UNIL | Université de Lausanne
Département de finance

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

FINANCE RESEARCH SEMINAR SUPPORTED BY UNIGESTION

"Exchange Rate Disconnect in General Equilibrium"

Prof. Oleg ITSKHOKI
(Princeton University)

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We propose a dynamic general equilibrium model of exchange rate determination, which simultaneously accounts for all major puzzles associated with nominal and real exchange rates. This includes the Meese-Rogoff disconnect puzzle, the PPP puzzle, the terms-of-trade puzzle, the Backus-Smith puzzle, and the UIP puzzle. The model has two main building blocks — the driving force (or the exogenous shock process) and the transmission mechanism — both crucial for the quantitative success of the model. The transmission mechanism — which relies on strategic complementarities in price setting, weak substitutability between domestic and foreign goods, and home bias in consumption — is tightly disciplined by the micro-level empirical estimates in the recent international macroeconomics literature. The driving force is an exogenous small but persistent shock to international asset demand, which we prove is the only type of shock that can generate the exchange rate disconnect properties. We then show that a model with this financial shock alone is quantitatively consistent with the moments describing the dynamic comovement between exchange rates and macro variables. Nominal rigidities improve on the margin the quantitative performance of the model, but are not necessary for exchange rate disconnect, as the driving force does not rely on the monetary shocks. We extend the analysis to multiple shocks and an explicit model of the financial sector to address the additional Mussa puzzle and Engel's risk premium puzzle.

Monday, June 11, 2018, 10:30-12:00
Room 126, Extranef building at the University of Lausanne

Exchange Rate Disconnect in General Equilibrium*

Oleg Itskhoki
itskhoki@princeton.edu

Dmitry Mukhin
mukhin@princeton.edu

December 10, 2017

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*We thank Ariel Burstein, Charles Engel, Robert Kollmann, Cédric Tille and Eric van Wincoop for insightful discussions, Mark Aguiar, Andy Atkeson, Adrien Auclert, V.V. Chari, John Cochrane, Giancarlo Corsetti, Javier Cravino, Michael Dooley, Konstantin Egorov, Emmanuel Farhi, Gita Gopinath, Pierre-Olivier Gourinchas, Galina Hale, Tarek Hassan, Hanno Lustig, Maury Obstfeld, Kenneth Rogoff, Esteban Rossi-Hansberg, Jesse Schreger, Jón Steinsson, John Taylor, Chris Tonetti, Jaume Ventura, Adrien Verdelhan, Mark Watson and seminar/conference participants for helpful comments, and Nastya Burya for outstanding research assistance.

1 Introduction

Exchange rate disconnect is among the most challenging and persistent international macro puzzles (see Obstfeld and Rogoff 2001). The term disconnect narrowly refers to the lack of correlation between exchange rates and other macro variables, but the broader puzzle is more pervasive and nests a number of additional empirical patterns, which stand at odds with conventional international macro models. We define the broader exchange rate disconnect to include:

1. Meese and Rogoff (1983) puzzle: *nominal* exchange rate follows a volatile random-walk-like process, which is not robustly correlated, even contemporaneously, with macroeconomic fundamentals (see also Engel and West 2005).
2. PPP puzzle (Rogoff 1996): *real* exchange rate tracks very closely the nominal exchange rate at most frequencies and, in particular, exhibits a similarly large persistence and volatility as the nominal exchange rate. Mean reversion, if any, takes a very long time, with half-life estimates in the range of 3-to-5 years, much in excess of conventional durations of price stickiness (see also Chari, Kehoe, and McGrattan 2002). A related Mussa (1986) puzzle emphasizes a stark change in the properties of the real exchange rate associated with a change in the monetary regime to/from a nominal exchange rate peg (see also Mussa 1990, Monacelli 2004).
3. *Terms of trade* are positively correlated with the real exchange rate, yet exhibit a markedly lower volatility, in contrast with the predictions of the standard models, suggesting a particular pattern of the *law of one price* violations (Atkeson and Burstein 2008). In addition, real exchange rate dynamics at most horizons is largely accounted for by the law of one price violations for tradable goods, while the contribution of the relative non-tradable prices is small (Engel 1999).
4. Backus and Smith (1993) puzzle: the international risk-sharing condition that relative consumption across countries should be strongly positively correlated with the real exchange rates (implying high relative consumption in periods of low relative prices) is sharply violated in the data, with a mildly negative correlation and a markedly lower volatility of relative consumption (see Kollmann 1995 and also Benigno and Thoenissen 2008).
5. Forward premium puzzle (Fama 1984), or the violation of the *uncovered interest rate parity* (UIP) condition: the UIP prediction that a relatively high interest rate should predict a nominal exchange rate devaluation is violated in the data with an opposite sign, yet a nearly zero R^2 (see also Engel 1996). A related set of puzzles explores further the dynamic comovement between interest rate differential and exchange rate changes, or risk premia (see Engel 2016, Valchev 2016).

We summarize the above puzzles as a set of moments characterizing comovement between exchange rates and macro variables (see Table 2), and use them as the quantitative targets in our analysis.

Existing general equilibrium international macro models either feature these puzzles, or attempt to address one puzzle at a time, often at the expense of aggravating the other puzzles, resulting in a lack of a unifying framework that exhibits satisfactory exchange rate properties. This is a major challenge for the academic and policy discussion, since exchange rates are the core prices in any international model, and failing to match their basic properties jeopardizes the conclusions one can draw from the analysis. In particular, would the conclusions in the vast literatures on currency unions, international policy spillovers and international transmission of shocks survive in a model with realistic exchange rate properties? Furthermore, what are the implications of such a model for the numerous micro-level empirical studies that treat exchange rate shocks as a source of exogenous variation?

The goal of this paper is to offer a unifying theory of exchange rates that can simultaneously account for all stylized facts introduced above. We emphasize two main features of our model. First, it relies on a specific *driving force* (that is, a shock process) behind nominal and real exchange rates, which does not have a simultaneous strong direct effect on consumption, output, prices and interest rates. Second, it relies on a *transmission mechanism*, which mutes the effect of volatile exchange rate fluctuations on local prices and quantities. Violation of either of these two properties would break the disconnect. While the literature has provided a lot of empirical evidence on the transmission mechanism, we currently lack direct empirical information on the details of the shock process, which could account for the *bulk* of exchange rate fluctuations. Under the circumstances, we adopt the following strategy. From the outset, we tightly discipline the transmission mechanism with the empirical estimates from the recent literature. In contrast, we initially impose no restriction on the nature of the shock process, and show theoretically that only one type of shocks can produce the exchange rate disconnect properties in general equilibrium.

In particular, as a diagnostic tool for shock selection, we consider a near-autarky behavior of the economy, and require that the shock process produces a volatile exchange rate behavior with a vanishing effect on the economy's quantities, prices and interest rates, as the economy becomes closed to trade. Indeed, in the limit of the closed economy, any exchange rate volatility (real or nominal) should be completely inconsequential for allocations. Not surprisingly, productivity and monetary shocks, as well as the majority of other shocks, violate this intuitive requirement. We show that the one shock that satisfies this requirement, and additionally produces the empirically relevant signs of comovement between exchange rates and macro variables (consumption and interest rates), is the shock to the international asset demand.[1] We then demonstrate how this shock can have a variety of microfoundations in the financial market, including noise trading with limits to arbitrage (e.g., Jeanne and Rose 2002), heterogeneous beliefs (e.g., Bacchetta and van Wincoop 2006) and financial frictions (e.g., Gabaix and Maggiori 2015), as well as time-varying risk premium (e.g., Alvarez, Atkeson, and Kehoe 2009, Colacito and Croce 2013, Farhi and Gabaix 2016).

Further, we show that the model with a single financial shock is consistent both *qualitatively* and *quantitatively* with the exchange rate disconnect properties. In particular, small persistent shocks to international asset demand result in a volatile random-walk-like behavior of both nominal and real exchange rates. As the economy becomes closed to international trade, this shock still generates volatile exchange rate fluctuations, which however have a vanishingly small effect on the rest of the economy. Furthermore, the transmission mechanism in the model ensures that exchange rates exhibit empirically relevant comovement properties with macro variables, even when the economy is open to international

[1] An exogenous foreign asset demand shock has been widely used in the portfolio models of the exchange rate (e.g. Kouri 1976, Blanchard, Giavazzi, and Sa 2005), and it is also isomorphic to a UIP shock, as for example in Devereux and Engel (2002), Kollmann (2005) and Farhi and Werning (2012).

trade in goods and assets. In particular, the transmission mechanism features four realistic ingredients:

1. significant *home bias* in consumption, consistent with the empirical trade shares in GDP, which limits the effects of expenditure switching on aggregate consumption, employment and output;
2. *pricing to market* and law of one price violations due to strategic complementarities in price setting, which limit the response of prices (terms of trade) to exchange rate movements;
3. *weak substitutability* between home and foreign goods, which limit the extent of expenditure switching conditional on the terms of trade movements; and
4. *monetary policy* that stabilizes domestic inflation (as opposed to a nominal exchange rate peg).

Interestingly, nominal rigidities are not an essential part of the transmission mechanism for generating a disconnect behavior, and therefore we omit them in the baseline model. Later, we generalize our analysis to an environment with nominal stickiness and conventional Taylor rules, and show the robustness of the quantitative properties of the model.

Furthermore, the results of a single-shock baseline model are robust to the introduction of additional shocks, including productivity and monetary shocks. We calibrate a multi-shock version of the model to match the weak correlations between exchange rates and macro variables. Using this calibrated model we conduct a variance decomposition of the equilibrium exchange rate volatility into the contribution of various types of shocks, and find that financial shock still accounts for the bulk of its variation, while both monetary and productivity shocks play limited roles. Since the structure of our model is rather standard, the transmission mechanism for monetary and productivity shocks is not different from a conventional international macro model. What sets our model apart, however, is the emphasis that monetary and productivity shocks cannot be the key drivers of the exchange rate, if the model is to feature the disconnect properties. At the same time, conventional shocks are still central in shaping the dynamics of other macroeconomic variables, resulting in no trade-off for our model in fitting the standard international business cycle moments (as in e.g. Backus, Kehoe, and Kydland 1992, 1994).

The tractability of the baseline model allows us to solve it in closed-form and emphasize four novel mechanisms. The first mechanism is the exchange rate determination, which emphasizes the interplay between equilibrium forces in the financial and goods markets. A small persistent increase in demand for foreign assets results in a sharp depreciation of the home currency and a slow but persistent appreciation thereafter in order to ensure equilibrium in the asset markets. Intertemporal budget constraint requires that these future appreciations are balanced out by an unexpected depreciation on impact. The more persistent is the shock, the larger is the initial depreciation, and thus the closer is the behavior of the nominal exchange rate to a random walk. Indeed, in our calibration, the equilibrium exchange rate is indistinguishable from a random walk in finite samples.

