

# FINANCE RESEARCH SEMINAR SUPPORTED BY UNIGESTION

## “Markets versus Mechanisms”

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### Abstract

We demonstrate limitations on the use of direct revelation mechanisms (DRMs) by corporations inhabiting economies with securities markets. Posting a standard DRM in an environment with a securities market endogenously increases the reservation value of the informed agent. If the informed agent rejects said DRM, then she convinces the market that she is uninformed, and she can trade aggressively sans price impact, generating large (off-equilibrium) trading gains. Due to this endogenous reservation value effect, using a DRM to screen out uninformed agents may be impossible. Even when screening is possible, refraining from posting a mechanism and instead relying on markets for information is optimal if the endogenous reservation value effect is sufficiently large. Finally, even if posting a DRM dominates relying on markets, outcomes are improved by introducing a search friction, which randomly limits the agent's ability to observe the DRM.

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# Markets vs. Mechanisms\*

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## Abstract

We demonstrate constraints on usage of direct revelation mechanisms (DRMs) by corporations inhabiting economies with securities markets. We consider a corporation seeking to acquire decision relevant information. Posting a standard DRM in an environment with a securities market endogenously increases the outside option of the informed agent. If the informed agent rejects said DRM, then she convinces the market that she is uninformed, and she can trade aggressively sans price impact, generating large (off-equilibrium) trading gains. Due to this endogenous outside option effect, using a DRM to screen out uninformed agents may be impossible. Even when screening is possible, refraining from posting a mechanism and instead relying on markets for information is optimal if the endogenous change in outside option value is sufficiently large. Finally, even if posting a DRM dominates relying on markets, outcomes are improved by introducing a search friction, which randomly limits the agent's ability to observe the DRM, forcing the firm to sometimes rely on markets for information.

Keywords: Market Microstructure, Mechanism Design

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“The greatest trick the devil ever pulled was convincing the world he didn’t exist.”

—Verbal Kint, *The Usual Suspects*

“The Devil’s greatest trick is to persuade you that he does not exist!”

—Charles Baudelaire, *The Generous Gambler*

## 1 INTRODUCTION

The provision of decision-relevant information to agents is critical for economic efficiency. Hayek (1945) extolled the virtues of markets in this regard, writing, “We must look at the price system as such a mechanism for communicating information...” More specifically, securities markets are commonly viewed as a vital source of information for firms making operating and real investment decisions. For example, Fama and Miller (1972) write, “(an efficient market) has a very desirable feature. In particular, at any point in time market prices of securities provide accurate signals for resource allocation...” Similarly, Fama (1976) writes, “An efficient capital market is an important component of a capitalist system... if the capital market is to function smoothly in allocating resources, prices of securities must be good indicators of value.”

Notwithstanding the ability of securities markets to convey information, economic theory would seem to suggest that firms have access to a superior source of information: direct revelation mechanisms (DRMs). After all, mechanism design theory informs us that any equilibrium outcome of some indirect revelation mechanism (IRM), can also be achieved by a DRM in which truth-telling is incentive compatible. Thus, one may view a firm using securities markets for information (denoted a *market-reliant* firm) as an IRM for eliciting information from some privately informed trader. But the revelation principle suggests that the firm can do at least as well by “hiring” said trader as part of a DRM. That is, rather than leaving the informed trader outside its boundaries, the firm can bring her inside and provide incentives through a DRM. In fact, economic theory points to another benefit to bringing an informed agent inside firm boundaries: insulation of uninformed shareholders from mispricing due to adverse selection. After all, under exclusivity (contractual or legal prohibitions on securities trading by insiders), an informed agent brought in-house cannot trade at the expense of uninformed shareholders who are forced to sell due to liquidity shocks.

Phrased differently, economic theory would seem to imply the existence of a Pareto-improving bargain to be struck between the market-reliant firm and the informed trader currently sitting outside its boundaries: The firm should compute what the trader is currently making in market gains and write a contract that pays her equal expected wages, aligns her interests with those of the firm, and screens out incompetents. Having done so, the firm should base corporate decisions directly on her (truthful) reports, as opposed to relying on securities prices. After all, market prices do not generally fully reveal the trader’s private information. In fact, it is the very noise in securities prices that is the underlying source of the informed outsider’s positive expected trading gains.

In this paper, we demonstrate an inherent limitation to the use of such contracts (DRMs)

by public corporations (“firms”) inhabiting economies with competitive securities markets. In particular, we show that for firms in such economies the set of feasible contracts may actually be empty. Moreover, even when the feasible set is non-empty, the firm may nevertheless find it optimal to refrain from posting a contract, instead relying exclusively on the securities market for information. Finally, we show that even in those instances where posting a contract dominates relying on the market, the firm can always achieve superior outcomes by stochastically limiting agents’ ability to observe the posted contract, and rely on markets for information when the contract is not observed.

In order to understand these results, it is important to first highlight one of the key properties of price formation in securities markets. When agents in the economy are aware that an informed agent may be trading in the securities market, a competitive market maker will set price in a way that is sensitive to order flow. In particular, when the informed agent trades, the price moves closer to the asset’s fundamental value, reducing the informed agent’s gains. Thus, the competitive securities market, and the price discipline it provides, naturally mitigates the informed agent’s rent.

Consider instead the nature of price discipline for a firm that opts to post a contract. In order to induce participation by the informed outsider, the firm must pay her an expected wage equal to her outside option—and this outside option value is equal to the expected trading gains she stands to capture if she foregoes the posted contract, leaving herself free to trade in the securities market. But note, since the posted contract is designed to induce participation by the informed agent, the market maker believes that the informed agent will accept the contract if she exists. Therefore, if the informed agent deviates, and leaves the contract sitting, the market maker will believe that no informed agent exists. Consequently, the market maker will not adjust the asset price in response to order flow, attributing observed orders to shareholders being hit by liquidity shocks. Thus, by foregoing the contract, the informed agent anticipates the ability to trade large volumes with zero price impact, dramatically increasing her outside option value. The key point here is that the very act of posting a contract fundamentally alters the nature of beliefs and price formation in the securities market—by rejecting a posted contract, the devil can convince the world she doesn’t exist.

The preceding paragraph illustrates starkly why Coasian bargaining breaks down, and why posting a contract may actually be self-defeating. In particular, once the firm posts the contract, the informed agent’s outside option value rises considerably since the disciplining effect of price impact vanishes. In some instances, the endogenous outside option value increases to the point where it is impossible to achieve the dual objectives of screening out incompetents and inducing informed participation. In order to screen out incompetents, the optimal contract rewards the agent when her advice proves correct and punishes the agent when her advice proves incorrect. When the outside option for the informed agent is sufficiently large, the reward that must be offered for correct advice is so large that even an uninformed agent is willing to accept the contract and make a guess, hoping to be right. In other instances, the endogenous increase in the outside option value may exceed the value provided by the expert’s information, which is the increase in

expected cash flow that naturally results when the firm bases its decision on an honest report from an informed insider rather than the noisy stock price. In this case, the outside option value (which is the expected wage bill under the contract) exceeds the combined benefit of correct production decisions along with the insulation of shareholders from the adverse selection costs associated with informed trading. In this case even if the mechanism is feasible, offering it is suboptimal. Our analysis completely characterizes the conditions under which it is optimal for the firm to refrain from posting a mechanism—instead relying exclusively on the securities market for information.

Intuitively, the analysis shows that market reliance dominates when the securities market, left to its own devices, provides powerful price discipline. This occurs when the probability that an informed agent exists is large and/or the probability of uninformed liquidity trading is low. Note, in this case, informed trading has a large price impact, which would vanish if the firm were to instead post a mechanism. Thus, precisely in this instance the endogenous increase in reservation value arising from posting the mechanism is largest, dominating the value of more accurate information provided by the mechanism.

We extend our analysis by next considering a firm that has access to a technology that can stochastically limit the informed agent's ability to observe the contract offer (e.g. scope of marketing). In particular, if the firm offers a contract, the agent observes the contract with a particular probability, which the firm is able to choose. Here we derive a complementary result: Even when parameters are such that the firm attains a higher value by offering the contract (with probability one) than when it does not (market-reliance), firm value is necessarily increased by introducing some limit on the informed agent's ability to observe the contract offer. Intuitively, if the agent does not always observe the contract, then the market maker can no longer be sure that a contract left sitting implies the absence of informed trading. Thus, even when the posted contract is rejected, the market maker will impose some degree of price discipline, which reduces the value of the informed agent's outside option. Due to this effect, starting from the case in which the contract is offered with probability one, firm value necessarily increases by limiting the informed agent's ability to observe the mechanism. The firm loses valuable information by doing so, but is more than compensated by the endogenous reduction in the agent's reservation value.

**Related literature.** With its focus on understanding the conditions under which corporations will, in equilibrium, rely on securities markets for information, our paper offers a novel contribution to diverse literatures assessing the efficiency of competitive markets and alternative allocative/incentive schemes. The key difference between our analytical framework and a large body of existing work is that we analyze the *interaction* between competitive financial markets and alternative allocative/incentive schemes. By way of contrast, Grossman and Stiglitz (1976) and Dow and Gorton (1997) analyze the allocative efficiency of financial markets when they do not interact with alternative information/incentive schemes. As such, their work is in the spirit of the comparative economic systems and mechanism design literatures, discussed below, both of which effectively put alternative incentive/allocative systems on different planets, with comparisons being performed.

In formal side-by-side comparisons, market-based information systems do not generally fare well,

creating a challenge for market advocates on the theory side, as well as a potential empirical puzzle regarding the apparent robustness of markets. For example, Lerner (1944), Taylor (1948), and Lange (1967) formally demonstrate the ability of centrally planned economies, left in isolation from market economies, to achieve equally efficient outcomes. More importantly, the revelation principle of Hurwicz (1973), Gibbard (1973), Hölmstrom (1979), Dasgupta, Hammond, and Maskin (1979), and Myerson (1979) informs us that indirect schemes, such as securities markets, cannot possibly achieve superior outcomes to a direct revelation mechanism when the latter is implemented in isolation.

Critically, our analysis departs from the standard mechanism design literature by analyzing the *interaction* between markets and mechanisms. In our model, all agents enjoy the option to trade in competitive financial markets, giving rise to endogenous outside option values for informed and uninformed agents. In this setting, we consider whether the principal will want to post a mechanism. In contrast, the mechanism design literature assumes that the principal has already decided to offer a mechanism and that the agent’s reservation value is identical for all feasible mechanisms that the principal may offer. By focusing on the design of optimal mechanisms under these assumptions, this literature abstracts away from any interaction between the mechanism and other institutions in the economy. Instead, we embed a corporation’s decision regarding whether to offer a mechanism within the broader institutional context of an economy with a securities market—fundamentally, in the setting we consider, it is the interaction between the mechanism and the stock market that generates an endogenous reservation value, limiting the mechanism’s feasibility and optimality.

Endogenous reservation values have been explored in a variety of other contexts. Tirole (2012) studies a government program to unfreeze markets plagued by adverse selection, where the decision not to participate in the program reveals information about the firm’s type to the market. Lizzeri (1999) develops a model of ratings in which the seller’s decision to avoid certification conveys information to potential buyers. A fundamental difference exists between our setting and other models with endogenous reservation values: mechanisms in other settings provide useful information which enhances the efficiency of the market. In our setting the market and the mechanism are fundamentally substitute sources of information. Thus, the central choice we analyze, markets versus mechanisms, has no analog in the existing endogenous reservation value literature. Jehiel and Moldovanu (2000) consider auctions in which agents’ values for winning and losing depend on their subsequent market interactions. Other mechanism design literature focuses on type-dependent outside options (e.g. Lewis and Sappington 1989, Jullien 2000) or outside options created endogenously from relationship specific investments (Rasula and Sonderegger 2010). To the best of our knowledge, ours is the first paper to explore the interaction between the securities market and a mechanism designed to bring expertise into the firm.

Our paper also provides a new perspective within the extant literature on the boundaries of the firm.<sup>1</sup> Williamson (1985) emphasizes the firm as a device for avoiding transaction costs. Grossman and Hart (1986) and Hart and Moore (1990) argue that firm boundaries allocate residual control

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<sup>1</sup>See Williamson (2002) and Gibbons (2005) for review articles.

rights optimally given the need for relationship specific investments. Other ideas include resolving incentive problems (e.g. Holmstrom 1999) and minimizing rent seeking (e.g. Klein 2000). In contrast, we analyze a corporation’s decision regarding whether to bring informed expertise inside the firm, via the contract, or to refrain from posting a mechanism and instead rely on market provision of information. Our analysis reveals a significant limitation on the use of mechanisms in market economies: the very act of offering the mechanism increases the agent’s reservation wage. In light of this limitation, we derive conditions under which it is either infeasible or suboptimal for the firm to offer a mechanism that brings the informed agent inside its boundaries.

A growing feedback-effect literature analyzes the interplay between the information contained in securities prices and economic decisions (Kahn and Winton 1998, Bond, Goldstein, and Prescott 2009, Bond and Goldstein 2015, Boleslavsky, Kelly, and Taylor 2017) (BKT). This literature establishes limitations on the information that may be gleaned from securities prices. For example, in BKT a short seller with private information that the state is bad knows that if short selling is too aggressive, the firm will infer the private information and take corrective action, eliminating the profit from trading on the information. Therefore, the short seller trades less aggressively, limiting the information contained in securities prices. As in this literature, our model has the feature that feedback between order flow and firm decisions results in dampened trade and limited price informativeness. Therefore, our key finding that market-reliance may be optimal—even if the mechanism is feasible—is even more striking. In contrast to our analysis, the feedback-effect literature assumes that expertise resides outside the boundaries of the firm. We depart from this literature by treating as endogenous the choice between outside (market-based) information production versus inside (managerial) information production.

In our analysis, if the agent’s potential liability is sufficiently large (relative to his reservation value), then the firm can design a contract to screen the agent’s expertise, elicit the agent’s private information, and act on this information efficiently, while just meeting the agent’s outside option. This result is reminiscent of Riordan and Sappington (1988), who consider a contracting problem with a verifiable public signal of the agent’s private information, deriving conditions under which the principal can implement the efficient production plan without paying the agent in excess of his reservation value. Cremer and McLean (1988) derive similar results in an auction with correlated valuations, deriving conditions under which the seller can design an auction mechanism that implements any feasible allocation rule, including one which fully extracts the buyers’ expected surplus. Of course, our primary focus is on the interaction between the mechanism and the market and its implications for the mechanism’s feasibility and optimality, which is absent in these analyses. Given the power of the mechanism in our setting, our result that the market may dominate is all the more surprising.

Section 2 gives the basics of the model. Section 3 develops a microstructure model of the securities market in which market activity is the sole source of information for the firm. Section 4 derives conditions under which posting a mechanism is feasible given the existence of a securities market and characterizes the optimal mechanism, assuming that the expert’s outside option is

exogenous. Section 5 derives conditions under which posting a mechanism is optimal given the existence of a securities market, allowing the participation constraint to change endogenously when the mechanism is posted. Section 6 considers an extension in which the firm uses a technology which limits an informed agent’s ability to observe the contract offer.

## 2 MODEL

We analyze the interaction between markets and mechanisms in the context of a canonical firm-level decision problem with an information asymmetry.

**Firm ownership.** Since we are particularly interested in the informational role of securities markets, we consider a widely-held public corporation with tradable shares. Initially, a set of ex ante identical risk neutral atomistic shareholders owns all outstanding equity. For brevity, these original shareholders are referred to as “the shareholders.” Each share entitles its holder to an infinitesimal share of the firm’s cash flow. The measure of outstanding shares and the measure of original shareholders are both normalized to 1, with each atomistic shareholder owning an atomistic share. The corporation is unlevered, ruling out distortions in decisions arising from conflicts of interest between debt and equity. The objective of the firm is to maximize the ex ante expected payoff the shareholders derive from their share, i.e. the ex ante value of the firm. The ex ante firm value consists of two parts. First, a share held to maturity is entitled to the firm’s cash flow accruing at the terminal date. Second, shareholders may be hit by liquidity shocks which force them to sell their stock in a competitive secondary market. Because markets may mistake uninformed liquidity selling for informed trading, the shareholders face potential adverse price impact causing shares to sell for less than expected cash flow (“fundamental value”). The expected underpricing of shares in the secondary market reduces the ex ante value of a share. In other words, underpricing is capitalized into the ex ante value of the firm’s equity.

**Firm decision.** The firm must choose between a risky action ( $R$ ) and a safe action ( $S$ ), and this decision must be sequentially rational. Heuristically, sequential rationality can be understood as arising from the fact that a public corporation has a fiduciary responsibility to its shareholders to select actions that are optimal given the information available to it.

The terminal cash flow of the firm under action  $R$  is a binary random variable  $\omega$  drawn from  $\{0, 1\}$ . Below, we refer to  $\omega$  as the *economic state*. All agents have common prior  $\Pr(\omega = 0) = q$ , where  $q \in (0, 1)$ . If the firm instead adopts action  $S$ , it is insulated from risk, receiving a sure terminal cash flow equal to  $1 - c$ , where  $c \in (0, 1)$ . Intuitively, one can interpret  $c$  as the cost of insulating the firm from the consequences of the bad economic state. Throughout the analysis, we assume  $c > q$ . Notice, given this maintained assumption, the action  $R$  would be optimal if the firm’s decision were to be based solely on prior beliefs.

