end{aligned}
$$

Thus, if $(\ell_j+1)\phi_j < (\ell_\$+1)\phi_\$$, then (27) always holds. Q.E.D.

Proof of Proposition 11. Since we assume that idiosyncratic TFP shocks are small, we will simply set $a_{j,t} = a_t$ is the future calculations. Defining

$$
\Delta_{i,t-1} \equiv \left(\frac{\bar{q}_i(\$)E_{t-1}\left[C_{t-1,t}^{-\gamma}\mathcal{P}_{\$,t-1,t}^{-1}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}\mathcal{P}_{\$,t-1,t}^{-\ell_i-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell_i}\right]}\right)^{\ell_i^{-1}}, \tag{43}
$$

equation (24) implies that

$$
b_{\$,t-1}(j) = \xi_j\Delta_{j,t-1}\,\mathcal{P}_{j,t-1}\,,
$$

and hence we get from (42) that

$$
\tilde{\psi}_j(b_{t-1}) = \frac{\Delta_{j,t-1}}{((1-\zeta_\$)(\Delta_{\$,t-1})^{\ell_\$+1})^{\frac{\phi_\$}{1+(\ell_\$+1)\phi_\$}}}
$$

whereas, (30) takes the form

$$\mathcal{P}_{\$,t-1,t} \;=\; \left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\Delta_{\$,t-1}\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}. \tag{44}$$

Thus, substituting (44) into (43), we get

$$\Delta_{\$,t-1}^{\frac{1}{1+(\ell_\$+1)\phi_\$}} \;=\; \left(\bar{q}_i(\$)E_{t-1}\left[\bar{C}_t^{-(\eta-1)^{-1}}\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{-(1+(\ell_\$+1)\phi_\$)^{-1}}\right]\Big/\right.$$
$$E_{t-1}\Big[\bar{C}_t^{-(\eta-1)^{-1}}\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{-(\ell_\$+1)(1+(\ell_\$+1)\phi_\$)^{-1}}$$
$$\left.\times\,(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell_\$}\Big]\right)^{\ell_\$^{-1}},$$

which gives

$$\Delta_{\$,t-1}^{\frac{1}{1+(\ell_\$+1)\phi_\$}} \;=\; \left(\frac{\bar{q}_i(\$)(1-\zeta_\$)^{\frac{\phi\ell_\$}{1+(\ell_\$+1)\phi}}E_{t-1}\left[\bar{C}_t^{-\frac{1}{\eta-1}+\frac{(\ell_\$+1)\phi}{1+(\ell_\$+1)\phi}\hat{\eta}}e^{\frac{(\ell_\$+1)\phi}{1+(\ell_\$+1)\phi}(\eta-1)a_t}\right]}{E_{t-1}\left[\bar{C}_t^{-\frac{1}{\eta-1}+\frac{(\ell_\$+1)\phi-\ell_\$}{1+(\ell_\$+1)\phi}\hat{\eta}}e^{\frac{(\ell_\$+1)\phi-\ell_\$}{1+(\ell_\$+1)\phi}(\eta-1)a_t}\right]}\right)^{\ell_\$^{-1}}.$$

Thus,

$$\Delta_{i,t-1} \;=\; \Delta_{\$,t-1}^{\frac{(\ell_\$+1)\phi_\$}{1+(\ell_\$+1)\phi_\$}}\left(\bar{q}_i(\$)E_{t-1}\left[C_{t-1,t}^{-\gamma}\left(\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-1}\right]\right.$$
$$\left.\times\,E_{t-1}\left[C_{t-1,t}^{-\gamma}\left(\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-\ell_i-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell_i}\right]^{-1}\right)^{\ell_i^{-1}},$$

Define

$$\tilde{C}_t \ \equiv \ \bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_t} \,.$$

Then,

$$\begin{aligned}
& E_{t-1}\left[C_t^{-\gamma}\left(\left(\left(e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-1}\right] \\
&= \ E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$}]
\end{aligned}$$

where we have defined

$$\hat{\ell}_\$ \ \equiv \ -\frac{1}{1+(\ell_\$+1)\phi_\$} \,.$$

Similarly,

$$\begin{aligned}
& E_{t-1}\left[C_t^{-\gamma}\left(\left(\left(e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-\ell_i-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell_i}\right] \\
&= \ E_{t-1}\left[C_t^{-\gamma}\tilde{C}_t^{\frac{(\ell_\$+1)\phi_\$(\ell_i+1)-\ell_i(1+(\ell_\$+1)\phi_\$)}{1+(\ell_\$+1)\phi_\$}}\right] \\
&= \ E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$(\ell_i+1)}] \,.
\end{aligned}$$

Thus,

$$\begin{aligned}
& \tilde{\psi}_i(b_{t-1}) \\
&= \ \bar{q}_i(\$)^{\ell_i^{-1}}\left(\frac{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$}]}{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$(\ell_i+1)}]}\right)^{\ell_i^{-1}} \,.
\end{aligned}$$

When leverage is small, we have

$$\bar{C}_t \;\approx\; \bar{C}_t^* \;=\; K\, e^{(\eta-1)(1-\alpha)^{-1}a_t}$$

for some $K > 0$, so that

$$\tilde{C}_t \;=\; K^{\hat{\eta}} e^{(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]}$$

and

$$C_t^{\gamma} \;=\; k\bar{C}_t^{(\eta-1)^{-1}} \;=\; kK^{(\eta-1)^{-1}} e^{(1-\alpha)^{-1}a_t}\,,$$

and hence, under the log-normal assumption, we get

$$\frac{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$}]}{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$(\ell_i+1)}]} \;=\; \frac{E_{t-1}\big[e^{[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$)(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]a_t}\big]}{E_{t-1}\big[e^{[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$(\ell_i+1))(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]a_t}\big]}$$
$$=\; e^{-\hat{\ell}_\$\ell_i(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}$$
$$\times\, e^{0.5[[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$)(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]^2-[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$(\ell_i+1))(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]^2](\sigma_{t-1}^a)^2}$$
$$=\; e^{-\hat{\ell}_\$\ell_i(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\, e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2\ell_i(\ell_i+2)(\sigma_{t-1}^a)^2}$$

where we have defined

$$\tilde{\eta} \;\equiv\; (\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\,.$$

Thus, default probability is given by

$$\bar{q}_i(\$)\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\, e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell_i+2)(\sigma_{t-1}^a)^2}\tilde{C}_t^{\hat{\ell}_\$}\right)^{\ell_i}$$

while losses due to debt overhang are given by

$$\bar{q}_i(\$)^{\ell_i^{-1}(\ell_i+1)}(1-\zeta_j)\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu^a_{t-1}}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell_i+2)(\sigma^a_{t-1})^2}\tilde{C}_t^{\hat{\ell}_\$}\right)^{\ell_i+1}$$

Expected welfare is then

$$\begin{aligned}
&(1-\gamma)^{-1}E_{t-1}[C_t^{1-\gamma}] \;\approx\; (1-\gamma)^{-1}E_{t-1}[(C_t^*)^{1-\gamma}]\\
&-\;\sum_i \kappa_i E_{t-1}[(C_t^*)^{-\gamma}\bar{q}_i(\$)^{\ell_i^{-1}(\ell_i+1)}(1-\zeta_j)\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu^a_{t-1}}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell_i+2)(\sigma^a_{t-1})^2}\tilde{C}_t^{\hat{\ell}_\$}\right)^{\ell_i+1}]\\
&=\;(1-\gamma)^{-1}E_{t-1}[(C_t^*)^{1-\gamma}]\\
&-\;\sum_i \kappa_i E_{t-1}[e^{-(1-\alpha)^{-1}a_t}\,\bar{q}_i(\$)^{\ell_i^{-1}(\ell_i+1)}(1-\zeta_j)\\
&\times\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu^a_{t-1}}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell_i+2)(\sigma^a_{t-1})^2}e^{\hat{\ell}_\$\tilde{\eta}a_t}\right)^{\ell_i+1}]
\end{aligned}$$

for some constants κ_i. Thus, utility losses are given by

$$\begin{aligned}
&E_{t-1}[e^{-(1-\alpha)^{-1}a_t}\,\bar{q}_i(\$)^{\ell_i^{-1}(\ell_i+1)}(1-\zeta_j)\\
&\times\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu^a_{t-1}}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell_i+2)(\sigma^a_{t-1})^2}e^{\hat{\ell}_\$\tilde{\eta}a_t}\right)^{\ell_i+1}]\\
&\approx\; e^{-(1-\alpha)^{-1}\mu_{t-1}}(1-0.5\sigma_{t-1}^2((\ell_i+1)(\hat{\ell}_\$^2\tilde{\eta}^2+2\hat{\ell}_\$\tilde{\eta}(1-\alpha)^{-1})+(1-\alpha)^{-2}))\,,
\end{aligned}$$

Q.E.D.