The second mechanism concerns the real exchange rate, and in particular the PPP and related puzzles, which are often viewed as the prime evidence in support of long-lasting real effects of nominal rigidities (as surveyed in Rogoff 1996). The alternative interpretation in the literature is that, given the moderate empirical durations of nominal prices, sticky price models are incapable of generating persistent PPP deviations observed in the data (see Chari, Kehoe, and McGrattan 2002). Both of these views adopt the baseline assumption that monetary shocks are the main drivers of the nominal exchange rate, and that nominal rigidity is the key part of the transmission mechanism into the real exchange rate. We suggest an entirely different perspective, which deemphasizes nominal rigidities, and instead shifts focus to the nature of the shock process. We argue that the behavior of the real exchange rates — both in the time series (PPP puzzle) and in the cross-section (see e.g. Kehoe and Midrigan 2008) — is not evidence in favor or against sticky prices, but is instead evidence against monetary shocks as the key source of exchange rate fluctuations. In contrast, we show that financial shocks drive both nominal and real exchange rates in concert, resulting in volatile and persistent behavior for both variables, thus reproducing the PPP puzzle. The only two relevant ingredients of the transmission mechanism for this result are the monetary policy rule, which stabilizes domestic inflation, and the home bias in consumption, which limits the response of consumer prices to exchange rate.

The third mechanism addresses the Backus-Smith puzzle, namely the comovement between consumption and the real exchange rate. Our approach crucially shifts focus from risk sharing (in the financial market) to expenditure switching (in the goods market) as the key force shaping this comovement. We show that expenditure switching robustly implies a negative correlation between relative consumption and the real exchange rate, as is the case in the data. Intuitively, an exchange rate depreciation increases global demand for domestic goods, which in light of the home bias requires an increase in domestic production and a reduction in domestic consumption. We show that this force is present in all models with expenditure switching and goods market clearing, yet it is usually dominated by the direct effect of shocks on consumption. With financial shock as the key source of exchange rate volatility, there is no direct effect, and expenditure switching is the only force affecting consumption, resulting in the empirically relevant direction of comovement.[2] Our transmission mechanism with substantial home bias and low pass-through into prices and quantities ensures that the movements in consumption are very mild, much smaller than those in exchange rates, as is the case in the data.

Lastly, we provide an explicit microfoundation of the financial shock in an extension of the model, in which risk-averse arbitrageurs intermediate international financial transactions and require a risk premium proportional to the size of their currency exposure. Without compromising the model's ability to match the main exchange rate moments, this extension results in an endogenous feedback from the net foreign asset position of the country into the risk premium, and allows the model to reproduce the non-monotonic dynamic comovement between UIP deviations and interest rates, emphasized recently by Engel (2016) and Valchev (2016). The model further implies that a policy commitment to an exchange rate peg has a coordination effect on the arbitrageurs, encouraging them to take larger positions and endogenously suppressing the volatility of the UIP deviations (as in Jeanne and Rose 2002). We show that this mechanism is important to account for an additional set of stylized facts associated with a switch in the monetary regime to/from an exchange rate peg, to which we collectively refer as the Mussa puzzle. As with the PPP puzzle, our explanation here emphasizes the nature of the shock driving the exchange rate and the monetary policy rule, rather than nominal rigidities (*cf.* Monacelli 2004, Kollmann 2005).

The rest of the paper is organized as follows. In Section 2, we describe the modeling framework

[2]The analytical tractability of our model allows us to establish the relationship between our results and those in the earlier Backus-Smith puzzle literature, in particular in Corsetti, Dedola, and Leduc (2008).

and prove that the international asset demand shocks is the only shock consistent with the exchange rate disconnect properties. We also discuss in this section various microfoundations for the origin of this financial shock. Section 3 then explores the qualitative and quantitative properties of the model with the financial shock alone, addressing in turn all of the exchange rate disconnect puzzles outlined in the beginning of this introduction. Along the way, we also provide a discussion of the relationship of our results to the existing literature. Section 4 then describes a number of extensions, including a full-fledged model with nominal rigidities and conventional Taylor rules, as well as a model with an explicit financial sector with noise traders and limits to arbitrage. In this section, we also allow for multiple sources of shocks and provide a variance decomposition of the exchange rate volatility into the contribution of these various shocks. Lastly, this section addresses the Mussa puzzle and the Engel risk premium puzzle. Section 5 discusses the implication of our results and concludes, while the appendix provides detailed derivations and proofs, as well as a number of additional extensions and results.

2 Modeling Framework and Shocks

We start with a flexible modeling framework that can nest most standard international macro models, which allows us in what follows to consider various special cases and extensions. There are two countries, home (Europe) and foreign (US, denoted with a $*$). Each country has its nominal unit of account, in which the local prices are quoted. In particular, the home wage rate is W_t euros and the foreign wage rate is W_t^* dollars. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, hence an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the euro (the home currency). We allow for a variety of shocks hitting the economy, and proxying in some cases for unmodelled market imperfections. We then explore which of these disturbances can account for the exchange rate disconnect, as we formally define it below in Section 2.2.

2.1 Model setup

Households A representative home household maximizes the discounted expected utility over consumption and labor:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\chi_t} \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{e^{\kappa_t}}{1+1/\nu} L_t^{1+1/\nu} \right), \tag{1}$$

where (χ_t, κ_t) are the utility shocks, σ is the relative risk aversion parameter, ν is the Frisch elasticity of labor supply, and our results are robust to alternative utility specifications, including the GHH utility without income effects on labor supply (see Appendix A.11). The flow budget constraint is given by:

$$P_t C_t + \frac{B_{t+1}}{R_t} + \frac{B_{t+1}^* \mathcal{E}_t}{e^{\psi_t} R_t^*} \le B_t + B_t^* \mathcal{E}_t + W_t L_t + \Pi_t - T_t + \Omega_t, \tag{2}$$

where P_t is the consumer price index, (B_t, B_t^*) are the quantities of the home and foreign bonds paying out next period one unit of the currency of the issuing country, and (R_t, R_t^*) are their discounts (i.e., $1/R_t$ and $1/R_t^*$ are their prices); Π_t are the dividends and T_t are lump-sum taxes. Lastly, ψ_t is a wedge between the effective return on foreign bonds for the home households and the foreign interest rate, driven by shocks in the international asset market and with the resulting profits of the financial sector $\Omega_t = \left(e^{-\psi_t} - 1\right)\frac{B^*_{t+1}\mathcal{E}_t}{R^*_t}$ reimbursed lump-sum to the households.[3]

The households are active in three markets. First, they supply labor according to the standard static optimality condition:

$$e^{\kappa_t} C_t^{\sigma} L_t^{1/\nu} = \frac{W_t}{P_t}, \tag{3}$$

where the preference shock κ_t can be alternatively interpreted as the *labor wedge*, playing an important role in the closed-economy business cycle literature and capturing the departures from the neoclassical labor market dynamics due to search frictions or sticky wages (see e.g. Shimer 2009). In addition, we denote $W_t \equiv e^{w_t}$ and interpret w_t as the shock to the nominal value of the unit of account, which captures monetary shocks in our framework.[4]

Second, the households choose their bond positions according to the dynamic optimality conditions:

$$1 = R_t\, \mathbb{E}_t \Theta_{t+1} \qquad \text{and} \qquad 1 = e^{\psi_t} R_t^*\, \mathbb{E}_t \left\{ \Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}, \tag{4}$$

$$\text{where} \qquad \Theta_{t+1} \equiv \beta e^{\Delta \chi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

is the stochastic discount factor. The time preference shock χ_{t+1}, as in Stockman and Tesar (1995), affects the consumption-savings decision and acts as an intertemporal demand shifter. The ψ_t shock acts instead as a demand shifter for the foreign-currency bond, and in Section 2.3 we discuss a number of microfounded financial models which result in a similar reduced form as (4).

Lastly, the households allocate their within-period expenditure between home and foreign goods:

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft},$$

and we assume the good demand is homothetic and symmetric, and given by:

$$C_{Ht} = (1-\gamma) e^{-\gamma \xi_t} h\left(\frac{P_{Ht}}{P_t} \right) C_t \qquad \text{and} \qquad C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{P_t} \right) C_t, \tag{5}$$

where ξ_t is the relative demand shock for the foreign good (as in Pavlova and Rigobon 2007) and γ captures the *home bias*, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see Obstfeld and Rogoff 2001). Note that the demand for the foreign good collapses to zero as $\gamma \to 0$. The function $h(\cdot)$ controls the curvature of the demand schedule, and satisfies $h'(\cdot) < 0$ and $h(1) = 1$. We denote its point elasticity by $\theta \equiv -\frac{\partial \log h(x)}{\partial \log x}\big|_{x=1}$. The demand formulation in (5) emerges from a homothetic and separable Kimball (1995) demand aggregator, as we show in

[3]We adopt the assumption that a risk-free bond is the only internationally-traded asset because we rely on a log-linearization for the analytical solution of the model. In Appendix A.6, we explore the role of asset market (in)completeness.

[4]In Section 4.1 we provide an explicit model with nominal wage stickiness, local-currency price stickiness and a conventional Taylor rule, which offers an example of one typical source of $(w_t, \kappa_t, \mu_t, \eta_t)$ shocks (with μ_t and η_t defined below).

Appendix A.2, where we also derive an explicit expression for the price index P_t.[5] In our analysis, we focus on the behavior of the economy around a symmetric steady state and make use of the following three properties of this demand system:

Lemma 1 (Properties of Demand) *In a symmetric steady state with $\xi = 0$ and $P = P_H = P_F$:*

(i) The expenditure share on foreign goods (the foreign share *for brief), defined as $\frac{P_{Ft}C_{Ft}}{P_t C_t}$, equals the home bias parameter γ.*

(ii) The log-linear approximation of the consumer price index around the steady state is given by (where small letters denote log deviations from the steady state):

$$p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}. \tag{6}$$

(iii) The log-linear approximation to demand (5) around the steady state is given by:

$$c_{Ht} = -\gamma\xi_t - \theta(p_{Ht} - p_t) + c_t \qquad \text{and} \qquad c_{Ft} = (1-\gamma)\xi_t - \theta(p_{Ft} - p_t) + c_t. \tag{7}$$

Therefore, $c_{Ft} - c_{Ht} = \xi_t - \theta(p_{Ft} - p_{Ht})$, and the elasticity of substitution between home and foreign goods, defined as $\frac{\partial \log(C_{Ft}/C_{Ht})}{\partial \log(P_{Ht}/P_{Ft})}$, equals the point elasticity of the demand schedule θ.

We show below that the values of γ (trade openness) and θ (elasticity of substitution between home and foreign goods) play a central role in the quantitative properties of the transmission mechanism.