**Shareholders.** Our treatment of the firm’s original shareholders follows the noise-trader setup commonly-adopted in the market microstructure literature, e.g Kyle (1985) and Glosten and Mil-



grom (1985). In particular, it is assumed that each original shareholder will hold their stock until maturity unless forced to liquidate. The probability of a liquidity shock is  $l \in (0, 1)$ . The arrival of the liquidity shock is observed only by the atomistic shareholders, creating noise in the stock market.

If a liquidity shock does indeed arrive, the fraction of original shareholders forced to sell is itself a random variable assumed to be uniformly distributed on  $[0, 1]$ . Aside from assuming a compact support for the liquidity shocks, the specific density function assumed has no bearing on the results other than simplifying the algebra. As in Faure-Grimaud and Gromb (2004), it is assumed that the shareholder’s liquidity sale arrives in the market as a batch.

**Outsider Agents.** A countably infinite number of identical agents exist, each of whom will never acquire any information. These agents are labeled *uninformed outsiders*. With probability  $a$  an additional outsider agent exists, and this agent is labeled as the *expert outsider* (or “expert” for brevity). The expert can privately observe the economic state  $\omega$  at the time it is determined by Nature. We refer to an expert who learns that the economic state is  $\omega$  as the “type- $\omega$  expert”. The expert’s existence is his private information: to others, the expert is indistinguishable from an uninformed outsider. We refer to the existence of the expert as the *information state*, denoted  $\eta \in \{0, 1\}$ : if  $\eta = 1$ , then the expert exists, and if  $\eta = 0$ , then the expert does not exist. The information state and the economic state are statistically independent.

The uninformed outsiders and the expert are risk neutral and seek to maximize their expected wealth. Each outsider (uninformed or expert) has wealth  $W \geq 1$  which is sufficient to cover any feasible short sale. In addition, each outsider has the ability to post a “bond” worth  $B \geq 0$  as part of any incentive contract signed with the firm. The bond represents the maximum amount the legal system can credibly extract from an agent, inclusive of reputation costs. In practice,  $B$  is a function of the legal system, the value attached to reputation, wealth, and the financial structure of a bonded agent.<sup>2</sup> In the interest of generality, we remain agnostic regarding the size of  $B$  aside from assuming it to be non-negative. This treatment allows us to analyze parametrically whether and how the existence of the stock market, as an outside option, serves to test the limits of bonding capability, undermining an otherwise viable mechanism.

**Mechanism.** The firm has an opportunity to publicly offer a mechanism to the outsider agents in an attempt to elicit information about the economic state  $\omega$ . Formally, the mechanism specifies a transfer to the firm’s advisor as a function of his report of the economic state and the realized terminal cash flow.

The mechanism is offered on a first-come first-serve basis. An outsider who takes up the firm’s offer becomes an inside advisor. To fix ideas, one can think of the mechanism as being a consulting contract or an employment contract to supervise the firm’s action. It is assumed that, as a firm insider, the advisor would be barred from trading the firm’s stock—an assumption consistent with

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<sup>2</sup>The parameter  $B$  can be less than one for a variety of reasons, e.g. limited liability and frictions in contract enforcement.

standard insider trader laws.

We show in section 7 that this “exclusivity assumption” is made without loss of generality. In particular, we show that even if it were legal to do so, the firm could not increase its ex ante value under a mechanism by granting its advisor the freedom to trade its shares. Intuitively, what dictates the required total payoff (wages plus trading gains at the expense of shareholders) to an advisor who takes up a given contract is the trading gain they stand to make if they leave the contract sitting. This off-equilibrium trading gain is invariant across all contracts acceptable to the expert outsider.

**Information Channels.** We separately consider two different channels by which the firm can acquire information about the economic state  $\omega$ . A firm will be said to be *mechanism-reliant* if it offers a mechanism to the outsider agents. The goal of a mechanism-reliant firm is to elicit direct reporting of the private information of the expert outsider, should such an agent exist. In contrast, a firm will be said to be *market-reliant* if it refrains from offering a mechanism to the outsider agents. Essentially, such a firm leaves the market to its own devices and relies only on the market for information about the economic state.

Sections 3 and 4 provide more-detailed information about the sequence of events in each setting. After characterizing the equilibrium in each setting, we determine whether the firm’s shareholders achieve higher ex ante value by posting a mechanism (mechanism-reliance) or by refraining from posting a mechanism (market-reliance).

### 3 MARKET-RELIANCE

This section considers a market-reliant firm—a firm that refrains from offering a mechanism to the outsider agents. Such a firm relies on the market for the provision of information. In particular, if an expert outsider does indeed exist, the market-reliant firm hopes that their trading will provide information about the economic state.

As will be shown formally below, market-reliance suffers two weaknesses relative to mechanism-reliance. First, the expert outsider anticipates that if his trades reveal information to the market, then the stock price will move closer to its fundamental value, reducing his trading gain. This effect is exacerbated because the expert outsider also anticipates that the firm may adjust its action in response to his trade. In particular, if the expert sells too aggressively after observing the true state to be  $\omega = 0$ , then the firm may infer the information and implement the safe action  $S$ . But if the firm implements the safe strategy, the expert’s private knowledge about the economic state becomes worthless since the firm’s terminal cash flow will be  $1 - c$  for sure. The expert’s incentive to mask his private information from both the market and the firm implies that stock market trading generally provides only a noisy signal of the economic state. Consequently, the market-reliant firm may implement the incorrect action even if the expert outsider exists.

The second weakness of market-reliance is that, with an expert outsider left free to trade in the stock market, the firm’s shareholders are exposed to adverse selection. In particular, if shareholders

are forced to sell due to a liquidity shock, then the market considers the possibility that the order reflects trading by an expert outsider with knowledge that the true economic state is 0. The resulting underpricing is capitalized into the ex ante share value.

**Timing.** If the firm is market-reliant (fails to post a mechanism) then the game unfolds in the following sequence.

1. *Information State.* The expert outsider's existence (information state  $\eta$ ) is realized.
2. *Economic State.* The economic state  $\omega$  is realized and is privately observed by the expert outsider if he exists.
3. *Liquidity Shock.* Any liquidity shock is realized.
4. *Market.* The firm's shares are traded in a stock market (described below) with orders observed by a competitive market maker and the firm.<sup>3</sup>
5. *Decision.* The firm chooses an action,  $S$  or  $R$ .
6. *Cash flows.* The firm's cash flows are revealed.

**Market.** The firm's shares are traded in an anonymous competitive stock market. As discussed above, the firm's shareholders submit a uniformly distributed *sell* order, if hit with a liquidity shock. In addition, each outsider agent  $i$  has discretion to submit a *sell* order of size  $t_i \in [-1, 1]$ , with a negative value for  $t_i$  representing a buy order.<sup>4</sup> Since the focus of the analysis is largely on the sell side, we denote positive values of  $t$  as sell orders, and negative values as buy orders. Of course, each outsider agent is free submit a sell order of size 0. We shall label an order of 0 as *inactivity*. Indeed, we will be particularly interested in the ability of a competitive stock market to deter trading by the uninformed outsiders, thus screening out incompetents.

A competitive market maker observes the countably infinite vector of submitted orders without observing each order's source. Thus, the market maker cannot observe whether a particular order in the vector is the result of liquidity selling by shareholders, a trade by an uninformed outsider, or a trade by the expert outsider. The market maker updates his beliefs about the economic state based on the vector of observed orders, adjusts the price of the stock to equal its expected terminal cash flow, and fills all orders at this price. Intuitively, one can think of the market maker as engaging in Bertrand competition with other market makers in order to fill the submitted order vector.

**Notation.** Let  $T$  denote the countably infinite vector consisting of the orders submitted by the shareholders and each outsider agent. Let  $T = \vec{0}$  denote an order vector consisting entirely of zeros, i.e. complete market inactivity. Let  $T = \vec{t}$  denote an order vector consisting of zeros along with a

<sup>3</sup>BKT show in a related model that payoffs and information flow are equivalent if the firm observes securities prices, but not transactions.

<sup>4</sup>As is standard, we abstract from institutional details of short-selling.

single non-zero sell order of size  $t$ . Such an order vector is an important case in the model, as only in this case is information not perfectly revealed by the order vector.

Let  $\chi(T)$  denote the probability assessment of the market maker and firm that the economic state is bad ( $\omega = 0$ ) given observed order vector  $T$ . Let  $\alpha(T)$  denote the probability that the firm follows the safe strategy  $S$  after observing order vector  $T$ .

Recall, the type- $\omega$  expert knows the economic state is  $\omega$ . With this in mind, let  $\Phi_\omega(\cdot)$ , denote the cumulative distribution function from which the type- $\omega$  expert draws his order. We denote the associated probability density function by  $\phi_\omega(\cdot)$ , using the Dirac  $\delta(\cdot)$  function to denote mass points. An uninformed outsider's trading strategy is denoted by the distribution function  $\Phi_U(\cdot)$ . Finally, let  $u_\omega^*$  denote the type- $\omega$  expert's expected trading profit at the time the market opens; that is,  $u_\omega^*$  is the expected trading profit for the expert outsider in economic state  $\omega$ .

## PRELIMINARY ANALYSIS

We derive conditions and present a number of simplifying results ahead of the main equilibrium characterization in the next section.

Consider the firm's choice between the safe and risky action following order vector  $T$ . If the firm chooses safe, its terminal cash flow is  $1 - c$  for sure. If the firm chooses risky, the expected cash flow is  $1 - \chi(T)$ . Therefore, any sequentially strategy for the firm must entail:

$$(1) \quad \alpha(T) = \begin{cases} 0 & \text{if } \chi(T) < c \\ [0, 1] & \text{if } \chi(T) = c \\ 1 & \text{if } \chi(T) > c \end{cases}$$

Intuitively, the firm will find it optimal to implement the safe strategy only if it assesses a sufficiently high probability that the true economic state is  $\omega = 0$ . Recalling that the risky action is optimal based upon prior information, with  $q < c$ , it is apparent that the firm will only implement the safe strategy if the observed order vector brings about a sufficiently large negative revision of beliefs.

Given order vector  $T$ , the competitive secondary market stock price must be equal to the expected terminal cash flow,

$$(2) \quad p(T) = [1 - \alpha(T)][1 - \chi(T)] + \alpha(T)(1 - c).$$

With probability  $\alpha(T)$ , the firm implements the safe action, with terminal cash flow equal to  $1 - c$  for certain. If the firm instead implements the risky action, expected cash flow is then  $1 - \chi(T)$ . Notice, the secondary market stock price reflects information about the economic state contained in the order flow, as well as the firm's optimal action given the order flow.

Consider now the expert outsider's expected trading gain in economic state  $\omega = 0$ , assuming he indeed exists. The expert knows that under the risky strategy a share will be worth 0. Thus, if the expert outsider submits a sell order of size  $t$ , and the realized order vector is  $T$ , then his realized

trading gain will be

$$(3) \quad \begin{aligned} u_0(t, T) &= t[p(T) - \alpha(T)(1 - c)] \\ &= t[1 - \chi(T)][1 - \alpha(T)]. \end{aligned}$$

Consider next the expert outsider's expected trading gain in economic state  $\omega = 1$ , assuming he indeed exists. If  $\omega = 1$ , the expert knows that under the risky strategy a share will be worth 1. Thus if he submits a sell order of size  $t$ , and the realized order vector is  $T$ , then his realized trading gain will be

$$(4) \quad \begin{aligned} u_1(t, T) &= t[p(T) - (1 - \alpha(T)) - \alpha(T)](1 - c) \\ &= -t\chi(T)[1 - \alpha(T)]. \end{aligned}$$

From the preceding two expressions we see that, all else equal, the expert's trading profit increases with his order size. Conversely, his trading gain decreases when order flow reveals more information to the market maker about the true economic state. Finally, we see that the expert's trading profit decreases with the probability that the firm implements the safe action. By implementing the safe action, the firm severs the link between the economic state and the firm's cash flow, rendering the expert's private knowledge of the economic state worthless.

Next, note that equations (3) and (4) imply the following:

**Lemma 3.1** (*Dominance*). *Given any belief function  $\chi(T) \in [0, 1]$  for the market maker and the firm, any firm strategy  $\alpha(T) \in [0, 1]$ , and any realization of the liquidity shock, in state zero (state one) the expert's profit from submitting a sell order  $t_S$  (buy order  $t_B$ ) is weakly larger than his payoff from inactivity, which is weakly larger than his payoff from submitting a buy order (sell order):*

$$t_B < 0 < t_S \Rightarrow u_0(t_B, T) \leq 0 \leq u_0(t_S, T) \quad \text{and} \quad u_1(t_S, T) \leq 0 \leq u_1(t_B, T).$$

In other words, for the type-0 expert, selling a positive amount is always weakly better than inactivity or buying, and for the type-1 expert, buying a positive amount is always weakly better than inactivity or selling.

**Equilibrium Trading Patterns.** We characterize equilibria that meet three intuitive conditions. First, we conjecture equilibria in which each uninformed outsider finds it optimal to be inactive, and verify in Lemma 3.3 that inactivity is indeed an optimal strategy for such agents. Intuitively, since uninformed outsiders have no private information, they should have no incentive to trade in the market, especially given that equilibrium prices tend to move against a trader.

Second, since Lemma 3.1 establishes that for the type-0 expert, selling a positive amount is always weakly better than inactivity or buying, we characterize equilibria in which the type-0 expert, should he exist, always sells ( $\Phi_0(0) = 0$ ). Third, since Lemma 3.1 establishes that for the type-1 expert, buying a positive amount is always weakly better than inactivity or selling, we characterize equilibria in which the type-1 expert always buys ( $\Phi_1(0) = 0$ ). In this case, any

equilibrium buy order must originate with the type-1 expert, so he cannot possibly earn a strictly positive expected trading gain. Thus, the type-1 expert outsider is indifferent over all possible buy orders. For ease of exposition, we assume the type-1 expert uses a mixed trading strategy supported on the entire interval of feasible buy orders:  $\phi_1(t) > 0$  for  $t \in [-1, 0)$ , with  $\Phi_1(0) = 0$ . If we were to instead consider equilibria in which the type-1 expert is inactive, analogous results would obtain regarding the potential dominance of market-reliance over mechanism-reliance.

**Beliefs.** In equilibrium, two classes of order vector perfectly reveal the economic state. First, an order vector containing two sell orders can be submitted to the market only if the liquidity shock arrives, the expert outsider exists and the economic state is zero. Consequently, any order vector containing two sell orders reveals the state to be  $\omega = 0$ . This induces the firm to implement the safe strategy with probability 1. Second, any order vector containing a buy order reveals that the economic state is  $\omega = 1$ . This induces the firm to implement the risky strategy with probability 1.

In contrast, complete market inactivity ( $T = \vec{0}$ ) reveals no information about the economic state. After all, the expert outsider is active if he exists. Thus, if there is no activity in the market, i.e.  $T = \vec{0}$ , then the market maker and firm infer that no expert exists, and their beliefs about the economic state are unaffected. That is,  $\chi(\vec{0}) = q$ .

We summarize this discussion in the following lemma.

**Lemma 3.2** (*Revealing Orders and Inactivity*). *In equilibrium, (i) any order vector  $T$  containing two sell orders reveals that the state is zero,  $\chi(T) = 1$  and induces a switch to the safe strategy,  $\alpha(T) = 1$ ; (ii) market inactivity does not affect beliefs  $\chi(\vec{0}) = q$  and induces the firm to select the risky strategy,  $\alpha(\vec{0}) = 0$ ; (iii) any order vector  $T$  containing a single buy order, or a buy order and a single sell order reveals that the state is one,  $\chi(T) = 0$  and induces the firm to select the risky strategy,  $\alpha(T) = 0$ .*

The preceding lemma gives the belief for all on-path order vectors, except for those containing a single sell order, which we consider next.

Consider finally beliefs following the arrival of an order vector containing all zeroes and a single sell order ( $\vec{t}$ ). When such an order vector arrives, the firm and market maker consider two possibilities: either (1) the expert outsider does not exist and the sell order is due to a liquidity shock or (2) the expert outsider exists, the economic state is 0, and no liquidity shock arrived. Bayes' rule implies updated beliefs are:

$$(5) \quad \chi(\vec{t}) = \frac{aq(1-l)\phi_0(t) + q(1-a)}{aq(1-l)\phi_0(t) + (1-a)}.$$

It is readily verified that  $\chi(\vec{t})$  is increasing in  $\phi_0(t)$ . It follows that if the type-0 trader places a sell order of size  $t$  with higher likelihood, beliefs will become more negative in response to the observation of order vector  $\vec{t}$ .

The preceding discussion concerned beliefs in response to order vectors on the equilibrium path. However, if one of the uninformed outsiders chooses to deviate from inactivity, then an

off-path order vector may be observed by the market maker and the firm, with off-path beliefs influencing an uninformed agent's expected payoff from deviating from the equilibrium. When a single uninformed agent deviates, he may generate a variety of off-path order vectors: three sell orders; two sell orders and one buy order; two buy orders and one sell order; or two buy orders. We adopt a simple convention for assigning beliefs to these off-path order vectors (others supporting the same equilibria are possible).