Proof of Proposition 12. We need to compute

$$IRP_{i,t} \;=\; \frac{e^{r_{i,t}}\mathrm{Cov}_t(M_{i,t,t+1},\mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t+1}]}\,.$$

We have

$$\mathcal{P}_{i,t,t+1} \;=\; \kappa_t\left(\left(e^{-(\eta-1)a_{\$,t}}\bar{C}_t^{-\hat{\eta}}\right)^{(\ell_i+1)\phi_i}\,e^{\varepsilon_{\$,t+1}}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}$$

Now, when leverage is small, we have that consumption is close to the frictionless one,

$$\bar{C}_t \sim e^{a_t(\eta-1)(1-\alpha)^{-1}}$$

Since $\bar{C}_t \sim (C_t^{\gamma})^{\eta-1}$, we get that

$$C_t^{-\gamma} = \bar{C}_t^{-(\eta-1)^{-1}} \sim e^{-(1-\alpha)^{-1}a_t}.$$

whereas

$$e^{-(\eta-1)a_t}\bar{C}_t^{-\hat{\eta}} \sim e^{-(\eta-1)a_t}\bar{C}_t^{-(\frac{\gamma^{-1}}{\eta-1}+\alpha-1)} = e^{-(\eta-1)a_t}e^{-a_t(\eta-1)(1-\alpha)^{-1}(\frac{\gamma^{-1}}{\eta-1}+\alpha-1)} = e^{-a_t(1-\alpha)^{-1}\gamma^{-1}}.$$

Thus, ignoring the monetary shock, we get that

$$\mathcal{P}_{i,t,t+1} \sim \left(e^{-a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell_i+1)\phi_i(1+(\ell_\$+1)\phi_\$)^{-1}}.$$

At the same time,

$$M_{i,t,t+1} = e^{-\beta}C_{t,t+1}^{-\gamma}\mathcal{P}_{i,t,t+1}^{-1} \sim e^{-(1-\alpha)^{-1}a_{t+1}}\left(e^{a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell_i+1)\phi_i(1+(\ell_\$+1)\phi_\$)^{-1}}$$

Our goal is to prove that

$$IRP_{i,t}+1 = \frac{E_t[M_{i,t,t+1}\mathcal{P}_{i,t,t+1}]}{E_t[M_{i,t,t+1}]E_t[\mathcal{P}_{i,t,t+1}]}$$
$$= \frac{E_t[e^{-(1-\alpha)^{-1}a_{t+1}}]}{E_t[e^{-(1-\alpha)^{-1}a_{t+1}}\left(e^{a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell_i+1)\phi_i(1+(\ell_\$+1)\phi_\$)^{-1}}]E_t[\left(e^{-a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell_i+1)\phi_i(1+(\ell_\$+1)\phi_\$)^{-1}}]}$$

is monotone increasing in ϕ_i. Define

$$x \equiv \gamma^{-1}(\ell_i+1)\phi_i(1+(\ell_\$+1)\phi_\$)^{-1},\ \tilde{a}_{t+1}=(1-\alpha)^{-1}a_{t+1}\,.$$

Then, we can rewrite it as

$$IRP_t(x) \ + \ 1 \ = \ \frac{E_t[e^{-\tilde{a}_{t+1}}]}{E_t[e^{-\tilde{a}_{t+1}(1-x)}]E_t[e^{-\tilde{a}_{t+1}x}]}\,.$$

Thus,

$$\frac{\partial}{\partial x}\log(IRP_t(x) \ + \ 1) \ = \ \frac{E_t[e^{-\tilde{a}_{t+1}x}\tilde{a}_{t+1}]}{E_t[e^{-\tilde{a}_{t+1}x}]} \ - \ \frac{E_t[e^{-\tilde{a}_{t+1}(1-x)}\tilde{a}_{t+1}]}{E_t[e^{-\tilde{a}_{t+1}(1-x)}]}$$

Making a change of measure $d\tilde{P} \ = \ e^{-\tilde{a}_{t+1}x}/E_t[e^{-\tilde{a}_{t+1}x}]$, we can rewrite the required inequality as

$$\tilde{E}_t[\tilde{a}_{t+1}] \ > \ \frac{\tilde{E}_t[e^{-\tilde{a}_{t+1}(1-2x)}\tilde{a}_{t+1}]}{\tilde{E}_t[e^{-\tilde{a}_{t+1}(1-2x)}]}\,,$$

which is equivalent to $\tilde{C}ov_t(e^{-\tilde{a}_{t+1}(1-2x)},\tilde{a}_{t+1})<0$. Q.E.D.

D Proofs for Exorbitant Duty

Proof. Denote by

$$\tilde{\ell}_{\$,t}$$

the public estimate of the future $\hat{\ell}_{\$,t}$.

Following the proof of Proposition 11, we get that utility losses are given by

$$
\begin{aligned}
&E_{t-1}[e^{-(1-\alpha)^{-1}a_t}\,\bar{q}_i(\$)^{\ell_i^{-1}(\ell_i+1)}(1-\zeta_j) \\
&\times\left(e^{-\tilde{\ell}_{\$,t-1}(\eta-1)[\tilde{\eta}(1-\alpha)^{-1}+1]\mu^a_{t-1}}\,e^{-0.5\tilde{\ell}^2_{\$,t-1}\tilde{\eta}^2(\ell_i+2)(\sigma^a_{t-1})^2}e^{\hat{\ell}_{\$,t}\tilde{\eta}a_t}\right)^{\ell_i+1}] \\
&\approx\; e^{-(1-\alpha)^{-1}\mu_{t-1}}(1-0.5\sigma^2_{t-1}((\ell_i+1)(\tilde{\ell}^2_{\$,t-1}(\ell_i+2)-\hat{\ell}^2_{\$,t}(\ell_i+1))\tilde{\eta}^2+2\hat{\ell}_{\$,t}\tilde{\eta}(1-\alpha)^{-1})+(1-\alpha)^{-2}))\,,
\end{aligned}
$$

Thus, CB objective is to minimize

$$
E\left[\sum_{t=1}^{\infty}e^{-\beta t}e^{-(1-\alpha)^{-1}\mu_{t-1}}\sigma^2_{t-1}((K_1(i)\tilde{\ell}^2_{\$,t-1}-K_2(i)\hat{\ell}^2_{\$,t})\tilde{\eta}^2+2K_3(i)\hat{\ell}_{\$,t}\tilde{\eta}(1-\alpha)^{-1})\right]
$$

under the linear dynamics

$$
\tilde{\ell}_{\$,t}\;=\;q_0\tilde{\ell}_{\$,t-1}+q_1\hat{\ell}_{\$,t}\,.
$$

Assuming that conditional mean and volatility of consumption growth are constant, we get that Q.E.D.

