# Labor Leverage, Coordination Failures, and Systematic Risk *

Matthieu Bouvard and Adolfo de Motta†

**Abstract**

We study an economy where demand spillovers make firms' production decisions strategic complements. Firms choose their operating leverage trading off higher fixed costs for lower variable costs. Operating leverage governs firms' exposures to an aggregate labor productivity shock. In equilibrium, firms exhibit excessive operating leverage as they do not internalize that an economy with higher aggregate operating leverage is more likely to fall into a recession following a negative productivity shock. Welfare losses coming from firms' failure to coordinate production are amplified by suboptimal risk-taking, which magnifies the impact of productivity shocks onto aggregate output.

*Key words: Operating leverage, labor leverage, coordination failure, global games, systematic risk.*

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†Matthieu Bouvard is from Toulouse School of Economics and Toulouse School of Management, Adolfo de Motta is from McGill University. Corresponding author: Adolfo de Motta, McGill University, 1001 Sherbrooke St W, Montreal, Quebec H3A 1G5, Canada; Phone: +1-514-398-4029; adolfo.demotta@mcgill.ca.

# 1 Introduction

Labor leverage is a first-order determinant of firms' risk as it creates a rigid cost structure that cannot be easily adjusted to demand shocks. For instance, Danthine and Donaldson (2002) note that the obligations originated by firms' labor force dwarf their financial obligations and provide evidence consistent with labor leverage generating undiversifiable risk. A subsequent stream of literature in asset pricing has shown that labor leverage has explanatory power for the pricing of equity and debt, both in the cross-section and in aggregate.[1] In this paper, we endogenize firms' labor leverage decisions and study how these firm-level choices aggregate into systemic risk. Specifically, aggregate labor leverage affects the likelihood that the economy falls into a Pareto-inferior equilibrium in which output is depressed and welfare is low. These episodes of coordination failure are interpreted as economic crises, in line with a long tradition in economics.[2] While these crises are self-fulfilling in nature, the global games approach suggests their occurrence is anchored to fundamentals through firms' exposure to a common productivity shock.[3] We illustrate how this transmission channel operates through labor leverage and evaluate its implications for the risk of recessions and more broadly for welfare.

We consider an imperfectly competitive economy with aggregate demand spillovers as in Murphy, Shleifer and Vishny (1989).[4] The economy is populated by consumers who supply labor to firms in different sectors. Labor is subject to a productivity shock, which is the only source of risk in this economy. Firms are collectively owned by consumers and use labor as their sole input to produce final goods. Each sector has a competitive fringe of firms with a constant returns to scale technology and a unique large firm with an increasing returns to scale technology. Increasing returns to scale generates strategic complementarities across sectors:

[1]See Merz and Yashiv (2007), Donangelo et al. (2019), and Faviloukis, Lin and Zhao (2020).

[2]See Cooper (1999) for an overview of strategic complementaries and multiple equilibria in economics.

[3]See Morris and Shin (1998), Morris and Shin (2000), Morris and Shin (2003), and Schaal and Taschereau-Dumouchel (2015).

[4]See Mian and Sufi (2014) for a recent paper that highlights the importance of the aggregate demand channel. It shows that the decline in aggregate demand driven by household balance sheet shocks accounts for 65% of the lost jobs from 2007 to 2009.

higher profits in one firm trickle down to consumers and stimulate their demand for goods in other sectors. Higher demand, in turn, allows firms in other sectors to leverage their increasing returns to scale technologies and increase their own profits.

Importantly, we allow large firms to adjust their labor cost structure ex ante. We interpret operating leverage as an enduring technological and organizational choice, consistent with the idea that adjustments to the size and structure of the labor force within a firm requires time and involves sizeable costs.[5] Specifically, at the initial date, each firm chooses its operating leverage trading off higher fixed labor costs for lower variable labor costs. Operating leverage determines the exposure of the firm's profits to the aggregate shock. On the one hand, when demand is high, the firm benefits from higher operating leverage as it can produce at lower average cost. On the other hand, higher leverage, by raising fixed labor costs, makes the firm more vulnerable to a drop in demand. At that final date, the productivity shock is realized and large firms set prices to maximize profits given the threat from the competitive fringe. A large firm, however, retains the option to leave the market following a negative productivity shock if it expects demand to be insufficient to cover its fixed costs.

As in Murphy, Shleifer and Vishny (1989), strategic complementarities between firms' production decisions do not necessarily generate coordination failures. To create scope for coordination failures, we make one additional assumption: large firms are subject to a moral hazard friction. This is consistent with the idea that in larger and more complex organizations, information frictions create room for opportunistic behavior. The presence of moral hazard requires large firms to pay a premium over the wage in the competitive fringe. Critically, these firms fail to internalize the demand spillovers that higher wages have on the rest of the economy. As a result, in the final date, the economy can be trapped in an equilibrium in which large firms exit and output is inefficiently low.

[5]For instance, in Chaney and Ossa (2013), firms organize production by choosing their division of labor, which trades-off fixed versus marginal costs. In their model, a greater division of labor is associated to higher operating leverage.

The main contribution of the paper is to show that coordination failures at the production stage contaminate ex ante risk-taking decisions and generate excessive operating leverage in aggregate. We derive this result in two steps. First, we study production decisions at the final date and show that when labor productivity falls below a threshold, coordination motives generate a sharp transition from a Pareto-optimal production outcome in which large firms reap the benefits of increasing returns to scale, to a Pareto-inferior outcome in which large firms cease to operate. Importantly, we also show that this endogenous operating threshold is higher when firms' aggregate operating leverage increases, that is, an increase in operating leverage increases the likelihood of a coordination failure.

We then turn to ex ante operating leverage decisions. Firms set their operating leverages taking into account their option to cease operations when demand is low. Because of this option, firms optimize leverage over the states in which they anticipate to be active. Coordination problems make it less likely that large firms survive a bad shock, which tilts each firm's optimization program towards higher-productivity states. In these states, the return to operating leverage is higher, so that aggregate operating leverage rises when the production-stage coordination problem worsens. We show that the equilibrium operating leverage is constrained inefficient: a social planer who takes the production-stage equilibrium as given could increase welfare by forcing all firms to lower their operating leverage. The welfare loss caused by excess operating leverage occurs through a shift of the output distribution towards high-productivity states that comes at the cost of higher downside risk and lower expected output. Intuitively, each firm chooses its operating leverage, taking the aggregate leverage as given. Hence, firms do not internalize that their collective decision to increase operating leverage pushes the coordination threshold up, making it more likely that the economy coordinates on a Pareto-inferior regime with depressed output.

The excessive operating leverage is inherently related to the existence of a coordination failure. To formally make this point, we also study a case without coordination failures, that is, a case in which we allow firms to coordinate on the Pareto-superior equilibrium at the production stage. As earlier, atomistic firms do not internalize that a collective decrease in leverage would widen the range of states in which production takes place. However, when firms coordinate on the Pareto-superior equilibrium, production decisions are optimal ex post and hence, the social value of producing in the marginal state (at the production threshold) is exactly zero. Therefore a marginal decrease in aggregate leverage would extend production to a state where it brings no social value. In that case, the equilibrium leverage is constrained efficient. By contrast, when there is a coordination failure at the production stage, there exists a region just below the production threshold where the net value of all firms switching from inaction to production is strictly positive.

Our paper builds on a stream of literature in economics that grounds low aggregate output realizations into coordination failures. Cooper and John (1988) provide a general framework to analyze economies in which agents' actions are strategic complements.[6] In our paper, demand complementarities originate from a combination of imperfect competition and increasing-return-to-scale technologies as in Shleifer and Vishny (1988), Kiyotaki (1988) and Murphy, Shleifer and Vishny (1989). Lamont (1995) shows that coordination failures arise with imperfect competition but constant-return-to-scale economy when firms can suffer from debt overhang. The common feature in these papers is the existence of multiple self-fulfilling equilibria that can be Pareto-ranked. While this illustrates how economies can be trapped into inferior low-output equilibria, equilibrium indeterminacy precludes a meaningful analysis of firms' ex ante risk-exposure decisions, the objective of our paper.

To overcome this problem, we take advantage of global games techniques. As shown in Carlsson and Van Damme (1993) introducing dispersed information in games of strategic complementarities can lead to a unique equilibrium prediction pinned down by the realization of an underlying economic fundamental. Chamley (1999) and Bebchuk and Goldstein (2011) use this

[6]See also Diamond (1982), Hart (1982), Weitzman (1982) and Blanchard and Kiyotaki (1987) for examples of related papers.

insight to study economies in which agents' investment decisions exhibit a generic form of complementarity. Schaal and Taschereau-Dumouchel (2015) introduce global games in a dynamic model with demand complementarities, and show how transitory shocks can trigger long-lasting periods of depressed investment and recession.[7] Guimaraes and Morris (2008) show that risk-averse agents' exposures to an underlying source of risk affect the outcome of the coordination game they play. Our contribution to this literature is to endogenize risk-taking, that is, agents' exposures to the underlying common risk-factor.[8]

Our paper is also related to a literature on operating leverage and firms' risk initiated by Lev (1974). Specifically, a stream of papers in finance focuses on labor leverage, building on the premise that labor is the main fixed component of firms' cost structures. Danthine and Donaldson (2002) note that labor generates obligations for firms that are of an order of magnitude greater than their financial obligations, and that the labor share is countercyclical. These empirical observations motivate a general equilibrium model in which labor leverage creates non-diversifiable risk for equity holders. Subsequent papers have shown labor leverage to be positively related to CAPM beta, stock volatility, credit risk and procyclicality of profits in a cross-section of firms (see, for instance, Donangelo et al., 2019 and Faviloukis, Lin and Zhao, 2020). Merz and Yashiv (2007) show that the shape and magnitude of labor adjustment costs effectively makes hiring conceptually similar to an investment.[9] These empirical findings lend support to the mechanism at play in the model where operating leverage is driven by labor costs, and constitutes a structural long-term choice that affects firm's exposure to aggregate risk.[10]

[7]Global games have been applied to other games of coordination, e.g., currency attacks Morris and Shin (1998), bank runs Goldstein and Pauzner (2005), financial markets Angeletos and Werning (2006), accounting Plantin, Sapra and Shin (2008), trading Edmans, Goldstein and Jiang (2015) and contracting Ozdenoren and Yuan (2017).

[8]Goldstein and Pauzner (2004) show that investors' individual incentives to diversify their portfolio across otherwise independent economies create a propagation channel for coordination failures.

[9]A number of papers document that unionization and employment protection laws increase operating leverage. For instance, Chen, Kacperczyk and Ortiz-Molina (2011) shows that unionization is positively related to various measures of operating leverage, and that unionization significantly increases the cost of equity. Simintzi, Vig, and Volpin (2015) provides evidence consistent with employment protection laws increasing operating leverage and crowding out financial leverage.

[10]Tuzel and Zhang (2017) documents that the cyclicality in wages of the local economy affects the risk of

Finally, there is a large literature that studies the relation between financial frictions and economic crises. (See, for instance, Bernanke and Gertler, 1989, Kiyotaki and Moore, 1997, Lorenzoni, 2008, Gorton and Ordoñez, 2014, and He and Kondor, 2016.) This literature highlights the role of financial frictions, e.g., credit constraints arising from contract incompleteness, in generating and amplifying crises. While the role of financial leverage in economic crises has been extensively studied, the role of operating leverage has received less attention. This is despite the fact that financial distress is typically preceded by economic distress, and despite the importance of firms' non-financial obligations in comparison to their financial obligations. For instance, for US non-financial corporations in 2018, the compensation of employees amounted to 17 times net interest payments.[11] The nature of operating and financial leverage are also markedly distinct: firms may change their capital structure by going to the capital market or through renegotiation, whereas operating leverage is an structural choice that cannot be adjusted in the short run. Moreover, firms can use financial markets to hedge against the costs associated to financial risk. In contrast, financial markets are of limited use in managing operating risk as hedging does not change the cost structure and therefore, whether firms create or destroy value. Our paper suggests that these fundamental technological and organizational choices at the firm level can have outsized aggregate effects in economies subject to coordination failures.

# 2 Model

We consider a unit interval of identical consumers with a utility function $\exp\left[\int_0^1 \ln x(q)dq\right]$ defined over a unit interval of goods indexed by $q$.[12] Consumers are endowed with $L$ units of labor each that they supply inelastically, so $L$ is also the total amount of labor in the economy.

firms operating in that economy by affecting their labor-induced operating leverage.

[11]In 2018, compensation of employees in nonfinancial corporate businesses amounted to \$5,946B while net interest payments amounted to \$338B. *Source:* Bureau of Economic Analysis.

[12]This utility function ensures that consumers' marginal utility of income will be constant across realizations of the aggregate shock $L$, that is, in the model consumers behave as risk-neutral vis-à-vis aggregate uncertainty.

$L$ is drawn from a uniform distribution over $[0, \overline{L}]$, and the realization of $L$ is interpreted as an economy-wide labor productivity shock, the only source of risk in this economy. To ensure the existence of equilibria in which agents' optimization problems have interior solutions, we take the upper bound, $\overline{L}$, to be large enough, in a sense we make precise in the Appendix.[13]

Each good $q$ is produced by a sector, and each sector consists of two types of firms. A competitive fringe of firms with a constant returns to scale technology in which one unit of output requires one unit of labour, and a unique firm, a *monopolist*, with access to an increasing returns to scale technology. To produce, monopolist $q$ incurs a fixed cost of $F(s_q) = \frac{s_q}{\alpha - s_q}$ units of labor and a constant marginal cost of $\alpha - s_q$ units of labor per unit of output. We assume $\alpha < 1$, and take $s_q \in [0, \alpha]$ to be a choice variable that captures *operating leverage*: by increasing $s_q$, monopolist $q$ reduces its marginal cost $\alpha - s_q$ at the expense of inflating its fixed cost $F(s_q)$.[14] The model is closed by assuming that consumers own all the profits and that each firm maximizes expected profits where the workers' wage per unit of labor is taken as the numeraire.[15,16]

Consistent with the literature on labor leverage surveyed in the introduction, we interpret leverage $s_q$ as a structural, long-term choice that regulates monopolist $q$'s cost structure in future production periods. As originally illustrated by Adam Smith's pin factory example, a key decision in the organization of production is the division of labor, which he argued to be limited by the market size. Chaney and Ossa (2013) provides a formalization Adam's Smith argument. In their model, firms choose their division of labor trading-off fixed and marginal costs. Specifically, more teams -a greater division of labor- is associated with higher fixed costs

[13]For most of our results, this assumption boils down to $\overline{L} > 2$.

[14]As we will discuss in Section 4.3, the optimization problem with respect to operating leverage $s_q$, which we study below, is not convex so local conditions are not sufficient for global optimality. The convex function we choose for $F(s_q)$ provides the necessary analytical tractability for studying large deviations to determine existence. However, as Proposition 7 in Section 4.3 shows, the economic intuitions of the paper hold for any generic increasing and convex function.

[15]In the model, because of risk-neutrality, maximizing expected profit is equivalent to maximizing firm value. See Shleifer (1986) for a discussion of this assumption.

[16]We can take the wage per unit of labor as the numeraire because, as will become clear below, the real wage is constant across realizations of the aggregate shock $L$.

but lower marginal costs. (Each team bears a fixed cost but having more teams allows each team to perform a narrower range of tasks around a core competency.) As the expected size of the market increases, firms choose a greater division of labor, that is, a production technology with higher operating leverage.