**Remark 3.1** (*Off-path Beliefs and Actions*). Consider an off-path order vector,  $T$ . If the number of buy orders is greater than or equal to the number of sell orders, then  $T$  reveals state 1,  $\chi(T) = 0$ , and the firm selects the risky action,  $\alpha(T) = 0$ . Otherwise,  $T$  reveals state 0,  $\chi(T) = 1$ , and the firm selects the safe action,  $\alpha(T) = 1$ .

These beliefs imply that each uninformed outsider prefers complete inactivity to either buying or selling. Intuitively, the possibility that trade originates with the informed expert moves beliefs closer to the fundamental value, generating an adverse price impact. Because an uninformed agent faces this adverse price without knowledge of the economic state, he cannot make money by participating in the market—price impact screens out incompetents. We have the following lemma:

**Lemma 3.3** (*Market Screening*) Given the equilibrium beliefs, an uninformed agent's expected profit from submitting any order to the market is weakly negative.

The equilibrium beliefs also allow us to derive the type-0 expert's expected profit from selling  $t$  shares. If the liquidity shock arrives along with his order, then the state is revealed to be  $\omega = 0$ . The firm will then find it optimal to implement the safe strategy, with the market maker setting the stock price at  $p = 1 - c$ , resulting in zero profit for the expert. With probability  $1 - l$  the order vector consists of a single sell order  $\vec{t}$  and the type-0 expert's trade determines beliefs. Consequently, when a type-0 expert outsider submits a sell order  $t$ , his expected trading profit is

$$(6) \quad E[u_0(t, T)] = t[1 - \chi(\vec{t})][1 - \alpha(\vec{t})](1 - l).$$

**Key Parameters.** Finally, we introduce two transformations of the model parameters that simplify subsequent exposition.

$$K \equiv \left( \frac{aq}{1-a} \right) \left( \frac{1-l}{l} \right), \quad J \equiv \frac{1-q}{1-c}.$$

Parameter  $K \in (0, \infty)$  is labeled *market informativeness*, since it captures the information content of an order vector containing a single sell order combined with zeroes. The numerator of  $K$  is the probability that a single sell order arrives due to the existence of an informed expert, economic state  $\omega = 0$ , and no liquidity shock. The denominator of  $K$  is the probability that a single sell order instead arrives due to a liquidity shock hitting the firm's shareholders, with the outside expert non-existent. Thus, the variable  $K$  will be high if  $a$  is high and  $l$  is low. If  $K$  is

indeed high, then the arrival of a single sell order is strongly informative that the true economic state is  $\omega = 0$ .

Parameter  $J \in (1, \infty)$  is the ratio of the firm's expected cash flow (under the prior) from implementing the risky and safe actions. As the cost  $c$  of implementing the safe strategy tends to 1,  $J$  tends to infinity, and the firm would only want to switch to the safe strategy if it were sure that  $\omega = 0$ . Conversely, as the cost  $c$  of implementing the safe strategy tends downward to  $q$ ,  $J$  tends to 1, and the firm would want to switch to the safe strategy given a small downward revision of its beliefs relative to the prior.

It is worth noting that  $J$  and  $K$  can be adjusted independently through changes in the underlying parameters  $(a, q, c, l)$ . Specifically, any market informativeness measure  $K \in (0, \infty)$  can be achieved via changes in  $a$  or  $l$  while holding  $J$  fixed. Similarly, any value of  $J \in (1, \infty)$  can be achieved without affecting  $K$  by choosing an appropriate value of  $c > q$ .

Finally, it will be convenient to rewrite the beliefs in equation (5) as a function of  $K$  as follows:

$$(7) \quad \chi(\vec{t}) = \frac{K\phi_0(t) + q}{K\phi_0(t) + 1}.$$

## EQUILIBRIUM WITH INFORMATION RENT

We begin the characterization of equilibria by first analyzing those in which the type-0 expert outsider expects an information rent,  $u_0^* > 0$ .

A number of observations are immediate. First note that if the type-0 expert is to make an information rent, he must use a proper mixed strategy. After all, if he were to submit one particular order with probability 1, then observing this order would reveal the true economic state to be  $\omega = 0$ . This would induce the firm to implement the safe strategy, resulting in zero profit. And given that the liquidity sales have no mass points, the same reasoning implies that the cumulative distribution function  $\Phi_0$  cannot contain any mass points.

Second, note that in an equilibrium with an information rent, the minimum sell order size on the support, call it  $m$ , must be greater than 0. After all, if  $m$  were 0, then the type-0 expert's trading gain would also be 0. Third, we may note that it must be the case that the mixing density vanishes at  $m$ , that is  $\phi_0(m) = 0$ . After all, equation (5) implies  $\chi(\vec{t}) = q$  for any  $t$  outside the trading support. If  $\phi_0(m)$  were to exceed 0, then order  $m$  would entail an adverse price impact ( $\chi(\vec{m}) > q$ ), and thus the expert would earn a higher expected trading gain by deviating to an order infinitesimally smaller than  $m$ . Finally, a similar argument rules out gaps in the trading support, since there would be a gain to deviating to a gap point, given that a gap point  $t$  would also have the property that  $\chi(\vec{t}) = q$ . Thus, in any equilibrium with information rents, the type-0 expert outsider must play a continuous mixed strategy with no mass points or gaps, as we summarize next.

**Lemma 3.4** (*Structure of Expert's Strategy*). *If the type-0 expert expects an information rent in equilibrium, then he plays a continuous mixed strategy with no mass points or gaps supported on interval  $[m, 1]$ , for some  $m > 0$ .*



From equation (6) we have the following indifference condition:

$$(8) \quad t \in [m, 1] \Rightarrow t[1 - \chi(\vec{t})][1 - \alpha(\vec{t})](1 - l) = u_0^*.$$

Ceteris paribus, the type-0 expert's trading gain is increasing in the size of his sell order. To maintain indifference, the gains from trading larger volume must be exactly offset by a combination of greater price impact (i.e. an increase in  $\chi(\cdot)$ ) and a higher probability of the firm switching to the safe strategy.

Small orders have low price impact, with the smallest order  $m$  in the support having no impact at all on beliefs, implying a stock price  $p = 1 - q$  provided no liquidity shock arrives. Thus, the type-0 expert's expected profit from selling  $m$  shares is  $m(1 - q)(1 - l)$ . Because he must be indifferent among all trades inside the support of his mixed strategy, this must also be the type-0 expert's expected profit  $u_0^*$ .

**Lemma 3.5** (*Expert's Profit*). *If the type-0 expert expects an information rent in equilibrium, then  $u_0^* = m(1 - q)(1 - l)$ .*

Equation (8) also implies that in any equilibrium with an information rent ( $u_0^* > 0$ ), the type-0 expert never submits an order  $t$  such that the observation of such an order would induce the firm to switch to the safe strategy with probability 1 ( $\alpha(\vec{t}) = 1$ ). After all, such an order would imply an expected trading profit of 0.

To complete the characterization, we must derive the equilibrium strategies and the firm's profit. The appendix establishes the following important results.

**Lemma 3.6** (*Structure of Firm's Strategy*). *In an equilibrium in which the type-0 expert expects an information rent,*

(i) *if  $m > \frac{1}{J}$ , then  $\alpha(\vec{t}) = 0$  for all  $t \in (0, 1]$ .*

(ii) *if  $m < \frac{1}{J}$ , then  $\alpha(\vec{t}) = 0$  for all  $t \in (0, Jm]$ , and  $\alpha(\vec{t}) \in (0, 1)$  for all  $t \in (Jm, 1]$ .*

The intuition for the preceding lemma is as follows. The impact on beliefs and prices is small for sell orders near the minimum size  $m$ . Here the firm sticks with the risky strategy (provided no liquidity shock arrives, fully revealing the bad state). The effect on beliefs will then increase with the size of the order. If  $m$  is small, then beliefs are affected over a wide interval, and sufficiently large orders reveal sufficient information to induce the firm to mix between the risky and safe strategies. If  $m$  is close to 1, then the interval of orders over which beliefs are affected is narrow. In this case, even the largest sell order by itself is insufficient to induce the firm to switch to the safe strategy with positive probability.

With the preceding argument in mind, consider the possibility of an equilibrium with large information rents, those with a large minimum sell order  $m > 1/J$ . Applying Lemma 3.6, in such an equilibrium the firm selects the risky action following every sell order (that arrives on its own).

That is,  $\alpha(\vec{t}) = 0$  for all  $t \in (0, 1]$ . Consequently, the type-0 expert outsider's indifference condition (8) is that for all  $t \in [m, 1]$ :

$$(9) \quad m(1 - q)(1 - l) = t[1 - \chi(\vec{t})](1 - l).$$

In equilibrium, beliefs following a single sell are given by (7). Substituting this into the preceding indifference condition we find that

$$(10) \quad m(1 - q)(1 - l) = t \left[ 1 - \frac{K\phi_0(t) + q}{K\phi_0(t) + 1} \right] (1 - l) \Rightarrow \phi_0(t) = \frac{t - m}{Km}.$$

Thus, the type-0 expert outsider exploits his private information by using a mixing density that increases linearly in the trade size  $t$ . To determine the equilibrium value of the minimum sell order size  $m$ , note that  $\phi_0(t)$  must integrate to 1. We have:

$$(11) \quad \int_m^1 \frac{t - m}{Km} dt = 1 \Rightarrow m = K + 1 - \sqrt{(K + 1)^2 - 1}.$$

Finally, from the preceding equation it follows that our initial conjecture that  $m > 1/J$  holds true if and only if the market informativeness measure  $K$  is sufficiently low, specifically  $K < \underline{K}$ , where

$$(12) \quad \underline{K} \equiv (J - 1)^2 / 2J.$$

Intuitively, when the market informativeness measure  $K$  is small, due to a low and/or  $l$  high, the type-0 expert's trading has a small impact on beliefs. This allows him to use an aggressive trading strategy featuring large sell orders, delivering a high equilibrium trading gain, without ever inducing the firm to switch to the safe strategy. We summarize this case below.

**Proposition 3.7** (*Low Informativeness*). *If  $K \in (0, \underline{K})$ , then an equilibrium in which the type-0 expert expects an information rent exists, characterized as follows. The type-0 expert submits a sell order drawn from density*

$$\phi_0^L(t) \equiv \frac{t - m_L}{Km_L},$$

*supported on interval  $[m_L, 1]$ , where*

$$m_L \equiv K + 1 - \sqrt{(K + 1)^2 - 1}.$$

*The firm selects the risky action in response to any single sell order,  $\alpha(\vec{t}) = 0$ . Beliefs and actions for all other order vectors are given in Lemma 3.2 and Remark 3.1.*

Second, consider the possibility of an equilibrium with small information rents, those with a small minimum sell order  $m < 1/J$ . From Lemma 3.6, the firm sticks with the risky strategy for sure on the interval  $[m, Jm]$ . Here the type-0 expert's mixed strategy is derived from equation (10). On the interval  $(Jm, 1]$ , the firm mixes between safe and risky actions. In order for mixing by the

firm to be sequentially rational, it must be that  $\chi(\vec{t}) = c$  over this interval. Combining this fact with (7) and (8), we conclude that

$$(13) \quad \text{for } t \in (Jm, 1], \quad m(1-q)(1-l) = t(1-c)[1-\alpha(\vec{t})](1-l), \quad \text{and} \quad \frac{K\phi_0(t)+q}{K\phi_0(t)+1} = c,$$

and hence,

$$(14) \quad \text{for } t \in (Jm, 1], \quad \alpha(\vec{t}) = 1 - \frac{Jm}{t}, \quad \text{and} \quad \phi_0(t) = \frac{J-1}{K}.$$

Notice, orders larger than  $Jm$  occur with a constant density. It follows that beliefs and prices are constant on this interval. Since prices are constant on this interval, the probability of the firm switching to the safe strategy must increase in the size of the sell order, as shown above, which just offsets the type-0 expert's temptation to submit larger orders. Finally,  $\phi_0(t)$  must integrate to 1. It follows:

$$(15) \quad \int_m^{Jm} \frac{t-m}{Km} dt + (1-Jm)(J-1)/K = 1 \Rightarrow m = \frac{2(J-K-1)}{J^2-1}.$$

Thus, our initial supposition that  $m < 1/J$  holds if and only if  $K > \underline{K}$ . Intuitively, we have a second class of equilibria, featuring lower trading gains if the price informativeness measure exceeds  $\underline{K}$ . However, if equilibrium information rents are to be strictly positive  $K$  cannot be too high. In particular, Lemma 3.4 requires that  $m > 0$ , and hence  $K < \bar{K}$ , where:

$$(16) \quad \bar{K} \equiv J-1.$$

We thus have the following proposition.

**Proposition 3.8** (*Intermediate Informativeness*). *If  $K \in (\underline{K}, \bar{K})$ , then an equilibrium in which the type-0 expert expects an information rent exists, characterized as follows. The type-0 expert submits a sell order drawn from density*

$$\phi_0^I(t) \equiv \begin{cases} \frac{t-m_I}{Km_I} & \text{if } t \in [m_I, Jm_I] \\ \frac{J-1}{K} & \text{if } t \in [Jm_I, 1] \end{cases}$$

supported on interval  $[m_I, 1]$ , where

$$m_I \equiv \frac{2(J-1-K)}{J^2-1}.$$

Following a single sell order,  $\alpha(\vec{t}) = 0$  if  $t < Jm_I$  and  $\alpha(\vec{t}) = 1 - Jm_I/t$  if  $t \in [Jm_I, 1]$ . Beliefs and actions for all other possible order vectors are given in Lemma 3.2 and Remark 3.1.

Note that the conditions in Propositions 3.7 and 3.8 are mutually exclusive.

Based on the preceding characterization, we can derive a simple expression for the ex ante value of the firm in an equilibrium with information rent, call it  $V_R$ . The value of a share ex ante is equal to expected cash flow less the expected trading losses of the shareholders. In turn, since the market maker breaks even in expectation, expected shareholder trading losses are just equal to the ex ante expectation of expert trading gains. Regarding expert trading gains, they only accrue if the expert indeed exists and if the state is bad. Thus, the ex ante expectation of shareholder trading losses is:

$$(17) \quad aq u_0^* = aqm(1 - q)(1 - l).$$

Consider next expected cash flow, focusing first on the high information rent equilibria. As an informative benchmark, consider a firm that had zero access to outside information, being forced to rely on prior beliefs. Such a firm would always play the risky strategy, generating expected cash flow  $1 - q$ . In contrast, in a high information rent equilibrium, the firm increases its expected cash flow by correctly shifting to the safe strategy in one (but only one) state of nature: an expert exists (probability  $a$ ); the state is bad (probability  $q$ ); and a fully revealing liquidity shock occurs (probability  $l$ ). In this same state of nature, the always-risky strategy would generate a cash flow of 0. In contrast, by following the market and switching to the safe strategy in this one state of nature, the firm gains an incremental cash flow equal to  $1 - c$ . It follows that expected cash flow in a high information rent equilibrium is:

$$(18) \quad 1 - q + \underbrace{aql(1 - c)}_{\text{mkt information}}$$

Consider next those equilibria with intermediate information rents, those in which the firm mixes between safe and risky following some realizations of the order flow vector. Here, the fact that the firm is indifferent between safe and risky for such order vectors implies that the conditional expectation of the cash flow is the same as if it had simply played the risky strategy for those order vectors. Thus, expected cash flow is still given by the preceding equation in the event of an intermediate-rent equilibrium in which the firm mixes (see the proof of Proposition 3.9 for a formal derivation).

Putting this analysis together, we have the following proposition:

**Proposition 3.9** (*Ex Ante Firm Value—Eq. With Rent*). *In an equilibrium in which type-0 expects an information rent, ex ante firm value is*

$$V_R \equiv 1 - q + aql(1 - c) - aq u_0^*.$$

To understand the limitations of relying on the market, it is useful to compare the expected firm value to the *first-best benchmark*, in which the firm has direct access to the same information that is available to an expert outsider, should he exist.<sup>5</sup> In this case, the firm would correctly switch to

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<sup>5</sup>In the first-best the firm has access to *all private information* in the economy. If no expert exists, the firm does

the safe action whenever the expert outsider exists and the economic state is bad, yielding ex ante firm value

$$V^* \equiv (1 - a)(1 - q) + aq(1 - c) + a(1 - q) \cdot 1 = 1 - q + aq(1 - c),$$

and hence,

$$\begin{aligned} (19) \quad V_R &= V^* - aq(1 - l)(1 - c) - aqu_0^* \\ &= V^* - aq(1 - l)[(1 - c) - m(1 - q)] \end{aligned}$$

The preceding equations illustrate clearly the two weaknesses associated with market-reliance. First, the firm does not make first-best decisions. In particular, the firm fails to switch to the safe strategy if an informed outsider exists and the economic state is bad, but no liquidity shock occurs to fully reveal this fact. Second, in this very same state of nature, the type-0 expert makes trading gains at the expense of shareholders, as if he always short-sells  $m$  shares at a price  $p = 1 - q$  versus a fundamental value of 0.