E Exorbitant Duty

Suppose that US reacts to an average of the world gap:

$$
\mathcal{P}_{\$,t,t+1}\;=\;\prod_i\left(1-\bar{L}_{t+1}(i)/\bar{L}^*_{t+1}(i)\right)^{\chi_i}
$$

This gives the fixed point equation

$$
\mathcal{P}_{\$,t-1,t}\;=\;\prod_i\left((1-\zeta_\$)e^{-(\ell_i+1)(\eta-1)a_{i,t}}\left(((1-\tau_\$)c+1)\Delta_{i,t-1}\,\mathcal{P}^{-1}_{\$,t-1,t}\bar{C}_t^{-\hat{\eta}}\right)^{\ell_i+1}\right)^{\chi_i}
$$

Thus,

$$\mathcal{P}_{\$,t-1,t} \;=\; q_* \prod_i (\Delta_{i,t-1} \tilde{C}_{i,t}^{-1})^{(\ell_i+1)\chi_i/(1+\bar{\chi})}$$

where

$$\tilde{C}_{i,t} \;=\; \bar{C}^{\hat{\eta}} e^{a_{i,t}(\eta-1)}$$

and where we still have

$$\begin{aligned}
\Delta_{i,t-1} \;&=\; \left(\frac{\bar{q}_i(\$) E_{t-1}\left[C_{t-1,t}^{-\gamma} \mathcal{P}_{\$,t-1,t}^{-1} \right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma} \mathcal{P}_{\$,t-1,t}^{-\ell_i-1} (\bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_{i,t}})^{-\ell_i} \right]} \right)^{\ell_i^{-1}} \\
&=\; \left(\frac{\bar{q}_i(\$) E_{t-1}\left[C_{t-1,t}^{-\gamma} (\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})} \right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma} (\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell_i+1)/(1+\bar{\chi})} (\bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_{i,t}})^{-\ell_i} \right]} \right)^{\ell_i^{-1}} \quad q_* \prod_j (\Delta_{j,t-1})^{(\ell_j+1)\chi_j/(1+\bar{\chi})}
\end{aligned}$$

where we have defined

$$\bar{\chi} \;=\; \sum_i (1+\ell_i)\chi_i, \;\; \bar{a}_t \;=\; \sum_i (1+\ell_i)\chi_i a_{i,t}/\bar{\chi}\,.$$

This gives the fixed point system for leverage. Defining

$$e^{Q_i} \;\equiv\; \left(\frac{E_{t-1}\left[C_{t-1,t}^{-\gamma} (\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})} \right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma} (\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell_i+1)/(1+\bar{\chi})} (\bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_{i,t}})^{-\ell_i} \right]} \right)^{\ell_i^{-1}} \quad q_*(i),$$

we get

$$\log \Delta_{i,t-1} \;=\; Q_i \;+\; \sum_j (\ell_j+1)\chi_j (1+\bar{\chi})^{-1} \log \Delta_{j,t-1}\,.$$

Multiplying by

$$(\ell_j + 1)\chi_j$$

and summing up, we get

$$\sum_j (\ell_j + 1)\chi_j \log \Delta_{j,t-1} \;=\; \sum_j (\ell_j + 1)\chi_j Q_j \;+\; \bar{\chi} \sum_j (\ell_j + 1)\chi_j (1 + \bar{\chi})^{-1} \log \Delta_{j,t-1}$$

so that

$$\sum_j (\ell_j + 1)\chi_j (1 + \bar{\chi})^{-1} \log \Delta_{j,t-1} \;=\; \sum_j (\ell_j + 1)\chi_j Q_j \,.$$

Thus,

$$\log \Delta_{i,t-1} \;=\; Q_i \;+\; \sum_j (\ell_j + 1)\chi_j Q_j \,.$$

Now, in the small shock approximation, we have

$$\begin{aligned}
&\log \bar{C}_t^{1-\alpha} \;\approx\; \log \bar{\kappa} \;+\; \log \sum_j \kappa_i e^{a_{j,t}(\eta-1)} \\
&\approx\; \bar{\mu}_{t-1}(\eta - 1) \;+\; \log(1 + \sum_j \kappa_j ((a_{j,t} - \mu_{t-1}) + 0.5(a_{j,t} - \mu_{t-1})^2)) \\
&\approx\; (\eta - 1)\tilde{a}_t \;+\; 0.5(\eta - 1)^2 v_t \,,
\end{aligned}$$

so that

$$C_t^{-\gamma} \;=\; \bar{C}_t^{-(\eta-1)^{-1}} \;\approx\; e^{-(1-\alpha)^{-1}(\tilde{a}_t \;+\; 0.5(\eta-1)v_t)}$$

Define also

$$\tilde{a}_t \;=\; \sum_j \kappa_j a_{j,t},\;\; \tilde{v}_{t-1} \;=\; E_{t-1}[\sum_j \kappa_j (a_{j,t}-\mu_{t-1})^2 \;-\; \left(\sum_j \kappa_j (a_{j,t}-\mu_{t-1})\right)^2],$$

and note that since v_t is of the order ε^2, only its expectation will matter for the approximate calculations below. Here, the weights κ_j are normalized to add up to one. Thus,

$$\begin{aligned}
e^{Q_i \ell_i} &= q_*(i)^{\ell_i} \frac{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell_i+1)/(1+\bar{\chi})}(\bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_{i,t}})^{-\ell_i}\right]} \\
&= q_*(i)^{\ell_i}\; E_{t-1}\left[e^{-(1-\alpha)^{-1}(\tilde{a}_t \,+\, 0.5(\eta-1)\tilde{v}_{t-1})}(e^{(1-\alpha)^{-1}\hat{\eta}((\eta-1)\tilde{a}_t \,+\, 0.5(\eta-1)^2\tilde{v}_{t-1})} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})}\right] \\
&\times E_{t-1}\Bigg[e^{-(1-\alpha)^{-1}(\tilde{a}_t \,+\, 0.5(\eta-1)\tilde{v}_{t-1})}(e^{(1-\alpha)^{-1}\hat{\eta}((\eta-1)\tilde{a}_t \,+\, 0.5(\eta-1)^2\tilde{v}_{t-1})})^{(\bar{\chi}-\ell_i)/(1+\bar{\chi})} \\
&\times (e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell_i+1)/(1+\bar{\chi})}(e^{(\eta-1)a_{i,t}})^{-\ell_i}\Bigg]^{-1} \\
&= q_*(i)^{\ell_i} e^{(1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell_i(1+\bar{\chi})^{-1}\tilde{v}_{t-1}} \frac{E_{t-1}[e^{\Gamma_1\tilde{a}_t+\Gamma_2\bar{a}_t}]}{E_{t-1}[e^{\Gamma_3(i)\tilde{a}_t+\Gamma_4(i)\bar{a}_t-(\eta-1)\ell_i a_{i,t}}]} \\
&= q_*(i)^{\ell_i} e^{\hat{\mu}_{i,t-1}} e^{(1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell_i(1+\bar{\chi})^{-1}\tilde{v}_{t-1}} \\
&\times \exp\Bigg(0.5(\Gamma_1^2-\Gamma_3^2)\tilde{\sigma}_{t-1}^2 + 0.5(\Gamma_2^2-\Gamma_4^2(i))\bar{\sigma}_{t-1}^2 - 0.5(\eta-1)^2\ell_i^2\sigma_{i,t-1}^2 \\
&+ \sigma_{t-1}(\bar{a},\tilde{a})(\Gamma_1\Gamma_2-\Gamma_3(i)\Gamma_4(i)) + \sigma_{t-1}(\bar{a},a_i)(\eta-1)\ell_i\Gamma_4(i) + \sigma_{t-1}(\tilde{a},a_i)(\eta-1)\ell_i\Gamma_3(i)\Bigg)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_1 &= -(1-\alpha)^{-1} + (1-\alpha)^{-1}\hat{\eta}(\eta-1)\bar{\chi}/(1+\bar{\chi}) \\
\Gamma_2 &= (\eta-1)\bar{\chi}/(1+\bar{\chi}) \\
\Gamma_3(i) &= -(1-\alpha)^{-1} + (1-\alpha)^{-1}\hat{\eta}(\eta-1)(\bar{\chi}-\ell_i)/(1+\bar{\chi}) \\
\Gamma_4(i) &= (\eta-1)(\ell_i+1)\bar{\chi}/(1+\bar{\chi})\,.
\end{aligned}$$