Production and prices Output is produced by a given pool of identical firms according to a Cobb-Douglas technology in labor L_t and intermediate inputs X_t:

$$Y_t = e^{a_t} L_t^{1-\phi} X_t^{\phi}, \tag{8}$$

where a_t is the productivity shock and ϕ is the elasticity of output with respect to intermediates, which determines the equilibrium expenditure share on intermediate goods. The presence of intermediates is *not* essential for the qualitative results, however, is needed to properly capture the degree of trade openness in our calibration. For analytical tractability, we focus on a constant-returns-to-scale production without capital, and show the robustness of our results to an extension with capital and adjustment costs in Appendix A.11.

Intermediates are the same bundle of home and foreign varieties as the final consumption bundle, and hence their price index is also given by P_t. Therefore, the marginal cost of production is:

$$MC_t = e^{-a_t}\left(\frac{W_t}{1-\phi}\right)^{1-\phi}\left(\frac{P_t}{\phi}\right)^{\phi}, \tag{9}$$

and the firms optimally allocate expenditure between labor and intermediates according to the following

[5] Note that the conventional CES demand is nested as a special case with $h(x) = x^{-\theta}$, yet we adopt a more general demand formulation, which allows to accommodate variable markups.

input demand conditions:

$$W_tL_t = (1-\phi)MC_tY_t \qquad \text{and} \qquad P_tX_t = \phi MC_tY_t. \tag{10}$$

The expenditure on intermediates X_t is further split between the domestic and foreign varieties, X_{Ht} and X_{Ft}, in parallel with the household consumption expenditure in (5). The total production of the domestic firms is divided between the home and foreign markets, $Y_t = Y_{Ht} + Y^*_{Ht}$, resulting in profits that are distributed to the domestic households:[6]

$$\Pi_t = (P_{Ht} - MC_t)\,Y_{Ht} + (P^*_{Ht}\mathcal{E}_t - MC_t)\,Y^*_{Ht}. \tag{11}$$

We postulate the following price setting:

$$P_{Ht} = e^{\mu_t}MC_t^{1-\alpha}P_t^{\alpha}, \tag{12}$$

$$P^*_{Ht} = e^{\mu_t+\eta_t}\big(MC_t/\mathcal{E}_t\big)^{1-\alpha}P_t^{*\alpha}, \tag{13}$$

where $\alpha \in [0,1)$ is the *strategic complementarity* elasticity, μ_t is the markup shock, and η_t is the law of one price (LOP) shock. Given these prices, the firms satisfy the resulting demand in both markets. Equations (12)–(13) are *ad hoc* yet general pricing equations, as the markup terms (together with a flexible choice of α) allow them to be consistent with a broad range of price setting models, including both monopolistic and oligopolistic competition models under both CES and non-CES demand. Furthermore, if the time path of (μ_t, η_t) is not restricted, these equations are also consistent with dynamic price setting models, and in particular the sticky price models (with either producer, local or dollar currency pricing).[7]

Strategic complementarities in price setting ($\alpha > 0$) reflect the tendency of the firms to set prices closer to their local competitors, a pattern which is both pronounced in the data and emerges in a variety of models (see Amiti, Itskhoki, and Konings 2016), and we emphasize in our analysis below its role for the international transmission of shocks. Appendix A.2 discusses a model of Kimball (1995) demand that is simultaneously consistent with both our choices of elasticity of substitution θ and strategic complementarity elasticity α. Lastly, we note that the violations of the law of one price:

$$\mathcal{Q}_{Ht} \equiv \frac{P^*_{Ht}\mathcal{E}_t}{P_{Ht}} = e^{\eta_t}\mathcal{Q}_t^{\alpha}, \qquad \text{where} \qquad \mathcal{Q}_t \equiv \frac{P^*_t\mathcal{E}_t}{P_t}, \tag{14}$$

arise either due to the LOP shock η_t (capturing, for example, local currency pricing) or due to $\alpha > 0$ (capturing pricing-to-market). The *real exchange rate* $\mathcal{Q}_t$ reflects the differences in the price levels across the two markets.

[6] We assume no entry or exit of firms, as our model is a medium-run one (for the horizons of up to 5 years), where empirically extensive margins play negligible roles (see e.g. Bernard, Jensen, Redding, and Schott 2009).

[7] Note that η_t can stand in for a trade cost shock, which plays a central role in the recent quantitative analyses of Eaton, Kortum, and Neiman (2015) and Reyes-Heroles (2016). A combination of η_t and ξ_t can also stand in for a world commodity price shock, acting as a wealth transfer between countries. These shocks are an important source of volatility for the commodity-exporting countries such as Canada, Australia, South America, Brazil and Chile (see e.g. Chen and Rogoff 2003).

Government uses lump-sum taxes to finance an exogenous stochastic path of government expenditure $G_t \equiv e^{g_t}$, where g_t is the government spending shock. For simplicity, we assume that government expenditure is allocated between the home and foreign goods in the same way as the final consumption in (5). The government then taxes households $T_t = P_t e^{g_t}$ to run a balanced budget, which in view of Ricardian equivalence is without loss of generality.

Foreign households are symmetric, except that the home (euro) bonds are not available to them, and their budget constraint is given by:

$$P_t^* C_t^* + \frac{B_{t+1}^{*F}}{R_t^*} \leq B_t^{*F} + W_t^* L_t^* + \Pi_t^* + T_t^*,$$

where B_t^{*F} are the holdings of the foreign (dollar) bond by foreign households.[8] The optimal savings decision of the foreign households is characterized by the Euler equation:

$$1 = R_t^* \, \mathbb{E}_t \Theta_{t+1}^*, \qquad \text{where} \qquad \Theta_{t+1}^* \equiv \beta e^{\Delta \chi_{t+1}^*} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}. \tag{15}$$

The foreign households supply labor and demand home and foreign goods according to the optimality condition parallel to (3) and (5) respectively. In particular, the goods demand by the foreign households is given by:

$$C_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h\left(\frac{P_{Ht}^*}{P_t^*} \right) C_t^* \qquad \text{and} \qquad C_{Ft}^* = (1-\gamma) e^{-\gamma \xi_t^*} h\left(\frac{P_{Ft}^*}{P_t^*} \right) C_t^*, \tag{16}$$

where ξ_t^* is the foreign demand shock for home goods. Lastly, the foreign firms are also symmetric, demand foreign labor and a composite intermediate good, and charge prices according to the counterparts of (12)–(13) with their own markup and LOP shocks μ_t^* and η_t^*, as we detail in Appendix A.3.

Equilibrium conditions ensure equilibrium in the asset, product and labor markets, as well as the intertemporal budget constraints of the countries. The labor market clears when L_t is consistent simultaneosly with labor supply in (3) and labor demand in (10), and symmetrically for L_t^* in foreign. The goods market clearing requires $Y_t = Y_{Ht} + Y_{Ht}^*$, where

$$Y_{Ht} = C_{Ht} + X_{Ht} + G_{Ht} = (1-\gamma) e^{-\gamma \xi_t} h\left(\frac{P_{Ht}}{P_t} \right) [C_t + X_t + G_t], \tag{17}$$

$$Y_{Ht}^* = C_{Ht}^* + X_{Ht}^* + G_{Ht}^* = \gamma e^{(1-\gamma)\xi_t^*} h\left(\frac{P_{Ht}^*}{P_t^*} \right) [C_t^* + X_t^* + G_t^*], \tag{18}$$

and symmetric conditions hold for $Y_{Ft} + Y_{Ft}^* = Y_t^*$. The bonds market clearing requires $B_t = 0$ for the home-currency bond, as it is in zero net supply and not traded internationally, and $B_t^* + B_t^{*F} = 0$ for the foreign-currency bond, which is in zero net supply internationally.

[8] We consider this asymmetric formulation between home and foreign for simplicity, and provide a symmetric version of the model with a financial sector in Section 4.2.

Table 1: Model parameters and shocks

Shocks		Parameters	
w_t	nominal wage rate (shock to the value of the unit of account)	$\beta = 0.99$	discount factor
a_t	productivity shock	$\sigma = 2$	relative risk aversion (inverse of IES)
g_t	government spending shock	$\nu = 1$	Frisch elasticity of labor supply
χ_t	intertemporal preference shock	$\gamma = 0.07$	foreign share (home bias) parameter
κ_t	labor wedge (sticky wages)	$\theta = 1.5$	elasticity of substitution
μ_t	markup shock (sticky prices)	$\alpha = 0.4$	strategic complementarity elasticity
η_t	law-of-one-price shock (local currency pricing)	$\phi = 0.5$	intermediate share
ξ_t	international good demand shock	$\rho = 0.97$	persistence of the shock
ψ_t	financial (international asset demand) shock		

Note: The left panel summarizes the shocks to the home economy, with foreign facing a symmetric set of shocks, apart from ψ_t^*, which with our structure is equivalent to χ_t^*. The right panel reports the baseline parameter values used in Sections 3–4.

Lastly, we combine the household budget constraint (2) with profits (11) and taxes to obtain the country budget constraint:

$$\frac{B_{t+1}^*\mathcal{E}_t}{R_t^*} - B_t^*\mathcal{E}_t = NX_t, \qquad \text{where} \qquad NX_t = \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - P_{Ft}Y_{Ft}. \tag{19}$$

NX_t is net exports of the home country (in home currency). Note that the relative price at which the home country exchanges its exports for imports is the *terms of trade*:

$$\mathcal{S}_t \equiv \frac{P_{Ft}}{P_{Ht}^*\mathcal{E}_t}. \tag{20}$$

This completes the description of the model environment and the equilibrium system, which we also summarize in Appendix A.3.

Shocks are summarizes in Table 1, along with the parameters of the model and their baseline values, which we use in Sections 3 and 4 for quantitative evaluation of the model. In general, we allow shocks to follow arbitrary joint stochastic processes with unrestricted patterns of cross-correlations. In this sense, our shocks are not primitive innovations, but rather disturbances to the equilibrium conditions of the model, akin to Chari, Kehoe, and McGrattan (2007) *wedges*.[9] We use them differently, however. Instead of accounting for the sources of variation in the macro variables, we prove two theoretical results characterizing which subsets of disturbances can and cannot result in an equilibrium *disconnect* behavior of the exchange rates, as defined below.

[9]For example, Eaton, Kortum, and Neiman (2015) is a recent study, which uses wedge accounting in the international context. Our approach differs in that we do not attempt to fully match macroeconomic time series, but instead focus on a specific theoretical mechanism, which accounts for a set of exchange rate disconnect moments within a parsimonious model. This is also what sets our paper apart from the international DSGE literature following Eichenbaum and Evans (1995).

2.2 Disconnect in the limit

This section uses the general modeling framework to prove two propositions, which narrow down the set of shocks that can be consistent with the empirical exchange rate disconnect properties. In particular, we study the behavior of the equilibrium system around the autarky limit, which we use as a diagnostic device. The autarky limit (as foreign share $\gamma \to 0$) is interesting for two reasons:

1. Autarky ($\gamma = 0$) offers a model of *complete exchange rate disconnect*. When countries are in autarky, the nominal exchange rate is of no consequence, and can take any values as an outcome of arbitrary sunspot equilibria. Therefore, it can be arbitrary volatile, yet have no relationship with any macro variables in the two economies (the Meese-Rogoff puzzle). Since price levels do not respond to this volatility, the real exchange rate comoves perfectly with these nominal exchange rate shocks, and as a result can exhibit arbitrary persistence (the PPP puzzle). This is possible because in autarky the real exchange rate does not affect allocations.