To capture the enduring quality of these long-term choices, we assume that each monopolist $q$ chooses $s_q$ at $t = 0$, before the realization of the productivity shock and the production decisions at $t = 1$. At $t = 1$, the final period, $L$ is realized, and all firms decide how much to produce and at which price to sell. Note that monopolist $q$ may decide not to produce upon learning $L$. In that case, it saves the fixed cost $F(s_q)$, but abandons production in sector $q$ to the competitive fringe. While in the model production only takes place at $t = 1$, a set-up in which firms also produce at $t = 0$ would produce qualitatively similar results. What is important for our results is that monopolists cannot respond to the productivity shock by quickly changing their production technologies and hence, as we will show, that monopolists' choice of operating leverage ex ante ($t = 0$) affects the probability of a production coordination failure ex post ($t = 1$).

Finally, we assume that production by monopolists requires monitoring that needs to be incentivized. Specifically, a subset of the workers hired by each monopolist perform supervisory functions, where each supervisor is responsible for a share of the firm's production. If a supervisor exerts monitoring effort, each unit produced under his supervision is functional but he incurs a personal cost $\beta$ per unit.[17] If a supervisor does not exert effort, units are defective and worthless to consumers. Quality is contractible, and we assume that $\alpha + \beta < 1$, so that the overall marginal cost is still lower when a monopolist produces than when the competitive fringe does. This specification is consistent with monopolists being larger, more sophisticated organizations in which division of labor and information frictions require supervisory tasks of a different nature than the tasks required in small fringe firms.[18] This monitoring friction is isomorphic

[17]Having a cost of effort proportional to production ensures that only aggregate effort within a firm, rather than the mass of supervisors or the scope of their assignments matters.

[18]See Garicano and Van Zandt (2012) for a review of models in which hierarchies and division of labor jointly emerge as a response to costly information processing.

to the assumption in Murphy, Shleifer and Vishny (1989), who posit that workers experience a disutility when working for monopolist firms rather than for firms in the competitive fringe.[19] The disutility in Murphy, Shleifer and Vishny (1989) is in the spirit of Rosenstein-Rodan (1943), capturing the idea that to bring farm laborers to work in a factory, a firm has to pay them a wage premium. While their specification is tailored to undeveloped economies where a "big push" can jump-start development, our specification has a more natural interpretation in developed economies where business cycles are amplified by coordination failures. As in Murphy, Shleifer and Vishny (1989), the role of this assumption is to generate equilibrium multiplicity and the possibility of a coordination failure at the production stage.[20] This allows us to study the relation between coordination failures in production at $t = 1$ and risk-taking decisions at $t = 0$, the objective of this paper. As will become clear below (Proposition 4), our results are not related to the monitoring friction per se, but to the possibility of a coordination failure.

# 3 Coordination failures and excess operating leverage

## 3.1 Demand externalities and equilibrium multiplicity

The analysis of the final period ($t = 1$) is analogous to the one in Murphy, Shleifer and Vishny (1989). A reader familiar with their analysis could jump to Lemma 1 below, which summarizes a result equivalent to the one in their paper, and which we take as a building block.

Each consumer buys the consumption basket $\{x(q)\}_{q\in[0,1]}$ that maximizes his utility given prices $\{p(q)\}_{q\in[0,1]}$ and his available income $y$:

$$\max_{\{x(q)\}_{q\in[0,1]}} \exp\left[\int_0^1 \ln x(q)dq\right] \quad \text{s.t.} \quad \int_0^1 p(q)x(q) = y. \tag{1}$$

At optimum, consumers spend equally on each good, that is, buy quantity $x(q) = \frac{y}{p(q)}$ of good

[19] Footnote 28 shows the equivalency between the two assumptions.

[20] The assumption that the probability of producing a functional unit without monitoring effort is zero implies that workers do not obtain an information rent when working for monopolist. If one instead assumes some form of efficiency wage or rent-sharing, the case for coordination failures would be made stronger. In Section 4 below, we discuss an extension in which the probability of producing a functional unit without monitoring effort is strictly positive and therefore, in which the supervisor perceives a rent.

$q$.

Monopolist $q$ sets price $p(q)$ to maximize profit given demand

$$x(q) = \begin{cases} \dfrac{y}{p(q)} & \text{if } p(q) \leq 1, \\ 0 & \text{if } p(q) > 1. \end{cases} \tag{2}$$

Note that the monopolist is priced out if it sets $p(q) > 1$: since the competitive fringe can produce one unit of good $q$ with one unit of labor whose wage is the numeraire, the price in the competitive fringe is equal to one. Given the demand function in (2), if monopolist $q$ is active, it sets the highest price that generates a positive demand, $p(q) = 1$, and produces $x(q) = y$ units of output. Because monitoring requires effort, supervisors have to be paid a premium over the wage in the competitive fringe (the numeraire). Since quality is contractible, and since the probability of producing a functional unit without monitoring effort is zero, promising a bonus $\beta$ conditional on delivering a functioning unit is sufficient to incentivize monitoring, and only compensates the supervisor for his monitoring cost. The profit of an active monopolist is then

$$\pi_q = (1 - \beta - \alpha + s_q)\, y - F(s_q). \tag{3}$$

Monopolist $q$ is active when $\pi_q \geq 0$. Consumption and production decisions endogenously determine aggregate income at $t = 1$. Specifically, aggregate income, or equivalently, aggregate output, is the sum of corporate profits and labor income,

$$y = \int_{q \in \mathcal{A}} [(1 - \beta - \alpha + s_q)\, y - F(s_q)] \;dq + L + \int_{q \in \mathcal{A}} \beta y \; dq, \tag{4}$$

where $\mathcal{A} \subset [0, 1]$ is the set of sectors in which a monopolist is active at $t = 1$. From (4), monitoring does not affect aggregate income $y$ for a given set of active monopolists $\mathcal{A}$: monitoring reduces the profit of each active monopolist by $\beta y$, but increases labor income by the same amount. Rearranging (4) we obtain

$$y = \frac{L - \int_{q \in \mathcal{A}} F(s_q) \; dq}{1 - \int_{q \in \mathcal{A}} [1 - \alpha + s_q] \; dq}. \tag{5}$$

From (5) we can derive the marginal impact on output $y$ of monopolist $i$ joining the set of active monopolists $\mathcal{A}$:[21]

$$dy = \frac{\pi_i(y) + \beta y}{1 - \int_{q\in\mathcal{A}}[1-\alpha+s_q]\ dq}\ di. \tag{6}$$

This expression captures an important intuition for production decisions at $t=1$: the contribution of the marginal active monopolist to output is that monopolist's profit $\pi_i(y)$ plus the paid wage premium $\beta y$ amplified by a multiplier $(1-\int_{q\in\mathcal{A}} 1-\alpha+s_q\ dq)^{-1}$, which is strictly greater than one as long as the set of active monopolists $\mathcal{A}$ has a positive mass. This multiplier captures a *demand externality*: when $\pi_i(y)+\beta y$ is positive, the profit and the wage premium stimulates demand for other goods, which allows other active monopolists to take advantage of their increasing returns to-scale technologies and increase their own profits.

Note that monopolist $i$, by becoming active, generates $\beta y$ as compensation for the supervisors' monitoring cost, in addition to profit $\pi_i$. Factoring in this cost in (6), the net impact of monopolist $i$'s decision to become active on consumers' utility is

$$\frac{\pi_i + \beta y}{1 - \int_{q\in\mathcal{A}}[1-\alpha+s_q]\ dq} - \beta y. \tag{7}$$

This expression shows how the demand multiplier $(1-\int_{q\in\mathcal{A}} 1-\alpha+s_q dq)^{-1}$ and the monitoring cost $\beta y$ combine to create a wedge between private and social incentives. The sign of a monopolist's profit $\pi_i$ and the sign of its impact on welfare (7) coincide if there is no multiplier, i.e., $\mathcal{A} = \{\emptyset\}$ so $(1-\int_{q\in\mathcal{A}} 1-\alpha+s_q dq)^{-1} = 1$, in which case (7) reduces to $\pi_i$. The misalignment of private and social incentives arises when $\beta > 0$ and a strictly positive mass of monopolists is active. In that case, monopolist $i$ internalizes the monitoring cost $\beta y$ through the wage premium it pays, but not the multiplier effect of that additional compensation on the demand to other active monopolists $\mathcal{A}$. Specifically, if $\pi_i < 0 < (\pi_i + \beta y)\,(1-\int_{q\in\mathcal{A}} 1-\alpha+s_q dq)^{-1} - \beta y$, it is individually optimal for monopolist $i$ to leave the market, yet its net effect on consumers' utility

[21]Assume a mass $n$ of monopolists with the same leverage $s_i$ become active, in addition to $\mathcal{A}$. Then (5) becomes $y(n) \equiv \frac{L-\int_{q\in\mathcal{A}} F(s_q)dq - nF(s_i)}{1-\int_{q\in\mathcal{A}} 1-\alpha+s_q dq - n(1-\alpha+s_i)}$, and (6) is just $y'(0)$ where we use (4) to simplify the expression.

if it were to produce (7) would be positive.

This misalignment creates scope for coordination failures. Consider the case in which all monopolists choose the same operating leverage $s$ at $t = 0$. Then combining (3) and (4), a monopolist does not operate if no other monopolist operates (if $\mathcal{A} = \{\emptyset\}$) when

$$L < \frac{F(s)}{1 - \alpha - \beta + s}. \tag{8}$$

Conversely, a monopolist operates if all other monopolists operate (if $\mathcal{A} = [0, 1]$) when

$$L > \frac{(1 - \beta)F(s)}{1 - \alpha - \beta + s}. \tag{9}$$

Therefore, if

$$L \in \left( \frac{(1 - \beta)F(s)}{1 - \alpha - \beta + s}, \frac{F(s)}{1 - \alpha - \beta + s} \right), \tag{10}$$

a high-output equilibrium in which all monopolists are active coexists with a low-output equilibrium in which the fringe takes over production. That second equilibrium is Pareto-dominated and embodies a coordination failure: each monopolist does not internalize that its individual decision to produce and pay a wage premium increases demand and hence, the other monopolists' profits from producing. We summarize this discussion in the next Lemma.

**Lemma 1** *If all monopolists choose the same operating leverage at $t = 0$, then at $t = 1$: (i) if $L > \frac{F(s)}{1-\alpha-\beta+s}$, all monopolists operate; (ii) if $L < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$, no monopolist operates; and (iii) if $L \in \left( \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, \frac{F(s)}{1-\alpha-\beta+s} \right)$, there is one equilibrium in which all monopolists operate and another Pareto-dominated equilibrium in which no monopolist operates.*

## 3.2 Global games and the production threshold

While the presence of multiple equilibria captures the idea of a coordination failure, the indeterminacy at the production stage at $t = 1$ precludes the analysis of the operating leverage decision at $t = 0$. To resolve this indeterminacy, we now depart from Murphy, Shleifer and Vishny (1989) and apply a global games treatment. This is done by introducing dispersed information: at $t = 1$ each monopolist $q$ observes a noisy signal of $L$, $l_q = L + \xi_q$, where $\xi_q$ is independent across monopolists and uniformly distributed on $[-\varepsilon, \varepsilon]$. One can show there is no loss of generality in restricting attention to threshold strategies in which monopolist $q$ operates at $t = 1$ if $l_q$ is above a threshold $l^*$. Then, for a given $L$, the mass of active monopolists is

$$n(L) \equiv \begin{cases} 1 & \text{if } L > l^* + \varepsilon \\ \frac{L+\varepsilon-l^*}{2\varepsilon} & \text{if } L \in [l^* - \varepsilon, l^* + \varepsilon] \\ 0 & \text{if } L < l^* - \varepsilon \end{cases} . \tag{11}$$

When observing $l_q = l^*$, monopolist $q$ must be indifferent between producing or not,

$$\frac{1}{2\varepsilon} \int_{l^*-\varepsilon}^{l^*+\varepsilon} (1 - \alpha - \beta + s) \frac{L - n(L)F(s)}{1 - n(L)(1 - \alpha + s)} - F(s)\; dL = 0, \tag{12}$$

which uniquely pins down $l^*$. Finally, a well-known property of global games is that equilibrium uniqueness carries over to the asymptotic case in which the noise in the signals vanishes, $\varepsilon \to 0$, and the information structure becomes arbitrarily close to common knowledge. In that case, solving for $l^*$ in (12) delivers the threshold on the underlying state $L$ above which production takes place,

$$L^T(s) \equiv \frac{F(s)}{1 - \alpha + s} + \frac{\beta F(s)}{(1 - \alpha - \beta + s) \ln\left(\frac{1}{\alpha - s}\right)}. \tag{13}$$

**Proposition 1** *If all monopolists choose operating leverage $s$ at $t = 0$ and $\varepsilon \to 0$, then all monopolists operate at $t = 1$ if $L \geq L^T(s)$ and no monopolist operates at $t = 1$ if $L < L^T(s)$.*

The production equilibrium in Proposition 1 has two important features. First, the equilibrium bears an economically intuitive relation to fundamentals: monopolists are more likely to operate for higher realizations of the productivity shock $L$. Notice that the production threshold belongs to the region in which multiple equilibria previously coexisted: $L^T(s) \in \left(\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, \frac{F(s)}{1-\alpha-\beta+s}\right)$. Specifically, the global games treatment selects the Pareto-inferior equilibrium for the lower realizations of the productivity shock, i.e., for $L \in \left(\frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, L^T(s)\right)$, thereby maintaining scope for coordination failures.

Second, the production threshold $L^T(s)$ is increasing in aggregate operating leverage $s$: a more levered economy needs a higher productivity shock to coordinate on the production equilibrium at $t = 1$, that is, operating leverage aggravates the coordination problem. Indeed, note that if monopolists choose zero operating leverage, there are no coordination failures at $t = 1$ as monopolists always produce, i.e., $L^T(0) = 0$. In other words, coordination failures will endogenously arise in the model because of monopolists' incentives to take on operating leverage.

Next, we build on Proposition 1 to analyze these incentives and to derive the equilibrium operating leverage at $t = 0$.

## 3.3 Equilibrium operating leverage

The distinct feature in our model is to allow each monopolist $q$ to calibrate its exposure to the aggregate shock $L$ through the choice of operating leverage $s_q$, that is, we endogenize risk-taking. To build intuition on the relation between operating leverage and risk, we start with two simple observations.

First, monopolist $q$ only produces at $t = 1$ if $\pi_q \geq 0$, which, from (3), requires demand to be high enough: $y \geq \frac{F(s_q)}{1-\alpha-\beta+s_q}$. This income threshold, $\frac{F(s_q)}{1-\alpha-\beta+s_q}$, increases with $s_q$: a monopolist with higher operating leverage needs higher demand to cover its fixed cost $F(s_q)$, and hence is more likely to exit the market at $t = 1$. The adverse effect of leverage can extend to states in which demand is high enough to allow the monopolist to operate, yet too low for the lower marginal costs to outweigh the higher fixed costs: $\frac{\partial \pi_q}{\partial s_q} < 0$ if $y < \frac{\alpha}{(\alpha-s_q)^2}$. However, as demand increases further, higher leverage becomes beneficial: $\frac{\partial \pi_q}{\partial s_q} > 0$ if $y > \frac{\alpha}{(\alpha-s_q)^2}$. Intuitively, operating leverage magnifies the impact of demand shocks on profits, $\frac{\partial \pi_q}{\partial y} = s_q$, and therefore is more valuable when demand is high.