Furthermore,

$$\bar{Q} \;=\; \sum_i (\ell_i+1)\chi_i Q_i$$

and

$$\mathcal{P}_{\$,t-1,t} \;=\; q_* \tilde{C}_t^{-\bar{\chi}/(1+\bar{\chi})} e^{\bar{Q}}$$

We have

$$\mathcal{B}_{j,t}(B_{t-1}) \;=\; ((1-\tau_j)c+1) b_{\$,t-1}(j) \mathcal{P}_{\$,t-1,t}^{-1} \mathcal{P}_{j,t-1,t}$$

with

$$b_{\$,t-1}(j) \;=\; \xi_j \Delta_{j,t-1} \mathcal{P}_{j,t-1}$$

Thus,

$$\Psi_{j,t} \;=\; \frac{\mathcal{B}_{j,t}\mathcal{P}_{j,t}^{-1}}{\xi_j e^{a_{j,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}} \;=\; \frac{\Delta_{j,t-1}\mathcal{P}_{\$,t-1,t}^{-1}}{e^{a_{j,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}} \;=\; \frac{e^{Q_i}\tilde{C}_t^{\bar{\chi}/(1+\bar{\chi})}}{\tilde{C}_t} \;=\; e^{Q_i} \tilde{C}_t^{-(1+\bar{\chi})^{-1}}\,.$$

Expected welfare is then

$$(1-\gamma)^{-1} E_{t-1}[C_t^{1-\gamma}] \;\approx\; (1-\gamma)^{-1} E_{t-1}[(C_t^*)^{1-\gamma}] \;-\; \sum_i \kappa_i E_{t-1}[(C_t^*)^{-\gamma}(1-\zeta_j)\,(\Psi_{j,t})^{\ell_i+1}]\,.$$

Thus, utility losses from country i debt overhang are given by

$$E_{t-1}[(C_t^*)^{-\gamma}(1-\zeta_j)\,(\Psi_{i,t})^{\ell_i+1}] \;=\; (1-\zeta_j) e^{Q_i(\ell_i+1)} E_{t-1}[(C_t)^{-\gamma} \tilde{C}_t^{-(1+\bar{\chi})^{-1}(\ell_i+1)}]$$

We have

$$
\begin{aligned}
&C_t^{-\gamma}\tilde{C}_t^{-(1+\bar{\chi})^{-1}(\ell_i+1)} \;\approx\; \big(e^{-(1-\alpha)^{-1}(\tilde{a}_t\;+\;0.5(\eta-1)\tilde{v}_{t-1})}\big)\big(\bar{C}^{\hat{\eta}}e^{a_{i,t}(\eta-1)}\big)^{-(1+\bar{\chi})^{-1}(\ell_i+1)}\\
&=\;\big(e^{-(1-\alpha)^{-1}(\tilde{a}_t\;+\;0.5(\eta-1)\tilde{v}_{t-1})}\big)\big(e^{(1-\alpha)^{-1}\hat{\eta}((\eta-1)\tilde{a}_t\;+\;0.5(\eta-1)^2\tilde{v}_{t-1})}e^{a_{i,t}(\eta-1)}\big)^{-(1+\bar{\chi})^{-1}(\ell_i+1)}\\
&=\;e^{-\tilde{a}_t(1-\alpha)^{-1}(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell_i+1))-0.5\tilde{v}_t(1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell_i+1))}\,e^{-a_{i,t}(\eta-1)(1+\bar{\chi})^{-1}(\ell_i+1)}\\
&=\;e^{-K_1(i)\tilde{a}_t-K_2(i)v_t-K_3(i)a_{i,t}}\,.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&E_{t-1}\big[(C_t)^{-\gamma}\tilde{C}_t^{-(1+\bar{\chi})^{-1}(\ell_i+1)}\big]\\
&\approx\;E_{t-1}\big[e^{-K_1(i)\tilde{a}_t-K_2(i)v_t-K_3(i)a_{i,t}}\big]\\
&\approx\;e^{-K_1(i)\tilde{\mu}_{t-1}-K_2(i)\tilde{v}_{t-1}-K_3(i)\mu_{t-1}+0.5(K_1(i)^2\tilde{\sigma}_{t-1}^2+K_3(i)^2\sigma_{i,t-1}^2+2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))}
\end{aligned}
$$

and hence the welfare loss is proportional to

$$
\begin{aligned}
&\sum_i(1-\zeta_i)e^{Q_i(\ell_i+1)}e^{-K_1(i)\tilde{\mu}_{t-1}-K_2(i)\tilde{v}_{t-1}-K_3(i)\mu_{t-1}+0.5(K_1(i)^2\tilde{\sigma}_{t-1}^2+K_3(i)^2\sigma_{i,t-1}^2+2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))}\\
&=\;\sum_i(1-\zeta_i)q_*(i)^{\ell_i+1}e^{\hat{\mu}_{i,t-1}(1+\ell_i^{-1})}e^{(1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell_i(1+\bar{\chi})^{-1}\tilde{v}_{t-1}(1+\ell_i^{-1})}\\
&\times\exp\Bigg(\bigg(0.5(\Gamma_1^2-\Gamma_3^2)\tilde{\sigma}_{t-1}^2+0.5(\Gamma_2^2-\Gamma_4^2(i))\bar{\sigma}_{t-1}^2-0.5(\eta-1)^2\ell_i^2\sigma_{i,t-1}^2\\
&+\sigma_{t-1}(\bar{a},\tilde{a})(\Gamma_1\Gamma_2-\Gamma_3(i)\Gamma_4(i))+\sigma_{t-1}(\bar{a},a_i)(\eta-1)\ell_i\Gamma_4(i)+\sigma_{t-1}(\tilde{a},a_i)(\eta-1)\ell_i\Gamma_3(i)\bigg)(1+\ell_i^{-1})\Bigg)\\
&\times e^{-K_1(i)\tilde{\mu}_{t-1}-K_2(i)\tilde{v}_{t-1}-K_3(i)\mu_{t-1}+0.5(K_1(i)^2\tilde{\sigma}_{t-1}^2+K_3(i)^2\sigma_{i,t-1}^2+2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))}
\end{aligned}
$$

Note that

$$
\tilde{v}_{t-1}\;=\;\sum_j\kappa_j\sigma_{j,t-1}^2\;-\;\tilde{\sigma}_{t-1}^2\,.
$$

Denoting

$$\tilde{q}_i \;\equiv\; \kappa_i(1-\zeta_i)q_*(i)^{\ell_i+1}\,,$$

we get that the volatility part of the welfare loss is (up to an additive constant) approximately given by

$$\sum_i \tilde{q}_i\Bigg((1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell_i(1+\bar{\chi})^{-1}(\sum_j \kappa_j\sigma^2_{j,t-1} \;-\; \tilde{\sigma}^2_{t-1})(1+\ell_i^{-1})$$
$$+\;\Bigg(0.5(\Gamma_1^2-\Gamma_3^2)\tilde{\sigma}^2_{t-1}+0.5(\Gamma_2^2-\Gamma_4^2(i))\bar{\sigma}^2_{t-1}-0.5(\eta-1)^2\ell_i^2\sigma^2_{i,t-1}$$
$$+\,\sigma_{t-1}(\bar{a},\tilde{a})(\Gamma_1\Gamma_2-\Gamma_3(i)\Gamma_4(i))+\sigma_{t-1}(\bar{a},a_i)(\eta-1)\ell_i\Gamma_4(i)+\sigma_{t-1}(\tilde{a},a_i)(\eta-1)\ell_i\Gamma_3(i)\Bigg)(1+\ell_i^{-1})$$
$$-\,0.5(\sum_j \kappa_j\sigma^2_{j,t-1} \;-\; \tilde{\sigma}^2_{t-1})(1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell_i+1))$$
$$+\;0.5(K_1(i)^2\tilde{\sigma}^2_{t-1}+K_3(i)^2\sigma^2_{i,t-1}+2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))\Bigg)$$
$$=\;\sum_j \Xi_j\sigma^2_{j,t-1} \;+\; \bar{\Xi}\bar{\sigma}^2_{t-1} \;+\; \tilde{\Xi}\tilde{\sigma}^2_{t-1} \;+\; \hat{\Xi}\tilde{\sigma}_{t-1}(\tilde{a}_t,\bar{a}_t) \;+\; \sigma_{t-1}(\bar{a}_t,\hat{a}_t) \;+\; \sigma_{t-1}(\tilde{a}_t,\;\check{a}_t)$$