2. Furthermore, away from autarky, the response of macro variables to exchange rate tends to increase together with the degree of openness γ, resulting in more volatile and less disconnected macroeconomic behavior (see Appendix Figure A1 for illustration). Therefore, if the economy does not exhibit exchange rate disconnect properties near autarky (for $\gamma \approx 0$), it is unlikely to feature them away from autarky (for $\gamma \gg 0$).[10] In addition, $\gamma \approx 0$ is not an unreasonable point of approximation from an empirical perspective, as we discuss in Section 3.

We now extend the autarky logic to study circumstances under which a near-closed economy features a *near-complete exchange rate disconnect*. While such continuity requirement may appear natural as the equilibrium dynamics is continuous in γ, it nonetheless offers a sharp selection criterion for exogenous shocks. This is because a limiting economy with $\gamma > 0$ acts as a refinement on equilibria when $\gamma = 0$, as it rules out the sunspot equilibria with volatile exchange rate dynamics.

We start by formalizing the notion of exchange rate disconnect in the autarky limit:

Definition 1 (Exchange rate disconnect in the limit) *Denote with* $\mathbf{Z}_t \equiv (W_t, P_t, C_t, L_t, Y_t, R_t)$ *a vector of all domestic macro variables (wage rate, price level, consumption, employment, output, interest rate) and with* $\varepsilon_t \equiv \mathbf{V}'\Omega_t + \mathbf{V}^{*\prime}\Omega_t^*$ *an arbitrary combination of shocks* $\Omega_t = \{w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t, \eta_t, \xi_t, \psi_t\}$. *We say that an open economy (with* $\gamma > 0$*) exhibits exchange rate disconnect in the autarky limit, if the impulse responses have the following properties:*

$$\lim_{\gamma \to 0} \frac{\mathrm{d}\mathbf{Z}_{t+j}}{\mathrm{d}\varepsilon_t} = 0 \quad \textit{for all } j \geq 0 \qquad \textit{and} \qquad \lim_{\gamma \to 0} \frac{\mathrm{d}\mathcal{E}_t}{\mathrm{d}\varepsilon_t} \neq 0. \tag{21}$$

A corollary of condition (21) *is that* $\lim_{\gamma \to 0}[\mathrm{d}\log \mathcal{E}_{t+j} - \mathrm{d}\log \mathcal{Q}_{t+j}]/\mathrm{d}\varepsilon_t = 0$ *for all* $j \geq 0$.

In words, a model, defined by its structure and the set of shocks, exhibits exchange rate disconnect in the autarky limit if the shocks have a vanishingly small effect on the macro variables, yet result in a volatile equilibrium exchange rate, as captured by the two conditions in (21). This property captures the

[10] The empirical literature finds that more open economies have less volatile exchange rates, even after controlling for country size and other characteristics (e.g., Hau 2002), which is a pattern reproduced by our model (see Section 3.1).

exchange rate disconnect in its narrow Meese-Rogoff sense. However, as the corollary points out, this property also implies the PPP-puzzle behavior for the real exchange rate, which in the limit comoves one-for-one with the nominal exchange rate.

Definition 1 immediately allows us to exclude a large number of candidate shocks:

Proposition 1 *The model of Section 2.1 cannot exhibit exchange rate disconnect in the autarky limit (21), if the combined shock ε_t in Definition 1 has a weight of zero on the subset of shocks $\{\eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t\}$.*

In other words, this proposition states that the shocks in $\Omega_t^{\varnothing} \equiv \{w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t\}$ together with their foreign counterparts, in any combinations and with arbitrary cross-correlations, cannot reproduce an exchange rate disconnect property even as the economy approaches autarky. We provide a formal proof in Appendix A.4, yet the intuition behind this result is straightforward: Any of the shocks in $\Omega_t^{\varnothing}$ will have a direct effect on real allocations, prices, and/or interest rates, and thus cannot result in a volatile exchange rate without having a direct effect on the macro variables of the same order of magnitude.[11] Therefore, as an economy subject to these shocks approaches autarky, the disconnect property (21) is necessarily violated. The proof of this proposition does not rely on the international risk sharing condition, and therefore this result is robust to the assumption about (in)completeness of the international asset markets.

Proposition 1 can be viewed as pessimistic news for both the International RBC and the New Open Economy Macro (NOEM) models of the exchange rate. It does *not* imply, however, that productivity cannot be an important source of exogenous shocks. Instead, it suggests that productivity shocks a_t are unlikely to be the dominant drivers of exchange rate movements if the model is to exhibit exchange rate disconnect.[12] The same applies to monetary shocks in a model with nominal rigidities, which we study in detail in Section 4.1.

We view Proposition 1 as a diagnostic tool suggesting that the shocks in $\Omega_t^{\varnothing}$ are unlikely to be successful at reproducing the empirical exchange rate behavior even away from the autarky limit. Therefore, we should first explore the other three types of shocks — namely, the LOP deviation (or trade cost) shock η_t, the international good demand shock ξ_t, and/or the financial shock ψ_t — as the likely key drivers of the exchange rate dynamics. The distinctive feature of these shocks is that they affect the equilibrium system exclusively through the *international equilibrium conditions*: ψ_t affects international risk sharing (see (39) below), while η_t and ξ_t affect the country budget constraint (19) through their impact on export prices (13) and export demand (18) respectively.[13] The impact of shocks to these

[11]Intuitively, the unit of account w_t shocks result in wage inflation, the markup μ_t shocks result in price inflation, the labor wedge κ_t shocks result in changes in either employment or consumption, the productivity a_t shocks result in changes in either employment or output, the government spending g_t shocks result in changes in either consumption or output, and the intertemporal preference shocks χ_t result in changes in the interest rate (see illustration in Figure A1). The formal proof in Appendix A.4 establishes further that no combination of these shocks can be consistent with the disconnect property (21).

[12]Productivity shocks can have two additional indirect effects, either acting as *news shocks* about future productivity or by affecting the *risk premium* (e.g., rare-disaster or long-run-risk shocks). As the direct effect of the productivity shock becomes vanishingly small relative to its indirect effects, this shock becomes similar to an Euler-equation shock in that it does not affect the static equilibrium conditions (see Appendix A.8). Proposition 1 nonetheless applies as long as the direct effect of the productivity shock is non-trivial.

[13]The ξ_t and η_t shocks are additionally featured in the goods market clearing (17)–(18) and in the price level (6), but in both cases their effect on these conditions is proportional to trade openness γ, and thus vanishes in the autarky limit.

equilibrium conditions on the macro variables is vanishingly small as the economy becomes closed to international trade in goods and assets, yet such shocks can have substantial effect on the equilibrium exchange rates and terms of trade even when γ is close to zero.

Proposition 1 does not allow us to discriminate between the remaining three types of shocks, as they all satisfy the autarky-limit disconnect condition (21). Yet, these shocks differ in the implied comovement between exchange rates and macro variables, which we now use as a further selection criterion. In particular, we explore the comovement between the exchange rates and respectively terms of trade, relative consumption, and the interest rate differential, near the autarky limit (as $\gamma \to 0$). Since these shocks are already consistent with the Meese-Rogoff and the PPP puzzles by virtue of Proposition 1, the additional moments correspond to the three remaining exchange rate puzzles. We prove the following result (see Appendix A.4):

Proposition 2 *Near the autarky limit (for $\gamma \to 0$), the international asset demand shock ψ_t is the only shock in $\{\eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t\}$ that simultaneously and robustly produces:*

(i) *a positive correlation between the terms of trade and the real exchange rate;*

(ii) *a negative correlation between relative consumption growth and real exchange rate depreciation;*

(iii) *deviations from the UIP and a negative Fama coefficient.*

The main conclusion is that both the LOP deviation (trade cost) shock η_t and the international good demand shock ξ_t produce the *counterfactual* comovement between the exchange rates and respectively relative consumption (the Backus-Smith puzzle) and interest rate differential (the Forward Premium puzzle). The financial shock ψ_t is instead consistent with both of these empirical patterns, as we explain in detail in Section 3.

To summarize, Propositions 1 and 2 explain why most shocks have a hard time at reproducing the empirical exchange rate properties, and hence why these properties are labeled as *puzzles* in the literature. These propositions favor the financial shock ψ_t as the likely shock to generate exchange rate disconnect in an equilibrium model. While these propositions are concerned with the autarky limit, the continuity of the model in trade openness γ suggests that the near-disconnect properties should hold for $\gamma > 0$, but small. In Sections 3, we explore the properties of the model with the ψ_t shock alone, away from the autarky limit, and in Section 4 we bring back the additional shocks.

2.3 Models of financial shock ψ_t

Since Propositions 1 and 2 favor the international asset demand shock ψ_t as the likely source of the exchange rate disconnect, we discuss here a number of microfoundations for the origins of this shock. In view of the parity condition between home and foreign bonds (arising from (4) upon log-linearization), the ψ_t is commonly referred to as the *UIP shock*:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t, \tag{22}$$

where $i_t - i_t^* = \log R_t - \log R_t^*$ and $e_t = \log \mathcal{E}_t$. It follows that the (uncovered) interest rate parity $i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$ deviates from zero by the magnitude of the financial shock ψ_t, which may have a number of origins explored in the macro-finance literature (see also Cochrane 2016):

1. Exogenous preference for international assets, where ψ_t is an *ad hoc* shock to the utility from holding the foreign bond, as a result of which domestic households are willing to hold the foreign asset with a negative excess return (UIP deviation), as in Dekle, Jeong, and Kiyotaki (2014). This approach is closely related to the non-optimizing *portfolio balance models* of the 1970-80s (e.g. Kouri 1976, 1983, Branson and Henderson 1985), revived recently by Blanchard, Giavazzi, and Sa (2005) and Gourinchas (2008).
2. Noise traders and limits to arbitrage in currency markets, as in Jeanne and Rose (2002), where all international trade in assets needs to go through risk-averse intermediaries, who are willing to be exposed to the currency risk only if they are offered a sufficient compensation in the form of a positive expected return. A noise trader shock ψ_t, thus, needs to be accommodated by a UIP deviation. In Section 4.2, we explore a general equilibrium version of this model and its implications for exchange rate disconnect and additional exchange rate puzzles.
3. A related class of models relies on financial frictions to generate upward sloping supply in the currency market, where ψ_t represents shocks to the risk-bearing capacity of the financial sector, for example a shock to the net worth of the financial intermediaries limiting the size of the positions that they can absorb (see e.g. Hau and Rey 2006, Brunnermeier, Nagel, and Pedersen 2009, Gabaix and Maggiori 2015, Adrian, Etula, and Shin 2015).
4. Incomplete information, heterogeneous beliefs and expectational errors in the currency market, as in Evans and Lyons (2002), Gourinchas and Tornell (2004) and Bacchetta and van Wincoop (2006), where ψ_t represents the deviations from the full-information rational expectations.
5. Time-varying risk premia models, where ψ_t corresponds to shocks to the *second moments* of the stochastic discount factor, such as rare disasters (Farhi and Gabaix 2016), long-run risk (Colacito and Croce 2013), or habits (Verdelhan 2010). For convenience, these models typically assume complete markets, while an alternative approach relies on modeling segmented markets, where the SDF shocks emerge from the participation margin (see e.g. Alvarez, Atkeson, and Kehoe 2009).