Second, shocks to demand $y$ are driven by shocks to labor productivity $L$, as is apparent in (5). Operating leverage affects the relation between monopolists' profits and this fundamental risk through two channels: $\frac{\partial \pi_q}{\partial L} = \frac{\partial \pi_q}{\partial y} \cdot \frac{\partial y}{\partial L}$. The first one, $\frac{\partial \pi_q}{\partial y} = s_q$, is the direct exposure of profits $\pi_q$ to demand $y$ discussed in the previous paragraph. The second one, $\frac{\partial y}{\partial L} = (1 - \int_{i \in \mathcal{A}} 1 - \alpha + s_i \; di)^{-1}$, captures that shocks to $L$ are amplified by the demand multiplier, which itself depends on all active monopolists' leverages, $\{s_i\}_{i\in\mathcal{A}}$. Hence, through demand externalities, a monopolist's leverage affects the transmission of productivity shock $L$ to aggregate demand $y$, and therefore the sensitivity of all other monopolists' profits to this productivity shock. Via this demand externality channel, firms' leverage decisions become interdependent and jointly determine aggregate risk.

We analyze operating leverage decisions focusing on symmetric equilibria. Specifically, suppose all monopolists choose non-cooperatively the same operating leverage $s$ at $t = 0$. From Proposition 1, aggregate income is then given by

$$y(s, L) \equiv \begin{cases} \frac{L-F(s)}{\alpha-s} & \text{if } L \geq L^T(s) \\ L & \text{if } L < L^T(s) \end{cases}. \tag{14}$$

Suppose monopolist $q$ chooses operating leverage $s_q$ while all other monopolists choose $s$. Monopolist $q$ produces when it makes a profit, that is, when

$$(1 - \alpha - \beta + s_q)\, y(s, L) - F(s_q) \geq 0, \tag{15}$$

or equivalently, when

$$L \geq \hat{L}(s_q, s) \equiv \begin{cases} \max\left\{(\alpha - s)\dfrac{F(s_q)}{1-\alpha-\beta+s_q} + F(s), L^T(s)\right\} & \text{if } s_q \geq s, \\ \min\left\{\dfrac{F(s_q)}{1-\alpha-\beta+s_q}, L^T(s)\right\} & \text{if } s_q < s. \end{cases} \tag{16}$$

Then $s^*$ is an equilibrium if and only if

$$s^* \in \underset{s_q}{\operatorname{argmax}} \int_{\min\{\hat{L}(s_q,s^*),\overline{L}\}}^{\overline{L}} (1 - \alpha - \beta + s_q)\, y(s^*, L) - F(s_q)\; dL. \tag{17}$$

The next proposition follows from this condition.

**Proposition 2** *There exists a unique symmetric equilibrium $s^*$ defined by*

$$\mathbb{E}\left[y | L \geq L^T(s^*)\right] - F'(s^*) = 0. \tag{18}$$

To understand Condition (18), note that, from (17), operating leverage $s_q$ affects monopolist $q$'s profit through two margins. The first one is an extensive margin: changing $s_q$ affects monopolist $q$'s production threshold $\hat{L}(s_q, s^*)$ at $t = 1$, that is, the lower bound of the integral in (17). The extensive margin, $\hat{L}(s_q, s^*)$, does not enter the monopolist's first order condition (18). Intuitively, at $\hat{L}(s^*, s^*) = L^T(s^*)$, monopolist $q$'s profit is strictly positive only because other monopolists produce as well. As $L$ falls below $L^T(s^*)$ and other monopolists stop producing, monopolist $q$'s profit from producing turns strictly negative, and the monopolist is better off stopping production as well. This holds when $s_q = s^*$, but also, by continuity, for a range of values of $s_q$ below $s^*$. Conversely, monopolist $q$ is better off producing at $L^T(s^*)$ for a range of values of $s_q$ above $s^*$. The rigidity of $\hat{L}(s_q, s^*)$ with respect to $s_q$ around $s^*$ reflects the strength of strategic complementarities around the production threshold, which underlines the coordination failure. Consequently, the only relevant margin is an intensive one: changing $s_q$ affects monopolist $q$'s profit taking the states in which it produces, i.e., the bounds of the integral in (17), as fixed. Condition (18) states that there are no marginal gains in terms of profits along that intensive margin when monopolist $q$ chooses $s^*$ and produces for $L$ above $L^T(s^*) = \hat{L}(s^*, s^*)$. Note that this marginal reasoning only provides a necessary condition for optimality. Specifically, gains along the extensive margin become non-negligible as $s_q$ moves further away from $s^*$. The proof of Proposition 2 in the Appendix shows that these large deviations can also be ruled out.

A key aspect in the above analysis is that the coordination failure at $t = 1$ is unaffected by a unilateral change in leverage, that is, the leverage and production decisions of a single monopolist have a negligible effect on the aggregate income and therefore, on the production decisions of all the other monopolists at $t = 1$. We now ask whether welfare could improve if a social planner were to choose all monopolists' leverage. Specifically, we define the optimal leverage $s^{opt}$ as the one maximizing consumers' expected utility at $t = 0$ given that monopolists behave at $t = 1$ as per Proposition 1. Our social optimum is therefore *constrained* efficient: the social planer chooses leverage at $t = 0$ but cannot avoid coordination failures at $t = 1$. Note finally that the extra labor income $\beta y$ earned by supervisors in states in which monopolists are active merely compensates them for their effort cost and therefore, labor income net of monitoring disutility always sums up to $L$. Consequently, maximizing consumers' expected utility amounts to maximizing their expected income from corporate profits:

$$s^{opt} \in \underset{s}{\text{argmax}} \int_{\min\{L^T(s),\overline{L}\}}^{\overline{L}} (1-\alpha-\beta+s)\, y(s,L) - F(s)\; dL. \tag{19}$$

**Proposition 3** *In the symmetric equilibrium, there is excessive operating leverage: $s^* > s^{opt}$.*

To understand why equilibrium leverage is excessive, consider the first order derivative of the social planner objective function in (19) with respect to $s$,[22]

$$\begin{aligned} &\int_{L^T(s)}^{\overline{L}} y(s,L) - F'(s)\; dL + \int_{L^T(s)}^{\overline{L}} (1-\alpha-\beta+s)\, \frac{\partial y}{\partial s}(s,L) dL \\ &-\frac{\partial L^T(s)}{\partial s}[(1-\alpha-\beta+s)\, y(s,L^T(s)) - F(s)]. \end{aligned} \tag{20}$$

This derivative has three terms. The first one corresponds to the impact of leverage on each monopolist's profit, given that a monopolist produces for $L \geq L^T(s)$. From (18), this term is zero at $s = s^*$: each monopolist optimizes along the intensive margin in equilibrium. The second term captures the demand externality that each monopolist exerts by stimulating demand $y$ for other active monopolists when $L \geq L^T(s)$. From (7), the equilibrium impact of monopolist $q$ on $y$ is $\pi_q + \beta y$ times a multiplier $(\alpha - s^*)^{-1}$ independent from $L$. The monopolist does not internalize that the equilibrium demand multiplier, $(\alpha - s^*)^{-1}$, amplifies the contribution of its profit to output. However, because the equilibrium demand multiplier is constant for $L \geq L^T(s^*)$, monopolist $q$, when choosing $s_q$ to maximize its expected profit $\pi_q$ along the intensive margin, also maximizes the expected externality it exerts on other monopolists across all states $L \geq L^T(s^*)$.[23] As a result, the second term in (20) also cancels out when evaluated at the equilibrium leverage: $\int_{L^T(s^*)}^{\overline{L}} \frac{\partial y}{\partial s}(s^*,L)dL = 0$.

[22]This is assuming that $s$ is such that $L^T(s) < \overline{L}$ which holds both in equilibrium and at the social optimum: it is neither individually nor socially optimal to choose $s$ at $t = 0$ such that monopolists never operate at $t = 1$.

[23]The equilibrium demand multiplier is 1 for $L < L^T(s^*)$ and $(\alpha - s^*)^{-1}$ for $L \geq L^T(s^*)$.

The equilibrium inefficiency goes through the impact of leverage on the lower bound of the monopolists' production region $L^T(s)$, which is captured by the third term in (20). As discussed earlier, if a monopolist were to *unilaterally* lower its leverage at the margin, it would not produce in the neighborhood below $L^T(s^*)$ at $t = 1$, as all other monopolists would remain inactive. However, if a social planner constrains *all* monopolists to change their leverage, coordination in production at $t = 1$ changes: $\frac{\partial L^T(s)}{\partial s} > 0$. Specifically, at the equilibrium leverage $s^*$, a marginal decrease in the leverage of all monopolists lead them to switch from inactivity to production just below $L^T(s^*)$. Because $L^T(s^*)$ belongs to the region in which monopolists being inactive is Pareto-dominated, inducing all of them to produce in that marginal state improves welfare by the corresponding aggregate profit $(1 - \alpha - \beta + s^*)\, y(s, L^T(s^*)) - F(s^*) > 0$. In other words, if monopolists could collectively commit to lower their leverage at $t = 0$, they would increase their expected profits and raise welfare by expanding the range of states in which they coordinate on the Pareto-superior production decision at $t = 1$. They, however, have no unilateral incentive to do so.

It follows from the above discussion, that the excessive leverage result in Proposition 3 is inherently related to the monopolists' failure to coordinate production below $L^T(s)$ at $t = 1$. In fact, our model establishes a direct relationship between the possibility of coordination failures at $t = 1$ and the efficiency of the risk-taking decision at $t = 0$. To see this, consider the equilibrium in Lemma 1 and instead of applying a global games treatment, suppose monopolists always coordinate on the Pareto-superior production equilibrium, that is, assume that for any common leverage $s$, firms produce at $t = 1$ if and only if $L \geq \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$.

**Proposition 4** *If firms coordinate on the Pareto-superior production equilibrium at $t = 1$ and a symmetric operating leverage equilibrium exits, it is unique and socially optimal.*

Proposition 4 highlights that constrained inefficiency in Proposition 3 is not related to the monitoring friction per se: equilibrium leverage is socially optimal with $\beta > 0$ as long as monopolists coordinate on the Pareto-superior equilibrium at $t = 1$. Hence, the possibility of coordination failures at $t = 1$, rather than their origin in itself, is what really matters for the excessive leverage result. In other words, the only meaningful role in the model of the monitoring cost $\beta$ is to create scope for these inefficient production outcomes at $t = 1$. Intuitively, absent coordination failures, the net social value of monopolists producing in the marginal state, i.e., at $L = \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$, is equal to zero. By contrast, the net social value of monopolists producing at $L^T(s^*)$ is strictly positive, which reflects the coordination failure. This strictly positive value at $L^T(s^*)$ creates the social benefit of lowering aggregate leverage to extend production below $L^T(s^*)$, which monopolists fail to internalize.

The inefficiency in Proposition 3 is a statement on the first moment of the output distribution: excessive operating leverage causes expected output to be lower in equilibrium than in the constrained optimum. Our model also shows that this loss in expected output compounds differential effects across states: output is actually higher than in the social optimum for high realizations of the productivity shock, but lower for low realizations. In addition, the transition from a Pareto-superior equilibrium with high production to the Pareto-inferior one with low production occurs through a larger drop in output in equilibrium than in the constrained optimum. This result is the combination of two forces: first, the economy is more levered in equilibrium than in the social optimum, $s^* > s^{opt}$, and second, the drop into a low-production regime occurs for a higher realization of the productivity shock, $L^T(s^*) > L^T(s^{opt})$.[24] Summing up, the welfare loss caused by excess leverage occurs through a shift of the output distribution towards high-productivity states that comes at the cost of higher downside risk and lower expected output.

A growing literature shows that operating leverage, and more specifically labor leverage, is a first-order determinant of firms' risks. For instance, Donangelo et al. (2019) and Faviloukis,

[24] The specific argument on which the proof for this result relies is that higher leverage expands the region in which low output is the byproduct of a coordination failure: $L^T(s) - (1-\beta)\frac{F(s)}{1-\alpha-\beta+s}$ is increasing in $s$ (See Appendix.)

Lin and Zhao (2020) study the effect of labor leverage on firms' risks cross-sectionally, Danthine and Donaldson (2002) study the effect of labor leverage on firms' risks in the aggregate, and Merz and Yashiv (2007) shows that incorporating firms' hiring decisions into a structural model of production with rigidities to labor adjustment helps explaining the volatility of asset prices. Tuzel and Zhang (2017) shows that the cyclicality in wages of the local economy –the metropolitan statistical area– affects the risk of firms operating in that economy by affecting their labor-induced operating leverage. This importance of the labor leverage channel for firms' risks lends support to the mechanism at play in our model and suggests that distortions to labor leverage are likely to have a material impact on output and ultimately welfare.

The link from excessive risk to a higher probability of falling into an inferior low-output equilibrium adds to a long tradition in economics that views economic crises as the product of regime switches between multiple equilibria sustained by strategic complementarities (see Cooper, 1999, for a review). Our model suggests that sharp downturns not only reflect transitions between high- and low-production equilibria ex post, but also inefficient risk decisions ex ante when firms face the prospect of coordination failures. The recent Great Depression is evidence that developed economies are not immune to deep economic crises (Schaal and Taschereau-Dumouchel, 2015), and while crises of that magnitude are not frequently observed, there is evidence that the mere possibility of such rare disasters significantly affects employment, output and asset prices (Gourio, 2012). The volatility induced by recessions is also harmful to developing economies. For instance, Hnatkovska and Loayza (2005) show that the volatility due to crises has outsized effects on the growth of developing countries relative to the volatility due to normal fluctuations.[25] More generally, our results relate to a stream of papers initiated by Ramey and Ramey (1995)'s finding that output volatility and output growth are negatively correlated.

The link between ex post coordination failures and ex ante excessive leverage also implies that public policy can have far-reaching effects. Proposition 4 suggests that government intervention

[25]See Loayza et al. (2007) for a review of the literature on the causes and effects of macroeconomic volatility in developing countries.

that mitigates the coordination problem at $t = 1$ has the related benefit of curbing leverage at $t = 0$. That is, governments that can credibly commit to stimulate their economy in low productivity states have a stabilizing effect by both, avoiding abrupt transitions between high- and low-production regimes ex post and inducing lower leverage ex ante. Importantly, government intervention at $t = 1$ can only be fully effective if it is anticipated at $t = 0$. That is, a lack of credibility to intervene in the future generates excess leverage ex ante, which in turn magnifies the need for government intervention ex post.[26]

Finally, our results also indicate that a public sector can be an efficient instrument to curb risk-taking and prevent crises. In our model, this would operate through several channels. First, a large public sector (in the context of our model, composed of several sectors) can mitigate the coordination problem at the production stage. Indeed, firms within the public sector acting in concert could internalize the effect of their production decisions on the production decisions in the rest of the economy.[27] In addition, mitigating the ex post coordination problem would feed back into lowering ex ante leverage, as shown in Proposition 4. Second, firms within a large public sector could also internalize the effect of their combined leverage decisions at $t = 0$ on the probability of a coordination failure at $t = 1$. Finally, complementarities in risk-taking can also play a role at $t = 0$: a decrease in leverage by firms within the public sector would expand production to lower productivity states in which leverage is less valuable and consequently, would tilt incentives in the rest of the economy towards lowering leverage. Overall, our discussion suggests that a public sector, by curving excessive risk-taking, can cause economies to perform worse during good economic times but also to have lower downside risk. Rodrik (1998) provides evidence consistent with the idea that government spending plays a risk-reducing role in economies exposed to a significant amount of external risk.

[26] Most developing countries are unable or unwilling to implement fiscal expansions during recessions. (See World Development Report 2014, Risk and Opportunity, Managing Risk for Development, World Bank, https://openknowledge.worldbank.org/handle/10986/16092.)

[27] There is a large literature that highlights the role of governments in preventing coordination failures in investment and production, e.g., Murphy, Shleifer and Vishny (1989) and Rodrik (1998).