### EQUILIBRIUM WITH NO INFORMATION RENT.

We now analyze the possibility of an equilibrium in which the type-0 expert outsider expects no information rent,  $u_0^* = 0$ .

To begin, note that equations (3) and (6) imply that in such an equilibrium  $\alpha(\vec{t}) = 1$  for all  $t \in (0, 1]$ . In particular, if  $u_0^* = 0$ , then each sell order  $t$  must generate zero expected utility, otherwise the type-0 expert would have a profitable deviation. Hence, for all  $t > 0$ , either  $\chi(\vec{t}) = 1$ , or  $\alpha(\vec{t}) = 1$ . But, from (1) it follows that  $\chi(\vec{t}) = 1$  implies  $\alpha(\vec{t}) = 1$ . Therefore, a no-rent equilibrium exists if and only if it is possible to find a trading strategy  $\Phi_0(t)$  such that the firm switching to the safe action is consistent with (1) for all order vectors consisting of a single sell order.

To characterize the no-rent equilibrium, we derive conditions under which a single sell order—of any size—induces a switch to the safe action. For this to be the case, the posterior following any single sell order must exceed the cost of switching:

$$(20) \quad c \leq \frac{K\phi_0(t) + q}{K\phi_0(t) + 1}.$$

Therefore, if the firm switches to the safe strategy in equilibrium for all sell orders, then for all  $t \in (0, 1]$ , the type-0 expert's trading density must satisfy:

$$(21) \quad \phi_0(t) \geq \frac{J - 1}{K}.$$

Of course, the trading density must also integrate to 1 on the unit interval, which would be impossible if  $K < J - 1$ . Conversely, if  $K \geq \bar{K} = J - 1$ , then many feasible mixing densities exist  


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 not acquire information about  $\omega$ .

which satisfy the preceding equation. That is, a multiplicity of no-rent equilibria exist if the market informativeness measure  $K$  is sufficiently high. Intuitively, high market informativeness can be understood as resulting from a sufficiently high probability ( $a$ ) of the expert outsider existing and a sufficiently low probability ( $l$ ) of a liquidity shock. Under such a parameter configuration, any sell order contains sufficient negative information to induce the firm to switch to the safe strategy. But when the firm is on such a hair-trigger the type-0 expert can never trade profitably based upon his private information.

Consider now the ex ante value of the firm in a no-rent equilibrium, again using as a benchmark  $V^*$ , the value of a firm that observes all available information in the economy. To begin, note that since the expert outsider's payoff is zero in both economic states, no adverse selection discount exists in a no-rent equilibrium. That is, ex ante firm value is here equal to the ex ante expectation of realized cash flow. Next, note that if the expert outsider exists, then the firm selects the risky action if  $\omega = 1$  and the safe action if  $\omega = 0$ . That is, if the expert outsider does indeed exist, then the firm always selects the correct action for each economic state. However, if the expert outsider does not exist, then the firm does not select same action that would be selected by a firm holding all information available in the economy. After all, in the absence of an informed expert, the only information available is the prior, and under the prior the risk action is optimal. However, in a no-rent equilibrium, the firm is on a hair-trigger. Thus, if the expert outsider does not exist, but a liquidity shock arrives, then the firm will mistakenly switch to the safe action. This mistake occurs with probability  $l(1 - a)$ , reducing the firm's expected cash flow by  $(1 - q) - (1 - c) = c - q$ . Compared to the benchmark in which the firm observes all available information ( $V^*$ ), in the no-rent equilibrium the firm switches to the safe action too often, resulting in lower firm value, with

$$\begin{aligned}
 V_{NR} &= V^* - l(1 - a)(c - q) \\
 (22) \quad &= 1 - q + l(1 - c)(1 - a) \left( \frac{K}{1 - l} - \bar{K} \right).
 \end{aligned}$$

Since  $K \geq \bar{K}$ , it follows from the second line above that the market delivers useful information to the firm, increasing firm value relative to the uninformed firm's expected cash flow.

The following proposition summarizes the analysis.

**Proposition 3.10** (*High Informativeness*). *If  $K \geq \bar{K} \equiv J - 1$ , then a multiplicity of equilibria in which  $u_0^* = 0$  exists. If  $K < \bar{K}$ , then no such equilibrium exists. In any such equilibrium,  $\phi_0(\cdot)$  satisfies condition (21), the firm switches to safe following any single sell order,  $\alpha(\vec{t}) = 1$  for all  $t \in (0, 1]$ , and expected firm value is*

$$V_{NR} = 1 - q + aq(1 - c) - l(1 - a)(c - q).$$

*Beliefs and actions for all other possible order vectors are given in Lemma 3.2 and Remark 3.1.*

**Market-Reliant Firm Value.** Figure 1 provides a summary of the equilibria in the market reliance case.

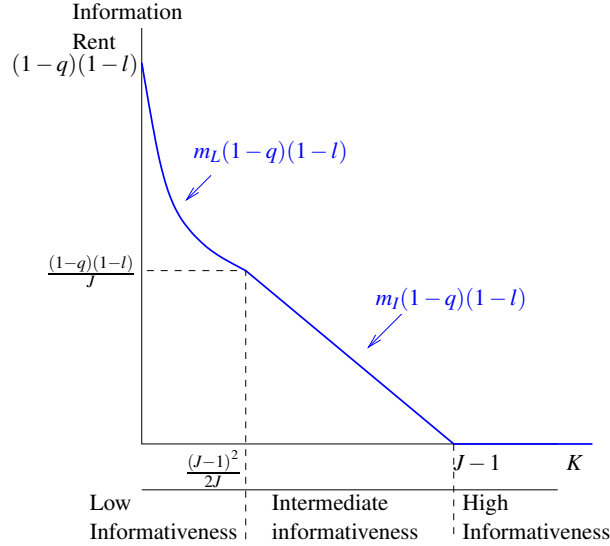


Figure 1: Information rent received by the type-0 expert.

Note that  $m$  and the information rent are continuous functions of  $K$ . To see this note that at the first threshold  $K = \underline{K}$ , the minimum trade sizes are equal,  $m_L = m_I = 1/J$ . Note further that at the second threshold,  $K = \bar{K}$ , the minimum trade size is zero,  $m_I = 0$ . Further, it is straightforward to show that  $m_L$  and  $m_I$  are decreasing functions of  $K$ .

Finally, for use later, we summarize the ex-ante value of the market-reliant firm as:

$$(23) \quad V_{MKT} \equiv \begin{cases} V^* - aq(1-l)(1-c) - aqm_L(1-q)(1-l) & \text{if } K \in [0, \underline{K}] \\ V^* - aq(1-l)(1-c) - aqm_I(1-q)(1-l) & \text{if } K \in [\underline{K}, \bar{K}] \\ V^* - l(1-a)(c-q) & \text{if } K \in (\bar{K}, \infty) \end{cases}$$

where  $m_L$  and  $m_I$  are defined in Propositions 3.7 and 3.8, respectively. Note that  $V_{MKT}$  is continuous in the underlying parameters.<sup>6</sup>

#### 4 THE CASE FOR MECHANISMS: EXOGENOUS RESERVATION VALUE

This section presents the case for the direct revelation mechanism (DRM) as devices for eliciting private information from an informed agent, when the participation constraint is exogenous. In particular, we show that given sufficient bonding capability, the firm can devise a mechanism in which the firm selects the first-best action, while pinning the expert outsider's payoff to his reservation value. Notably, we show that the firm will be able to achieve this outcome despite the

<sup>6</sup>To see this note that  $m$  is continuous in  $K$  and that  $K = \bar{K} \Rightarrow aq(1-l)(1-c) = l(1-a)(c-q)$ .

requirement that the firm allocates the mechanism to the first willing agent, and the firm's fiduciary responsibility, both of which limit its commitment power.

**Timing.** If the firm offers a mechanism, then the game unfolds in the following sequence.

1. *Information State.* The expert's existence (information state) is realized.
2. *Mechanism Offered.* Without observing the expert's existence, the firm publicly offers a mechanism to the countably infinite pool of outsider agents. The mechanism specifies a vector of wages  $w_{r\varphi}$ , where  $w_{r\varphi}$  is the advisor's wage following report  $r \in \{0, 1\}$  and realized terminal cash flow  $\varphi \in \{0, 1 - c, 1\}$ . Each outsider decides whether he would like to participate in the mechanism. The mechanism is assigned to the first agent who indicates that he is willing to participate, thereby becoming the firm's advisor. Acceptance of the mechanism is publicly observed.
3. *Economic State.* The economic state is realized and is privately observed by the expert if he exists.
4. *Reporting.* If any agent agreed to participate, then this advisor privately issues a report of the economic state to the firm. If no agent agreed to participate, then this step is skipped.
5. *Liquidity Shock.* The liquidity shock is realized.
6. *Market.* Orders are anonymously submitted to the market. The market maker and firm observe the countably infinite order vector.
7. *Decision.* The firm chooses an action,  $S$  or  $R$ .
8. *Cash flows.* The firm's cash flows are revealed. If the firm hired an advisor, then it makes transfers as specified by the mechanism.

Note that the sequence of events is identical to the game played under market-reliance, except that here a mechanism is publicly posted and accepted/rejected prior to trading in the stock market.

The analysis in this section proceeds as follows. We first derive a set of constraints that are *necessary* for a mechanism to deliver a higher ex ante firm value than the firm can achieve by relying on the market. We then characterize conditions under which the necessary conditions are consistent, and we solve for the optimal mechanism(s) among those satisfying the necessary conditions. In the next section, derive conditions under which the mechanism dominates the market.

The expert's reservation value is directly relevant for the firm's mechanism design problem. Let  $\underline{u}$  represent the expert's continuation payoff from rejecting the posted mechanism. In this section, we treat  $\underline{u}$  as exogenous and assume that  $\underline{u} > qu_0^*$  and  $\underline{u} > 0$ . In the next section, we derive the expert's endogenous equilibrium continuation value, showing that  $\underline{u}$  is the same for all mechanisms that beat market-reliance and that it satisfies the assumed inequalities. We will also show that an



uninformed agent's continuation payoff from rejecting the firm's mechanism offer is zero, which we take as given in this section.

Recall, fiduciary duty requires that the firm's behavior is sequentially rational. Since the firm cannot commit to future actions, the Revelation Principle does not apply directly. However, we establish an analogous result in Lemmas 4.1 and 4.3. Some formalities are first necessary. To this end, let  $\chi_r$  be the firm's belief that the state is  $\omega = 0$  following report  $r \in \{0, 1\}$ . As a normalization, let us label the reports so that  $\chi_1 \leq q \leq \chi_0$ .<sup>7</sup> Let  $\rho_r$  be the probability that the firm selects the risky action following report  $r$ . Let  $x \in \{0, 1\}$  denote the expert's participation decision, where  $x = 1$  represents the decision to participate.<sup>8</sup> Let  $\gamma_\omega$  be the probability that the advisor sends report  $r = 1$  in economic state  $\omega$ . Finally, let  $d$  be the probability that each uninformed outsider agrees to participate in the mechanism.

Any mechanism that outperforms market-reliance must have certain properties. First, any such mechanism must be rejected by the uninformed outsiders and accepted by the expert outsider if he exists. After all, the uninformed outsiders are countably infinite, and the mechanism is assigned to the first willing agent. Thus, a mechanism that does not screen out the uninformed will almost surely be accepted by an uninformed outsider, and hence, cannot deliver useful information about the economic state. The firm would therefore watch the market for information, and both the firm and the market maker anticipate that the expert will be active in the market if he exists. Thus, offering a mechanism that fails to screen out incompetents cannot do better than market-reliance. Following the same logic, any mechanism that fails to induce participation by the expert also cannot do better than market-reliance. We have the following lemma.

**Lemma 4.1** (*Screening*). *If a mechanism delivers higher ex ante firm value than market-reliance, then it must screen out uninformed agents and induce participation by the expert if he exists,  $d = 0$  and  $x = 1$ .*

Second, any mechanism that does better than market-reliance has the property that it “grants the expert real authority” in the sense that  $\rho_1 = 1$  and  $\rho_0 = 0$ . That is, with probability 1 the firm implements the risky (safe) action following the advisor's report that  $\omega = 1$  ( $\omega = 0$ ). To see why this must be the case, recall report-contingent beliefs regarding the bad state are such that  $\chi_1 \leq q \leq \chi_0$ . Because the firm selects the risky action under the prior and  $r = 1$  conveys good news about the economic state, sequential rationality implies the firm surely implements the risky action following  $r = 1$ . If the firm were to also select the risky action following  $r = 0$ , then the firm *always* implements the risky action. But then the expected cash flow is  $1 - q$ , which is less than the expected cash flow under market-reliance. Furthermore, if the firm mixes between safe and risky following  $r = 1$ , then it must be indifferent between them. Therefore, its expected cash flow is the same as if it always selects risky, which is  $1 - q$ . To see this formally, note that the firm mixes only

<sup>7</sup>The Law of Iterated Expectations requires  $\Pr(r = 0)\chi_0 + \Pr(r = 1)\chi_1 = \Pr(\omega = 0) = q$ . Therefore one posterior belief must be weakly smaller than the prior and the other weakly larger.

<sup>8</sup>For brevity, we abstract from mixing by the expert in his participation decision in this section. Section 6 considers an extension that is formally equivalent to a setting in which the expert mixes in the participation decision.

if  $\chi_0 = c$ , and hence,

$$\begin{aligned} E[\varphi] &= \Pr(r = 0)[\rho_0(1 - \chi_0) + (1 - \rho_0)(1 - c)] + \Pr(r = 1)(1 - \chi_1) \\ &= \Pr(r = 0)[\rho_0(1 - \chi_0) + (1 - \rho_0)(1 - \chi_0)] + \Pr(r = 1)(1 - \chi_1) \\ &= \Pr(r = 0)(1 - \chi_0) + \Pr(r = 1)(1 - \chi_1) = 1 - q, \end{aligned}$$

where the last equality follows from the Law of Iterated Expectations. Therefore, if the firm does not select the safe action with probability one following report 0, then its expected cash flow is smaller than under market-reliance. At the same time, the mechanism must induce participation by the expert. Thus, the mechanism must offer the expert an ex ante expected transfer of at least  $au > aqu_0^*$ . That is, the expected transfer under the mechanism exceeds the expected adverse selection cost under market reliance. Thus, a mechanism that does not grant the expert real authority results in lower ex ante firm value than does market-reliance.

We thus have the following lemma.

**Lemma 4.2** (*Delegated Decision*). *If a mechanism delivers higher ex ante firm value than market-reliance, then it must delegate the decision to the expert,  $\rho_0 = 0$  and  $\rho_1 = 1$ .*

Third, any mechanism that achieves higher value than under market reliance induces the expert to report truthfully with probability 1. After all, if it is sequentially rational for the firm to follow the expert, with  $\rho_0 = 0$  and  $\rho_1 = 1$  (see preceding lemma) then it must be that  $\chi_0 \geq c > q > \chi_1$ . Therefore, it must be that the expert tells the truth with positive probability (i.e. he cannot strictly prefer to lie):  $\gamma_1 > 0$  and  $\gamma_0 < 1$ . These conditions imply two constraints on wages. First, to ensure  $\gamma_1 > 0$ , it must be that  $w_{11} \geq w_{01-c}$ . Second, to ensure  $\gamma_0 < 1$ , it must be that  $w_{01-c} \geq w_{10}$ . Furthermore, as we show in the Appendix, in any mechanism that delivers a higher payoff than market reliance (consistent with Lemma 4.1 and 4.2), these two constraints on wages hold with strict inequality, and so the expert strictly prefers to report truthfully.

**Lemma 4.3** (*Truthful Reporting*). *If a mechanism screens out uninformed agents and induces participation by the expert (as in Lemma 4.1) and delegates the decision to the expert (as in Lemma 4.2), then the expert's unique sequentially rational strategy is to report truthfully with probability 1,  $\gamma_0 = 0$  and  $\gamma_1 = 1$ .*

It follows that any mechanism that delivers the firm a higher expected payoff than market-reliance induces the expert to report truthfully with probability one. That is, screening out incompetents and inducing participation by the expert are more stringent conditions than the expert's truth-telling.

To understand this intuitively, recall that the mechanism must be designed to screen out the uninformed outsiders and induce participation by the expert. This has implications for the wage structure. Given that the firm always follows the expert's recommendation (Lemma 4.2), to screen out the uninformed outsider the firm must punish both a report of state 0, and an *incorrect* report

of state 1. Furthermore, because an incorrect report of state 1 could only be generated by an uninformed outsider while a report of state 0 could be generated by an honest expert, an incorrect report of state 1 must be punished more severely than a report of state 0. Simultaneously, in order to induce participation by the expert, the firm must reward a *correct* report of state 1, thereby offsetting the possibility of punishment in expectation. Thus, if the expert lies by reporting  $r = 0$  in state 1, the expert sacrifices a reward in order to obtain a punishment, which is clearly suboptimal. Conversely, by reporting  $r = 1$  in state 0, the expert sacrifices a small punishment in order to obtain a larger one, which is, again, clearly suboptimal.