Given our focus on the set of moments characterizing the empirical exchange rate disconnect, all the approaches to modeling the financial shock ψ_t listed above are isomorphic, as they all result in a version of equation (22), and hence we cannot distinguish between them. The specific models of ψ_t can be discriminated based on a richer set of asset market moments, e.g. a term structure of carry trade returns or a comovement of exchange rate with returns across various asset classes (e.g., see Farhi et al. 2009 and Lustig and Verdelhan 2016). In particular, in Section 4.2 we address an additional Engel (2016) risk premium puzzle in the context of a specific financial model of the ψ_t shock that we adopt.

Note that while our model relies on a violation of the UIP, it is silent in general whether the *covered interest rate parity* (CIP) is violated simultaneously.[14] The exogenous asset demand and financial friction models typically imply CIP violations. In contrast, CIP holds in the risk-premia-based models, and in particular it holds in the limits-to-arbitrage model that we study in Section 4.2.

[14] See e.g. Du, Tepper, and Verdelhan (2016) and Jiang, Krishnamurthy, and Lustig (2017) for the empirical analysis of CIP violations over time.

3 Baseline Model of Exchange Rate Disconnect

Our baseline model features a single shock — the financial shock ψ_t — and the transmission mechanism, which emphasizes home bias in expenditure (low γ), strategic complementarities in price setting ($\alpha > 0$), and weak substitutability between home and foreign goods ($\theta > 1$, but small). The other parameters, including the intertemporal elasticity of substitution (IES) and the elasticity of labor supply, prove to be less consequential for the results, as we discuss below. Even more surprisingly, nominal rigidities turn out to be of little importance for generating the quantitative exchange rate disconnect properties in response to a ψ_t shock. Therefore, our baseline model does not feature any nominal rigidities, with both wages and prices set *flexibly*. We further assume that monetary authorities adopt policy rules that fully stabilize local wage inflation at zero, that is $W_t \equiv 1$ and $W_t^* \equiv 1$.[15]

The model of this section is analytically tractable (upon log-linearization), and all our results can be easily obtained with pen and paper, which allows us to fully explore the intuition behind various mechanisms. At the same time, we emphasize the quantitative objective of this section. That is, our goal is to establish whether a simple one-shock model can be *quantitatively* consistent with a rich set of moments describing the comovement between exchange rates and macro variables. In doing so, we tie our hands from the start, and calibrate the parameters of the model on which we have direct and reliable empirical evidence. In particular, we set $\gamma = 0.07$ to be consistent with the 0.28 trade (imports plus exports) to GDP ratio of the United States, provided the intermediate input share $\phi = 0.5$.[16] We further use the estimate of Amiti, Itskhoki, and Konings (2016) of the elasticity of strategic complementarities $\alpha = 0.4$, which is also in line with much of the markup and pass-through literature and corresponds to the own cost shock pass-through elasticity of $1 - \alpha = 0.6$ (see survey in Gopinath and Itskhoki 2011). In contrast, the value of the elasticity of substitution between home and foreign goods θ is more contested. We follow here the recent estimates of Feenstra, Luck, Obstfeld, and Russ (2014) and set $\theta = 1.5$, which is also the number used in the original calibrations of Backus, Kehoe, and Kydland (1994) and Chari, Kehoe, and McGrattan (2002).[17]

For concreteness, we assume the financial shock ψ_t follows an exogenous AR(1) process:

$$\psi_t = \rho\psi_{t-1} + \varepsilon_t, \tag{23}$$

with persistence $\rho \in [0, 1]$ and variance of innovations given by σ_ε^2. For our quantitative analysis, we assume that ψ_t shocks are small, but persistent. While ψ_t shocks are not directly observable, the model

[15]In Section 4.1, we extend our analysis to allow for nominal rigidities, conventional Taylor rules and multiple sources of shocks. We show, in particular, that our results are *not* very sensitive to the extent of nominal rigidities in wage and price setting, as long as monetary authorities follow rules that target local price or wage inflation.

[16]This value of the trade-to-GDP ratio is also characteristic of the other large developed economies (Japan and the Euro Zone). Appendix A.3.1 derives the relationship between the value of the trade-to-GDP ratio and the value of γ (steady state imports-to-expenditure ratio), which we set to be four times smaller. Intuitively, imports in a symmetric steady state are half of total trade (imports plus exports), and GDP (final consumption) is about one half of the total expenditure with the other half allocated to intermediate inputs ($\phi = 0.5$). This value of ϕ is consistent with both aggregate input-output matrices and firm level data on intermediate expenditure share in total sales. The decomposition of gross exports for the U.S., the E.U. and Japan in Koopman, Wang, and Wei (2014) suggests this proportion holds for trade flows as well.

[17]The *macro* elasticity of substitution between the aggregates of home and foreign goods is indeed the relevant elasticity for our analysis, while the estimates of the *micro* elasticity at more disaggregated levels are typically larger (around 4). The quantitative performance of our model does not deteriorate significantly for elasticities of substitution as high as 3.

implies that ρ equals the equilibrium persistence of the interest rates. We, therefore, set $\rho = 0.97$. For the remaining parameters, we set the relative risk aversion $\sigma = 2$, the Frish elasticity of labor supply $\nu = 1$ and a quarterly discount factor $\beta = 0.99$, as we summarize in Table 1.[18]

3.1 Equilibrium exchange rate dynamics

The model admits a convenient recursive structure, which allows us to solve for the relationship between exchange rate and macro variables (prices, quantities and interest rates) using static equilibrium conditions, as we do in turn in Sections 3.2–3.5 and Appendix A.5. We start, however, with the characterization of the equilibrium exchange rate dynamics.

The equilibrium exchange rate process is shaped by the interplay between the country budget constraint (19) and international asset market equilibrium, as captured by the UIP condition (22). Using the static equilibrium conditions, we solve for the equilibrium relationships between net exports, interest rates and exchange rate, which allows us to rewrite (22) and the log-linearized version of (19) as:[19]

$$\mathbb{E}_t \Delta e_{t+1} = -\frac{1}{1+\gamma\lambda_1}\psi_t, \tag{24}$$

$$\beta b^*_{t+1} - b^*_t = nx_t = \gamma\lambda_2 e_t. \tag{25}$$

The coefficients λ_1 and λ_2 depend on the structural model parameters, other than β and ρ. We have $\lambda_1, \lambda_2 > 0$ under mild parameter restrictions, such as the Marshall-Lerner condition, which we assume are satisfied (see Appendix A.5.1). Equation (25) reflects simply that net foreign assets accumulate with trade surpluses, which in turn increase with exchange rate depreciations, and more so the more open is the country to international trade. Equation (24), in turn, suggests that a financial shock $\psi_t > 0$ — e.g., an increase in demand for foreign assets — requires an expected appreciation of the home currency to balance the depressed relative demand for domestic bonds.[20]

We solve the dynamic system (24) and (25), together with the shock process (23), to obtain:

Proposition 3 (Equilibrium exchange rate process) *In the baseline model with the financial shock $\psi_t \sim$ AR(1) with persistence ρ and innovation ε_t, the equilibrium nominal exchange rate e_t follows an ARIMA(1,1,1), or equivalently $\Delta e_t \sim$ ARMA(1,1), with an AR root ρ and a non-invertible MA root $1/\beta$:*

$$\Delta e_t = \rho \Delta e_{t-1} + \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\left(\varepsilon_t - \frac{1}{\beta}\varepsilon_{t-1}\right). \tag{26}$$

This describes the unique equilibrium exchange rate path, and non-fundamental solutions do not exist.

[18] We show robustness to alternative parameter values in Appendix A.5.2. Furthermore, our qualitative results require only a weak parameter restriction, easily met in conventional calibrations, as we discuss in Appendix A.5.1.

[19] In Appendix A.3, we log-linearize the equilibrium system around a symmetric steady steady, and by default use small letters for log deviations. Since the NFA position B^*_t and net exports NX_t are zero in a symmetric steady state, we denote $b^*_t \equiv \frac{\bar{\mathcal{E}}}{\bar{P}_H \bar{Y}} B^*_t$ and $nx_t \equiv \frac{1}{\bar{P}_H \bar{Y}} NX_t$.

[20] Observe from (22) that $\psi_t > 0$ must be accommodated either by $i_t - i^*_t > 0$, or by $\mathbb{E}_t \Delta e_{t+1} < 0$ (expected appreciation), both of which moderate the increased demand for the foreign bond. We show in Section 3.5 that both effects occur in equilibrium, with the relative importance of the interest rate adjustment decreasing in country openness. In the autarky limit, the interest rates do not move, and we have $\mathbb{E}_t \Delta e_{t+1} = -\psi_t$, consistent with (24) as $\gamma \to 0$.

An analytical proof of this proposition in Appendix A.5 relies on the Blanchard and Kahn (1980) solution method, while here we offer a more intuitive explanation. Asset market equilibrium condition (24) determines the expected path of the future exchange rate changes, as emphasize by Engel and West (2005), and can be iterated forward to obtain:

$$e_t = \lim_{T\to\infty}\left\{\mathbb{E}_t e_{t+T} + \frac{1}{1+\gamma\lambda_1}\sum\nolimits_{j=0}^{T}\mathbb{E}_t\psi_{t+j}\right\} = \mathbb{E}_t e_\infty + \frac{1}{1+\gamma\lambda_1}\frac{\psi_t}{1-\rho}. \tag{27}$$

In contrast to Engel and West (2005), however, the general version of the risk sharing (or UIP) condition features no discounting, and therefore admits a multiplicity of no-bubble solutions parametrized by the long-run expectation $\mathbb{E}_t e_\infty$. Nonetheless, the equilibrium path of the exchange rate is uniquely pinned down by the intertemporal budget constraint, which obtains from (25) by iterating forward and imposing the No Ponzi Game Condition on net foreign assets $\lim_{T\to\infty}\beta^T b^*_{t+T+1} = 0$. This pins down the only budget-consistent long-run expectation, and hence the instantaneous level of the exchange rate e_t, according to (26).[21] Indeed, any deviation from these values of e_t and $\mathbb{E}_t e_\infty$ would shift the whole path of the exchange rate, and hence all trade surpluses on the right-hand side of (25), violating the intertemporal budget constraint. This general equilibrium discipline on the exchange rate determination is what distinguishes our solution from that in Engel and West (2005), as we discuss in detail in Appendix A.7.