Here,

$$
\begin{aligned}
\Xi_j &= -\tilde{q}_j 0.5(\eta-1)^2\ell_j^2(1+\ell_j^{-1}) + 0.5\tilde{q}_j K_3(j)^2 \\
&+ \kappa_j \sum_i \tilde{q}_i \left((1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell_i(1+\bar{\chi})^{-1}(1+\ell_i^{-1}) - (1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell_i+1))\right) \\
\bar{\Xi} &= \sum_i \tilde{q}_i 0.5(\Gamma_2^2 - \Gamma_4^2(i))(1+\ell_i^{-1}) \\
\tilde{\Xi} &= \Big(-\sum_i \tilde{q}_i (1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell_i(1+\bar{\chi})^{-1}(1+\ell_i^{-1}) \\
&+ \sum_i \tilde{q}_i 0.5(\Gamma_1^2 - \Gamma_3^2 + K_1(i)^2)(1+\ell_i^{-1}) \\
&+ \sum_i \tilde{q}_i 0.5(1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell_i+1))\Big) \\
\hat{\Xi} &= \sum_i \tilde{q}_i(\Gamma_1\Gamma_2 - \Gamma_3(i)\Gamma_4(i))(1+\ell_i^{-1}) \\
\hat{a}_t &= \sum_i \tilde{q}_i(\eta-1)\ell_i\Gamma_4(i)(1+\ell_i^{-1})a_{i,t} \\
\check{a}_t &= \sum_i \tilde{q}_i[(\eta-1)\ell_i\Gamma_3(i)(1+\ell_i^{-1}) + K_1(i)K_3(i)]a_{i,t}\,.
\end{aligned}
$$

Out interest is in the dependence on the coefficients defining $\bar{a}_t$. This is the only place the exorbitant duty coefficients enter the welfare. This part of welfare can be rewritten as

$$
\bar{\Xi}\bar{\sigma}_{t-1}^2 \;+\; \sigma_{t-1}(\bar{a}_t, \hat{\Xi}\tilde{a}_t + \hat{a}_t)\,.
$$

Here,

$$
\begin{aligned}
\hat{\Xi}\tilde{a}_t + \hat{a}_t &= \sum_i \left(\hat{\Xi}\kappa_i + \tilde{q}_i(\eta-1)\ell_i\Gamma_4(i)(1+\ell_i^{-1})\right) a_{j,t} \\
&= \sum_i \left(\hat{\Xi}\kappa_i + \tilde{q}_i(\eta-1)\ell_i(\eta-1)(\ell_i+1)\bar{\chi}/(1+\bar{\chi})(1+\ell_i^{-1})\right) a_{j,t}
\end{aligned}
$$

with

$$
\begin{aligned}
\hat{\Xi} &= \sum_i \tilde{q}_i(\Gamma_1\Gamma_2 - \Gamma_3(i)\Gamma_4(i))(1+\ell_i^{-1}) \\
&= (\eta-1)\bar{\chi}/(1+\bar{\chi})(1-\alpha)^{-1}\sum_i \tilde{q}_i\Bigg((-1+\hat{\eta}(\eta-1)) \\
&- (-1+\hat{\eta}(\eta-1)(\bar{\chi}-\ell_i)/(1+\bar{\chi}))(\ell_i+1)\Bigg)(1+\ell_i^{-1}) \\
&= (\eta-1)\bar{\chi}/(1+\bar{\chi})(1-\alpha)^{-1}\sum_i \tilde{q}_i\left(\ell_i+\hat{\eta}(\eta-1)\frac{1-\ell_i(\bar{\chi}-(\ell_i+1))}{1+\bar{\chi}}\right)(1+\ell_i^{-1})
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{\Xi} &= \sum_i \tilde{q}_i 0.5(\Gamma_2^2 - \Gamma_4^2(i))(1+\ell_i^{-1}) \\
&= -((\eta-1)\bar{\chi}/(1+\bar{\chi}))^2\sum_i \tilde{q}_i 0.5\ell_i(\ell_i+2)
\end{aligned}
$$

Our result follows then from the following general lemma.

Lemma 17 *Consider the minimization problem*

$$
\min_A\{\mathrm{Cov}_{t-1}(\sum_i A_i a_{i,t}, \sum_i \Psi_i a_{i,t}) - 0.5\mathrm{Var}_{t-1}(\sum_i A_i a_{i,t})
$$

over the unit simplex

$$
A_i \geq 0,\ \sum_i A_i = 1\,.
$$

then, the solution to this problem is given by the following: there exists a threshold Ψ_* such that $A_i = 0$ if and only if $\Psi_i/\sigma_i\ >\ \Psi_*$.

a

Proof. We have

$$\mathrm{Cov}_{t-1}\Big(\sum_i A_i a_{i,t}, \sum_i \Psi_i a_{i,t}\Big) \;=\; \sum_i A_i\big(\Psi_i\sigma_i^2+\rho\sigma_i\sum_{j\neq i}\sigma_j\Psi_j\big) \;=\; \sum_i A_i\sigma_i(\Psi_i\sigma_i(1-\rho)+\rho\bar{\Psi})$$

with

$$\bar{\Psi} = \sum_j \sigma_j\Psi_j\,.$$

Thus, the first order Kuhn-Tucker condition takes the form

$$\Psi_i\sigma_i(1-\rho)+\rho\bar{\Psi} \;-\sum_{j\neq i}\sigma_i\sigma_j\rho A_j \;-\; \lambda \;\geq\; 0$$

when the constraint $A_i \geq 0$ binds, and and

$$\Psi_i\sigma_i(1-\rho)+\rho\bar{\Psi} \;-\; \sigma_i^2 A_i - \sum_{j\neq i}\sigma_i\sigma_j\rho A_j \;-\; \lambda \;=\; 0$$

when $A_i > 0$. Here, λ is the Lagrange multiplier for the constraint $\sum_i A_i = 1$. That is, $A_i = 0$ for all countries for which

$$\Psi_i\sigma_i(1-\rho)+\rho\bar{\Psi} \;-\; \sigma_i\sum_j\sigma_j\rho A_j \;-\; \lambda \;\geq\; 0\,,$$

while the interior solution is for

$$\Psi_i\sigma_i(1-\rho)+\rho\bar{\Psi} \;-\; \sigma_i^2(1-\rho)A_i - \sigma_i\sum_j\sigma_j\rho A_j \;-\; \lambda \;=\; 0$$

This gives

$$A_i \;=\; \frac{\Psi_i\sigma_i(1-\rho)+\rho\bar{\Psi}-\sigma_i\bar{A}-\lambda}{\sigma_i^2(1-\rho)}$$

and hence

$$\bar{A} \;=\; \sum_{j\in\mathcal{J}} \sigma_j A_j \;=\; \sum_j$$

Q.E.D.

F Additional Results

G Mixture of LC and \$

We first state the following extension of the Theorem 8 for the case of firms borrowing both in local currency and in dollars.