Lemma 4.2 allows us to focus on mechanisms in which the advisor's wage depends only on the firm's terminal cash flow, not on his report. To see this, note that whenever the advisor reports  $r = 1$  the firm implements the risky action which implies wage  $w_{11-c}$  is irrelevant. Similarly, whenever the advisor reports  $r = 0$ , the firm selects the safe strategy which implies  $\{w_{00}, w_{01}\}$  are irrelevant. We thus need only focus on wages  $\{w_{01-c}, w_{11}, w_{10}\}$ , which can be written as a function only of the realized cash flow. Therefore, in what follows we drop the first subscript (the report) from the agent's wage.

Lemmas 4.1-4.3, along with the agents' liability limit, imply that any mechanism which delivers a higher payoff than market reliance must satisfy the following system of constraints.

$$w_{1-c} \leq 0 \quad (\text{SC0})$$

$$qw_0 + (1 - q)w_1 \leq 0 \quad (\text{SC1})$$

$$qw_{1-c} + (1 - q)w_1 \geq \underline{u} \quad (\text{PC})$$

$$w_i \geq -B \quad \forall i \in \{0, 1, 1 - c\}. \quad (\text{BOND})$$

Constraint (SC0) ensures that an uninformed outsider prefers to reject the mechanism, rather than accept and report  $r = 0$ . Similarly, (SC1) rules out an uninformed agent participating and reporting  $r = 1$ .<sup>9</sup> Constraint (PC) ensures that the expert outsider is willing to participate in the mechanism if he indeed exists, anticipating that he will report the state truthfully; from Lemma 4.3, we know that the expert's only sequentially rational strategy is truthful reporting (with probability 1) in any mechanism that delivers a higher payoff than market-reliance. The constraints in (BOND) reflect the expert's limited liability. We refer to the set of constraints as  $\mathcal{S}$ . Because (SC0), (SC1), and (PC) are imposed by the mechanism's need to screen out uninformed outsiders and screen in the expert, we refer to  $\mathcal{S}$  as the *screening constraints*. If the screening constraints are mutually consistent, we say that *screening is feasible*, and we refer to a mechanism that satisfies  $\mathcal{S}$  as a *feasible mechanism*.<sup>10</sup>

The screening constraints demonstrate an important tension between the mechanism's ability to attract the expert and its ability to screen out uninformed outsiders. In order to meet the expert's participation constraint, the firm needs to ensure that a particular linear combination of  $w_{1-c}$  and

<sup>9</sup>(SC0) and (SC1) also ensure that an uninformed agent would rather reject than accept and then report randomly.

<sup>10</sup>Note that we define feasibility in the sense of the existence of a mechanism which potentially delivers a payoff in excess of market-reliance.

$w_1$  is sufficiently large. However, increasing  $w_{1-c}$  makes it more attractive for an uninformed agent to accept and report  $\omega = 0$ , while increasing  $w_1$  makes it more attractive for an uninformed agent to accept and report  $\omega = 1$ . The temptation for an uninformed agent to report  $\omega = 1$  can be offset by reducing  $w_0$ , thereby generating a punishment for incorrectly reporting that the state is good. However, the firm's ability to punish is restricted by the agent's limited liability, suggesting that screening is not always feasible. This intuition is confirmed by the following proposition.

**Proposition 4.4** (*Feasible Screening*). *If  $\underline{u} > qB$ , then screening is infeasible and every mechanism does no better than market-reliance.*

We now find the optimal mechanism assuming liability is large enough that screening is feasible. The firm's objective is to maximize the ex ante value of a share (or equivalently, total firm value) subject to  $\mathcal{S}$ . In any feasible mechanism, ex ante firm value is

$$(1-a)(1-q) + a[(1-q) + q(1-c)] - a[qw_{1-c} + (1-q)w_1].$$

The first term is the firm's expected payoff if no expert exists, the second term is the firm's expected payoff from its decision if the expert exists, and the third term is the expected wage bill.<sup>11</sup> No adverse selection cost is capitalized into share value, because on the equilibrium path the expert always signs on as the firm's advisor if he exists and is thus prohibited from trading in the market. With no informed trading, shareholders hit with a liquidity shock are able to sell at fundamental value.

The next proposition gives a simple condition for the existence of a feasible mechanism and characterizes the optimal mechanism.

**Proposition 4.5** (*Optimality*). *If  $\underline{u} \leq qB$ , then any feasible mechanism in which (PC) holds with equality is optimal, maximizing ex ante firm value subject to  $\mathcal{S}$ . In every optimal mechanism, (PC) holds with equality, project selection is first best, and ex ante firm value is*

$$V_{DRM} = (1-a)(1-q) + a[(1-q) + q(1-c) - \underline{u}] = 1 - q + aq(1-c) - a\underline{u} = V^* - a\underline{u}.$$

*The following mechanism is feasible and optimal whenever  $\underline{u} \leq qB$ :*

$$(w_0, w_{1-c}, w_1) = \left( -B, 0, \frac{\underline{u}}{1-q} \right).$$

The feasible and optimal mechanism presented in Proposition 4.5 is intuitive. First, since an uninformed outsider can hide ignorance by always reporting  $\omega = 0$ , thereby inducing the firm to implement the safe strategy, the optimal mechanism offers a wage payment of zero if the adviser recommends this course of action. The optimal mechanism also features maximum punishment, with  $w_0 = -B$ , since a wrong report reveals the adviser to be uninformed. The wage  $w_1$  is set so

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<sup>11</sup>Note that Lemmas 4.2 and 4.3 imply that the firm always selects the correct action in each state whenever the expert exists.

that the expert outsider’s participation constraint is just binding. Finally, under this mechanism the expert reports truthfully with probability 1, and thus it is sequentially rational for the firm to follow the expert’s advice. This mechanism has a natural interpretation as a compensation contract which penalizes the advisor for a low cash flow, neither rewards nor punishes for a moderate cash flow, and rewards for high cash flow.

The preceding mechanism (as well as every optimal mechanism) delivers the firm the same payoff as a “buyout” by the expert. If the expert owned the entire firm, then he would always select the optimal action in each economic state, which would deliver him a payoff of  $(1 - q) + q(1 - c)$ . However, by acquiring the firm, the expert would forego his outside option, worth  $\underline{u}$ . Hence, the expert would be willing to pay up to  $(1 - q) + q(1 - c) - \underline{u}$ . Thus, the firm’s ex ante value in any optimal mechanism is *as if* every shareholder sells his share to the expert, should he exist, at the largest price that the expert is willing to pay. Note however that the buyout mechanism is only feasible in a strict subset of cases in which the mechanism given in Proposition 4.5 is feasible ( $\underline{u} \leq qB$ ).

The mechanism is extremely powerful—provided the expert has sufficient bonding capability. An optimal mechanism: allows the firm to select the best action in each state of nature; eliminates adverse selection costs, and pins the expert to his outside option. That is, the ex ante firm value under an optimal direct revelation mechanism is

$$(24) \quad V_{DRM} = V^* - a\underline{u}.$$

The same outcome would arise in an economy without private information, where the firm directly observes each agent’s type, hires the expert just by meeting his outside option, and directly observes the expert’s signal of the economic state.

## 5 MARKETS VS. MECHANISMS: ENDOGENOUS RESERVATION VALUE

The remainder of this section determines conditions under which the firm optimally abstains from posting a mechanism, relying instead on the market to guide its decision. Recall, it is optimal for the firm to offer the mechanism whenever it generates a higher expected firm value than market-reliance. Thus, the firm may choose not to offer the mechanism for two distinct reasons. First, it could be that screening is infeasible in equilibrium, in which case the firm offers no payments and relies exclusively on the market for information. Second, it could be that screening is feasible, but expected firm value is strictly higher under market-reliance. We consider both possibilities below.

Recall, in the previous section we derived a set of necessary conditions for a mechanism to outperform the market, found a necessary and sufficient condition for these conditions to be mutually consistent, and characterized the optimal mechanism(s). Critically, the analysis of the preceding section treated  $\underline{u}$ , the expert’s reservation value for participating in the mechanism (or his continuation payoff from rejecting), as exogenous, consistent with the standard mechanism design approach. This subsection determines the equilibrium value of  $\underline{u}$  generated in the securities market economy.

In order for a mechanism to do better than market-reliance, it must induce the expert to participate if he exists (see Lemma 4.1). Therefore, if a mechanism is indeed offered, and no agent agrees to participate, in equilibrium the firm and market maker infer that no expert exists ( $\eta = 0$ ). Thus, whenever a single sell order arrives (i.e., an on-path order vector), the market maker and firm attribute the single order to the arrival of the liquidity shock. Thus, the firm sticks with the risky action, and the market maker sets the price to the expected cash flow under the risky strategy,  $p = 1 - q$ . Crucially, the market maker sets this price *regardless of the sell order size*: when the mechanism is left sitting, the arrival of a single sell order has no price impact.

When the mechanism is rejected, the arrival of two sell orders is an off-path event, as is the arrival of a buy order. Consistent with Remark 3.1, the market maker interprets the arrival of two sell orders as confirmation of economic state zero, and interprets the arrival of a buy order (paired with zero or one sell orders) as confirmation of economic state one. This assumption about the formation of off-path beliefs minimizes  $\underline{u}$ , giving the mechanism the best possible chance to beat the market. Furthermore, it immediately implies that an uninformed outsider's expected trading gain is zero when the mechanism is rejected.

In light of the preceding discussion consider the optimal strategy of an expert who deviates by rejecting the mechanism. If  $\omega = 1$ , there is zero profit to be made from any buy order, because a buy order is interpreted as confirmation of  $\omega = 1$  and is cleared at price 1. However, if  $\omega = 0$ , the expert earns maximal expected profits by selling the maximal feasible amount,  $t = 1$ . After all, if a liquidity shock arrives, then the market maker and firm will infer that  $\omega = 0$ , the firm will switch to the safe strategy, price will be set at fundamental value  $(1 - c)$ , leaving the expert with zero profit. However, if no liquidity shock arrives, then the secondary market price will be set at  $p = 1 - q$  for any feasible sell order  $t \leq 1$ . Thus, if the expert were to deviate by rejecting a posted optimal mechanism, his maximal expected trading profit in state  $\omega = 0$  is equal to  $(1 - q)(1 - l)$ . We thus have the following lemma.

**Lemma 5.1** (*Reservation Values*). *If a mechanism satisfying the screening constraints is posted, then an uninformed outsider's reservation value is 0, and the expert's reservation value is*

$$(25) \quad \underline{u} = q(1 - q)(1 - l).$$

It is readily verified that the expert's endogenous reservation value for participating in the mechanism exceeds the reservation value that would be posited if one were to ignore the effect of the posted mechanism on trading opportunities. That is, the reservation value for participating in the mechanism exceeds the trading gain the expert stands to capture in a market when no mechanism has been posted ( $qu_0^*$ ). In particular:

$$(26) \quad \underline{u} = q \cdot 1(1 - q)(1 - l) > q \cdot m(1 - q)(1 - l) = qu_0^*.$$

By posting a mechanism that screens for expertise, the firm allows the expert to make large trading gains in the market, by giving him an opportunity to convince the market maker and firm that

he doesn't exist. In contrast, when no mechanism is posted, the expert is constrained to trade less aggressively, e.g. selling  $m < 1$  units due to concern over negative price impact. Notice, the preceding inequality also informs us that the expected wage bill under the optimal mechanism ( $\underline{u}$ ) exceeds the adverse selection selection discount under market-reliance ( $qu_0^*$ ).

### 5.1 MARKETS VS. MECHANISMS: FEASIBILITY

Recall from Lemma 4.4 that screening is feasible only if  $qB \geq \underline{u}$ . The next proposition follows immediately from (25).

**Proposition 5.2** (*Markets vs. Mechanisms I*). *In equilibrium, screening is feasible only if*

$$(27) \quad B \geq (1 - q)(1 - l).$$

*Otherwise, the firm offers no mechanism and relies exclusively on the market for information.*

To appreciate the significance of the preceding result, suppose outsiders have bonding capability  $B$  such that:

$$(28) \quad \underbrace{q(1 - q)(1 - l)}_{\underline{u}} > qB > \underbrace{qm(1 - q)(1 - l)}_{qu_0^*}.$$

In this case, with  $B > u_0^*$ , the feasible set for the mechanism design problem would appear to be non-empty if one were to naively posit that the expert's reservation value is not affected by the posting of the mechanism. However, with an endogenous reservation value, the feasible set is empty due to the increase in (off-equilibrium) informed trading gains that arise from the posting of the mechanism.

In other words, the existence of stock market trading opportunities generates a link between the expert's decision to participate in the mechanism and his reservation value, and this link can cause an otherwise optimal mechanism to become infeasible. Intuitively, if a mechanism is posted, an informed investor can capture especially large trading gains by deviating and rejecting the mechanism. In order to counter this strong temptation to deviate, the mechanism-reliant firm must offer a large reward for correct advice ( $w_1$ ). But in so doing, the firm also increases the temptation of incompetents to take up the mechanism. If the trading gains following a deviation become sufficiently high (equation 25), accomplishing the dual tasks of eliciting expert participation in the mechanism and the screening out of incompetents becomes impossible.

### 5.2 MARKETS VS. MECHANISMS: OPTIMALITY

The remainder of the analysis assumes that the bonding capability of the outsider agents ( $B$ ) is sufficiently large to ensure that screening is feasible.

Combining (24) and (25), we find that the ex ante share price of a firm posting a mechanism is

$$(29) \quad V_{DRM} = V^* - aq(1 - q)(1 - l),$$

while equation (23) gives the ex ante share price of a market-reliant firm. Therefore, we turn next to a case-by-case comparison of the ex ante stock prices attained by mechanism-reliant versus market-reliant firms.

**No-Rent Equilibrium.** When the market informativeness parameter is high, ( $K > \bar{K} = J - 1$ ), the market-reliant equilibrium generates no rent for the type-0 expert, and the firm switches to the safe action if any sell order arrives without an accompanying buy order. Equation (23) implies the ex ante value of the firm in this equilibrium is

$$V_{MKT} = V_{NR} = V^* - l(1 - a)(c - q).$$

Comparing the expressions for firm value, we find that

$$(30) \quad \begin{aligned} V_{MKT} \geq V_{DRM} &\iff l(1 - a)(c - q) \leq aq(1 - q)(1 - l) \\ &\iff \frac{J - 1}{J} \leq K \end{aligned}$$

The preceding inequality illustrates the fundamental tradeoff between the market and mechanism when the market informativeness measure is high. Specifically, the right side of the first line of (30) captures the expected wage bill for a firm posting a mechanism, which reflects the high reservation value of an expert outsider. The left side of the inequality captures the cost of relatively less efficient production of a market-reliant firm, which mistakenly switches to the safe action in response to an uninformative sell order generated by a liquidity shock. Since  $K > \bar{K} = J - 1$  and  $J > 1$ , the second line of (30) implies immediately that expected firm value is higher under market-reliance,  $V_{MKT} > V_{DRM}$ . Thus, even if screening for expertise using a mechanism is feasible, doing so is suboptimal in the no-rent case.

**Proposition 5.3** (*Markets vs. Mechanisms II: High Informativeness*). *If  $K > \bar{K}$ , then the ex ante value of a market-reliant firm is strictly larger than the ex ante value of a mechanism-reliant firm.*

Before proceeding to the cases of moderate and low  $K$ , it is worth discussing why, at an intuitive level, the market looks especially attractive relative to the mechanism when  $K$  is high. Recall, that  $K$  is high when there is a high probability ( $a$ ) of an expert outsider existing and/or a low probability of a liquidity shock ( $l$ ). In this case, the firm's production decision under market-reliance is efficient, except that it incorrectly switches to the safe strategy in the event of a liquidity shock combined with non-existence of the expert. But this inefficient production decision only occurs with probability  $l(1 - a)$ , which is low when  $a$  is high or  $l$  is low (i.e., when  $K$  is high). Thus, the first advantage of the mechanism, better information about the economic state, is not large. Furthermore, when



$K$  is large, the expert's trading profit under market-reliance,  $qu_0^* = 0$ , and the market imposes no adverse selection cost. Thus, the *endogenous increase in the reservation value generated by the mechanism posting is largest in this case*. Thus, the cost of posting the mechanism is as large as possible, and the slightly better information provided by the mechanism is not enough to overcome it. Indeed, we can interpret (30) as:

$$V_{MKT} \geq V_{DRM} \iff \text{Cash Flow Loss} \leq \underline{u} - qu_0^* = \text{Change in Reservation Value} .$$

As described above, the left hand side of the inequality is small, and the right hand side is large when  $K$  is large.