An increase in demand for foreign assets (i.e., $\varepsilon_t > 0$ in (26)) results in an instantaneous depreciation of the home currency, while also predicting an expected appreciation according to (24), akin to the celebrated overshooting mechanism of Dornbusch (1976). This exchange rate path ensures both equilibrium in the financial market (via expected appreciation) and balanced country budget (via instantaneous depreciation). [22] Lastly, note that the equilibrium exchange rate process (26) is shaped by parameters ρ and β, and depends on the other parameters of the model only through the proportional volatility scaler $1/(1+\gamma\lambda_1)$. In particular, since $\lambda_1 > 0$, the exchange rate volatility increases as the economy becomes more closed to international trade, and it is maximized in the autarky limit.

Equation (24) suggests departures from the random walk behavior and implies predictability of the nominal exchange rate. We now explore how pronounced are these properties quantitatively and whether the equilibrium exchange rate process predicted by our model can be consistent with the empirical near-random-walk behavior:[23]

[21]Iterating (25) forward and applying the NPGC, we obtain $b^*_t + \gamma\lambda_2\sum_{j=0}^{\infty}\beta^j e_{t+j} = 0$, which *a fortiori* holds in expectations. Combining this with (27), which implies $\mathbb{E}_t e_{t+j} = \mathbb{E}_t e_\infty + \frac{1}{1+\gamma\lambda_1}\frac{1}{1-\rho}\rho^j\psi_t$, we obtain the solution for the level of exchange rate as a function of the shock ψ_t and the state variable b^*_t (the predetermined NFA position):

$$e_t = \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\psi_t - \frac{1}{\gamma\lambda_2}(1-\beta)b^*_t.$$

Lastly, combining with (25), we express $(\Delta e_t, \Delta b^*_{t+1})$ as a function of exogenous innovations to the financial shock ψ_t.

[22]Alternatively, both the intertemporal budget constraint and the no-arbitrage condition in financial markets can be expressed as functions of the path of the *real* (rather than nominal) exchange rate. As we show in the next section, real and nominal exchange rates respond in tandem to the financial shock ψ_t, allowing us to characterize the equilibrium dynamics in terms of the nominal exchange rate.

[23]Indeed, there exists empirical evidence on the departure of the exchange rate process from a pure random walk (see e.g. Lustig, Stathopoulos, and Verdelhan 2016, Eichenbaum, Johannsen, and Rebelo 2017, as well as our discussion in Section 4.2).

Proposition 4 (Near-random-walk behavior) *The equilibrium exchange rate process (26) becomes indistinguishable from a random walk as $\beta\rho \to 1$. In particular, the following properties hold:*[24]

1. *The autocorrelation of exchange rate changes becomes arbitrary close to zero:*

$$\lim_{\beta\to 1} \frac{\mathrm{cov}(\Delta e_{t+1}, \Delta e_t)}{\mathrm{var}(\Delta e_t)} = -\frac{1-\rho}{2} \xrightarrow[\rho\to 1]{} 0.$$

2. *The contribution of the predictable component to the variance of Δe_{t+1} shrinks to zero, or equivalently the unpredictable component (innovation) fully dominates the variance of Δe_{t+1}:*

$$\lim_{\beta\to 1} \frac{\mathrm{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\mathrm{var}(\Delta e_{t+1})} = \frac{1+\rho}{2} \xrightarrow[\rho\to 1]{} 1.$$

3. *The volatility of the exchange rate innovation (surprise) becomes unboundedly large relative to the volatility of the financial shock:*

$$\lim_{\beta\to 1} \frac{\mathrm{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\mathrm{var}(\psi_{t+1})} = \frac{1}{(1+\gamma\lambda_1)^2} \frac{1+\rho}{1-\rho} \xrightarrow[\rho\to 1]{} \infty.$$

The results in Proposition 4 derive directly from the exchange rate process in (26), and we provide the formal algebra behind them in Appendix A.5. A simple way to see why the exchange rate process in (26) approaches a random walk is to rewrite it using the lag operator:

$$(1-\rho L)\Delta e_t = \frac{1}{1+\gamma\lambda_1} \frac{\beta}{1-\beta\rho} \big(1-\beta^{-1}L\big)\varepsilon_t. \tag{28}$$

As both β and ρ become close to 1, the lag operators on the two sides of the equation cancel out, and the process converges to a random walk. Intuitively, a more persistent financial shock results in a more persistent expected appreciation with a larger effect on the cumulative discounted value of future net exports, especially when β is high. Therefore, when β and ρ are both large, a given shock ε_t requires a large contemporaneous jump in the exchange rate e_t to balance the intertemporal budget constraint of the country. In other words, the innovation to e_t is large relative to the innovation to $\mathbb{E}_t\Delta e_{t+1}$, and hence the unexpected component of the exchange rate changes dominates the expected component, making the process more like a random walk.

We now explore the extent to which Proposition 4 provides an accurate approximation of the exchange rate properties away from the $\beta\rho \to 1$ limit. In particular, Figure 1 plots the impulse response of the exchange rate (left panel) and the finite-sample variance decomposition of exchange rate changes (right panel) for our baseline case with $\beta = 0.99$ and $\rho = 0.97$. The left panel illustrates that in response to a financial shock ε_t the model produces a large depreciation $\Delta e_t > 0$ on impact, followed immediately by small and persistent expected appreciations in all future periods $\mathbb{E}_t\Delta e_{t+j} < 0$ for $j \geq 1$,

[24]Note that we first take the $\beta \to 1$ limit and then consider the $\rho \to 1$ limit. This is because for $\beta < 1$ and $\rho = 1$ the unconditional second moments are not well-defined for Δe_t, as it becomes an integrated process. This sequential limit provides a good quantitative approximation to our baseline case in which $\rho = 0.97 < \beta = 0.99$. For the last two moments, we assume that $\{\varepsilon_t, \varepsilon_{t-1}, \ldots\}$ is part of the information set at time t when constructing $\mathbb{E}_t\Delta e_{t+1}$, even though the MA root of the e_t process is non-invertible, hence offering a conservative lower bound for our results.

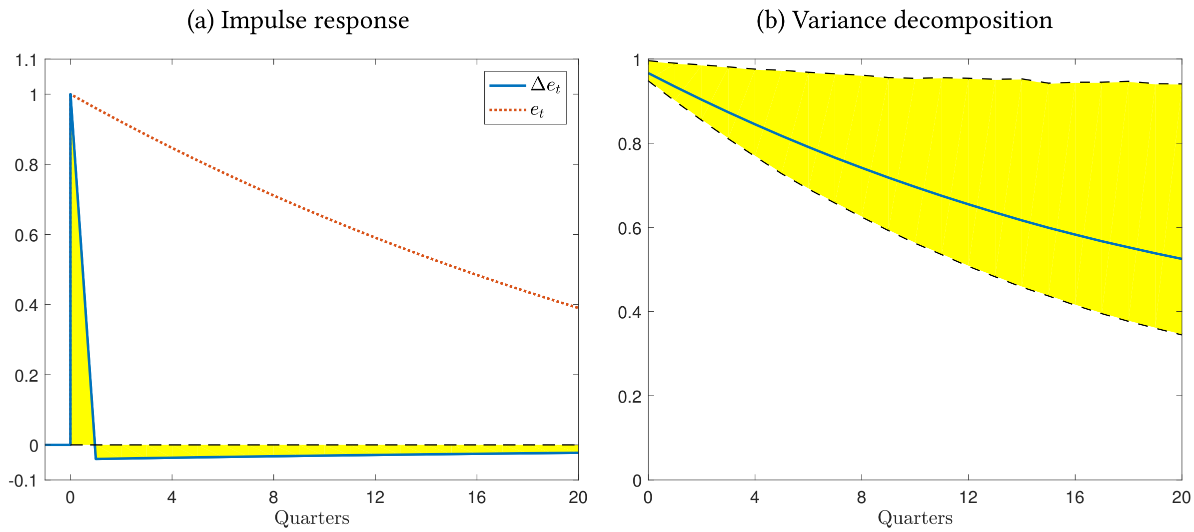

Figure 1: Impulse response and variance decomposition of Δe_t

Note: baseline model with $\rho = 0.97$ and $\beta = 0.99$. Left panel: impulse response to ε_0 of Δe_t (solid blue) and e_t (dashed red, equal to the area under the blue curve). Right panel: the contribution of the unexpected component $(\Delta_k e_{t+k} - \mathbb{E}_{t-1}\Delta_k e_{t+k})$ to the variance of the exchange rate change $\Delta_k e_{t+k} \equiv e_{t+k} - e_{t-1}$ at various horizons $k \geq 0$ (the $k = 0$ case corresponds to the second moment in Proposition 4). The shaded area is the 95% bootstrap confidence interval in a sample of 120 quarters.

with Δe_t about 25 times larger than $\mathbb{E}_t \Delta e_{t+1}$ in absolute value. The right panel further shows that the unexpected component dominates the variance of the exchange rate at all horizons. In the limit, the expected future appreciations become arbitrary small relative to the size of the devaluation on impact, and thus the impulse response converges to that of a white noise for Δe_t (or equivalently, random walk for e_t). In our baseline calibration, the autocorrelation of Δe_t has a median estimate of -0.02 and is not statistically different from zero in 30-year-long samples (see Appendix Table A1).[25]

To summarize, we find that with a conventional value of the discount factor β and the value of $\rho = 0.97$, the model reproduces the volatile near-random-walk behavior of the nominal exchange rate observed in the data.

3.2 Real exchange rate and the PPP puzzle

We next explore the equilibrium dynamics of the real exchange rate (RER) and the associated purchasing power parity (PPP) puzzle, which we broadly interpret as the close comovement between the nominal and the real exchange rates, and a volatile near-random-walk behavior of both variables. As emphasized by Rogoff (1996), the high volatility of RER is at odds with the productivity shocks, including shocks to the relative productivity of non-tradables, while the high persistence of RER (3–5 year half-lives) is at odds with the monetary shocks given conventional price durations.

[25] The near-random-walk behavior of the exchange rate is a joint equilibrium outcome with a persistent evolution of the net foreign asset position b_t^*, which follows an AR(1) process in first differences. Nonetheless, the response of b_t^* to the financial shock is quantitatively muted when the economy is relatively closed to international trade (i.e., γ is low). See Appendix A.5.