Theorem 18 *Suppose that $q_i(i) = q_i(\$)$. Then, issuing in a mixture of local currency and dollars is optimal if and only if*

$$\frac{\bar{q}_i(j,\$)}{\bar{q}_i(\$)} - 1 \;\leq\; \frac{\mathrm{Cov}_t^{\$}\left(\left(\frac{\Omega_{i,t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell_i}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[\left(\frac{\Omega_{i,t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell_i}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]} \tag{45}$$

for all $j = 1, \cdots, N$.

Proof of Theorem 18. The standard Kuhn-Tucker conditions that borrowing only in LC and dollars is optimal if and only if

$$
\begin{aligned}
& E_t\left[M_{i,t,t+1}\left[\left(1-(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right](1-q_i(j)) \\
& + E_t\left[M_{i,t,t+1}\left[\left(-\ell_i(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i-1}\Omega_{i,t+1}^{-1}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\mathcal{B}_{t+1}(B_t)\right](1-q_i(\$)) \\
& - (1+c(1-\tau_i))E_t\left[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}\right] \\
& + E_t\Bigg[M_{i,t,t+1}(\ell_i+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}(1+c(1-\tau_i))\mathcal{E}_{j,i,t+1} \\
& - \ell_i\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}(1+c(1-\tau_i))\mathcal{E}_{j,i,t+1}\Bigg] \leq 0
\end{aligned}
$$

for all j with the identity for $j = i, \$$. This inequality can be rewritten as

$$
\bar{q}_i(j,\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\mathcal{E}_{j,i,t+1}\right]} \;\leq\; 1 \;=\; \bar{q}_i(\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell_i}\mathcal{E}_{\$,i,t+1}\right]}
$$

and the first claim follows.

For the second claim, we get the system

$$
\begin{aligned}
1 &= \bar{q}_i(\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)\mathcal{E}_{\$,i,t+1}\right]} \\
1 &= \bar{q}_i(\$)\frac{E_t[M_{i,t,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)\right]}
\end{aligned}
$$

whereby

$$
\mathcal{B}_{t+1}(B_t) \;=\; (1+c(1-\tau_i))\left(B_t(i) \;+\; B_t(\$)\mathcal{E}_{\$,i,t+1}\right)
$$

Thus, we get the system

$$
\begin{aligned}
E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}]B_t(i) \ + \ E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(\$) \ &= \ \tilde{q}_i(\$)E_t[M_{i,t,t+1}] \\
E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(i) \ + \ E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]B_t(\$) \ &= \ \tilde{q}_i(\$)E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]
\end{aligned}
$$

where we have defined

$$
\tilde{q}_i(\$) = \bar{q}_i(\$)/(1+c(1-\tau_i))\,.
$$

Thus,

$$
\begin{pmatrix} B_t(i) \\ B_t(\$) \end{pmatrix} \ = \ \tilde{q}_i(\$)\Delta_t^{-1}\begin{pmatrix} E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2] & -E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}] \\ -E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}] & E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}] \end{pmatrix}\begin{pmatrix} E_t[M_{i,t,t+1}] \\ E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}] \end{pmatrix}.
$$

where

$$
\Delta_t \ = \ E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}] - (E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}])^2
$$

Thus,

$$
\frac{B_t(i)}{B_t(\$)\mathcal{E}_{t,\$,i}} \ = \ \frac{-\mathrm{Cov}_t^{\$}(\Omega_{i,t+1}^{-1}\mathcal{E}_{t,t,+1,\$,i}, \ \mathcal{E}_{t,t,+1,\$,i}^{-1})}{\mathrm{Cov}_t^{\$}(\Omega_{i,t+1}^{-1}, \ \mathcal{E}_{t,t,+1,\$,i}^{-1})}\,.
$$

Substituting from (26), we get

$$
\frac{B_t(i)}{B_t(\$)\mathcal{E}_{t,\$,i}} \ = \ \frac{-\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}{\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}\,.
$$

In the small variance approximation, we that's get

$$
\frac{B_t(i)}{B_t(\$)\mathcal{E}_{t,\$,i}} \ \approx \ \frac{\sigma_{\varepsilon,\$}^2 - \sigma_{\varepsilon,i,\$} + \alpha_i\sigma_c^2(i) + \alpha_{\$}^2\sigma_c^2(\$) - (\alpha_{\$} + \alpha_i\alpha_{\$})\sigma_c(i,\$)}{\sigma_{\varepsilon,i}^2 - \sigma_{\varepsilon,i,\$} + (1-\alpha_i)(\alpha_{\$}\sigma_c(i,\$) - \alpha_i\sigma_c^2(i))}
$$

where $\sigma_c(i)^2 = \mathrm{Var}_t[\log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}})]$ and $\sigma_c(i,\$) = \mathrm{Cov}_t[\log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}}), \log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{\$,t+1}})]$.
Q.E.D.

G.1 Investment and Debt Overhang

In this section, we assume that the firm can pay a cost of

$$h_i(1+\beta^{-1})^{-1} k_{i,t}^{\beta^{-1}+1},$$

to increase the lower bound of the support of the idiosyncratic shock distribution. Namely, upon having selected $k_{i,t}$, the firm gets $Z_{i,t}$ drawn from the distribution with the density $\phi_i(z) \;=\; (1+k_{i,t})\ell_i z^{\ell_i-1}$ on $[q_{i,t}, 1]$ with $q_{i,t} = (1-(1+k_{i,t})^{-1})^{\ell_i^{-1}}$.

The firm is then solving

$$\begin{aligned} &-h_i\Omega_{i,t}(\beta+1)^{-1}k_i^{\beta+1} + k_i\Omega_{i,t}\int_{\Psi_{i,t}}^1 z\phi_i(z)dz \\ &= \; -h_i\Omega_{i,t}(\beta^{-1}+1)^{-1}k_i^{\beta^{-1}+1} + (k_i+1)\Omega_{i,t}\ell_i(\ell_i+1)^{-1}(1-\Psi_{i,t}^{\ell_i+1}). \end{aligned}$$

Solving the optimization problem gives

$$k_{i,t} = \; \left(h_i^{-1}\ell_i(\ell_i+1)^{-1}(1-\Psi_{i,t}^{\ell_i+1})\right)^{\beta}.$$

In particular, absent debt overhang,

$$k_{i,t} \;=\; k_i^* \;=\; \left(h_i^{-1}\ell_i(\ell_i+1)^{-1}\right)^{\beta}.$$

Note that here $1+k_i = (1-q_i^{\ell_i})^{-1}$.

Then, redefining

$$G_i(\Psi_{i,t}) = \ell_i(\ell_i+1)^{-1}\left(1+\left(h_i^{-1}\ell_i(\ell_i+1)^{-1}(1-\Psi_{i,t}^{\ell_i+1})\right)^{\beta}\right)$$
$$\times\left((\zeta_i-1)\Psi_{i,t}^{\ell_i+1}+1-\zeta_i\left(\left(1-\left(1+\left(h_i^{-1}\ell_i(\ell_i+1)^{-1}(1-\Psi_{i,t}^{\ell_i+1})\right)^{\beta}\right)^{-1}\right)\right)^{\ell_i^{-1}+1}\right),$$

we get the same equilibrium equation.

