

Stochastic Processes in Domains with Boundaries and Some of Their Financial Applications

Alexander Lipton

SilaMoney, MIT & EPFL

In this talk we consider two connected problems:

First, we study the classical problem of the first passage hitting density of an Ornstein-Uhlenbeck process. We give two complementary (forward and backward) formulations of this problem and provide semi-analytical solutions for both. The corresponding problems are comparable in complexity. By using the method of heat potentials, we show how to reduce these problems to linear Volterra integral equations of the second kind. For small values of t we solve these equations analytically by using Abel equation approximation; for larger t we solve them numerically. We also provide a comparison with other known methods for finding the hitting density of interest, and argue that our method has considerable advantages and provides additional valuable insights.

Second, we study the non-linear diffusion equation associated with a particle system where the common drift depends on the rate of absorption of particles at a boundary. We provide an interpretation as a structural credit risk model with default contagion in a large interconnected banking system. Using the method of heat potentials, we derive a coupled system of Volterra integral equations for the transition density and for the loss through absorption. An approximation by expansion is given for a small interaction parameter. We also present a numerical solution algorithm and conduct computational tests.

Joint work with V. Kaushansky and C. Reisinger, University of Oxford

References:

<https://arxiv.org/abs/1810.02390>

<https://arxiv.org/abs/1808.05311>