We start by rewriting the definition of RER in (14) in logs as:

$$q_t \equiv e_t + p_t^* - p_t. \tag{29}$$

Combining the definition of the price level (6) with the price setting equations for domestic and foreign goods, we obtain (see Appendix A.3.2):

$$p_t = \left(w_t - \frac{1}{1-\phi} a_t \right) + \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma} q_t, \tag{30}$$

where $(w_t - \frac{1}{1-\phi}a_t)$ is the domestic nominal unit labor cost and $\frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t$ captures the effect of the cost of foreign goods (both final and intermediate) on the price of the domestic consumption bundle.[26]

A long tradition in international macro literature models the real exchange rate as the relative price of non-tradable goods (e.g., the Balassa-Samuelson effect). This approach is, however, at odds with the empirical decomposition in Engel (1999) that finds a negligible role for the relative non-tradable prices in shaping the dynamics of the real exchange rate. We, therefore, abstract from explicitly modeling the relative prices of non-tradables. As a result, the price level in (30) depends on the overall degree of home bias γ, but *not* on how the domestic expenditure is split between tradables and non-tradables.

Combining (30) and its foreign counterpart for p_t^* with the definition of q_t in (29), we solve for the relationship between nominal and real exchange rates:

$$\left[1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right] q_t = e_t + \left((w_t^* - w_t) - \frac{1}{1-\phi}(a_t^* - a_t) \right). \tag{31}$$

Intuitively, when γ is small, the real exchange rate approximately equals the differential unit labor costs in the two countries. In contrast, as the home bias disappears ($\gamma \to 1/2$), we have $q_t \to 0$, and the purchasing power parity holds in the limit. Recall that in our baseline model we switch off productivity shocks (that is, $a_t - a_t^* \equiv 0$) and assume that monetary policy fully stabilizes wage inflation (namely, $w_t = w_t^* \equiv 0$), and therefore (31) implies the following result:

Proposition 5 (Real exchange rate) *In the baseline model, the relationship between the real exchange rates q_t and the nominal exchange rate e_t is given by:*

$$q_t = \frac{1}{1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} e_t. \tag{32}$$

Therefore, q_t is perfectly correlated with e_t, follows the same ARIMA process with a proportionally smaller innovation, and $\lim_{\gamma \to 0}(q_t - e_t) = 0$. The estimated AR(1) coefficient for q_t increases towards 1 with sample size, and hence the corresponding half-life estimate increases without bound.

Thus, in response to a financial shocks ψ_t, the nominal and real exchange rates are perfectly correlated, equally persistent, and the real exchange rate is somewhat less volatile:

[26] Note that the expression for p_t does not depend on the strategic complementarity elasticity α, which controls the extent of exchange rate pass-through. With firms symmetric in α, low pass-through into import prices is exactly offset by the domestic firm price adjustment (for further analysis see Burstein and Gopinath 2012, Amiti, Itskhoki, and Konings 2016).

$$\frac{\mathrm{std}(\Delta q_t)}{\mathrm{std}(\Delta e_t)} = \frac{1}{1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} < 1,$$

with this gap vanishing as the economy becomes closed to international trade ($\gamma \to 0$). Quantitatively, under our baseline parameterization with $\gamma = 0.07$, the real exchange rate is three-quarters as volatile as the nominal exchange rate. We show in Section 4.1 that with nominal rigidities this relative volatility is close to 1, as in the data. However, we emphasize here that even the simple version of our model without any price or wage stickiness can account for the bulk of the empirical properties of the real exchange rate. In particular, if one were to fit an AR(1) process for the real exchange rate, as is conventionally done in the PPP puzzle literature surveyed by Rogoff (1996), one would be challenged to find evidence of mean reversion and would infer very long half-lives for the real exchange rate process. In Appendix Figure A3, we show that under our baseline parametrization with $\rho = 0.97$, the model reproduces the 3-to-5 year half-lives of the real exchange rate in finite (30-year-long) samples.[27]

In summary, our model is consistent with the PPP puzzle in that it reproduces the close comovement between the nominal and the real exchange rates. Furthermore, this is achieved without any reliance on nominal rigidities. A natural question then is why the PPP puzzle posed such a challenge to the literature? Equations (29)–(31) offer an explanation. The close comovement between q_t and e_t suggests that the price levels p_t and p_t^* should, in turn, move little with the nominal exchange rate e_t (see (29)). The PPP puzzle literature has largely focused on one conceptual possibility, namely that price levels move little due to nominal rigidities, assuming monetary shocks to be the main drivers of the nominal exchange rate. The issue with this approach is that monetary shocks necessarily imply cointegration between relative nominal variables — $(w_t - w_t^*)$, $(p_t - p_t^*)$ and e_t — which results in mean reversion in the real exchange rate q_t. The speed of this mean reversion is directly controled by the duration of nominal price stickiness, which is empirically insufficient to generate long half lives, characteristic of the real exchange rate.

We focus on the other conceptual possibility, namely that prices are largely disconnected from exchange rates, or in other words the low exchange rate pass-through into CPI inflation even in the long run, due to substantial home bias (small γ; see (30)), as is the case empirically. Importantly, this mechanism requires that the main drivers of the exchange rate are not productivity or monetary shocks, which drive a wedge between nominal and real exchange rates independently of the extent of the home bias, as reflected by $(a_t - a_t^*)$ and $(w_t - w_t^*)$ terms in (31). Instead, the shock we focus on is the financial shock ψ_t, and it drives no wedge between nominal and real exchange rates, even in the long-run. Home bias is thus the only relevant part of the transmission mechanism, leaving nominal rigidities, real rigidities (α), or the extent of expenditure switching (θ) largely irrelevant.[28] The mechanism does rely, however, on the monetary policy rule. We show in Section 4.1 that our results are robust to conventional Taylor rules, but are sensitive to a switch to an exchange rate peg.

[27] While real exchange rate follows an integrated ARIMA(1,1,1) process in the baseline model, Section 4.2 offers an extension with a financial sector, in which it follows a mean-reverting ARMA(2,1), yet the two are indistinguishable in finite samples.

[28] As a result, our model is also consistent with the Kehoe and Midrigan (2008)'s finding of the missing correlation between price durations and RER persistence across sectors, which is evidence against monetary shocks as the key driver of the nominal exchange rate rather than against sticky prices as the transmission mechanism.

3.3 Exchange rates and the terms of trade

In this section we explore the joint properties of the equilibrium (CPI-based) real exchange rate, terms of trade and producer prices. As emphasized by Atkeson and Burstein (2008), the conventional models imply a counterfactually volatile terms of trade and producer prices relative to consumer prices. In the data, consumer- and producer-based real exchange rates are equally volatile, while the terms of trade are substantially more stable — about two-to-three times less volatile than the real exchange rate.

The results in this section follow from two equilibrium relationships between real exchange rates (RER) and the terms of trade (ToT):

$$q_t = (1-\gamma)q_t^P - \gamma s_t, \tag{33}$$

$$s_t = q_t^P - 2\alpha q_t, \tag{34}$$

where $s_t = p_{Ft} - p_{Ht}^* - e_t$ is the log of the ToT in (20) and $q_t^P \equiv p_{Ft}^* + e_t - p_{Ht}$ is the log producer-price RER. Intuitively, (33) reflects that the relative consumer prices q_t differ from the relative producer prices q_t^P by the relative price of imports s_t. Equation (34), in turn, states that the terms of trade reflect the relative producer prices adjusted for the law of one price deviations of exports (recall from (14) that the log LOP deviation equals αq_t).

Conventional models, without strategic complementarities ($\alpha = 0$), and thus without LOP deviations, imply that the ToT equal producer RER, and both are more volatile than the consumer RER:

$$s_t = q_t^P = \frac{1}{1-2\gamma} q_t.$$

Intuitively, consumer prices are less volatile than producer prices as they smooth out the relative price fluctuations by combining home and foreign goods into the consumption bundle (i.e., a diversification argument). This is, however, empirically counterfactual, and as explained by Atkeson and Burstein (2008) is not necessarily the case in the models with *pricing to market* (PTM), arising, for example, from strategic complementarities in price setting ($\alpha > 0$). Combining (33) and (34) together, we arrive at:

Proposition 6 (Real exchange rate and the terms of trade) *The real exchange rates and the terms of trade are linked by the following equilibrium relationships:*

$$q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma} q_t \qquad \textit{and} \qquad s_t = \frac{1-2\alpha(1-\gamma)}{1-2\gamma} q_t. \tag{35}$$

The empirical patterns $\mathrm{std}(\Delta s_t) \ll \mathrm{std}(\Delta q_t) \approx \mathrm{std}(\Delta q_t^P)$ *and* $\mathrm{corr}(\Delta s_t, \Delta q_t) > 0$ *obtain when strategic complementarities in price setting are significant, but not too strong:* $\frac{\gamma}{1-\gamma} \ll \alpha < \frac{1}{2(1-\gamma)}$.

When $\alpha > \frac{\gamma}{1-\gamma}$, the model reproduces the empirically relevant case, in which RER q_t is considerably more volatile than the ToT s_t. Pricing to market ($\alpha > 0$) smoothes out the response of ToT to changes in the producer prices q_t^P, and hence makes ToT less volatile, as export prices mimic the local competition. Very strong PTM, $\alpha > \frac{1}{2(1-\gamma)}$, just like local currency pricing (LCP), can turn the positive correlation between ToT and RER into negative, which is empirically counterfactual, as emphasized by Obstfeld and Rogoff (2000). For the intermediate values, $\alpha \lesssim 0.5$, the performance of the model is good on both margins. Additionally, a low value of γ ensures that $\mathrm{std}(\Delta q_t) \approx \mathrm{std}(\Delta q_t^P)$, as is the case empirically. Under our baseline parameterization with $\gamma = 0.07$ and $\alpha = 0.4$, the terms of trade are about a third as volatile as the real exchange rate, yet still positively correlated, exactly as in the data.[29] We further explore these quantitative implications of the model in Appendix Figure A4.

3.4 Exchange rate and consumption: the Backus-Smith puzzle

We now study the relationship between aggregate consumption and the real exchange rate, which in our model is shaped by the product and factor markets, and in particular the *expenditure switching* forces, as opposed to the *risk sharing* in the financial market emphasized by the Backus-Smith condition.

First, we combine labor supply (3) and labor demand (10) to express labor market clearing as:

$$\sigma\nu c_t + y_t = -\frac{\nu+\phi}{1-\phi}\frac{\gamma}{1-2\gamma}q_t, \tag{36}$$

where the real wage $(w_t - p_t)$ on the right-hand side is expressed using (30) as a decreasing function of q_t. Indeed, real depreciation (high q_t) depresses real wage through the increased cost of foreign goods. Equation (36) characterizes the locus of allocations consistent with equilibrium in the labor market: low real wage and high consumption reduce labor supply and, hence, output y_t.