G.2 Long-lived Firms With Long-Term Debt

Suppose that firms live for two periods and produce only in period $t+2$. We assume that firms can only issue two-period debt when they are born at time t. The time-$t+1$ continuation is then given by

$$V_{t+1} = E_{t+1}[M_{i,t+1,t+2}\max\{\Pi_{t+2}(i,\omega) - \mathcal{B}_{t+2}(B_t),0\}]$$
$$= E_{t+1}\left[M_{i,t+1,t+2}\left[-\mathcal{B}_{t+2}(B_t)\left(1-\left(\frac{\mathcal{B}_{t+2}(B_t)}{\Omega_{i,t+2}}\right)^{\ell_i}\right)\right.\right.$$
$$\left.\left.+\,\Omega_{i,t+2}\ell_i(\ell_i+1)^{-1}\left(1-\left(\frac{\mathcal{B}_{t+2}(B_t)}{\Omega_{i,t+2}}\right)^{\ell_i+1}\right)\right]\right]$$
$$= E_{t+1}[M_{i,t+1,t+2}\Omega_{i,t+2}\ell_i(\ell_i+1)^{-1}] - E_{t+1}[M_{i,t+1,t+2}\mathcal{B}_{t+2}(B_t)]$$
$$+ (\ell_i+1)^{-1}E_{t+1}\left[M_{i,t+1,t+2}\frac{(\mathcal{B}_{t+2}(B_t))^{\ell_i+1}}{(\Omega_{i,t+2})^{\ell_i}}\right]$$

Thus, the time-t firm value is given by

$$V_t = E_t[M_{i,t,t+1}\max\{V_{t+1} - \mathcal{B}_{t+1}(B_t)-A_{t+1}Z_{t+1}^{-1},0\}]$$
$$= E_t[M_{i,t,t+1}(V_{t+1} - \mathcal{B}_{t+1}(B_t) - A_{t+1}Z_{i,t+1}^{-1})\mathbf{1}_{A_{t+1}Z_{t+1}^{-1}<V_{t+1} - \mathcal{B}_{t+1}(B_t)}]$$
$$= E_t\left[M_{i,t,t+1}(V_{t+1}-\mathcal{B}_{t+1}(B_t))\left(1-\left(\frac{A_{t+1}}{V_{t+1} - \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\right]$$
$$- E_t\left[M_{i,t,t+1}A_{t+1}\,\ell_i(\ell_i-1)^{-1}\left(1-\left(\frac{A_{t+1}}{V_{t+1} - \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i-1}\right)\right]$$

For simplicity, we assume that $\rho_i = 0$. Then, the total issued debt price is given by

$$\begin{aligned}
\Delta \;&=\; E_t\Bigg[\mathbf{1}_{Z_{t+1}>F_{t+1}}\Bigg[\Bigg\{M_{i,t,t+1}c_1\sum_j \mathcal{E}_{j,i,t+1}B_{j,t} \\
&+\; M_{i,t,t+2}\left(1-\left(\frac{\mathcal{B}_{t+2}(B_t)}{\Omega_{i,t+2}}\right)^{\ell_i}\right)(1+c_2)\sum_j \mathcal{E}_{j,i,t+2}B_{j,t}\Bigg\}\Bigg]\Bigg] \\
&=\; E_t[M_{i,t,t+1}(1-\tau)^{-1}\mathcal{B}_{t+1}(B_t)\left(1-\left(\frac{A_{t+1}}{V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&+\; E_t[M_{i,t,t+2}\frac{(1+c_2)}{1+c_2(1-\tau)}\mathcal{B}_{t+2}(B_t)\left(1-\left(\frac{A_{t+1}}{V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\left(1-\left(\frac{\mathcal{B}_{t+2}(B_t)}{\Omega_{i,t+2}}\right)^{\ell_i}\right)]
\end{aligned}$$

Differentiating, we get

$$\begin{aligned}
&\frac{\partial}{\partial B_{j,t}}\Delta \\
&=\; E_t[M_{i,t,t+1}c_1\mathcal{E}_{j,i,t+1}\left(1-\left(\frac{A_{t+1}}{V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&+\; E_t[M_{i,t,t+2}(1+c_2)\mathcal{E}_{j,i,t+2}\left(1-\left(\frac{A_{t+1}}{V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\left(1-\left(\frac{\mathcal{B}_{t+2}(B_t)}{\Omega_{i,t+2}}\right)^{\ell_i}\right)] \\
&-\; E_t[M_{i,t,t+2}\frac{(1+c_2)}{1+c_2(1-\tau)}\mathcal{B}_{t+2}(B_t)\left(1-\left(\frac{A_{t+1}}{V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\ell_i(\mathcal{B}_{t+2}(B_t))^{\ell_i-1}(\Omega_{i,t+2})^{-\ell_i} \\
&\times(1+c_2(1-\tau))\mathcal{E}_{j,i,t+2}] \\
&+\; E_t[M_{i,t,t+2}\frac{(1+c_2)}{1+c_2(1-\tau)}\mathcal{B}_{t+2}(B_t)\ell_i A_{t+1}^{\ell_i}(V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)}(\frac{\partial}{\partial B_{j,t}}V_{t+1}-c_1(1-\tau)\mathcal{E}_{j,i,t+1}) \\
&\times\left(1-\left(\frac{\mathcal{B}_{t+2}(B_t)}{\Omega_{i,t+2}}\right)^{\ell_i}\right)] \\
&+\; \ell_i E_t[M_{i,t,t+1}(1-\tau)^{-1}\mathcal{B}_{t+1}(B_t)A_{t+1}^{\ell_i}(V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)}(\frac{\partial}{\partial B_{j,t}}V_{t+1}-c_1(1-\tau)\mathcal{E}_{j,i,t+1})]\,.
\end{aligned}$$

We are assuming that $\ell_i > 2$. Now, assuming that the cost of issuance is sufficiently high, so that leverage is sufficiently small, we will keep the first order approximation in $B_{j,t}$ and hence we will drop all terms involving B_t^2 :

$$
\begin{aligned}
&\frac{\partial}{\partial B_{j,t}}\Delta \\
&= \; E_t[M_{i,t,t+1}c_1\mathcal{E}_{j,i,t+1} \left(1-\left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&+ \; E_t[M_{i,t,t+2}(1+c_2)\mathcal{E}_{j,i,t+2} \left(1-\left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&+ \; E_t[M_{i,t,t+2}\frac{(1+c_2)}{1+c_2(1-\tau)}\mathcal{B}_{t+2}(B_t)\ell_i A_{t+1}^{\ell_i}(V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)}(\frac{\partial}{\partial B_{j,t}}V_{t+1}-c_1(1-\tau)\mathcal{E}_{j,i,t+1})] \\
&+ \; \ell_i E_t[M_{i,t,t+1}(1-\tau)^{-1}\mathcal{B}_{t+1}(B_t)A_{t+1}^{\ell_i}(V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)}(\frac{\partial}{\partial B_{j,t}}V_{t+1}-c_1(1-\tau)\mathcal{E}_{j,i,t+1})] \; + \; O(B_t^2)
\end{aligned}
$$

where

$$
V_{t+1} \; \approx \; E_{t+1}[M_{i,t+1,t+2}(\Omega_{i,t+2}\ell_i(\ell_i+1)^{-1}-\mathcal{B}_{t+2}(B_t))]
$$

At the same time,

$$
\begin{aligned}
\frac{\partial}{\partial B_{j,t}}\mathcal{B}_{t+2}(B_t) \; &= \; (1+c_2(1-\tau))\mathcal{E}_{j,i,t+2} \\
\frac{\partial}{\partial B_{j,t}}V_{t+1} \; &= \; -E_{t+1}[M_{i,t+1,t+2}\left(1-\frac{(\mathcal{B}_{t+2}(B_t))^{\ell_i}}{(\Omega_{i,t+2})^{\ell_i}}\right)(1+c_2(1-\tau))\mathcal{E}_{j,i,t+2}] \\
\approx \; & -E_{t+1}[M_{i,t+1,t+2}(1+c_2(1-\tau))\mathcal{E}_{j,i,t+2}]
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\partial}{\partial B_{j,t}} V_t \\
&= \; E_t[M_{i,t,t+1}(\frac{\partial}{\partial B_{j,t}} V_{t+1} \; - \; \frac{\partial}{\partial B_{j,t}} \mathcal{B}_{t+1}(B_t)) \mathbf{1}_{A_{t+1} Z_{t+1}^{-1} < V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}] \\
&= \; E_t[M_{i,t,t+1}(\frac{\partial}{\partial B_{j,t}} V_{t+1} \; - \; \frac{\partial}{\partial B_{j,t}} \mathcal{B}_{t+1}(B_t)) \left(1 - \left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&= \; E_t\Bigg[M_{i,t,t+1}\Bigg(- M_{i,t+1,t+2}(1 + c_2(1-\tau))\mathcal{E}_{j,i,t+2} \; + \; M_{i,t+1,t+2} \frac{(\mathcal{B}_{t+2}(B_t))^{\ell_i}}{(\Omega_{i,t+2})^{\ell_i}} (1 + c_2(1-\tau))\mathcal{E}_{j,i,t+2} \\
&- \; c_1(1-\tau)\mathcal{E}_{j,i,t+1}\Bigg) \left(1 - \left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\Bigg] \\
&= \; E_t\Bigg[M_{i,t,t+1}\Bigg(- M_{i,t+1,t+2}(1 + c_2(1-\tau))\mathcal{E}_{j,i,t+2} \; - \; c_1(1-\tau)\mathcal{E}_{j,i,t+1}\Bigg) \left(1 - \left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\Bigg] \\
&+ \; O(B_t^2)\,.
\end{aligned}
$$