# 4 Discussion

## 4.1 Sources of coordination failure

As discussed above, the role of the monitoring friction is to generate equilibrium multiplicity and the possibility of a coordination failure at the production stage. Our assumption is isomorphic to one in Murphy, Shleifer and Vishny (1989) but it also lends itself to a natural extension in which workers perceive a rent.[28] Suppose that when supervisors do not exert effort, there is now a small but strictly positive probability $p$ that the product is functional. In that case, if supervisors are protected by limited liability, incentivizing monitoring requires leaving each supervisor with a pay equal to $\beta/(1-p) > \beta$ per unit, which is strictly higher than the supervisor's monitoring cost. The severity of the moral hazard friction, i.e., the magnitude of this rent, aggravates equilibrium inefficiencies through two channels. First, a higher rent makes monopolists more likely to coordinate on the non-productive equilibrium at $t = 1$. This is because monopolists internalize the rent as a real cost, while it merely is a redistribution of surplus from shareholders to workers. The higher incidence of coordination failures at $t = 1$ lowers consumers' expected utility keeping operating leverage constant. Second, the incremental production inefficiency at $t = 1$ induces monopolists to increase their operating leverage at $t = 0$. Intuitively, more frequent coordination failures make all monopolists less likely to survive a bad shock, which increases the weight of high-productivity states in each monopolist's leverage optimization problem. As a result, equilibrium operating leverage moves further away from the social optimum, amplifying the ex post production inefficiency and depressing welfare. The following proposition formalizes this discussion.

**Proposition 5** *In any symmetric equilibrium, there is excessive operating leverage, and an increase in $p$ increases operating leverage and decreases welfare.*

[28] In our set-up profits monopolist $q$ makes profit $\pi(x_q) = p_q x_q(1-\beta) - [(\alpha - s_q)x_q - F(s_q)]$. If we rewrite $\pi'(x_q) = p_q x_q - \frac{1}{1-\beta}[(\alpha - s_q)x_q - F(s_q)]$, $\frac{\beta}{1-\beta}\left(= \frac{1}{1-\beta} - 1\right)$ would correspond to the wage premium in Murphy, Shleifer and Vishny (1989).

Proposition 5 establishes that rents increase operating leverage and decrease welfare by creating a misalignment between firms' objective functions and consumers' welfare. Note that this misalignment aggravates the inefficiency in risk-taking but is not necessary to generate it. As Proposition 3 shows, even when rents are absent ($p = 0$) and maximizing aggregate corporate profits coincide with maximizing consumers' welfare (see equation 19), the coordination problem between firms persists, which makes production and operating leverage decisions suboptimal.

Finally, Murphy, Shleifer and Vishny (1989) provide other mechanisms beyond moral hazard that lead to equilibrium multiplicity in the presence of demand externalities, for instance, when firms invest to generate future labor savings. Therefore, while our modelling choice is both technically convenient as well economically relevant –information frictions are pervasive in large organizations–, it is not the only way to generate a coordination failure in the context of the modelled economy.[29]

## 4.2 State-dependent multiplier

In our model, the inefficiency operates through the impact that monopolists' leverage decisions have on the probability of a coordination failure, that is, along an extensive margin. However, monopolists' leverage decisions also affect each other's profitability in every state $L \geq L^T(s^*)$ in which they produce, that is, along an intensive margin. So far, the incentives of monopolists and the social planer have been aligned along the intensive margin because the demand multiplier is constant across all states in which monopolists produce. (See equation 20 and the discussion that follows.) Shleifer and Vishny (1988) show that a misalignment of public and private incentives to produce can arise along the intensive margin if the demand multiplier is not constant across states. By shutting down this channel, our model is designed to isolate the effect of ex post coordination failures on ex ante risk-taking (Proposition 3). We explore next how a misalignment along the intensive margin would affect leverage decisions.

[29]As emphasized by Proposition 4, the excessive operating leverage at $t = 0$ is caused by the coordination failure at $t = 1$ in itself, rather than by the specific origin of the coordination failure.

As will become clear below, one way to endogenously make the demand multiplier state-contingent is to introduce heterogeneity among monopolists.[30] Specifically, we assume that all monopolists have the same fixed cost, $F(s_q) = \frac{s_q}{\alpha - s_q}$ but a proportion $n_i$ of monopolists have marginal cost $\alpha + \Delta_i - s_q$, where $i = 1, 2$, $n_i + n_j = 1$, $\alpha + \Delta_i < 1$, and $\Delta_1 \neq \Delta_2$. To isolate the effect of heterogeneity on operating leverage decisions, we rule out coordination failures at the production stage by assuming that $\beta = 0$. The next proposition characterizes the efficiency of the equilibrium operating leverage at $t = 0$.[31]

**Proposition 6** *Any symmetric equilibrium in which type-i monopolists choose $s_i^*$ is constrained inefficient: the equilibrium features insufficient operating leverage in that increasing operating leverage for the monopolists with the lowest production threshold increases welfare.*

To understand the intuition behind Proposition 6, note first that, since there are no coordination failures at $t = 1$ ($\beta = 0$), production is ex post efficient.[32] As a result, monopolists' and social planer's incentives to take on operating leverage are aligned along the extensive margin. However, private and public incentives no longer align along the intensive margin. While in the main model all monopolists have the same operating threshold, here, the production threshold depends on the monopolists' type. Assume without loss of generality that type-$i$ monopolists have a lower production threshold $L_i^T$ than type-$j$ monopolists $L_j^T$, that is, for $L \in [L_i^T, L_j^T)$ only type-$i$ monopolists operate while for $L \in [L_j^T, \overline{L}]$ both types operate. This implies that the demand multiplier is no longer constant across states in which a monopolist is active: it is equal to $(1 - n_i(1 - \alpha - \Delta_i + s_i^*))^{-1}$ for $L \in [L_i^T, L_j^T)$ and $(1 - n_i(1 - \alpha - \Delta_i + s_i^*) - n_j(1 - \alpha - \Delta_j + s_j^*))^{-1}$

[30] The effect of firms' heterogeneity on the demand multiplier is as in Shleifer and Vishny (1988). However, their model studies inefficiencies at the production stage, and relies on firms making production decisions before the productivity shock $L$ is realized. In our model, $L$ is known when firms make production decisions, but unknown when firms choose leverage.

[31] As earlier, we rule out corner solutions by assuming that $\overline{L}$ is large enough (greater than $2/\alpha$ in this case).

[32] In the main model, the net impact of monopolist $i$'s decision to become active on consumers' utility (equation 7) equals $\pi_i \left(1 - \int_{q \in \mathcal{A}} 1 - \alpha + s_q dq\right)^{-1}$ when $\beta = 0$. This implies that there is no longer a wedge between private and social incentives to operate at $t = 1$: the sign of a monopolist's profit $\pi_i$ is the same as the sign of its net impact on welfare. This logic extends to the case with heterogeneous firms and no coordination failures.

for $L \in [L_j^T, \overline{L}]$. Since $(1-n_i(1-\alpha-\Delta_i+s_i^*)-n_j(1-\alpha-\Delta_j+s_j^*))^{-1} > (1-n_i(1-\alpha-\Delta_i+s_i^*))^{-1}$, a marginal unit of profit has a higher welfare impact when $L \in [L_j^T, \overline{L}]$ than when $L \in [L_i^T, L_j^T)$. As a result, maximizing expected profit is no longer equivalent to maximizing expected welfare. Relative to a monopolist, a social planner has a higher willingness to shift profits from $[L_i^T, L_j^T)$ to $[L_j^T, \overline{L}]$ because the welfare impact per unit of profit is higher in $[L_j^T, \overline{L}]$ than in $[L_i^T, L_j^T)$. This is exactly what higher operating leverage achieves.[33]

Overall, contrasting the insufficient leverage result in Proposition 6 with our main result of excessive leverage in Proposition 3 stresses that coordination failures have a distinct directional effect on ex ante risk-taking decisions. While the analysis in the current section suggests that risk-taking decisions can be distorted in more than one way, the importance of the excessive leverage result of Proposition 3 is commensurate to the role that coordination failures play in explaining crises and recessions, as discussed at the end of Section 3. In that sense, the inefficiencies in Propositions 3 and 6 speak to different concerns about the distribution of output across states of the economy. In Proposition 6, insufficient leverage prevents the economy from fully exploiting demand spillovers after a positive productivity shock, while in Proposition 3, excessive leverage makes the economy too exposed to a negative productivity shock. To the extent that recessions and crises inflict material and long-lasting damages, that second channel ought to be of particular significance.

## 4.3 Operating leverage specification

In the paper, firms decide their cost structure by trading-off higher fixed labor costs for lower variable costs. Specifically, monopolist $q$ incurs a fixed cost of $F(s_q) = \frac{s_q}{\alpha - s_q}$ units of labor and a constant marginal cost of $\alpha - s_q$ units of labor per unit of output, where $s_q \in [0, \alpha]$ is choice variable that captures operating leverage. The function $F(s_q)$ is increasing, convex, and tends

[33]While increasing $s_i^*$ given $s_j^*$ increases welfare (Proposition 6), $s_j^*$ is optimal given $s_i^*$. Intuitively, the equilibrium demand multiplier is constant across all states in which type-$j$ monopolists are active (i.e., the demand multiplier is $(1 - n_i(1 - \alpha - \Delta_i + s_i^*) - n_j(1 - \alpha - \Delta_j + s_j^*))^{-1}$ for all $L \in [L_j^T, \overline{L}]$). Therefore, type-$j$ monopolists, maximizing expected profit in $L \in [L_j^T, \overline{L}]$ is equivalent to maximizing expected welfare.

to infinity as $s_q \to \alpha$, which guarantees that the equilibrium operating leverage $s^*$ is smaller than $\alpha$ (and hence, that variable costs are positive).

Because the monopolist's optimization problem with respect to $s_q$ in 17 is not convex, local optimality conditions are not sufficient for global optimality, and large deviations also need to be ruled out. The function we chose for $F(s_q)$, $\frac{s_q}{\alpha - s_q}$, provides the necessary analytical tractability for studying large deviations to determine existence. (See the proof of Proposition 2 in the Appendix.) However, the local necessary condition that characterizes the equilibrium in Proposition 2 holds for a generic increasing and convex function. Moreover, around this local equilibrium condition, there is excessive operating leverage, as in Proposition 3. The logic is also identical to the one previously discussed for Proposition 3: at the equilibrium leverage $s^*$, a marginal decrease in the leverage of all monopolists lead them to switch from inactivity to production just below $L^T(s^*)$, i.e., $\frac{\partial L^T(s^*)}{\partial s} > 0$, which improves welfare. This highlights, once again, that the excessive leverage result is driven by the monopolists' failure to coordinate production below $L^T(s^*)$ at $t = 1$, and not by the chosen functional form for the fixed cost. The following proposition formalizes this discussion.

**Proposition 7** *Consider a generic increasing and convex function $F(.)$ any symmetric interior equilibrium $s^*$ at $t = 1$ satisfies condition (18) in Proposition 2, and a collective marginal decrease in operating leverage around $s^*$ increases welfare.*

# 5 Conclusion

Coordination failures are often invoked to explain periods of recession and crises in which economies appear to be trapped in down-cycles of low output, trading and investment. In essence, the literature views economies as interdependent systems in which agents need to coordinate their investment and consumption decisions. We show that these economies feature excessive operating leverage as firms do not internalize the effect that their risk choices have on the probability of the economy suffering a coordination failure. The excessive operating leverage increases downside risk and causes expected output to be lower than in the social optimum. The loss in expected output compounds differential effects across states: there is a shift in output towards high-productivity states that comes at the cost of more frequent and severe economic crises. The analysis has implications for public policy: a countercyclical policy that stimulates the economy in low productivity states has a stabilizing effect by both, avoiding coordination failures ex post and curving excessive risk ex ante.

While our model emphasizes the role of demand externalities because of its economic importance, strategic complementaries across economic agents have been shown to arise through a broad variety of channels, e.g., thick market externalities, technological complementarities, and imperfect information. These other sources of strategic complementaries can also generate multiple equilibria and create scope for coordination failures. In general, we expect the same economic forces to apply in set-ups with these alternative sources. Indeed, as long as risk increases the probability of a coordination failure and agents do not internalize this effect, agents will tend to engage in excessive risk-taking.

Strategic complementarities are notably important in the financial system as illustrated by recurrent financial crises. Specifically, its liquidity transformation role combined with the fact that financial institutions form a highly interconnected network makes financial systems particularly vulnerable to coordination failures. Our analysis suggest that a concentrated financial system is more stable in that financial institutions internalize, to a larger extent, the effect that their risk choices have of the probability of a systemic financial crisis. Alternatively, it also suggests that a more disperse financial system requires stricter regulation to curve excessive risk-taking. The analysis of these effects in the specific context of an economy with a financial system is an interesting avenue for future research.

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# Appendix

To ensure the existence of equilibria in which agents' optimization problems have interior solutions, we assume throughout this Appendix that the upper bound $\overline{L}$ is large enough, in a sense we make precise in the proofs below. For most of our results, this assumption boils down to $\overline{L} > 2$.

## Proof of Proposition 1

Suppose all firms choose $s$, and firm $q$ observes a noisy signal of $L$, i.e., a signal $l_q = L + \xi_q$ where $\xi_q$ is uniform on $[-\varepsilon, \varepsilon]$. Suppose firm $q$ produces iff $l_q > l^*$. Then, for a given realization of $L$, the number of firms that produce is

$$n(L) \equiv \begin{cases} 1 & \text{if } L > l^* + \varepsilon \\ \frac{L+\varepsilon-l^*}{2\varepsilon} & \text{if } L \in [l^* - \varepsilon, l^* + \varepsilon] \\ 0 & \text{if } L < l^* - \varepsilon \end{cases} \tag{A.1}$$

The firm with signal $l^*$ must be indifferent, i.e.,

$$\frac{1}{2\varepsilon}\int_{l^*-\varepsilon}^{l^*+\varepsilon}(1-\alpha-\beta+s)\frac{L-n(L)F}{1-n(L)(1-\alpha+s)} - F\; dL = 0. \tag{A.2}$$

Consider the following change of variable: $z = \frac{L+\varepsilon-l^*}{2\varepsilon} \Leftrightarrow L = 2\varepsilon z + l^* - \varepsilon$. (A.2) becomes

$$\frac{1}{2\varepsilon}\int_0^1 \left\{(1-\alpha-\beta+s)\frac{2\varepsilon z + l^* - \varepsilon - n(2\varepsilon z + l^* - \varepsilon)F}{1-n(2\varepsilon z + l^* - \varepsilon)(1-\alpha+s)} - F\right\} 2\varepsilon dz = 0 \tag{A.3}$$

$$\Leftrightarrow \quad \int_0^1 (1-\alpha-\beta+s)\frac{2\varepsilon z + l^* - \varepsilon - zF}{1-z(1-\alpha+s)} - F\; dz = 0. \tag{A.4}$$

Taking the limit when $\varepsilon \to 0$ and defining $L^T(s)$ as the limit of $l^*$ when $\varepsilon \to 0$, equation A.4 writes

$$\int_0^1 (1-\alpha-\beta+s)\frac{L^T(s) - zF}{1-z(1-\alpha+s)} - F\; dz = 0 \tag{A.5}$$

$$\Leftrightarrow \quad \frac{1-\alpha-\beta+s}{1-\alpha+s}\int_0^1 \frac{(1-\alpha+s)L^T(s) - z(1-\alpha+s)F}{1-z(1-\alpha+s)}dz - F = 0 \tag{A.6}$$

$$\Leftrightarrow \quad \frac{1-\alpha-\beta+s}{1-\alpha+s}\int_0^1 \frac{(1-\alpha+s)L^T(s) - F}{1-z(1-\alpha+s)}dz - \frac{\beta}{1-\alpha+s}F = 0 \tag{A.7}$$

$$\Leftrightarrow \quad -\frac{1-\alpha-\beta+s}{(1-\alpha+s)^2}\left[(1-\alpha+s)\,L^T(s) - F\right]\ln(\alpha-s) - \frac{\beta}{1-\alpha+s}F = 0. \tag{A.8}$$