**Equilibrium With Information Rent.** We now turn to the case of  $K < \bar{K}$ , implying the market-reliant equilibrium delivers the expert an information rent. From equation (23), the expected value of the market reliant firm equals

$$V_{MKT} = V_R = V^* - aq(1-l)(1-c) - aq(1-q)(1-l)m^* ,$$

where  $m^*$  is the minimum trade size in the support of the type-0 expert's equilibrium mixed strategy. Comparing the expressions for ex ante firm value, we find that

$$V_{MKT} \geq V_{DRM} \iff aq(1-l)(1-c) \leq aq(1-l)(1-q)(1-m^*) .$$

The preceding equation again reveals the fundamental tradeoff between markets and mechanisms, a tradeoff between production efficiency and relative implementation costs. The left side of the equation reflects the fact that even if the investor is informed, the market-reliant firm incorrectly fails to switch to the safe strategy in the bad state absent a fully revealing liquidity shock, with the output loss equal to  $1 - c$ . The right side of the equation reflects the difference in relative implementation costs. Specifically, the type-0 expert is forced to trade less aggressively if the firm does not post a mechanism, with her minimum sell size equal to  $m$  shares. In contrast, if a mechanism is posted, a type-0 expert anticipates the possibility of being able to deviate and trade aggressively, selling one share with zero price impact. As argued above, this deviation gain represents the informed investor's opportunity cost of participating in the mechanism and is reflected in the expected wage bill for the mechanism-reliant firm. Thus, the right hand side again represents the change in the reservation value resulting from posting a mechanism.

Dividing both sides by  $aq(1-q)(1-l)$ , we find that

$$(31) \quad V_{MKT} \geq V_{DRM} \iff \frac{1}{J} \leq 1 - m^* .$$

Here the right hand side is the change in the reservation value and the left hand side is the loss of cash flow from market-reliance, both normalized by the wage. Recall that  $m^*$  is a continuous and decreasing function of  $K$ , satisfying  $m(0) = 1$  and  $m(\bar{K}) = 0$ . The next result follows immediately.

**Proposition 5.4** (*Markets vs. Mechanisms III: Low and Intermediate Informativeness*). *Suppose  $K < \bar{K}$ . For each  $J$ , there exists a threshold for market informativeness  $\hat{K}(J) < \bar{K}$  such that the ex ante value of the market-reliant firm is strictly larger than the ex ante value of a mechanism-reliant firm if and only if  $K > \hat{K}(J)$ . Furthermore,  $\hat{K}(J)$  is decreasing in  $J$ .*

Recall that the market informativeness parameter  $K$  is the likelihood ratio that a sell order (cum zero order) originates with type-0 expert as opposed to arising from the liquidity shock. A larger value of  $K$  increases the attractiveness of the market since trades by the type-0 expert then reveal more information, reducing adverse selection costs to the atomistic shareholders. Proposition 5.4 further indicates that as parameter  $J$  increases, the region of  $K$  values for which the market-reliant firm value exceeds the mechanism reliant firm value expands. Recall that in the intermediate and low informativeness cases, the market-reliant firm does not switch to the safe action often enough relative to the mechanism-reliant firm. In particular, if the expert exists, the economic state is bad, and no liquidity shock arrives, the mechanism-reliant firm switches to the safe action while the market-reliant firm either does not switch at all, or mixes. Because the firm would only mix if its payoff from safe and risky action is identical, its payoff is the same as if it always played risky. Thus, by not switching to the safe strategy, the firm's expected cash flow is zero in the bad state, instead of  $1 - c$ . Parameter  $J$  is high precisely when the probability of the bad state is low and when the switching cost is high, and hence, the expected loss from not switching is relatively small. That is, when  $J$  is high, there is a small loss of expected cash flow arising from relying on the market for information.

Surprisingly, market reliance can be optimal in a wide variety of circumstances. Proposition 5.4 shows that even when incorrect decisions result in large losses of cash flow, market-reliance is still optimal if the endogenous change in reservation value is sufficiently high. In turn, the change in reservation is large when trades are informative and the market has little adverse selection costs. Conversely, even in markets plagued by adverse selection, market-reliance is optimal if the cash flow under market-reliance is not too much lower than the mechanism.

Figure 2 summarizes our discussion of the trade-off between mechanism and market-reliance. After normalizing by  $aq(1-q)(1-l)$ , equation (31) reveals that the tradeoff between market-reliance and mechanism reliance depends on the relationship between the normalized difference in expected cash flows,  $1/J$ , and the normalized change in the expert's reservation value,  $1 - m^*$ . The left panel shows that the expected wage cost of the mechanism exceeds the adverse selection cost of market-reliance, both normalized by the expected wage. The difference between the two lines is the endogenous increase in the reservation wage. The right panel is the difference in expected cash flows, again normalized by the wage. The mechanism delivers higher cash flows. As  $K$  increases, the cash flows of both the mechanism and market reliance increase, but the normalized difference is constant. Clearly for  $K$  sufficiently small the mechanism delivers greater firm value, and for  $K$  sufficiently close to  $J - 1$ , market reliance delivers greater firm value.

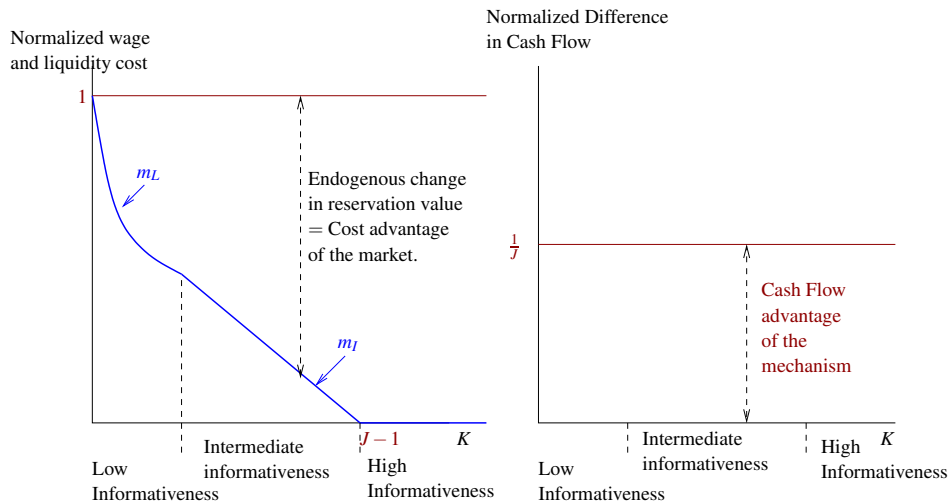


Figure 2: Left panel: Normalized expected reservation wage of the mechanism and adverse selection cost of market reliance. Right panel: normalized difference in expected cash flow between the mechanism and market-reliance.

## 6 SEARCH FRICTION

In the preceding sections we revealed a weakness of mechanisms as a source of information in an economy with a securities market: in order to elicit useful advice, a mechanism must induce an expert to participate with probability one if he exists, but such a mechanism results in a high endogenous reservation value for the expert. Indeed, by rejecting such a mechanism, the expert can convince the market that trade is uninformed, which allows him to trade in the market with no price impact. This line of reasoning suggests that it might be possible to improve upon the mechanism by designing a mechanism that leaves some doubt regarding the expert’s existence, even if no one accepts the contract.

In light of the preceding reasoning, suppose now that the firm introduces a search friction into the process of offering the mechanism, which limits the expert’s ability to observe the posted mechanism. That is, the firm can determine, in a way observable to the entire economy, a probability  $\pi \in [0, 1]$  that the contract will be observed by the expert if he exists.<sup>12</sup>

For example, by advertising the mechanism less widely or for less time, the firm can reduce the probability the expert will see it. Notice that this technology subsumes the market-reliant and mechanism-reliant firm as special cases in which  $\pi = 0$  and  $\pi = 1$ . That is, in our previous analysis, the firm’s only choices were  $\pi \in \{0, 1\}$ , and we now allow the firm to choose a value of  $\pi$  between these two extremes. To highlight this difference, in this section we refer to  $\pi = 0$  and  $\pi = 1$  as “pure market-reliance” and “pure mechanism-reliance”, respectively.

As before, the market maker can observe whether the mechanism is accepted. However, the

<sup>12</sup>The model in this section is formally equivalent to the setting of the preceding sections, except that the expert plays a mixed strategy in his acceptance decision. That is, if the expert is indifferent between accepting the mechanism and rejecting, then he accepts with probability  $\pi \in (0, 1)$ .

market maker cannot determine whether the expert saw the mechanism. Therefore, in contrast to the preceding analysis, if the firm offers a mechanism that would be acceptable to the expert and this mechanism is rejected, the market maker does not infer that no expert exists, because there is a positive probability that the expert may simply not have seen the mechanism offer.

Anticipating, we will show that the firm *always* prefers to add a search friction: pure mechanism-reliance ( $\pi = 1$ ) can always be improved upon by introducing a search friction ( $\pi < 1$ ).

**Mechanism Design.** To begin, note that the choice of  $\pi \in (0, 1]$  affects the market maker's belief about the expert's existence if the mechanism is not accepted, which is described in more detail below. Therefore, the choice of  $\pi$  can change the pricing rule used by the market maker if the mechanism is not accepted, which, in turn, affects the expert's reservation value in the mechanism. Therefore, we now express the expert's reservation value in the mechanism as  $\underline{u}(\pi)$ .

Given the reservation value  $\underline{u}(\pi)$ , the mechanism design problem facing the firm is identical in form to the one considered in Section 4. In particular, Lemmas 4.1, 4.2, and 4.3 are not affected by the firm's choice of  $\pi$ .<sup>13</sup> Because the choice of  $\pi$  only affects the expert's reservation value, we can apply the results of Section 4 to draw three conclusions. First, a mechanism that does better than market-reliance (i.e.,  $\pi = 0$ ) must simultaneously screen out incompetents, induce the expert to accept if he exists and observes the offer, and incentivize truthful reporting of the economic state, thereby satisfying the screening constraints  $\mathcal{S}$ . Second, a mechanism satisfying  $\mathcal{S}$  exists if and only if  $qB \geq \underline{u}(\pi)$ . Because  $\underline{u}(\pi) < 1$ , a sufficiently large value of  $B$  implies that screening is feasible for all possible  $\pi$ , and we maintain this assumption throughout this section. Third, any feasible mechanism that also satisfies the expert's participation constraint with equality maximizes ex ante firm value subject to  $\mathcal{S}$ . Thus, if the firm offers such a mechanism *and it is accepted*, then the firm expects to make the correct production decision in each state, while paying the expert  $\underline{u}(\pi)$ .

As in Section 4, if the mechanism is accepted, then the expert is barred from subsequently trading in the market; if the mechanism is rejected, then the adverse selection cost is zero. Therefore if the mechanism is accepted, then firm value is

$$V_{ACC}(\pi) = (1 - q)1 + q(1 - c) - \underline{u}(\pi).$$

**Asset Market.** We now analyze the market when the mechanism is not accepted. First, note that if the posted mechanism is not accepted, the market maker and firm Bayesian update that the expert is less likely to exist. In particular, with probability one the mechanism is not accepted if the expert does not exist. With probability  $1 - \pi$  the mechanism is not accepted if the expert does exist. Therefore, if the posted mechanism is not accepted, the market maker and firm's posterior belief that the expert exists is given by:

$$\hat{a}(\pi) = \frac{a(1 - \pi)}{a(1 - \pi) + 1 - a}.$$

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<sup>13</sup>The term "market-reliance" used in the lemmas refers to  $\pi = 0$ .

Next, consider how beliefs about the economic state respond to trade in the market. First, note that Lemma 3.2 continues to hold in the current setting. Indeed, for all  $\pi \in [0, 1]$  we know: the arrival of two sell orders reveals that the economic state is 0;<sup>14</sup> the absence of market activity implies that the expert does not exist; and an on-path order containing a buy reveals economic state is 1.

However, the formula for beliefs following a single sell order given in (7) must be modified to reflect the fact that the expert may not have seen the mechanism offer. In particular, if the mechanism is not accepted, the prior belief that an expert exists is revised down to  $\widehat{a}(\pi)$ . Thus, for a given value of  $\pi$ , the Bayesian update that the economic state is bad following the arrival of a single sell order is

$$(32) \quad \chi_\pi(\vec{t}) = \frac{\widehat{a}(\pi)q(1-l)\phi_0(t) + (1-\widehat{a}(\pi))ql}{\widehat{a}(\pi)q(1-l)\phi_0(t) + (1-\widehat{a}(\pi))l}.$$

As in Section 3, dividing through by  $(1-\widehat{a}(\pi))l$  allows us to rewrite this belief as

$$(33) \quad \chi_\pi(\vec{t}) = \frac{K_\pi\phi_0(t) + q}{K_\pi\phi_0(t) + 1},$$

where

$$K_\pi \equiv \frac{\widehat{a}(\pi)q}{1-\widehat{a}(\pi)} \frac{1-l}{l}.$$

Simplifying slightly, we find that

$$K_\pi = \frac{aq}{1-a} \frac{1-l}{l} (1-\pi) = K(1-\pi).$$

Intuitively, an increase in  $\pi$  is equivalent to a reduction in the market informativeness parameter. Indeed, the mechanism induces the expert to accept (if he observes the offer), and the expert is barred from trading if he does so. Therefore, if the offer is more likely to be observed, then the expert is less likely to be in the market following the mechanism's rejection, reducing market informativeness. However, unlike the analysis of Section 4, where rejecting the mechanism convinces the market maker that no expert exists, for  $\pi < 1$  the market maker and firm continue to believe that the expert may exist even after the mechanism's rejection.

With this modification to the beliefs following a single sell order, the equilibrium conditions are identical to those in Section 3. In particular, the firm's sequentially rational action choice is still described by (1), the market price is still given by (2), the expert's expected payoff of submitting order  $t$  is still given by (6). It follows that for a given value of  $\pi$ , the equilibrium following rejection of the mechanism is identical to the one characterized in Section 3, with  $K_\pi$  replacing  $K$ .

In order to establish the main result of this section—that pure mechanism-reliance is never

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<sup>14</sup>Strictly speaking in the case of  $\pi = 1$ , i.e. when the expert always accepts the mechanism if he exists, the arrival of two sell orders is off-path. The convention for off-path beliefs described in Remark 3.1 nevertheless implies that such an order vector reveals economic state zero.

optimal—we only need to consider values of  $\pi$  above a certain threshold. In particular, note that

$$\pi > \max\{1 - \underline{K}/K, 0\} \Rightarrow K_\pi < \underline{K}.$$

Thus, for any  $K$ , a  $\pi$  sufficiently close to one ensures that  $K_\pi < \underline{K}$ , and we focus on such  $\pi$  for the rest of the section. In this case, the equilibrium in the market following the mechanism's rejection is characterized by Proposition 3.7 with  $K_\pi$  replacing  $K$ . From the equilibrium characterization, the type-0 expert's expected profit following rejection of the mechanism is

$$u_0^*(\pi) = m_L(K_\pi)(1 - q)(1 - l),$$

where Proposition 3.7 defines  $m_L(\cdot)$ . Therefore, if he doesn't see the mechanism (or deliberately rejects it) the expert expects a trading profit of  $qu_0^*(\pi)$ . But the expert's reservation value in the mechanism is simply his expected trading profit following rejection, and thus,

$$\underline{u}(\pi) = qu_0^*(\pi).$$

Because  $\pi$  is sufficiently large so that  $K_\pi < \underline{K}$ , the expert expects a positive information rent from trading in the market, and therefore his reservation value for the mechanism is also positive. Furthermore, because  $K_\pi$  is decreasing in  $\pi$  and  $m_L(\cdot)$  is also decreasing in  $K$ , the expert's reservation value  $\underline{u}(\pi)$  is *increasing* in  $\pi$ . Intuitively, if the expert is more likely to observe the mechanism, he is less likely to be in the market; consequently, his trades have a smaller price impact, increasing his expected trading profit.

Next, we use the results in Section 3 to calculate firm value if the mechanism is not accepted. In particular, whenever the expert expects an information rent following rejection of the mechanism, expected firm value is given by Lemma 3.9 (with  $\hat{a}(\pi)$  replacing  $a$ ). Thus, if the mechanism is not accepted, expected firm value is

$$V_{REJ}(\pi) = 1 - q + \hat{a}(\pi)ql(1 - c) - \hat{a}(\pi)qu_0^*(\pi).$$

If the mechanism is not accepted, the market informativeness parameter is  $K_\pi < \underline{K}$ , and so the firm switches to safe whenever two sell orders arrive and sticks with risky otherwise. Two sell orders arrive if the expert exists (probability  $\hat{a}(\pi)$ ) and a liquidity shock is realized (probability  $l$ ). In this case, the firm expects to pay an adverse selection cost of  $qu_0^*(\pi)$  whenever the expert exists. Given that the mechanism was not accepted, the expected adverse selection cost is  $\hat{a}(\pi)qu_0^*(\pi)$ .

**Ex Ante Firm Value.** We decompose the ex ante value of the firm into two parts, based on whether the mechanism is accepted or not:

$$V(\pi) = a\pi V_{ACC}(\pi) + [a(1 - \pi) + 1 - a]V_{REJ}(\pi).$$

The first term is the probability that the mechanism is accepted in equilibrium multiplied by

expected firm value conditional on acceptance, while the second term is the probability that the mechanism is not accepted multiplied by expected firm value conditional on the mechanism not being accepted. Substituting the preceding expressions, we find

$$(34) \quad V(\pi) = 1 - q + qa[\pi + (1 - \pi)l](1 - c) - am_L(K_\pi)q(1 - q)(1 - l).$$

To understand this expression, note that if the expert exists, then he expects a surplus of  $qu_0^*(\pi)$ , whether or not he sees the mechanism. If he doesn't observe the mechanism, then this payoff comes in the form of trading gains in the market, and if he does, then this payoff comes in the form of an expected wage payment. Conversely, regardless of whether the expert observes the mechanism, if the expert exists, then the firm expects to pay  $qu_0^*(\pi)$  either as a wage or as an adverse selection cost. Thus, from an ex ante perspective, offering a mechanism with search friction  $\pi$  costs the firm  $aqu_0^*(\pi)$ .