Thus, Kuhn-Tucker conditions take the form

$$
\begin{aligned}
&0 \; \geq \; (1 - q_i(j)) \frac{\partial}{\partial B_{j,t}} \Delta \; + \; \frac{\partial}{\partial B_{j,t}} V_t \\
&\approx \; (1 - q_i(j)) \Bigg(E_t[M_{i,t,t+1} c_1 \mathcal{E}_{j,i,t+1} \; \left(1 - \left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&+ \; E_t[M_{i,t,t+2}(1 + c_2)\mathcal{E}_{j,i,t+2} \left(1 - \left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)] \\
&+ \; E_t[M_{i,t,t+2} \frac{(1 + c_2)}{1 + c_2(1-\tau)} \mathcal{B}_{t+2}(B_t) \ell_i A_{t+1}^{\ell_i} (V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)} (\frac{\partial}{\partial B_{j,t}} V_{t+1} - c_1(1-\tau)\mathcal{E}_{j,i,t+1})] \\
&+ \; \ell_i E_t[M_{i,t,t+1}(1-\tau)^{-1} \mathcal{B}_{t+1}(B_t) A_{t+1}^{\ell_i} (V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)} (\frac{\partial}{\partial B_{j,t}} V_{t+1} - c_1(1-\tau)\mathcal{E}_{j,i,t+1})] \Bigg) \\
&+ \; E_t\Bigg[M_{i,t,t+1}\Bigg(- M_{i,t+1,t+2}(1 + c_2(1-\tau))\mathcal{E}_{j,i,t+2} \; - \; c_1(1-\tau)\mathcal{E}_{j,i,t+1}\Bigg) \left(1 - \left(\frac{A_{t+1}}{V_{t+1} \; - \; \mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\Bigg]
\end{aligned}
$$

That is,

$$\tau c E_t\left[(M_{i,t,t+1}\mathcal{E}_{j,i,t+1}+M_{i,t,t+2}\mathcal{E}_{j,i,t+2})\left(1-\left(\frac{A_{t+1}}{V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t)}\right)^{\ell_i}\right)\right]$$
$$\leq\; E_t[M_{i,t,t+2}\frac{(1+c_2)}{1+c_2(1-\tau)}\mathcal{B}_{t+2}(B_t)\ell_i A_{t+1}^{\ell_i}(V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)}(E_{t+1}[M_{i,t+1,t+2}(1+c_2(1-\tau))\mathcal{E}_{j,i,t+2}] -$$
$$+\;\ell_i E_t[(M_{i,t,t+1}(1-\tau)^{-1}\mathcal{B}_{t+1}(B_t)+\frac{(1+c_2)}{1+c_2(1-\tau)}M_{i,t,t+2}(1-\tau)^{-1}\mathcal{B}_{t+2}(B_t))A_{t+1}^{\ell_i}(V_{t+1}\;-\;\mathcal{B}_{t+1}(B_t))^{-(\ell_i+1)}$$

where for simplicity we have set issuance costs to zero.

G.3 Optimal Debt Portfolio Composition

Proposition 19 *Suppose that $\ell_i = 1$. Then there exists a subset $\Xi_i \subset \{1, \cdots, N\}$ such that the optimal leverage vector $B_t(i) = (B_{j,t}(i))$ has $B_{j,t}(i) > 0$ if and only if $j \in \Xi_i$. This optimal vector $B_t(\Xi_i)$ is given by*

$$\mathcal{E}_{\Xi_i,i,t}B_t(\Xi_i) \;=\; \mathcal{A}_{i,t}^{-1}(\tilde{c}_{i,0}(\Xi_i)e^{-r_t(\Xi_i)})$$

where

$$\mathcal{A}_{i,t} \;=\; \Big(((\tilde{c}_{i,0}(j)+\tilde{c}_{i,0}(k))+1)E_t\left[M_{\$,t,t+1}\mathcal{E}_{j,t,t+1}\mathcal{E}_{k,t,t+1}(\mathcal{E}_{i,t,t+1}\Omega_{i,t+1})^{-1}\right]\Big)_{j,k\in\Xi_i}$$

Issuing only in dollars is optimal if and only if

$$\max_j\left(\frac{\tilde{c}_{i,0}(j)}{1+\tilde{c}_{i,0}(\$)+\tilde{c}_{i,0}(j)}\frac{E_t\left[M_{\$,t,t+1}\mathcal{E}_{j,t,t+1}\right]}{E_t\left[M_{\$,t,t+1}\mathcal{E}_{j,t,t+1}(\mathcal{E}_{i,t,t+1}\Omega_{i,t+1})^{-1}\right]}\right)\;\leq\;\frac{\tilde{c}_{i,0}(\$)}{1+2\tilde{c}_{i,0}(\$)}\frac{E_t\left[M_{\$,t,t+1}\right]}{E_t\left[M_{\$,t,t+1}(\mathcal{E}_{i,t,t+1}\Omega_{i,t+1})^{-1}\right]}\,.$$

If issue costs are independent of the issuance currency, then dollar is the dominant debt currency if and only if

$$\mathrm{Cov}_t^{\$}((\mathcal{E}_{i,t,t+1}\Omega_{i,t+1})^{-1}, \mathcal{E}_{j,t,t+1}) \;\geq\; 0$$

for all j.

Proof. Define $\tilde{B}_{k,t} = \mathcal{E}_{k,i,t}B_{k,t}$. Differentiating w.r.t. $\tilde{B}_{k,t}$ gives first order conditions

$$c_{i,0}(k)E_t\left[M_{i,t,t+1}\mathcal{E}_{k,i,t+1}\right] - \sum_j \tilde{B}_{j,t}(c_{i,0}(j)+c_{i,0}(k))E_t\left[M_{i,t,t+1}\mathcal{E}_{j,i,t,t+1}\mathcal{E}_{k,i,t,t+1}\Omega_{i,t+1}^{-1}\right]$$
$$-\sum_j B_{j,t}E_t\left[M_{i,t,t+1}\mathcal{E}_{j,i,t,t+1}\mathcal{E}_{k,i,t,t+1}\Omega_{i,t+1}^{-1}\right]$$

Thus, defining the matrix

$$\mathcal{A}_{i,t} \;\equiv\; ((c_{i,0}(j)+c_{i,0}(k))+1)E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t,t+1}\mathcal{E}_{k,i,t,t+1}\Omega_{i,t+1}^{-1}]$$

we get

$$B_t \;=\; \mathcal{A}_{i,t}^{-1}(c_{i,0}e^{-r_t})$$

where e^{-r_t} is the vector nominal rates in the different currencies. The last result follows from standard Kuhn-Tucker conditions. Q.E.D.

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c/o University of Geneva, Bd. Du Pont d'Arve 42, CH-1211 Geneva 4
T +41 22 379 84 71, rps@sfi.ch, www.sfi.ch