Therefore

$$L^T(s) = \frac{F}{1-\alpha+s} + \frac{\beta F}{(1-\alpha-\beta+s)\ln\left(\frac{1}{\alpha-s}\right)}. \tag{A.9}$$

It is apparent that the RHS of (A.4) is increasing in $l^*$, which guarantees the uniqueness of an equilibrium in threshold strategies for any $\varepsilon$. Now, iterated deletion of strictly dominated strategies ensure global uniqueness in a setting with global strategic complementarities, e.g., Morris and Shin, 2003. One potential concern here is that monopolists' actions are not complement if $L$ is small enough. Specifically, letting $n$ be the fraction of monopolists who produce,

$$\frac{\partial \pi}{\partial n} \geq 0 \Leftrightarrow L \geq \frac{F(s)}{1-\alpha+s}. \tag{A.10}$$

Note however, that

$$\frac{F(s)}{1-\alpha+s} < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}, \tag{A.11}$$

i.e., the region where the firms' actions are not strategic complement is strictly smaller than the upper bound of the lower-dominance region. Therefore if

$$2\varepsilon < \frac{(1-\beta)F(s)}{1-\alpha-\beta+s} - \frac{F(s)}{1-\alpha+s}, \tag{A.12}$$

a private signal $l_q$ consistent with $L \geq \frac{(1-\beta)F(s)}{1-\alpha-\beta+s}$ rules out $L < \frac{F(s)}{1-\alpha+s}$, i.e., ensures that $L$ is in the region where actions are complement. This, in turn, allows to apply the standard iterated deletion of strictly dominated strategies. It follows that the threshold equilibrium derived in (A.4) is the unique equilibrium for $\varepsilon$ small enough, hence when $\varepsilon \to 0$. *Q.E.D.*

## Proof of Proposition 2

**Part I. Proof of Proposition 2: Local Conditions.**

Suppose monopolist $q$ chooses $s_q$ and all other monopolists choose $s^* \in (0, \alpha)$.

(a) If no other monopolist operate, monopolist $q$ operates if

$$L > L_-(s_q) \equiv \frac{F(s_q)}{1-\alpha-\beta+s_q}. \tag{A.13}$$

Since (i) $\alpha + \beta < 1$, $L'_{-}(s_q) > 0$; (ii) $\beta > 0$, $L_{-}(s^*) > L^T(s^*)$ (from equation A.9); and (iii) $L_{-}(0) = 0 < L^T(s^*)$, there is a unique $s_{-}$ such that $L_{-}(s_{-}) = L^T(s^*)$ and $s_{-} < s^*$. If $L < L^T(s^*)$ and $s_q < s_{-}$, $q$ operates iff $L > L_{-}(s_q)$. If $L < L^T(s^*)$ and $s_q > s_{-}$, $q$ never operates.

(b) If all other monopolist operate, monopolist $q$ operates if

$$L > L_{+}(s_q) \equiv (\alpha - s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*), \tag{A.14}$$

Since (i) $L'_{+}(s_q) > 0$; (ii) $\beta > 0$, $L_{+}(s^*) < L^T(s^*)$ (from equation A.9); and $\lim_{s\to\alpha} L_{+}(s) = +\infty$, there is a unique $s_{+} < \alpha$ such that $L_{+}(s_{+}) = L^T(s^*)$ and $s_{+} > s^*$. If $L > L^T(s^*)$ and $s_q > s_{+}$, $q$ operates iff $L > L_{+}(s_q)$. If $L > L^T(s^*)$ and $s_q < s_{+}$, $q$ always operates.

(a) and (b) implies that

$$L \geq \hat{L}(s_q, s^*) \equiv \begin{cases} (\alpha - s^*)\dfrac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*) & \text{if } s_q \geq s_{+}, \\ L^T(s^*) & \text{if } s_q \in (s^*, s_{+}), \\ L^T(s^*) & \text{if } s_q \in (s_{-}, s^*), \\ \dfrac{F(s_q)}{1-\alpha-\beta+s_q} & \text{if } s_q \leq s_{-}. \end{cases} \tag{A.15}$$

The monopolist optimization problem writes (see equation 17)

$$\max_{s_q} \int_{\min\{\hat{L}(s_q,s^*),\overline{L}\}}^{\overline{L}} (1-\alpha-\beta+s_q)\, y(s^*, L) - F(s_q)\; dL, \tag{A.16}$$

and therefore, since the optimal $s_q$ is such that $\hat{L}(s_q, s^*) < \overline{L}$, and since (A.15) implies that $\left|\frac{\partial \hat{L}(s_q,s^*)}{\partial s_q}\right|_{s_q=s^*} = 0$, the first-order condition for an interior solution to the monopolist optimization problem writes

$$E\left[y|L \geq L^T(s^*)\right] - F'(s^*) = 0 \tag{A.17}$$

$$\Leftrightarrow \left(\frac{\overline{L} + L^T(s^*)}{2} - F(s^*)\right)\frac{1}{\alpha - s^*} - F'(s^*) = 0 \tag{A.18}$$

$$\Leftrightarrow \overline{L} = \frac{1}{\alpha - s^*}\left[2(\alpha + s^*) - \frac{s^*}{1-\alpha+s^*} - \frac{\beta s^*}{(1-\alpha-\beta+s^*)\ln\left(\frac{1}{\alpha-s^*}\right)}\right] \tag{A.19}$$

If $s^*=0$, the RHS of (A.19) is equal to 2. If $s^*\to\alpha$, the RHS of (A.19) tends to $+\infty$. It follows that if $\overline{L}>2$, (A.19) has at least one solution. Rewrite, (A.19) as

$$\alpha\overline{L}=2\alpha+s^*\underbrace{\left(2+\overline{L}-\frac{1}{1-\alpha+s^*}-\frac{\beta}{(1-\alpha-\beta+s^*)\ln\left(\frac{1}{\alpha-s^*}\right)}\right)}_{\equiv f(s^*)} \tag{A.20}$$

Since $f'(.)>0$ and $\lim_{s\to\alpha}f(s)=1+\overline{L}>0$ implies there exists $\hat{s}$ (possibly equal to 0) such that $f(s)>0$ iff $s>\hat{s}$. $\overline{L}>2$ implies that at $s=0$, the RHS is lower than $2\alpha$ and therefore, $s\in[0,\hat{s})$ cannot be a solution to (A.20) since for $s\in[0,\hat{s})$, the RHS is also lower than $2\alpha$. If $s>\hat{s}$, then $f(s)>0$ and $f'(s)>0$ imply that the RHS of (A.20) is strictly increasing. Therefore (A.20) has a unique solution. Hence, if there exists a symmetric equilibrium, this equilibrium is unique, interior, i.e., $s^*\in(0,\alpha)$, and defined by the f.o.c. in A.17.

The first-order condition in A.17 gives a necessary condition for a symmetric equilibrium to exists as otherwise firm $q$ would have incentive to deviate locally from $s^*$. Next we analyze the possibility of non-local deviations from $s^*$ by firm $q$.

**Part II. Proof of Proposition 2: Large Deviations**

**Consider first a deviation to* $s_q<s^*$.

Reminder: let $s_-\in(0,s^*]$ be the unique solution to $L_-(s_-)=L^T(s^*)$, where $L_-(.)$ is defined in (A.13). If $\beta>0$ then $s_-<s^*$, and

- if $s_q\leq s_-$, then $q$ invests iff $L\geq L_-(s_q)=\frac{F(s_q)}{1-\alpha-\beta+s_q}$,

- if $s_q>s_-$, then $q$ invests iff $L\geq L^T(s^*)$.

Monopolist $q$ solves

$$\pi(s_q)=\int_{L^T(s^*)}^{\overline{L}}(1-\alpha-\beta+s_q)\frac{L-F(s^*)}{\alpha-s^*}-F(s_q)\,dL+\int_{\min\{L_-(s_q),L^T(s^*)\}}^{L^T(s^*)}(1-\alpha-\beta+s_q)L-F(s_q)\,dL \tag{A.21}$$

and the first derivative with respect to $s_q$ is

$$\frac{\partial \pi}{\partial s_q}(s_q) = \int_{L^T(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*} - F'(s_q)\ dL + \int_{\min\{L_-(s_q), L^T(s^*)\}}^{L^T(s^*)} L - F'(s_q)\ dL. \tag{A.22}$$

Two observations:

1. If $s_q \in [s_-, s^*)$, $\pi'(s_q)$ simplifies to

$$\int_{L^T(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*} - F'(s_q)\ dL > 0 \tag{A.23}$$

   since $F'' > 0$. Therefore, monopolist $q$ does not deviate an $s_q \in [s_-, s^*)$.

2. $\pi'(.)$ is continuous (though not differentiable at $s_-$), which implies that when $s_q \to s_-$ from below, $\pi'(s_q) > 0$.

Suppose now $s_q < s_-$.

$$\begin{aligned} \pi''(s_q) &= -\int_{L_-(s_q)}^{\overline{L}} F''(s_q)\ dL - [L_-(s_q) - F'(s_q)]\, L'_-(s_q) & \text{(A.24)} \\ &= -[\overline{L} - L_-(s_q)]F''(s_q) + \frac{[F'(s_q) - L_-(s_q)]^2}{1 - \alpha - \beta + s_q}, & \text{(A.25)} \end{aligned}$$

where we are using (and will use repeatedly) that

$$L'_-(s_q) = \frac{(1 - \alpha - \beta + s_q)F'(s_q) - F(s_q)}{(1 - \alpha - \beta + s_q)^2} = \frac{F'(s_q) - L_-(s_q)}{1 - \alpha - \beta + s_q}. \tag{A.26}$$

Next, we calculate the sign of the second derivative $\pi''(s)$ in equation A.25. Since $F''(s_q) > 0$, sign($\pi''(s_q)$)=sign($\pi''(s_q)/F''(s_q)$)

$$\frac{\pi''(s_q)}{F''(s_q)} = -\overline{L} + \underbrace{L_-(s_q) + \frac{[F'(s_q) - L_-(s_q)]^2}{(1 - \alpha - \beta + s_q)F''(s_q)}}_{\equiv G(s_q)} \tag{A.27}$$

$$\begin{aligned} G'(s_q) = \ & \frac{F'(s_q) - L_-(s_q)}{1 - \alpha - \beta + s_q} & \text{(A.28)} \\ & + \frac{2[F'(s_q) - L_-(s_q)][F''(s_q) - L'_-(s_q)](1 - \alpha - \beta + s_q)F''(s_q)}{[F''(s_q)(1 - \alpha - \beta + s_q)]^2} \\ & - \frac{[F'(s_q) - L_-(s_q)]^2\,[F'''(s_q)(1 - \alpha - \beta + s_q) + F''(s_q)]}{[F''(s_q)(1 - \alpha - \beta + s_q)]^2}. \end{aligned}$$

Since $F'(s_q) > L_-(s_q)$, $G'(s_q)$ has the sign of

$$
\begin{aligned}
& [F''(s_q)]^2(1-\alpha-\beta+s_q) && \text{(A.29)}\\
& +2[F''(s_q)-L_-'(s_q)](1-\alpha-\beta+s_q)F''(s_q) \\
& -[F'(s_q)-L_-(s_q)]\left[F'''(s_q)(1-\alpha+s_q)+F''(s_q)\right] \\
= \;& 3[F''(s_q)]^2(1-\alpha-\beta+s_q)-2[F'(s_q)-L_-(s_q)]F''(s_q) && \text{(A.30)}\\
& -[F'(s_q)-L_-(s_q)]\left[F'''(s_q)(1-\alpha-\beta+s_q)+F''(s_q)\right] \\
= \;& 3[F''(s_q)]^2(1-\alpha-\beta+s_q) && \text{(A.31)}\\
& -[F'(s_q)-L_-(s_q)][F'''(s_q)(1-\alpha-\beta+s_q)+3F''(s_q)] \\
= \;& \frac{12\alpha^2}{(\alpha-s_q)^6}(1-\alpha-\beta+s_q)-\frac{6\alpha^2}{(\alpha-s_q)^6}(1-\alpha-\beta+s_q) && \text{(A.32)}\\
& +\frac{6\alpha s_q}{(\alpha-s_q)^5}-\frac{6\alpha^2}{(\alpha-s_q)^5}+\frac{6\alpha s}{(\alpha-s)^4(1-\alpha-\beta+s_q)},
\end{aligned}
$$

which, in turn has the sign of

$$g(s_q) \equiv \alpha(1-\alpha-\beta+s_q)^2-(1-\alpha-\beta)(\alpha-s_q)^2 \tag{A.33}$$

There are two possible cases:

(I) $2\alpha+\beta-1 \leq 0$

Then $g(.)$ is concave, $g(\alpha) > 0$ and $g(0) \geq 0$, therefore, $g(.) > 0$ for $s_q \in (0, s_-)$. It follows that $\pi''$ is increasing for $s_q = 0$ and is strictly increasing for $s \in (0, s_-)$.

(II) $2\alpha+\beta-1 > 0$ Then $g(.)$ is convex, $g(\alpha) > 0$ and $g(0) < 0$. It follows that $\pi''(.)$ is first decreasing, then increasing on $[0, \alpha)$.

**N.B.**: $\pi'(0) > 0$ *and* $\pi''(s_-) < 0$ *is then sufficient to rule out the deviation.*

Indeed, in (I), $\pi''(s_-) < 0$ and $\pi''$ increasing imply $\pi''(.) < 0$ on $[0, s_-)$. Then since $\pi'(s_-) > 0$, $\pi'(.) > 0$ on $[0, s_-)$.

In (II), $\pi''(s_-) < 0$ implies either $\pi''(.) < 0$ on $[0, s_-)$ (then back to previous case), or $\pi''(.)$ is first positive and then negative. Then since $\pi'(0) > 0$ and $\pi'(s_-) > 0$, $\pi'(.) > 0$ on $[0, s_-)$.