Next, note that the firm only switches to the safe action in two circumstances: 1) when the mechanism is accepted and the state is bad, or 2) when the mechanism is rejected and two sell orders arrive. Therefore, whenever the economic state is good the firm selects the risky action, yielding payoff 1. Furthermore, if the firm fails to switch to safe in the bad economic state, then its payoff is 0. Whenever the firm does switch to safe, its payoff is  $1 - c$ . Combining these observations gives the expected cash flow embedded in (34).

The expression for expected firm value in (34) reveals the fundamental tradeoff inherent in the firm's choice of search friction,  $\pi$ . By increasing  $\pi$ , the expert is more likely to observe the mechanism if he exists, increasing the probability that the firm selects the correct action in each state. At the same time, by offering the mechanism more often the firm makes it less likely that the expert is in the market following the mechanism's rejection. This reduces price impact in the market, which makes informed trading more profitable. This effect simultaneously increases the adverse selection cost and the expert's expected wage.

Differentiating  $V(\pi)$ , we find that

$$V'(\pi) = qa(1 - l)(1 - c) + am'_L(K_\pi)q(1 - q)(1 - l)K,$$

and hence,

$$(35) \quad V'(\pi) < 0 \iff m'_L(K_\pi) < -\frac{1}{KJ}.$$

A straightforward calculation reveals that  $m'_L(K_\pi)$  is an increasing function of  $K_\pi$  and that  $m'_L(0) = -\infty$ .<sup>15</sup> Hence, for  $K_\pi$  sufficiently close to zero, (35) is satisfied. Therefore, ex ante firm value is decreasing in  $\pi$  for  $\pi$  sufficiently close to one. We therefore have the following result.

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<sup>15</sup>First note that for  $m'_L(x) = \frac{-m_L(x)}{\sqrt{(x+1)^2 - 1}}$ , and  $m_L(0) = 1$ . Hence,  $\lim_{x \rightarrow 0} m'_L(x) = -\infty$ . Next, note that  $m''_L(x) = \frac{1}{[(x+1)^2 - 1]^{\frac{3}{2}}} > 0$ .

**Proposition 6.1** (*Search Friction*). *If the firm has the ability to choose the probability  $\pi \in [0, 1]$  with which the expert observes the mechanism if he exists, then pure mechanism-reliance,  $\pi = 1$ , is suboptimal for all possible model parameters.*

In other words, even if the model parameters are such that pure mechanism reliance generates greater firm value than pure market reliance, firm value increases further by introducing a search friction, which sometimes causes the firm to rely on the market for information. The search friction reduces the expert's equilibrium reservation value by more than the drop in the firm's cash flow resulting from the expert sometimes not seeing the mechanism, forcing the firm to rely on the imperfect information provided by the market.

## 7 DISCUSSION/ALTERNATIVE ASSUMPTIONS

We have shown that the existence of a securities market has a fundamental effect on the mechanism design problem. After all, the market is the source of the underlying (endogenous) outside option value in the mechanism design problem. The particulars of the mechanism affect the value of information to an informed trader in the securities market, and therefore her outside option value. For example, Section 6 showed that designing a mechanism so that an informed expert sometimes failed to observe the contract resulted in a lower outside option for the informed trader, thus reducing the cost of information under the mechanism.

In the interest of realism (insider trading is generally prohibited) and in order to give the mechanism the best possible hearing, Section 3 adopted the assumption that an informed investor who agrees to the DRM cannot trade in the market (exclusivity). It is readily apparent that the firm cannot improve upon the DRM by (non-randomly) posting some "incentive scheme" that does not impose exclusivity. To see this, note that the firm makes first-best production decisions under the DRM, so any alternative incentive scheme cannot increase expected cash flow. Consider next the expected cost of information. Without exclusivity, the costs of information are equal to the expected wage bill plus the expected trading gains of the firm's informed insider who has now been freed from exclusivity. But note, in order to induce the informed expert to accept this new incentive scheme, the sum of her expected wages and trading gains must not fall below her reservation value. But this reservation value is just equal to the trading gains an informed investor can expect if all agents form the belief that no informed agent exists. This is just equal to  $\underline{u}$  (equation 25), which is equal to the cost of information (expected wage bill) under the DRM with exclusivity imposed.

## 8 CONCLUDING REMARKS

When awarding the Nobel Prize in Economic Sciences, the Royal Swedish Academy generally takes great pains to highlight the numerous real-world applications to which a given contribution has been put, or to trumpet the successes of a given framework in helping to better understand empirical regularities or existing institutional arrangements. The 2007 prize for mechanism design theory was notable in that here the Academy went to some effort to explain that the theory is not intended to



be positive: “While direct mechanisms are not intended as descriptions of real-world institutions, their mathematical structure makes them relatively easy to analyze.” In a similar vein, in his Nobel lecture, Eric Maskin (2008) positioned mechanism design theory primarily as a normative theory:

The theory of mechanism design can be thought of as the “engineering” side of economic theory. Much theoretical work, of course, focuses on *existing* [his italics] economic institutions. The theorist wants to explain or forecast the economic or social outcomes that these institutions generate. But in mechanism design theory, the direction of inquiry is reversed. We begin by identifying our desired outcome or social goal. We then ask whether an appropriate institution (mechanism) could be designed to attain that goal.

In this paper we showed how securities markets may impose limits on the usage of mechanisms by corporations. After all, posting a DRM meeting an informed agent’s participation constraint generates a high endogenous reservation value since rejecting said DRM (deviating) convinces markets no informed agent exists, allowing aggressive informed trading sans price impact. The DRM-reliant firm must pay expected wages equaling this high outside option value, implying high costs of information. For the market-reliant firm, information acquisition costs (paid via uninformed shareholder trading losses) are necessarily lower, since price impact naturally limits informed trading gains when agents know informed parties have been left outside firm boundaries, and left free to trade. However, this reduction in information acquisition costs must be weighed against the concomitant reduction in information quality associated with reliance on noisy securities prices.

In our framework the firm considers two alternative sources of information: the mechanism and the market. One might expect similar results to apply in other mechanism design settings where a market provides an alternative source of information. Regardless, the use of securities prices and hired consultants for information is ubiquitous, and so a theory of markets versus mechanisms in this setting is an important step forward.

## REFERENCES

- Boleslavsky, R., D. L. Kelly, and C. R. Taylor, 2017, “Selloffs, Bailouts, and Feedback: Can Asset Markets Inform Policy?,” *Journal of Economic Theory*, 169, 294–343.
- Bond, P., and I. Goldstein, 2015, “Government Intervention and Information Aggregation by Prices,” *Journal of Finance*, 70, 2777–2812.
- Bond, P., I. Goldstein, and E. S. Prescott, 2009, “Market Based Corrective Actions,” *Review of Financial Studies*, 23, 781–820.
- Cremer, J., and R. P. McLean, 1988, “Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions,” *Econometrica*, pp. 1247–1257.

- Dasgupta, P., P. Hammond, and E. Maskin, 1979, "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility," *Review of Economic Studies*, 46, 185–216.
- Dow, J., and G. Gorton, 1997, "Stock market efficiency and economic efficiency: is there a connection?," *The Journal of Finance*, 52, 1087–1129.
- Fama, E. F., 1976, *Foundations of Finance: Portfolio Decisions and Securities Prices*, Basic, New York.
- Fama, E. F., and M. H. Miller, 1972, *The Theory of Finance*, Holt, Rinehart, and Winston, New York.
- Faure-Grimaud, A., and D. Gromb, 2004, "Public Trading and Private Incentives," *The Review of Financial Studies*, 17, 985–1014.
- Gibbard, A., 1973, "Manipulation of Voting Schemes: A General Result," *Econometrica*, pp. 587–601.
- Gibbons, R., 2005, "Four formal(izable) theories of the firm?," *Journal of Economic Behavior and Organization*, 58, 200–245.
- Glosten, L. R., and P. R. Milgrom, 1985, "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics*, 14, 71–100.
- Grossman, S. J., and O. D. Hart, 1986, "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691–719.
- Grossman, S. J., and J. E. Stiglitz, 1976, "Information and Competitive Price Systems," *American Economic Review*, 66, 246–253.
- Hart, O., and J. Moore, 1990, "Property Rights and the Nature of the Firm," *Journal of Political Economy*, 98, 1110–1158.
- Hayek, F. A., 1945, "The Use of Knowledge in Society," *American Economic Review*, 35, 519–530.
- Hölmstrom, B., 1979, "Moral Hazard and Observability," *Bell Journal of Economics*, pp. 74–91.
- Hölmstrom, B., 1999, "The firm as a Subeconomy," *Journal of Law, Economics, and Organization*, 15, 74–102.
- Hurwicz, L., 1973, "The Design of Mechanisms for Resource Allocation," *American Economic Review*, 63, 1–30.
- Jehiel, P., and B. Moldovanu, 2000, "Auctions with Downstream Interaction among Buyers," *The RAND Journal of Economics*, 31, 768–791.

- Jullien, B., 2000, "Participation Constraints in Adverse Selection Models," *Journal of Economic Theory*, 93, 1–47.
- Kahn, C., and A. Winton, 1998, "Ownership Structure, Speculation, and Shareholder Intervention," *Journal of Finance*, 53, 99–129.
- Klein, B., 2000, "Fisher-General Motors and the Nature of the Firm," *Journal of Law and Economics*, 43, 105–141.
- Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315–1335.
- Lange, O., 1967, *Essays on Economic Planning*, Asian Publishing House, Bombay, second edition edn.
- Lerner, A., 1944, *The Economics of Control*, MacMillan, New York.
- Lewis, T., and D. Sappington, 1989, "Countervailing incentives in agency problems," *Journal of Economic Theory*, 49, 294–313.
- Lizzeri, A., 1999, "Information Revelation and Certification Intermediaries," *The RAND Journal of Economics*, 30, 214–231.
- Maskin, E., 2008, "Mechanism Design: How to Implement Social Goals," *American Economic Review*, 98, 567–576.
- Myerson, R. B., 1979, "Incentive Compatibility and the Bargaining Problem," *Econometrica*, pp. 61–73.
- Rasula, I., and S. Sonderegger, 2010, "The Role of the Agent's Outside Options in Principal-Agent Relationships," *Games and Economic Behavior*, 68, 781–788.
- Riordan, M. H., and D. E. Sappington, 1988, "Optimal Contracts with Public Ex Post Information," *Journal of Economic Theory*, 45, 189–199.
- Taylor, F., 1948, "The Guidance of Production in the Socialist State," in Oskar Lange, and Frederic Taylor (ed.), *On the Economic Theory of Socialism*, University of Minnesota Press, Minneapolis.
- Tirole, J., 2012, "Overcoming adverse selection: How public intervention can restore market functioning," *The American Economic Review*, 102, 29–59.
- Williamson, O. E., 1985, *The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting*, Collier Macmillan, New York.
- Williamson, O. E., 2002, "The Theory of the Firm as Governance Structure: From Choice to Contract," *The Journal of Economic Perspectives*, 16, 171–195.

## APPENDICIES

## A PROOFS FROM SECTION 3

**Proof.** (Lemma 3.3). Suppose an uninformed agent deviates from the equilibrium by submitting order  $t$ . Let  $T_-$  be the order vector submitted by the other market participants, i.e.  $T_-$  is formed from the overall order vector  $T$  by deleting the uninformed agent's order. If the uninformed agent submits order  $t$  and the realization of the overall order vector is  $T$ , then the uninformed agent's belief just before the order is executed is  $\chi(T_-)$ , as in Lemma 3.2 and equation (7). That is, after the overall order vector is realized, the uninformed agent updates his beliefs based on the trading activity of the other market participants. Meanwhile, the market maker's belief is  $\chi(T)$ , as in Lemma 3.2, equation (7), and Remark (3.1). That is, the market maker updates his beliefs based on the entire vector of submitted orders. The market price is set as in equation (2). Therefore, if the uninformed agent submits order  $t \in [-1, 1]$  and the overall order vector is  $T$ , then the uninformed agent expects profit  $u_N(t, T) = t(p(T) - \chi(T_-)) = t(\chi(T_-) - \chi(T))(1 - \alpha(T))$ . We show that for any  $t \neq 0$ , and any realization of the expert's trade and the liquidity shock,  $u_N(t, T) \leq 0$ .

Suppose an uninformed agent deviates from the equilibrium and submits a buy order,  $t_B < 0$ .

*Case 1:* the uninformed agent's buy order is the only one submitted. The overall order vector  $T$  consists of a single buy order,  $t_B$ . From Lemma 3.2,  $\chi(T) = 0$  and  $\alpha(T) = 0$ . Meanwhile,  $T_- = 0$ . From Lemma 3.2 it follows that  $\chi(T_-) = q$ . Hence,  $u_N(t_B, T) = t_B q$ . Because  $t_B < 0$  it follows that  $u_N(t_B, T) < 0$ .

*Case 2:* the uninformed agent's buy order  $t_B < 0$  and a single sell order,  $t_S$  arrive. The overall order vector  $T$  consists of a single buy order and a single sell order. From Lemma 3.2 it follows that  $\chi(T) = 0$  and  $\alpha(T) = 0$ . Meanwhile,  $T_- = t_S$ . Hence,  $\chi(T_-) = \chi(\vec{t}_S) \geq q$  (consult equation (7)). Hence,  $u_N(t, T) = t_B \chi(\vec{t}_S)$ . Because  $t_B < 0$  it follows that  $u_N(t_B, T) < 0$ .

*Case 3:* two buy orders arrive. From Remark 3.1 it follows that  $\chi(T) = 0$  and  $\alpha(T) = 0$ . Meanwhile,  $T_-$  is a single buy order. Hence,  $\chi(T_-) = 0$  (consult Lemma (3.2)). Hence,  $u_N(t, T) = 0$ .

*Case 4:* the uninformed agent's buy order  $t_B < 0$  and two sell orders arrive. The overall order vector  $T$  consists of a single buy order and two sell orders. From Lemma 3.2 it follows that  $\chi(T) = 1$  and  $\alpha(T) = 1$ . Hence,  $u_N(t, T) = 0$ .

*Case 5:* two buy orders and a single sell order arrive. From Remark 3.1 it follows that  $\chi(T) = 0$  and  $\alpha(T) = 0$ . Meanwhile,  $T_-$ , consists of one buy order and one sell order. Hence,  $\chi(T_-) = 0$  (consult Lemma (3.2)). Hence,  $u_N(t, T) = 0$ .

This exhausts the possible cases.

Suppose an uninformed agent deviates from the equilibrium and submits a sell order,  $t_S > 0$ .

*Case 1:* the uninformed agent's sell order is the only one submitted. The overall order vector  $T$  consists of a single sell order,  $t_S$ . From equation 7,  $\chi(T) \geq q$  and  $\alpha(T) \in [0, 1]$ . Meanwhile,  $T_- = 0$ . From Lemma 3.2 it follows that  $\chi(T_-) = q$ . Hence,  $u_N(t_S, T) = t_S(q - \chi(T))(1 - \alpha(T))$ . Hence,  $u_N(t_S, T) \leq 0$ .

*Case 2:* the uninformed agent's sell order  $t_S > 0$  and a single buy order arrive. The overall order vector  $T$  consists of a single buy order and a single sell order. From Lemma 3.2 it follows that  $\chi(T) = 0$  and  $\alpha(T) = 0$ . Meanwhile,  $T_-$  is a single buy order. Hence,  $\chi(T_-) = 0$  and  $\alpha(T) = 0$  (consult Lemma (3.2)). Hence,  $u_N(t, T) = 0$ .

*Case 3:* two sell orders arrive. From Lemma 3.2 it follows that  $\chi(T) = 1$  and  $\alpha(T) = 1$ . Hence,  $u_N(t, T) = 0$ .

*Case 4:* three sell orders arrive. From Remark 3.1 it follows that  $\chi(T) = 1$  and  $\alpha(T) = 1$ . Hence,  $u_N(t, T) = 0$ .

*Case 5:* two sell orders and a single buy order arrive. From Remark 3.1 it follows that  $\chi(T) = 1$  and  $\alpha(T) = 1$ . Hence,  $u_N(t, T) = 0$ .

This exhausts the possible cases.

Thus, if an uninformed agent deviates from the equilibrium by buying or selling in the market, his expected profit is weakly negative for any possible realization of the order vector. ■

**Proof.** (Lemma 3.4). *Step 1:* We show that  $\Phi_0(\cdot)$  has no mass points. Because the order flow generated by the liquidity shock is uniformly distributed, if  $\Phi_0(\cdot)$  has a mass point at  $t$ , then  $\chi(\vec{t}) = 1$ . Consequently, the expert's expected payoff of submitting this order given in (6) is zero, and hence, the expert's equilibrium payoff in state zero must also be zero, contradicting  $u_0^* > 0$ .

*Step 2:* We show that the support of  $\Phi_0(\cdot)$  has no gaps. In particular, we show that if  $t_1$  is inside the support of  $\Phi_0(\cdot)$  and  $t_2 \in (t_1, 1]$ , then  $t_2$  is also inside the support of  $\Phi_0(\cdot)$ . To derive a contradiction, suppose not. Because  $t_2$  is outside the support of  $\Phi_0(\cdot)$ ,  $\chi(t_2) = q$  and hence  $\alpha(\vec{t}_2) = 0$ . Meanwhile,  $t_1$  is inside the support of  $\Phi_0(\cdot)$ , and hence  $\chi(\vec{t}_1) \geq q$ . Therefore,  $t_2(1 - \chi(\vec{t}_2))(1 - l) > t_1(1 - \chi(\vec{t}_1))(1 - \alpha(\vec{t}_1))(1 - l)$ , a contradiction.

*Step 3:* We show that  $m > 0$ . Suppose  $m = 0$ . For any  $\epsilon > 0$ , type-0 expert's indifference condition (8) requires  $u_0^* = \epsilon(1 - \chi(\vec{\epsilon}))(1 - \alpha(\vec{\epsilon}))(1 - l)$ . Because  $(1 - \chi(\vec{\epsilon}))(1 - \alpha(\vec{\epsilon}))(1 - l) < 1$ , it follows that  $u_0^* < \epsilon$  for any  $\epsilon > 0$ , contradicting  $u_0^* > 0$ . ■

**Proof.** (Lemma 3.5). *Step 1.* We show that  $u_0 \geq m(1 - q)(1 - l)$ . Lemma 3.4 implies that for  $t < m$ ,  $\phi_0(t) = 0$ , and thus equation (7) implies that  $\chi(t) = q$  and thus  $\alpha(t) = 0$  for  $t < m$ . Consequently, the expected payoff from submitting order  $t \in (0, m)$  is  $t(1 - q)(1 - l)$ . Thus, the expert does not prefer to deviate to an order outside the support and  $u_0 \geq m(1 - q)(1 - l)$ .

*Step 2.* We show that  $u_0 \leq m(1 - q)(1 - l)$ . Consider  $t = m + \epsilon$ . Because this order is inside the support,  $u_0 = (m + \epsilon)(1 - \chi(m + \epsilon))(1 - \alpha(m + \epsilon))(1 - l)$ . From Lemma 3.4 and equation (7),  $\chi(m + \epsilon) \geq q$ . Also,  $\alpha(m + \epsilon) \geq 0$ . Hence,  $u_0 \leq (m + \epsilon)(1 - q)(1 - l)$  for all  $\epsilon > 0$ . It follows that  $u_0 \leq m(1 - q)(1 - l)$ .

Combining Steps 1 and 2, we find that  $u_0 = m(1 - q)(1 - l)$ . ■

**Proof.** (Lemma 3.6)

Proof of part (i). For  $t < m$ ,  $\chi(\vec{t}) = q$ , and hence  $\alpha(\vec{t}) = 0$ . From (8),

$$t \in [m, 1] \Rightarrow \chi(\vec{t}) = 1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))}.$$

Because  $m > 1/J$  and  $t(1-\alpha(\vec{t})) \leq 1$ ,

$$1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))} < 1 - \frac{1-c}{t(1-\alpha(\vec{t}))} \leq c.$$

Hence,  $\chi(\vec{t}) < c$  for all  $t \in [0, 1]$ . Thus, (1) implies  $\alpha(\vec{t}) = 0$  for all  $t \in [0, 1]$ .

Proof of part (ii). For  $t < m$ ,  $\chi(\vec{t}) = q$ , and hence  $\alpha(\vec{t}) = 0$ . From (8),

$$t \in [m, 1] \Rightarrow \chi(\vec{t}) = 1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))}.$$

Consider  $t \in [m, Jm)$ . Because  $1 - \alpha(\vec{t}) \in [0, 1]$ ,

$$\chi(\vec{t}) = 1 - \frac{m(1-q)}{t(1-\alpha(\vec{t}))} \leq 1 - \frac{m(1-q)}{t},$$

and because  $t < Jm$ ,

$$1 - \frac{m(1-q)}{t} < 1 - \frac{1-q}{J} = c.$$

Hence, if  $t \in [m, Jm)$ , then  $\chi(\vec{t}) < c$ . Thus, (1) implies  $\alpha(\vec{t}) = 0$  for  $t \in [m, Jm)$ . Consider  $t \in (Jm, 1]$  (note that by assumption  $Jm < 1$ ). To derive a contradiction, assume that  $\alpha(\vec{t}) = 0$ . From (8),

$$\chi(\vec{t}) = 1 - \frac{m(1-q)}{t}.$$

Because  $t > Jm$ ,

$$\chi(\vec{t}) = 1 - \frac{m(1-q)}{t} > 1 - \frac{(1-q)}{J} = c.$$

Hence, if  $\alpha(\vec{t}) = 0$ , then  $\chi(\vec{t}) > c$ . But (1) implies that  $\alpha(\vec{t}) = 1$ , a contradiction. Hence, for  $t \in (Jm, 1]$ , it must be that  $\alpha(\vec{t}) > 0$ . Next, note that if  $\alpha(\vec{t}) = 1$  for  $t \in (Jm, 1]$ , then  $E[u_0(t, T)] = 0$ . Because  $t$  is inside  $(Jm, 1] \subset [m, 1]$ , it follows that  $u_0^* = 0$ , a contradiction. Hence, if  $t \in (Jm, 1]$ , then  $\alpha(\vec{t}) < 1$ . ■

**Proof.** (Lemma 3.9). We provide a calculation for the ex ante expected cash flow. The rest of the argument is in the text. Conditional on order vector  $T$ , the firm's expected cash flow is

$$(1 - \chi(T))(1 - \alpha(T)) + \alpha(T)(1 - c) = 1 - \chi(T) + \alpha(T)(\chi(T) - c).$$

Hence, ex ante firm value is

$$E[1 - \chi(T)] + E[\alpha(T)(\chi(T) - c)],$$

where the expectation is taken with respect to the distribution of the equilibrium order flow vector. Because  $1 - \chi(T)$  is the probability of economic state zero conditional on order vector  $T$ , the Law of Iterated Expectations implies that  $E[1 - \chi(T)] = 1 - q$ . From Lemma 3.2,  $\alpha(T) = 0$  if a buy order arrives or the market is inactive, and hence, in these cases  $\alpha(T)(\chi(T) - c) = 0$ . From Lemma 3.6 when a single sell order arrives, the firm either selects risky  $\alpha(T) = 0$  or it mixes,  $\alpha(T) \in (0, 1)$ . If the firm mixes, then (1) requires  $\chi(T) = c$ . Thus, whenever a single sell order arrives,  $\alpha(T)(\chi(T) - c) = 0$ . Finally, if two sell orders arrive, then  $\chi(T) = 1$  and  $\alpha(T) = 1$ , and hence,  $\alpha(T)(\chi(T) - c) = 1 - c$ . Therefore,  $E[\alpha(T)(\chi(T) - c)] = aql(1 - c)$ . ■

## B PROOFS FROM SECTION 4

**Proof.** (Lemma 4.2) (i)  $\rho_1 = 1$  follows from  $\chi_1 \leq q$  and the firm's sequential rationality. (ii). Note that any mechanism that beats remaining unadvised must induce the expert to participate and screen out uninformed agents and has  $\rho_1 = 1$ . Hence, the expected payoff to the firm in any such mechanism is

$$(1 - a)(1 - q) + a \Pr(r = 0) \{ (1 - \chi_0)\rho_0 + (1 - \rho_0)(1 - c) \} + \\ a \Pr(r = 1)(1 - \chi_1) - aU,$$

where  $U \geq \underline{u}$  is the expert's expected wage. Suppose  $\rho_0 = 1$ . Using the Law of Iterated Expectations, the firm's payoff simplifies,

$$(1 - a)(1 - q) + a \Pr(r = 0)(1 - \chi_0) + a \Pr(r = 1)(1 - \chi_1) - aU = \\ (1 - a)(1 - q) + a(1 - q) - aU = 1 - q - aU.$$

Suppose  $\rho_0 \in (0, 1)$ . Firm's sequential rationality requires  $\chi_0 = c$ , and hence, the firm's payoff simplifies to

$$(1 - a)(1 - q) + a \Pr(r = 0) \{ (1 - \chi_0)\rho_0 + (1 - \rho_0)(1 - \chi_0) \} + a \Pr(r = 1)(1 - \chi_1) - aU = \\ (1 - a)(1 - q) + a \Pr(r = 0)(1 - \chi_0) + a \Pr(r = 1)(1 - \chi_1) - aU = \\ (1 - a)(1 - q) + a(1 - q) - aU = \\ (1 - q) - aU.$$

Note that the transition from the second to the third line uses the Law of Iterated Expectations. Note that  $U \geq \underline{u} > qu_0^*$ . Thus, the cost of offering the mechanism  $aU$  exceeds the adverse selection cost under market-reliance,  $aqu_0^*$ . Finally, consider the firm's cash flow under market-reliance. If

$K < \bar{K}$ , then the firm's cash flow is  $1 - q + aql(1 - c) > 1 - q$ . If  $K > \bar{K}$ , then it is  $1 - q + aq(1 - c) - l(c - q)(1 - a)$ . Note that the firm's cash flow can be rewritten  $1 - q + aql(1 - c) + l(1 - c)(1 - a)(K - \bar{K}) > 1 - q$ . Thus, the firm's expected cash flow is larger under market reliance. Simultaneously, the adverse selection cost under market reliance is smaller than the expected wage bill under the mechanism. ■

**Proof.** (Lemma 4.3). Claim 1: *If a mechanism delivers the firm a higher expected payoff than market-reliance, then it cannot be the case that  $\gamma_0 = \gamma_1 = 1$ .* If  $\gamma_0 = \gamma_1 = 1$ , then  $\chi_1 = q$  and any value of  $\chi_0$  is consistent with Bayes' rule. Because the expert always reports  $r = 1$  in equilibrium, the firm always implements the risky action, and hence expected firm value is  $1 - q - aU$ , where  $U \geq \underline{u}$  is the expert's expected wage. This is smaller than expected firm value under market-reliance, as shown in the proof of Lemma 4.2.

Claim 2: *If a mechanism delivers the firm a higher expected payoff than market-reliance, then  $w_{11} \geq w_{01-c}$  and  $w_{01-c} \geq w_{10}$ .* From Lemma 4.2,  $\rho_0 = 1$ , and hence,  $\chi_0 \geq c$ . From Bayes' Rule,

$$\chi_0 = \frac{q(1 - \gamma_0)}{q(1 - \gamma_0) + (1 - q)(1 - \gamma_1)}.$$

From Claim 1,  $\chi_0$  is well defined. Hence,

$$(36) \quad \chi_0 \geq c \iff q(1 - c)\gamma_0 + c - q \leq c(1 - q)\gamma_1.$$

First we show  $w_{11} \geq w_{01-c}$ . Note that

$$\begin{aligned} q(1 - c)\gamma_0 + c - q \leq c(1 - q)\gamma_1 &\Rightarrow \\ c - q \leq c(1 - q)\gamma_1 &\Rightarrow \\ \gamma_1 \geq \frac{c - q}{c(1 - q)} &> 0. \end{aligned}$$

Thus,  $\gamma_1 > 0$ , which implies the expert must report truthfully in state 1 with positive probability. Thus, the expert's expected payoff of reporting truthfully in state 1 must be at least as large as his expected payoff of lying, and hence  $w_{11} \geq w_{01-c}$ .

Next we show  $w_{01-c} \geq w_{10}$ . Suppose that  $\gamma_0 = 1$ . Substituting into (36),

$$q(1 - c) + c - q \leq c(1 - q)\gamma_1 \Rightarrow \gamma_1 \geq 1.$$

Hence,  $\gamma_0 = 1$  implies  $\gamma_1 = 1$ , contradicting Claim 1. Hence,  $\gamma_0 < 1$ , which implies that the expert must report truthfully in state 0 with positive probability. Thus, the expert's expected payoff of reporting truthfully in state 0 must be at least as large as his expected payoff of lying, and hence  $w_{01-c} \geq w_{10}$ .

Claim 3: *If a mechanism delivers the firm a higher expected payoff than market-reliance, then  $w_{11} > w_{01-c}$  and  $w_{01-c} > w_{10}$ .* From Lemma 4.1, any mechanism which achieves higher value than



market reliance screens out uninformed outsiders and requires participation of the expert. These constraints are:

$$(SC0) \quad w_{01-c} \leq 0$$

$$(SC1) \quad qw_{10} + (1 - q)w_{11} \leq 0$$

$$(PC) \quad q[\gamma_0 w_{10} + (1 - \gamma_0)w_{01-c}] + (1 - q)[\gamma_1 w_{11} + (1 - \gamma_1)w_{01-c}] \geq \underline{u}$$

Constraint (SC0) ensures that uninformed outsiders prefer to reject the mechanism over accepting and reporting  $r = 0$ , (SC1) ensures that uninformed outsiders prefer to reject the mechanism over accepting and reporting  $r = 1$ , and (PC) ensures that an expert prefers to participate (if he exists).

Next, note that Claim 2 ensures  $w_{11} \geq w_{01-c}$ . Therefore, either  $w_{11} > w_{01-c}$  which implies  $\gamma_1 = 1$ , or  $w_{11} = w_{01-c}$ . In either case (PC) reduces to:

$$q[\gamma_0 w_{10} + (1 - \gamma_0)w_{01-c}] + (1 - q)w_{11} \geq \underline{u}$$

Analogously, either  $\gamma_0 = 1$  or  $w_{01-c} = w_{10}$  in which case (PC) reduces further to

$$(PC) \quad qw_{01-c} + (1 - q)w_{11} \geq \underline{u}.$$

Note that subtracting (SC0) from (PC) yields

$$qw_{01-c} + (1 - q)w_{11} - w_{01-c} \geq \underline{u} \Rightarrow w_{11} \geq \frac{\underline{u}}{1 - q} - w_{01-c} > 0,$$

where the last inequality follows because  $\underline{u} > 0$  and  $w_{01-c} \leq 0$ . Hence,  $w_{11} > 0 \geq w_{01-c}$ . Note further that subtracting (SC1) from (PC) yields

$$qw_{01-c} + (1 - q)w_{11} - (qw_{10} + (1 - q)w_{11}) \geq \underline{u} \Rightarrow w_{01-c} \geq w_{10} + \frac{\underline{u}}{q} \Rightarrow w_{01-c} > w_{10},$$

where the last inequality follows from  $\underline{u} > 0$ .

Claim 4: *The expert's unique sequentially rational reporting strategy is  $\gamma_0 = 0$  and  $\gamma_1 = 1$ .* Follows immediately from Claim 3. ■

**Proof.** (Proposition 4.4). We show that (SC0), (SC1), (PC), and (BOND) imply  $\underline{u} \leq qB$ . Subtracting (SC0) from (PC) yields  $(1 - q)w_1 \geq \underline{u}$ . Substituting into (SC1) we find that  $w_0 \leq -\underline{u}/q$ . Hence, (BOND) implies that  $\underline{u}/q \leq B$ , and hence  $\underline{u} \leq qB$ . ■

**Proof.** (Proposition 4.5). In the text, we argued that in any feasible mechanism, expected firm value is

$$(1 - a)(1 - q) + a[(1 - q) + q(1 - c)] - a[qw_{1-c} + (1 - q)w_1].$$

Thus, the firm would like to minimize expected compensation,  $qw_{1-c} + (1 - q)w_1$ , but (PC) requires  $qw_{1-c} + (1 - q)w_1 \geq \underline{u}$ . Hence, any feasible mechanism in which (PC) holds with equality is optimal,

yielding payoff

$$(1 - a)(1 - q) + a[(1 - q) + q(1 - c)] - a\underline{u} = 1 - q + aq(1 - c) - a\underline{u}.$$

We show that (SC0), (SC1), (PC), and (BOND) imply  $\underline{u} \leq qB$ . Subtracting (SC0) from (PC) yields  $(1 - q)w_1 \geq \underline{u}$ . Substituting into (SC1) we find that  $w_0 \leq -\underline{u}/q$ . Hence, (BOND) implies that  $\underline{u}/q \leq B$ , and hence  $\underline{u} \leq qB$ . ■

## C EQUILIBRIUM SELECTION

In this section we consider a perturbed version of the model in which the probability of switching to the safe action cannot exceed  $\bar{\alpha} \in (0, 1)$ . We then consider the limit as  $\bar{\alpha} \rightarrow 1$ . We show that if  $K < \underline{K}$  then the equilibrium converges to the unique equilibrium in Proposition 3.7, and if  $K \in (\underline{K}, \bar{K})$ , then the equilibrium converges to the unique equilibrium in Proposition 3.8. Finally, if  $K > \bar{K}$ , then the limiting equilibrium has two properties: it generates no rent for the type-0 expert, and the type-0 expert is active on the sell side of the market with probability one. Thus, the limiting equilibrium is in the class characterized in Proposition 3.10 and, as such, yields firm value  $V_{NR}$ .

**Proposition C.1** *Suppose*

**Proof.** . ■