*Sufficient conditions for $\pi'(0) > 0$ and $\pi''(s_-) < 0$:*

i) $\pi'(0) > 0$ is true if $\overline{L}$ is large enough, for instance $\overline{L} > \frac{2}{\alpha}$ is sufficient.

ii) $\pi''(s_-) < 0$ :

From (A.19), if $\overline{L} \to +\infty$ then $s^* \to \alpha$, and $\overline{L}(\alpha - s^*) \to 3\alpha$. In addition, if $\overline{L} \to +\infty$ then $L^T(s^*) \to +\infty$ which implies $L_-(s_-) \to +\infty$, which implies $s_- \to \alpha$. Finally, by definition, $L_-(s_-) = L^T(s^*)$:

$$L_-(s_-)(\alpha - s^*) = \frac{s_-}{(\alpha - s_-)(1 - \alpha - \beta + s_-)}(\alpha - s^*) = L^T(s^*)(\alpha - s^*) \quad \text{(A.34)}$$

$$\Leftrightarrow \quad \frac{s_-}{1 - \alpha - \beta + s_-}\overline{L}(\alpha - s^*) = L^T(s^*)(\alpha - s^*)\overline{L}(\alpha - s_-) \quad \text{(A.35)}$$

Using, $\lim_{\overline{L}\to+\infty} L^T(s^*)(\alpha - s^*) = \alpha$, $\lim_{\overline{L}\to+\infty} \overline{L}(\alpha - s^*) = 3\alpha$ and $\lim_{\overline{L}\to+\infty} s_- = \alpha$ yields

$$\lim_{\overline{L}\to+\infty} \overline{L}(\alpha - s_-) = \frac{3\alpha}{1 - \beta} \quad \text{(A.36)}$$

Next, from (A.25):

$$\begin{aligned} \pi''(s_-) &= -[\overline{L} - L_-(s_-)]F''(s_-) + \frac{[F'(s_-) - L_-(s_-)]^2}{1 - \alpha - \beta + s_-} && \text{(A.37)} \\ &= -\left[\overline{L} - \frac{s_-}{(\alpha - s_-)(1 - \alpha - \beta + s_-)}\right]\frac{2\alpha}{(\alpha - s_-)^3} && \text{(A.38)} \\ &\quad + \frac{1}{1 - \alpha - \beta + s_-}\left[\frac{\alpha}{(\alpha - s_-)^2} - \frac{s_-}{(\alpha - s_-)(1 - \alpha - \beta + s_-)}\right]^2, \end{aligned}$$

which has the sign of

$$\begin{aligned} &-\left[\overline{L}(\alpha - s_-) - \frac{s_-}{(1 - \alpha - \beta + s_-)}\right](2\alpha) + && \text{(A.39)} \\ &+\frac{1}{1 - \alpha - \beta + s_-}\left[\alpha - \frac{(\alpha - s_-)s_-}{(1 - \alpha - \beta + s_-)}\right]^2, \end{aligned}$$

which tends to

$$-(3\frac{\alpha}{1 - \beta} - \frac{\alpha}{1 - \beta})2\alpha + \frac{\alpha^2}{1 - \beta} = -\frac{3\alpha^2}{1 - \beta} < 0, \quad \text{(A.40)}$$

when $\overline{L} \to +\infty$. Therefore there exists $\hat{L}$ such that if $L > \hat{L}$, $\pi''(s_-) < 0$.

This concludes the proof that monopolist $q$ does not deviate to $s_q \in [0, s_-)$.

**Consider now a deviation to* $s_q > s^*$.

Monopolist $q$ continues if $L \geq \max\{(\alpha - s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*), L^T(s^*)\}$. The monopolist will not deviate to a $s_q > s^*$ such that $(\alpha - s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*) \leq L^T(s^*)$ as (A.17) implies that

$$s^* = \arg\max_{s_q} \int_{L^T(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*}(1 - \alpha - \beta + s_q) - F(s_q)\, dL = 0. \tag{A.41}$$

Consider a large deviation such that $(\alpha - s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q} + F(s^*) > L^T(s^*)$ The firm would solve

$$\max_{s_q} \frac{1}{\overline{L}} \int_{(\alpha-s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q}+F(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*}(1 - \alpha - \beta + s_q) - F(s_q)\, dL, \tag{A.42}$$

which yields the following first derivative

$$\frac{1}{\overline{L}} \int_{(\alpha-s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q}+F(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*} - F'(s_q)\, dL, \tag{A.43}$$

which also writes as

$$\frac{\overline{L} - (\alpha - s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q} - F(s^*)}{\overline{L}} \left[\frac{1}{2}\frac{\overline{L} - F(s^*)}{\alpha - s^*} + \underbrace{\frac{1}{2}\frac{F(s_q)}{1 - \alpha - \beta + s_q} - F'(s_q)}_{\equiv\Psi(s_q)}\right]. \tag{A.44}$$

Therefore since $\overline{L} > (\alpha - s^*)\frac{F(s_q)}{1-\alpha-\beta+s_q} - F(s^*)$ (i.e., monopolist will never deviate to a $s_q$ where never produces):

$$sign(\text{first derivative}) = sign\left\{\frac{1}{2}\frac{\overline{L} - F(s^*)}{\alpha - s^*} + \Psi(s_q)\right\}. \tag{A.45}$$

Since $L^T(s^*) > (\alpha - s^*)\frac{F(s^*)}{1-\alpha-\beta+s^*} + F(s^*)$ and, from (A.17),

$$\frac{1}{2}\frac{\overline{L} - F(s^*)}{\alpha - s^*} + \frac{1}{2}\frac{L^T(s^*) - F(s^*)}{\alpha - s^*} - F'(s^*) = 0, \tag{A.46}$$

then

$$\frac{1}{2}\frac{\overline{L} - F(s^*)}{\alpha - s^*} + \Psi(s^*) < 0. \tag{A.47}$$

Also

$$\frac{1}{2}\frac{\overline{L}-F\left(s^{*}\right)}{\alpha-s^{*}}+\Psi(0)=\frac{1}{2}\frac{\overline{L}-F\left(s^{*}\right)}{\alpha-s^{*}}-\frac{1}{\alpha}>\frac{1}{2\alpha}\left(\overline{L}-2\right)>0, \tag{A.48}$$

where the first inequality follows from

$$\frac{\overline{L}-F\left(s^{*}\right)}{\alpha-s^{*}}>\frac{\overline{L}}{\alpha}\Rightarrow\alpha F\left(s^{*}\right)<s^{*}\overline{L}\Rightarrow\frac{\alpha}{\alpha-s^{*}}<\overline{L}, \tag{A.49}$$

which holds since at $\overline{L}$ monopolists make a profit (i.e., $y(s^{*},\overline{L})=\frac{\overline{L}-F(s^{*})}{\alpha-s^{*}}>\frac{\overline{L}}{\alpha}=y(0,\overline{L})$) and from (A.17),

$$\frac{1}{2}\overline{L}+\frac{1}{2}L^{T}(s^{*})=\frac{\alpha+s^{*}}{\alpha-s^{*}}\Rightarrow\overline{L}>\frac{\alpha+s^{*}}{\alpha-s^{*}}>\frac{\alpha}{\alpha-s^{*}}. \tag{A.50}$$

From (A.47) and (A.48), $\Psi(0)>\Psi(s^{*})$, and therefore, $\Psi'(\widehat{s})<0$ for some $\widehat{s}<s^{*}$. Notice also that

$$\begin{aligned}
\frac{\partial\Psi(s_q)}{\partial s_q} &= \frac{1}{2}\frac{s_q^2+\alpha\left(1-\beta-\alpha\right)}{(1-\alpha-\beta+s_q)^2(\alpha-s_q)^2}-\frac{2\alpha}{(\alpha-s_q)^3} && \text{(A.51)}\\
&= \frac{\left(s_q^2+\alpha\left(1-\beta-\alpha\right)\right)\left(\alpha-s_q\right)-4\alpha(1-\alpha-\beta+s_q)^2}{2(1-\alpha-\beta+s_q)^2(\alpha-s_q)^2} && \text{(A.52)}\\
&= \frac{-s_q^3-3\alpha s_q^2-9\alpha\left(1-\beta-\alpha\right)s_q+9\alpha^2\left(1-\beta\right)-4\alpha\left(1-\beta\right)^2-5\alpha^3}{2(1-\alpha-\beta+s_q)^2(\alpha-s_q)^2} && \text{(A.53)}
\end{aligned}$$

which implies that if $\Psi'(\widehat{s})<0$ then $\Psi'(s_q)<0$ for all $s_q>\widehat{s}$, and therefore, that $\Psi(s^{*})>\Psi(s_q)$ for all $s_q>s^{*}$. Since $\Psi(s^{*})>\Psi(s_q)$ for all $s_q>s^{*}$, from (A.47) it follows that

$$sign(\text{first derivative})=sign\left\{\frac{1}{2}\frac{\overline{L}-F\left(s^{*}\right)}{\alpha-s^{*}}+\Psi(s_q)\right\}<0\text{ for all }s_q>s^{*}, \tag{A.54}$$

and therefore, there are no incentives to deviate to an $s_q>s^{*}$ such that $(\alpha-s^{*})\frac{F(s_q)}{1-\alpha-\beta+s_q}+F(s^{*})>L^{T}(s^{*})$. *Q.E.D.*

## Proof of Proposition 3

The social planner maximizes over

$$s^{opt}\in\underset{s}{\operatorname{argmax}}\int_{\min\{L^{T}(s),\overline{L}\})}^{\overline{L}}(1-\alpha-\beta+s)\frac{L-F(s)}{\alpha-s}-F(s)\;dL \tag{A.55}$$

Since at the optimum $L^T(s) < \overline{L}$, the first-order derivative writes

$$\int_{L^T(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s)\, dL + (1 - \alpha - \beta + s) \int_{L^T(s)}^{\overline{L}} \frac{\partial}{\partial s} \left( \frac{L - F(s)}{\alpha - s} \right) dL \tag{A.56}$$
$$- \left[ (1 - \alpha - \beta + s) \frac{L^T(s) - F(s)}{\alpha - s} - F(s) \right] \frac{\partial L^T(s)}{\partial s}$$

Consider the three above terms in turn

- We have shown in the proof of Proposition 2 (local conditions) that $\int_{L^T(s)}^{\overline{L}} \frac{L-F(s)}{\alpha-s} - F'(s)$ is 0 at $s^*$ and strictly negative for $s > s^*$.

- Next,

$$\int_{L^T(s)}^{\overline{L}} \frac{\partial}{\partial s} \left( \frac{L - F(s)}{\alpha - s} \right) = \frac{1}{\alpha - s} \int_{L^T(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s), \tag{A.57}$$

  which, as above, is 0 at $s^*$ and strictly negative for $s > s^*$.

- Finally, $\frac{\partial L^T(s)}{\partial s} > 0$ and, at the operating threshold, i.e., at $L = L^T(s)$, firms make a strictly positive profit:

$$(1 - \alpha - \beta + s) \frac{L^T(s) - F(s)}{\alpha - s} - F(s) > 0 \tag{A.58}$$

  for any $s$.

It follows, that first-order derivative in (A.56) is negative for any $s \geq s^*$, that is, in equilibrium there is excessive risk-taking. *Q.E.D.*

## Proof of Proposition 4

Assume that for any (common) leverage $s$, monopolists coordinate on the Pareto-superior equilibrium in the production game, that is, from (9) monopolists produce at $t = 1$ if and only if

$$L \geq L^+(s) \equiv (1 - \beta) \frac{F(s)}{1 - \alpha - \beta + s}.$$

Consider a deviation by monopolist $q$: monopolist $q$ chooses $s_q$ when all other monopolists choose leverage $s$. Monopolist $q$ produces if and only if

$$L \geq L^+(s_q, s) \equiv \begin{cases} \min\left\{\dfrac{F(s_q)}{1-\alpha-\beta+s_q}, L^+(s)\right\} & \text{if } s_q < s \\ (\alpha - s)\dfrac{F(s_q)}{1-\alpha-\beta+s_q} + F(s) & \text{if } s_q \geq s \end{cases} \tag{A.59}$$

The f.o.c. of $s_q$ when all other monopolists choose $s^*$ writes:

$$\frac{\partial}{\partial s_q} \int_{L^+(s_q,s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*}(1-\alpha-\beta+s_q) - F(s_q)\, dL \Big|_{s_q=s^*} = 0. \tag{A.60}$$

Since $L^+(s,s) = L^+(s)$, and for any $s$,

$$\frac{L^+(s) - F(s)}{\alpha - s}(1-\alpha-\beta+s) - F(s) = 0, \tag{A.61}$$

a marginal change in $s_q$ around $s^*$ only affects the integral in (A.60) through its integrand.

Therefore (A.60) writes:

$$\int_{L^+(s^*)}^{\overline{L}} \frac{L - F(s^*)}{\alpha - s^*} - F'(s^*)\, dL = 0. \tag{A.62}$$

$$\Leftrightarrow \left(\frac{\overline{L} + L^+(s^*)}{2} - F(s^*)\right)\frac{1}{\alpha - s^*} - F'(s^*) = 0 \tag{A.63}$$

$$\Leftrightarrow \overline{L} = 2(\alpha - s^*)F'(s^*) + \left(2 - \frac{1-\beta}{1-\alpha-\beta+s^*}\right)F(s^*) \tag{A.64}$$

If $s^* = 0$, the (A.64) is equal to 2. If $s^* \to \alpha$, the RHS of (A.64) tends to $+\infty$. It follows that if $\overline{L} > 2$, (A.64) has at least one solution. Rewrite, (A.64) as

$$\overline{L}\alpha = 2\alpha + s^* \underbrace{\left(\overline{L} + 2 - \frac{1-\beta}{1-\alpha-\beta+s^*}\right)}_{\equiv h(s^*)} \tag{A.65}$$

Since $h'(.) > 0$ and $\lim_{s\to\alpha} h(s) = \overline{L} + 1 > 0$, there exists $\hat{s}$ (possibly equal to 0) such that $h(s) > 0$ iff $s > \hat{s}$. $\overline{L} > 2$ implies that at $s = 0$, the RHS is lower than $2\alpha$ and therefore, $s \in [0, \hat{s})$ cannot be a solution to (A.65) since for $s \in [0, \hat{s})$, the RHS is also lower than $2\alpha$. If

$s > \hat{s}$, then $h(s) > 0$ and $h'(s) > 0$ imply that the RHS of (A.65) is strictly increasing. Therefore (A.65) has a unique solution. Hence, if there exists a symmetric equilibrium, this equilibrium is unique, interior, i.e., $s^* \in (0, \alpha)$, and defined by the f.o.c. in A.62.

As explained in the main text, maximizing consumers' expected utility amounts to maximizing their expected income from corporate profits. Therefore, the social planner's problem maximizes

$$\max_s \frac{1}{\overline{L}} \int_{\min\{L^+(s), \overline{L}\}}^{\overline{L}} y(s, L)(1 - \alpha - \beta + s) - F(s)\, dL, \tag{A.66}$$

where, as earlier, $y(s, L) = \frac{L - F(s)}{\alpha - s}$. Since at the social optimum $s^{**}$, $L^+(s^{**}) < \overline{L}$, and using (A.61) again, the f.o.c of (A.66) writes

$$\frac{1}{\overline{L}} \int_{L^+(s^{**})}^{\overline{L}} \frac{\partial y(s^{**}, L)}{\partial s}(1 - \alpha - \beta + s^{**}) + y(s^{**}, L) - F'(s^{**})\, dL = 0 \tag{A.67}$$

$$\Leftrightarrow \int_{L^+(s^{**})}^{\overline{L}} \frac{-F'(s^{**}) + y(s^{**}, L)}{(\alpha - s^{**})}(1 - \alpha - \beta + s^{**}) + y(s^{**}, L) - F'(s^{**})\, dL = 0 \tag{A.68}$$

$$\Leftrightarrow \int_{L^+(s^{**})}^{\overline{L}} y(s^{**}, L) - F'(s^{**})\, dL = 0 \tag{A.69}$$

Therefore (A.69) holds at $s^{**} = s^*$, i.e., the equilibrium leverage satisfies the first-order condition of the social planner. Notice also that (i) as $s \to \alpha$ then $L^+(s) \to +\infty$ and the objective function in (A.66)$\to 0$; and (ii) if $\overline{L} > 2$, the first derivative (see A.69) evaluated at $s^{**} = 0$ is positive:

$$sign\left\{\int_{L^+(0)}^{\overline{L}} y(0, L) - F'(0)\, dL\right\} = sign\left\{\int_0^{\overline{L}} \frac{L}{\alpha} - \frac{1}{\alpha} dL\right\} = sign\left\{\frac{\overline{L}}{2} - 1\right\} > 0, \tag{A.70}$$

then there is at interior optimum $s^{**} \in (0, \alpha)$, which needs to satisfy f.o.c. (A.69). Since (A.62) and coincide (A.69), and since we showed above that there is a unique interior solution, $s^*$, to (A.62), it follows that $s^{**} = s^*$. In words, if there is a symmetric equilibrium, it is unique and maximizes welfare. *Q.E.D.*

## Proof of Proposition 5

Define $\beta' \equiv \frac{\beta}{1-p}$. Monopolist $q$ profits are

$$\pi_q = (1 - \beta' - \alpha + s_q)\, y - F(s_q). \tag{A.71}$$

Given a set $\mathcal{A} \subset [0,1]$ of active monopolists, aggregate income at $t=1$ is the sum of profits and labor income,

$$y = \int_{q\in\mathcal{A}} (1-\beta'-\alpha+s_q)\, y - F(s_q)\ dq + L + \int_{q\in\mathcal{A}} \beta' y\ dq. \tag{A.72}$$

From (A.72), monitoring does not affect aggregate income $y$ for a given set of active monopolists $\mathcal{A}$: monitoring reduces the profit of each active monopolist by $\beta' y$, but increases labor income by the same amount. Hence, the expression for $y$ obtained from (A.72) is as in (equation 4),

$$y = \frac{L - \int_{q\in\mathcal{A}} F(s_q)\ dq}{1 - \int_{q\in\mathcal{A}} 1-\alpha+s_q\ dq}. \tag{A.73}$$

Consider the case in which all monopolists choose the same operating leverage $s$ at $t=0$. Then

combining (A.71) and (A.72), a monopolist does not operate if no other monopolist operates (if $\mathcal{A} = \{\emptyset\}$) when

$$L < \frac{F(s)}{1-\alpha-\beta'+s}. \tag{A.74}$$

Conversely, a monopolist operates if all other monopolists operate (if $\mathcal{A} = [0,1]$) when

$$L > \frac{(1-\beta')\,F(s)}{1-\alpha-\beta'+s}. \tag{A.75}$$

Therefore, if

$$L \in \left(\frac{(1-\beta')F(s)}{1-\alpha-\beta'+s}, \frac{F(s)}{1-\alpha-\beta'+s}\right), \tag{A.76}$$

Notice that (3) (8) (9) and (10) are identical to (A.71) (A.74) (A.75) and (A.76) are identical except that the first four equations depend on a parameter $\beta$ and the last four of parameter $\beta'$. Therefore, an equivalent to Proposition 1 still holds, that is, if monopolists choose $s$, they operate at $t=1$ if $L \geq L^T_{\beta'}(s)$ where

$$L^T_{\beta'}(s) \equiv \frac{F(s)}{1-\alpha+s} + \frac{\beta' F(s)}{(1-\alpha-\beta'+s)\ln\left(\frac{1}{\alpha-s}\right)}. \tag{A.77}$$

Therefore, aggregate income is then given by

$$y_{\beta'}(s,L) \equiv \begin{cases} \frac{L-F(s)}{\alpha-s} & \text{if } L \geq L^T_{\beta'}(s) \\ L & \text{if } L < L^T_{\beta'}(s) \end{cases}. \tag{A.78}$$

Suppose monopolist $q$ chooses operating leverage $s_q$ while all other monopolists choose $s$. Monopolist $q$ produces when it makes a profit, that is, when

$$(1-\alpha-\beta'+s_q)\, y_{\beta'}(s,L) - F(s_q) \geq 0, \tag{A.79}$$

or equivalently, when

$$L \geq \hat{L}_{\beta'}(s_q,s) \equiv \begin{cases} \max\left\{(\alpha-s)\dfrac{F(s_q)}{1-\alpha-\beta'+s_q} + F(s), L^T_{\beta'}(s)\right\} & \text{if } s_q \geq s, \\ \min\left\{\dfrac{F(s_q)}{1-\alpha-\beta'+s_q}, L^T_{\beta'}(s)\right\} & \text{if } s_q < s. \end{cases} \tag{A.80}$$

Then $s^*_{\beta'}$ is an equilibrium if and only if

$$s^*_{\beta'} \in \underset{s_q}{\operatorname{argmax}} \int_{\min\{\hat{L}_{\beta'}(s_q,s^*_{\beta'}),\overline{L}\}}^{\overline{L}} (1-\alpha-\beta+s_q)\, y_{\beta'}(s^*_{\beta'},L) - F(s_q)\; dL. \tag{A.81}$$

Notice that (17) and (A.81) identical except that the first equation depend on a parameter $\beta$ and the second equation depends on parameter $\beta'$. Therefore, one can show that an equivalent to Proposition 2 holds: There exists a unique symmetric equilibrium $s^*_{\beta'}$ defined by

$$\mathbb{E}\left[y|L \geq L^T_{\beta'}(s^*_{\beta'})\right] - F'\left(s^*_{\beta'}\right) = 0. \tag{A.82}$$

The social planer chooses leverage at $t=0$ but cannot avoid coordination failures at $t=1$(hence, it is a *constrained* social optimum):

$$\begin{aligned} s^{opt}_{\beta'} \;\in\; & \underset{s}{\operatorname{argmax}} \int_{\min\{L^T_{\beta'}(s),\overline{L}\}}^{\overline{L}} \left(1-\alpha-\beta'+s\right) y_{\beta'}(s,L) - F(s)\; dL + \\ & + \int_{\min\{L^T_{\beta'}(s),\overline{L}\}}^{\overline{L}} \left(\beta'-\beta\right) y_{\beta'}(s,L)\; dL. \end{aligned} \tag{A.83}$$

Notice that the social planer's optimization problem in (A.83) is different from (19). In (19) the extra labor income $\beta y$ earned by supervisors in states in which monopolists are active merely compensates them for their effort cost and therefore, labor income net of monitoring disutility always sums up to $L$. Consequently, in (19) maximizing consumers' expected utility amounts to maximizing their expected income from corporate profits. However, in (A.83) the extra labor income $\beta' y$ earned by supervisors in states in which monopolists are active more than compensates them for their effort cost $\beta y$. Consequently, in (A.83) maximizing consumers' expected utility amounts to maximizing the sum of the expected income from corporate profits plus the expected supervisors' rent.

Since at the optimum $L^T_{\beta'}(s) < \overline{L}$, the first-order derivative of (A.83) writes

$$\begin{aligned}
&\int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s)\, dL + (1 - \alpha - \beta' + s) \int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{\partial}{\partial s}\left(\frac{L - F(s)}{\alpha - s}\right)\, dL \\
&- \left[(1 - \alpha - \beta' + s)\frac{L^T_{\beta'}(s) - F(s)}{\alpha - s} - F(s)\right] \frac{\partial L^T_{\beta'}(s)}{\partial s} \\
&+(\beta - \beta') \int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{\partial}{\partial s}\left(\frac{L - F(s)}{\alpha - s}\right)\, dL \qquad \text{(A.84)} \\
&- \left[(\beta - \beta')\frac{L^T_{\beta'}(s) - F(s)}{\alpha - s})\right] \frac{\partial L^T_{\beta'}(s)}{\partial s}
\end{aligned}$$

which can be rewritten as

$$\int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s)\, dL + (1 - \alpha - \beta + s) \int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{\partial}{\partial s}\left(\frac{L - F(s)}{\alpha - s}\right)\, dL \qquad \text{(A.85)}$$

$$- \left[(1 - \alpha - \beta + s)\frac{L^T_{\beta'}(s) - F(s)}{\alpha - s} - F(s)\right] \frac{\partial L^T_{\beta'}(s)}{\partial s} \qquad \text{(A.86)}$$

Consider the above three terms in turn:

- Similarly to what we showed in the proof of Proposition 2, one can show that $\int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{L-F(s)}{\alpha-s} - F'(s)$ is 0 at $s^*_{\beta'}$ and strictly negative for $s > s^*_{\beta'}$.

- Next,

$$\int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{\partial}{\partial s}\left(\frac{L - F(s)}{\alpha - s}\right) = \frac{1}{\alpha - s} \int_{L^T_{\beta'}(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s), \qquad \text{(A.87)}$$

  which, as above, is 0 at $s^*_{\beta'}$ and strictly negative for $s > s^*_{\beta'}$.

- Finally, $\frac{\partial L^T_{\beta'}(s)}{\partial s} > 0$ and, at the operating threshold, i.e., at $L = L^T_{\beta'}(s)$, firms make a strictly positive profit:

$$(1 - \alpha - \beta + s)\frac{L^T_{\beta'}(s) - F(s)}{\alpha - s} > (1 - \alpha - \beta' + s)\frac{L^T_{\beta'}(s) - F(s)}{\alpha - s} - F(s) > 0 \qquad \text{(A.88)}$$

  for any $s$.

It follows, that first-order derivative in (A.86) is negative for any $s \geq s^*_{\beta'}$, that is, in equilibrium there is excessive risk-taking.

Similarly to equation (A.20), which defines $s^*$, the following equation defines $s^*_{\beta'}$:

$$\alpha\overline{L} = 2\alpha + s^*_{\beta'} \underbrace{\left( 2 + \overline{L} - \frac{1}{1-\alpha+s^*_{\beta'}} - \frac{\beta'}{(1-\alpha-\beta'+s^*_{\beta'})\ln\left(\frac{1}{\alpha-s^*_{\beta'}}\right)} \right)}_{\equiv f(s^*_{\beta'})} \tag{A.89}$$

Since, $\frac{\beta'}{1-\alpha-\beta'+s}$ is increasing in $\beta'$ for $1-\alpha+s$ and since $\beta'$ is increasing in $p$, then the RHS is decreasing in $p$ given $s^*_{\beta'}$. Since $f'(.) > 0$ the RHS is increasing in $s^*_{\beta'}$ given $\beta'$ (i.e., given $p$). Therefore, $s^*_{\beta'}$ is increasing in $p$.

Let $W(s,p)$ be the welfare function where (see A.83)

$$W(s,p) \equiv \int_{\min\{L^T_{\beta'}(s),\overline{L}\}}^{\overline{L}} (1-\alpha-\beta+s)\, y_{\beta'}(s,L) - F(s)\; dL \tag{A.90}$$

Since $L^T_{\beta'}(s)$ depends on $\beta'$ for any $s$ (see equation A.77) and since $s^*_{\beta'}$ depends on $\beta'$ (see equation A.89) then:

$$\begin{aligned}
\frac{dW(s^*_{\beta'},p)}{dp} &= \frac{\partial W(s^*_{\beta'},p)}{\partial s^*_{\beta'}}\frac{\partial s^*_{\beta'}}{\partial p} + \frac{\partial W(s^*_{\beta'},p)}{\partial p} && \text{(A.91)}\\
&= \frac{\partial W(s^*_{\beta'},p)}{\partial s^*_{\beta'}}\frac{\partial s^*_{\beta'}}{\partial p} && \text{(A.92)}\\
&\quad - \left[\left(1-\alpha-\beta+s^*_{\beta'}\right) y_{\beta'}(s^*_{\beta'}, L^T_{\beta'}(s^*_{\beta'})) - F(s^*_{\beta'})\right] \left.\frac{\partial L^T_{\beta'}(s)}{\partial p}\right|_{s=s^*_{\beta'}}
\end{aligned}$$

where we have used the fact that $\min\{L^T_{\beta'}(s^*_{\beta'}),\overline{L}\} = L^T_{\beta'}(s^*_{\beta'})$, i.e., monopolists will not choose an operating leverage that never allows them to produce.

Note that (i) we have proved that $\left.\frac{\partial W'(s,p)}{\partial s}\right|_{s=s^*_{\beta'}} < 0$ (i.e., the first-order derivative in (A.86) is negative for any $s \geq s^*_{\beta}$) and that $s^*_{\beta'}$ is increasing in $p$, therefore, the first term in the RHS in (A.92) is negative; (ii) at the threshold, $L^T_{\beta'}(s^*_{\beta'})$, monopolists make a profit and since $\beta < \beta'$, this means that

$$\left(1-\alpha-\beta+s^*_{\beta'}\right) y_{\beta'}(s^*_{\beta'}, L^T_{\beta'}(s^*_{\beta'})) - F(s^*_{\beta'}) > 0; \tag{A.93}$$

and (iii) for $1-\alpha+s>0$, $L^T_{\beta'}(.)$ is increasing in $p$ given $s$ (i.e., $\frac{\beta'}{1-\alpha-\beta'+s}$ is increasing in $p$). (i), (ii), and (iii) imply that

$$\frac{dW(s^*_{\beta'},p)}{dp}<0, \tag{A.94}$$

that is, welfare decreases in $p$. *Q.E.D*

## Proof of Proposition 6

Assume without loss of generality throughout the proof that in equilibrium $L^T_1 < L^T_2$, that is, for $L \in [L^T_1(s^*_1), L^T_2(s^*_1,s^*_2))$ only type-1 monopolists operate while for $L \in [L^T_2(s^*_1,s^*_2),\overline{L}]$ both types operate. (See the NOTE at the end of the proof.)

- If both types of monopolists operate:

$$\text{Income: } y(s_1,s_2,L) \equiv \frac{L-n_1F(s_1)-n_2F(s_2)}{1-n_1(1-\alpha-\Delta_1+s_1)-n_2(1-\alpha-\Delta_2+s_2)} \tag{A.95}$$

$$\text{Profits of type-}i\text{ firms: } \pi_i(s_1,s_2,L) \equiv y(s_1,s_2,L)(1-\alpha-\Delta_i+s_i)-F(s_i) \tag{A.96}$$

  The operating threshold for type-2 monopolists if type-1 monopolists operate, $L^T_2(s_1,s_2)$, satisfies:

$$\pi_2(s_1,s_2,L^T_2(s_1,s_2))=0 \tag{A.97}$$

- If only type-1 monopolists operate:

$$\text{Income: } y(s_1,L) \equiv \frac{L-n_1F(s_1)}{1-n_1(1-\alpha-\Delta_1+s_1)} \tag{A.98}$$

$$\text{Profits of type-}i\text{ firm: } \pi_i(s_1,L) = \frac{L-n_1F(s_1)}{1-n_1(1-\alpha-\Delta_1+s_1)}(1-\alpha-\Delta_i+s_i)-F(s_i) \tag{A.99}$$

  The operating threshold for type-1 monopolists if type-2 monopolists do not operate, $L^T_1(s_1)$, satisfies:

$$\pi_1(s_1,L^T_1(s_1))=0 \tag{A.100}$$

**Claim 1** *Let* $(s_1^*, s_2^*)$ *be a symmetric equilibrium in which all type-*1 *monopolists choose* $s_1^*$ *and all type-*2 *monopolists choose* $s_2^*$ *then*

$$E[y|L > \ L_1^T(s_1^*)] - F'(s_1^*) = 0 \tag{A.101}$$

$$E[y|L > \ L_2^T(s_1^*, s_2^*)] - F'(s_2^*) = 0. \tag{A.102}$$

*Proof of Claim 1*:

Consider the local incentives to deviate of a type-1 monopolist $q$ taking the operating leverage of all the other monopolists as given.

- Consider first, the local incentives to deviate to $s_q < s_1^*$. The profit for type-1 monopolist $q$ writes:

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L)\,(1-\alpha-\Delta_1+s_q) - F(s_q)dL \tag{A.103}$$
$$+ \int_{L_1^T(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*, L)\,(1-\alpha-\Delta_1+s_q) - F(s_q)dL$$
$$+ \int_{\frac{F(s_q)}{1-\alpha-\Delta_1+s_q}}^{L_1^T(s_1^*)} L\,(1-\alpha-\Delta_1+s_q) - F(s_q)dL.$$

  The first derivative of (A.103) w.r.t. $s_q$ writes:

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L) - F'(s_q)dL \tag{A.104}$$
$$+ \int_{L_1^T(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*, L) - F'(s_q)dL + \int_{\frac{F(s_q)}{1-\alpha-\Delta_1+s_q}}^{L_1^T(s_1^*)} L - F'(s_q)dL$$

  For $s_1^*$ to be an equilibrium, it must hold that at $s_q = s_1^*$ the f.o.c. in (A.104) must be greater or equal to zero, that is, since $L_1^T(s_1^*) = \frac{F(s_1^*)}{1-\alpha-\Delta_1+s_1^*}$:

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*, L) - F'(s_1^*)dL + \int_{L_1^T(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*, L) - F'(s_1^*)dL \geq 0 \tag{A.105}$$
$$\Leftrightarrow E[y|L \geq L_1^T(s_1^*)] - F'(s_1^*) \geq 0. \tag{A.106}$$

- Consider now, the local incentives to deviate to $s_q > s_1^*$. The profit for type-1 monopolist

$q$ writes:

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*,L)\,(1-\alpha-\Delta_1+s_q)-F(s_q)dL \tag{A.107}$$
$$+\int_{\frac{1-n_1(1-\alpha-\Delta_1+s_1^*)}{1-\alpha-\Delta_1+s_q}F(s_q)+n_1F(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*,L)\,(1-\alpha-\Delta_1+s_q)-F(s_q)dL$$

The first derivative of (A.107) w.r.t. $s_q$ writes:

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*,L)-F'(s_q)dL \tag{A.108}$$
$$+\int_{\frac{1-n_1(1-\alpha-\Delta_1+s_1^*)}{1-\alpha-\Delta_1+s_q}F(s_q)+n_1F(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*,L)-F'(s_q)dL$$

For $s_1^*$ to be an equilibrium, it must hold that at $s_q=s_1^*$ the f.o.c. in (A.108) must be smaller or equal to zero, that is, since

$$\left.\frac{1-n_1(1-\alpha-\Delta_1+s_1^*)}{1-\alpha-\Delta_1+s_q}F(s_q)+n_1F(s_1^*)\right|_{s_q=s_1^*}=L_1^T(s_1^*)=\frac{F(s_1^*)}{1-\alpha-\Delta_1+s_1^*} \tag{A.109}$$

then,

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*,s_2^*,L)-F'(s_1^*)dL+\int_{L_1^T(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*,L)-F'(s_1^*)dL\leq 0 \tag{A.110}$$
$$\Leftrightarrow E[y|L\geq L_1^T(s_1^*)]-F'(s_1^*)\leq 0 \tag{A.111}$$

Combining the two necessary conditions (A.106) and (A.111):

$$E[y|L>L_1^T(s_1^*)]-F'(s_1^*)=0 \tag{A.112}$$

A similar argument yields the a necessary condition for a symmetric equilibrium for type-2 monopolists:

$$E[y|L>L_2^T(s_1^*,s_2^*)]-F'(s_2^*)=0 \tag{A.113}$$

This completes the proof of the Claim *1.*

Assuming that $L_1^T(s_1) < L_2^T(s_1,s_2)$, the social planner chooses $s_1$ and $s_2$ to maximize expected profits plus labor income, that is, to maximize $W(s_1,s_2)$ where

$$
\begin{aligned}
W(s_1,s_2) \equiv\, & n_1 \int_{L_2^T(s_1,s_2)}^{\overline{L}} y(s_1,s_2)\left(1-\alpha-\Delta_1+s_1\right) - F(s_1)dL \\
& + n_2 \int_{L_2^T(s_1,s_2)}^{\overline{L}} y(s_1,s_2)\left(1-\alpha-\Delta_2+s_2\right) - F(s_2)dL \\
& + n_1 \int_{L_1^T(s_1)}^{L_2^T(s_1,s_2)} y(s_1)\left(1-\alpha-\Delta_1+s_1\right) - F(s_1)dL + \int_0^{\overline{L}} LdL
\end{aligned}
\tag{A.114}
$$

The first derivatives of $W(s_1,s_2)$ write:

$$
\begin{aligned}
\frac{\partial W(s_1,s_2)}{\partial s_1} =\, & n_1 \int_{L_2^T(s_1,s_2)}^{\overline{L}} y(s_1,s_2) - F'(s_1)dL \\
& + \left[n_1\left(1-\alpha-\Delta_1+s_1\right) + n_2\left(1-\alpha-\Delta_2+s_2\right)\right] \int_{L_2^T(s_1,s_2)}^{\overline{L}} \frac{\partial y(s_1,s_2)}{\partial s_1} dL \\
& + n_1 \int_{L_1^T(s_1)}^{L_2^T(s_1,s_2)} y(s_1) - F'(s_1)dL \\
& + n_1 \int_{L_1^T(s_1)}^{L_2^T(s_1,s_2)} \frac{\partial y(s_1)}{\partial s_1}\left(1-\alpha-\Delta_1+s_1\right)dL
\end{aligned}
\tag{A.115}
$$

and

$$
\begin{aligned}
\frac{\partial W(s_1,s_2)}{\partial s_1} \;=\;\; & n_2 \int_{L_2^T(s_1,s_2)}^{\overline{L}} y(s_1,s_2) - F'(s_2)dL \\
& + \left[n_1\left(1-\alpha-\Delta_1+s_1\right) + n_2\left(1-\alpha-\Delta_2+s_2\right)\right] \int_{L_2^T(s_1,s_2)}^{\overline{L}} \frac{\partial y(s_1,s_2)}{\partial s_2} dL.
\end{aligned}
\tag{A.116}
$$

From (A.95),

$$
\frac{\partial y(s_1,s_2)}{\partial s_i} = \frac{n_i[y(s_1,s_2) - F'(s_i)]}{1-n_1(1-\alpha-\Delta_1+s_1) - n_2(1-\alpha-\Delta_2+s_2)} \text{ for } i=1,2 \tag{A.117}
$$

and from (A.98),

$$
\frac{\partial y(s_1)}{\partial s_1} = \frac{n_1[y(s_1) - F'(s_1)]}{1-n_1(1-\alpha-\Delta_1+s_1)}. \tag{A.118}
$$

Therefore, the first derivatives can be rewritten as

$$
\frac{\partial W(s_1,s_2)}{\partial s_1} = \frac{n_1 \int_{L_2^T(s_1,s_2)}^{\overline{L}} y(s_1,s_2) - F'(s_1)dL}{1-n_1(1-\alpha-\Delta_1+s_1) - n_2(1-\alpha-\Delta_2+s_2)} + \frac{n_1 \int_{L_1^T(s_1)}^{L_2^T(s_1,s_2)} y(s_1) - F'(s_1)dL}{1-n_1(1-\alpha-\Delta_1+s_1)} \tag{A.119}
$$

and

$$\frac{\partial W(s_1, s_2)}{\partial s_2} = \frac{n_2 \int_{L_2^T(s_1,s_2)}^{\overline{L}} y(s_1, s_2) - F'(s_2)dL}{1 - n_1(1 - \alpha - \Delta_1 + s_1) - n_2(1 - \alpha - \Delta_2 + s_2)} \tag{A.120}$$

From Claim (**1**), at $(s_1, s_2) = (s_1^*, s_2^*)$,

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*) - F'(s_1^*)dL + \int_{L_1^T(s_1^*)}^{L_2^T(s_1^*,s_2^*)} y(s_1^*) - F'(s_1^*)dL = 0, \tag{A.121}$$

and using (A.119) and (A.121):

$$\left.\frac{\partial W(s_1, s_2)}{\partial s_1}\right|_{(s_1,s_2)=(s_1^*,s_2^*)} = \frac{n_1 \int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*) - F'(s_1^*)dL}{1 - n_1(1 - \alpha - \Delta_1 + s_1) - n_2(1 - \alpha - \Delta_2 + s_2)}$$

$$-\frac{n_1 \int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*) - F'(s_1^*)dL}{1 - n_1(1 - \alpha - \Delta_1 + s_1)} > 0 \tag{A.122}$$

In words, at $(s_1, s_2) = (s_1^*, s_2^*)$, increasing $s_1$ increases welfare.

From Claim (**1**), at $(s_1, s_2) = (s_1^*, s_2^*)$,

$$\int_{L_2^T(s_1^*,s_2^*)}^{\overline{L}} y(s_1^*, s_2^*) - F'(s_2^*)dL = 0, \tag{A.123}$$

and using (A.119) and (A.121):

$$\left.\frac{\partial W(s_1, s_2)}{\partial s_2}\right|_{(s_1,s_2)=(s_1^*,s_2^*)} = 0 \tag{A.124}$$

In words, at $(s_1, s_2) = (s_1^*, s_2^*)$, a marginal change in $s_2$ does not change welfare.

NOTE: Since $\overline{L} > 2/\alpha$, $s_i^* \in (0, \alpha)$, which implies that $(s_1^*, s_2^*)$ are such that $L_1^T = L_2^T$ cannot be an equilibrium. Indeed, Any equilibrium requires

$$E[y|L > L_2^T] - F'(s_2^*) = 0 \tag{A.125}$$

$$E[y|L > L_1^T] - F'(s_1^*) = 0 \tag{A.126}$$

and, if $L_1^T = L_2^T$, (A.125) and (A.126) imply that $s_1^* = s_2^* (= s^*)$. However, since $\Delta_1 \neq \Delta_2$, $L_1^T(s^*) \neq L_2^T(s^*)$, that is, one type of monopolists would have incentive to start producing for a lower threshold than the other type of monopolists. *Q.E.D.*

## Proof of Proposition 7

**Part I: Local Necessary Equilibrium Condition.**

Note first, that the Proof of Proposition **1** does not make use of the specific functional form for $F(s_q)$ and therefore, the operating threshold $L^T(s)$ in Proposition **1** holds for a generic function $F(s_q)$.

Suppose monopolist $q$ chooses $s_q$ while all other monopolists choose $s^* \in (0, \alpha)$. If $s_q = s^*$ then, at $L = L^T(s^*)$ monopolist $q$ makes a strictly positive profit if all other monopolist operate and makes a strictly negative profit if no other monopolist operates. This implies that we can define an interval $[s^* - \tau, s^* + \tau]$ for some $\tau > 0$ such that if $s_q \in [s^* - \tau, s^* + \tau]$ monopolist $q$ produces if and only if $L \geq L^T(s^*)$. Local optimality requires that

$$s^* \in \underset{s_q \in [s^*-\tau, s^*+\tau]}{\text{argmax}} \int_{L^T(s^*)}^{\overline{L}} (1 - \alpha - \beta + s_q)\, y(s^*, L) - F(s_q)\ dL, \tag{A.127}$$

and therefore, the local necessary condition for an interior solution to the monopolist optimization problem writes

$$E\left[y|L \geq L^T(s^*)\right] - F'\left(s^*\right) = 0. \tag{A.128}$$

**Part II: Excessive Operating Leverage.**

The social planner maximizes over

$$s^{opt} \in \underset{s}{\text{argmax}} \int_{\min\{L^T(s), \overline{L}\})}^{\overline{L}} (1 - \alpha - \beta + s)\frac{L - F(s)}{\alpha - s} - F(s)\ dL \tag{A.129}$$

Since at the optimum $L^T(s) < \overline{L}$, the first-order derivative writes

$$\begin{aligned} &\int_{L^T(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s)\ dL + (1 - \alpha - \beta + s)\int_{L^T(s)}^{\overline{L}} \frac{\partial}{\partial s}\left(\frac{L - F(s)}{\alpha - s}\right)\ dL \\ &- \left[(1 - \alpha - \beta + s)\frac{L^T(s) - F(s)}{\alpha - s} - F(s)\right]\frac{\partial L^T(s)}{\partial s} \end{aligned} \tag{A.130}$$

Consider the three above terms in turn

- The local necessary condition in (A.128) implies that $\int_{L^T(s)}^{\overline{L}} \frac{L-F(s)}{\alpha-s} - F'(s)$ is 0 at any interior symmetric equilibrium $s^*$.

- Next,

$$\int_{L^T(s)}^{\overline{L}} \frac{\partial}{\partial s}\left(\frac{L - F(s)}{\alpha - s}\right) = \frac{1}{\alpha - s}\int_{L^T(s)}^{\overline{L}} \frac{L - F(s)}{\alpha - s} - F'(s), \tag{A.131}$$

  which, as above, is 0 at $s^*$.

- Finally, at the operating threshold, i.e., at $L = L^T(s)$, firms make a strictly positive profit:

$$(1 - \alpha - \beta + s)\frac{L^T(s) - F(s)}{\alpha - s} - F(s) > 0 \tag{A.132}$$

  for any $s$. Therefore, if $\frac{\partial L^T(s^*)}{\partial s} > 0$ , the first-order derivative in (A.130) is strictly negative at $s = s^*$, that is, a marginal decrease in $s$ around $s^*$ increases welfare.

To complete the proof, we next show $\frac{\partial L^T(s^*)}{\partial s} > 0$.

The local necessary equilibrium condition in (A.128) writes,

$$\frac{y(L^T(s^*), s^*) + y(\overline{L}, s^*)}{2} = F'(s^*) \tag{A.133}$$

Since $y(L, s^*)$ is increasing in $L$, it follows that

$$y(L^T(s^*), s^*) < F'(s^*) \tag{A.134}$$

From (A.5), $L^T(s^*)$ is implictly given by the indifference condition

$$\int_0^1 (1 - \alpha - \beta + s^*)\hat{y}(L^T(s^*), s^*, z) - F(s^*)\ dz = 0 \tag{A.135}$$

where

$$\hat{y}(L, s, z) \equiv \frac{L - zF(s)}{1 - z(1 - \alpha + s)}. \tag{A.136}$$

$\frac{\partial \hat{y}(L,s,z)}{\partial z}(L^T(s^*), s^*, .)$ has the sign of $(1 - \alpha + s^*)L^T(s^*) - F(s^*)$ which, from (13) is strictly positive. It follows that for any $z \in (0, 1)$,

$$\hat{y}(L^T(s^*), s^*, z) < \hat{y}(L^T(s^*), s^*, 1) = y(L^T(s^*), s^*) < F'(s^*) \tag{A.137}$$

where the last inequality just repeats (A.134).

Next, for any $z \in (0,1)$, $\frac{\partial \hat{y}(L,s,z)}{\partial s}(L^T(s^*), s^*, .)$ has the sign of $y(L^T(s^*), s^*, z) - F'(s^*)$ which, from (A.137), is strictly negative.

Finally, total differentiation of the indifference condition (A.135) with respect to $s^*$ yields

$$\begin{aligned} & \int_0^1 \hat{y}(L^T(s^*), s^*, z) - F'(s^*)\ dz \\ + & \int_0^1 (1-\alpha-\beta+s^*)\frac{\partial \hat{y}(L,s,z)}{\partial s}(L^T(s^*), s^*, .)\ dz \\ + & \int_0^1 (1-\alpha-\beta+s^*)\frac{\partial \hat{y}(L,s,z)}{\partial L}(L^T(s^*), s^*, .)\frac{\partial L^T(s^*)}{\partial s^*}\ dz = 0. \end{aligned} \tag{A.138}$$

(A.137) implies that the first line of this equation is strictly negative. We have shown $\frac{\partial \hat{y}(L,s,z)}{\partial s}(L^T(s^*), s^*, .) < 0$ for any $z \in (0,1)$ which implies that the second line is also negative. Finally $\frac{\partial \hat{y}(L,s,z)}{\partial L}(L^T(s^*), s^*, .) > 0$ for any $z \in (0,1)$. Therefore $\frac{\partial L^T(s^*)}{\partial s^*} > 0$, which concludes our proof.