Second, we look at the product market clearing condition, which derives from (17)–(18), and upon log-linearization results in:[30]

$$y_t = (1-\phi)\big[(1-\gamma)c_t + \gamma c_t^*\big] + \phi\big[(1-\gamma)y_t + \gamma y_t^*\big] + \gamma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \phi\right]q_t. \tag{37}$$

This equation characterizes the locus of allocations consistent with equilibrium in the product market. Note that in the closed economy ($\gamma = 0$), we simply have $y_t = c_t$, which holds in log deviations independently of the intermediate share ϕ. In open economy, home production y_t is split between final consumption and intermediate use at home and abroad, as reflected by the first two terms on the right-hand side of (37). The remaining term in the real exchange rate q_t combines the positive effect of expenditure switching from foreign to home goods and the negative effect of substitution away from the intermediate inputs towards local labor. In particular, the *expenditure switching effect* acts to increase demand for domestic output y_t when home exchange rate depreciates (q_t increases), and this effect is shaped by $\theta(1-\alpha)$, the product of the exchange rate pass-through into prices $(1-\alpha)$ and the elasticity of substitution θ.

Now combining (36)–(37) together with their foreign counterparts, we can solve for the equilibrium relationship between the relative consumption $c_t - c_t^*$ and the real exchange rate q_t (see Appendix Figure A2 for a simple geometric solution):

[29] As in Atkeson and Burstein (2008), the lower volatility of the terms of trade results from the markup adjustment by exporting firms. We check in our quantitative analysis that this does not imply counterfactually volatile aggregate profits.

[30] To derive (37), we solve out the real wage and relative prices as a function of the real exchange rate. In particular, in Appendix A.5 we show that the exchange rate pass-through into relative prices is determined by $(1-\alpha)$, as $p_{Ht} - p_t = -(1-\alpha)\frac{\gamma}{1-2\gamma}q_t$ and $p_{Ht}^* - p_t^* = -(1-\alpha)\frac{1-\gamma}{1-2\gamma}q_t$.

Proposition 7 (Backus-Smith resolution) *In the baseline model with financial shocks ψ_t only, the relative home consumption declines with real depreciation, according to the following equilibrium relation:*

$$c_t - c_t^* = -\gamma\kappa_q q_t, \quad \textit{where} \quad \kappa_q \equiv \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu + \frac{\nu+\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}}{(1-\phi)\big[1+\sigma\nu\big(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\big)\big]}\frac{2}{1-2\gamma} > 0. \tag{38}$$

The effect of the real exchange rate on the relative consumption vanishes as the economy becomes closed to international trade ($\gamma \to 0$). For $\gamma > 0$, the elasticity of the relative consumption with respect to real exchange rate increases (in absolute terms) in the strength of the expenditure switching effect, $\theta(1-\alpha)$.

It follows from Proposition 7 that our baseline model robustly reproduces a negative correlation between relative consumption and real exchange rate, both in levels and in growth rates. That is, our model predicts that consumption is low when prices are low, in relative terms across countries. This violates efficient international risk sharing, predicted by the celebrated Backus-Smith condition, yet is consistent with the patterns in the data (see cross-country estimates in Benigno and Thoenissen 2008). Furthermore, this property of our model stands in stark contrast with predictions of both productivity-driven IRBC models and monetary-shock-driven New Keynesian (NOEM) models, even when those models feature incomplete asset markets.

What is most striking about this result is that we have derived (38) using solely labor and product market clearing conditions, which are completely ubiquitous in international general equilibrium models. Indeed, the negative relationship between consumption and real exchange rate is a *robust* feature of the expenditure switching mechanism. Real exchange rate depreciation switches expenditure towards home goods, and in order to clear the markets home output needs to rise and home consumption, in view of the home bias, needs to fall. A natural question then is what makes our model different?

There are two key features of our model that allow it to produce the empirical negative correlation between consumption and the real exchange rate. First, we shift the determination of consumption from asset to product markets. Indeed, in complete market models (with CRRA utility), partial equilibrium in the asset market requires a positive correlation between relative consumption and real exchange rate: $\sigma(c_t - c_t^*) = q_t$, as an outcome of the optimal international risk-sharing. Our model instead features incomplete markets and, more importantly, a *risk-sharing shock* ψ_t, which implies the following relationship between consumption growth and real appreciation in expectations:

$$\mathbb{E}_t\big\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}\big\} = \psi_t. \tag{39}$$

Furthermore, the equilibrium relationship between consumption and exchange rate (38) is fully determined in product market, without reference to asset market and risk sharing.[31]

Second, our model ensures that the key force shaping the comovement between consumption and

[31] Asset markets matter in general equilibrium, as both consumption and exchange rate dynamics need to be consistent with asset market clearing and equilibrium interest rates (see Section 3.5). Equation (38), however, is a static equilibrium condition, which holds state-by-state and hence determines the correlation between consumption and real exchange rate independently of their dynamic processes. This is possible because the financial shock ψ_t does not directly affect the goods market, and thus is absent from (38).

the real exchange rate is expenditure switching, emphasized in equation (38). This is in contrast with the IRBC and NOEM models, where real depreciation is associated with increased supply of domestic output—either due to a high productivity shock or a stimulating effect of a monetary easing shock—while expenditure switching is only a byproduct. As a result, real depreciation in these models is associated with an empirically counterfactual domestic consumption boom. The mechanism in our financial model is different. A real depreciation is not caused by increased supply of domestic goods, but instead by increased demand for foreign assets. Therefore, the only effect on the real economy is indirect, induced by expenditure switching arising from a real depreciation, which causes a decline in real wages and consumption.[32]

Lastly, we comment on the quantitative implications of Proposition 7. A salient feature of the data is a much greater volatility of the exchange rate relative to other macroeconomic variables, and in particular consumption. From (38), we see that the relative volatility of consumption tends to zero as the economy becomes closed to international trade ($\gamma \to 0$). Outside this limit, for our baseline parameterization $\mathrm{std}(\Delta c_t)/\mathrm{std}(\Delta q_t) = 0.15$. That is, consumption is about 6 times less volatile than the real exchange rate, in line with the empirical magnitudes (for example, Chari, Kehoe, and McGrattan (2002) target a ratio of 5).[33] We conclude that the model is not only consistent with the negative correlation between consumption and real exchange rate, but also reproduces quantitatively their relative volatilities.

Alternative mechanisms in the literature Naturally, all explanations of the Backus-Smith puzzle must relax the straitjacket of the international risk-sharing condition $\sigma(c_t - c_t^*) = q_t$, either by assuming incomplete markets (e.g. Corsetti, Dedola, and Leduc 2008, Benigno and Thoenissen 2008), or by departing from separable CRRA utility (e.g. Colacito and Croce 2013, Karabarbounis 2014). The analytical tractability of our model, coupled with the conventional product and labor market structure, allows us to shed light on the mechanisms in other papers in relationship to our mechanism.

For concreteness, we focus on the model with productivity shocks only, as is the case in much of the Backus-Smith puzzle literature. This case results in the following equilibrium relationship:

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t, \tag{40}$$

where $\kappa_q > 0$ is given in (38) and $\kappa_a > 0$ is derived in the appendix (see (A49)). This is the sense in which the same expenditure switching effect of the real exchange rate on consumption is still present in the models with other shocks, but these shocks also have a direct effect in product and labor markets. Importantly, κ_a does not go to zero with $\gamma \to 0$, and therefore, the direct effect always dominates the expenditure switching effect when economies are relatively closed to international trade. While our

[32] While we solved for equilibrium consumption given the exchange rate, one can also do the reverse. Relationship (38) can then be interpreted as a *Keynes transfer effect*: a financial shock makes home households postpone their consumption resulting in a lower relative demand for home goods, which requires an exchange rate depreciation to clear the goods market (see e.g. Pavlova and Rigobon 2008, Caballero, Farhi, and Gourinchas 2008).

[33] We further find that this quantitative performance of the model is not very sensitive to the specific values of the relative risk aversion σ and Frish elasticity of labor supply ν, but is mostly sensitive to trade openness γ and the combined expenditure switching elasticity $\theta(1-\alpha)$, as we illustrate in the Appendix Figure A4 and Appendix A.5.2.

mechanism in (38) emphasizes the expenditure switching as the key source of the relationship between consumption and real exchange rate, other papers have it only as a feedback mechanism, typically not strong enough to overturn the direct effects of the product market shocks.

Equilibrium relationship (40) makes clear the two possible ways in which the Backus-Smith puzzle can be resolved. First, this occurs when exchange rate dynamics is shaped by a shock with a small direct effect on consumption relative to its indirect (expenditure switching) effect through exchange rate. Financial shocks emphasized in this paper, as well as news shocks about future productivity or long-run risk shocks in Colacito and Croce (2013) operate in this fashion. Second, if the direct effect is strong and consumption increases with productivity, the Backus-Smith puzzle can be resolved if relative prices also increase with productivity (i.e., $\partial q_t/\partial a_t < 0$). This may occur due to Balassa-Samuelson forces (e.g., Benigno and Thoenissen 2008), persistent productivity growth rates and/or low elasticity of substitution between home and foreign goods ($\theta < 1$), as in Corsetti, Dedola, and Leduc (2008). These alternative mechanisms are, however, at odds with other exchange rate puzzles, including Meese-Rogoff and PPP puzzles discussed above. See Appendix A.8 for two illustrations and a further discussion.

3.5 Exchange rate and interest rates: the UIP Puzzle

Finally, we explore the equilibrium properties of interest rates, and in particular the UIP puzzle. Home and Foreign Euler equations for local bonds in (4) and (15) result together in:

$$i_t - i_t^* = -\gamma\lambda_1 \mathbb{E}_t \Delta e_{t+1}, \tag{41}$$

where coefficient $\lambda_1 > 0$ under a mild parameter restriction (Appendix A.5 provides a derivation, which makes use of the equilibrium relationships between consumption and prices and the exchange rate). Combining (41) with the UIP condition (22) yields both the expression for the expected devaluation (24) used in Section 3.1 and the following solution for the equilibrium interest rate differential:

$$i_t - i_t^* = \frac{\gamma\lambda_1}{1+\gamma\lambda_1}\psi_t. \tag{42}$$

A demand shock for foreign bond ψ_t raises the interest rate differential $i_t - i_t^*$ to equilibrate the asset market. Further, (42) implies that the interest rate differential, like ψ_t, follows an AR(1) process with persistence ρ, and in addition with volatility declining towards zero in the closed economy limit ($\gamma \to 0$).

Making use of our characterization in Proposition 3, we now study the joint properties of the interest rates and the nominal exchange rate (with similar relationships also holding in real terms):

Proposition 8 (Exchange rate and interest rates) *The Fama regression, i.e. the projection of the exchange rate change Δe_{t+1} on the interest rate differential $(i_t - i_t^*)$, has a negative coefficient $\beta_F = -1/(\gamma\lambda_1) < 0$. Furthermore, around the $\beta\rho \to 1$ limit:*

(i) the R^2 in the Fama regression *becomes arbitrary small;*

(ii) the volatility of $(i_t - i_t^*)$ relative to Δe_{t+1} becomes arbitrary small;

(iii) the persistence of Δe_{t+1} relative to $(i_t - i_t^*)$ becomes arbitrary small;

(iv) the Sharpe ratio of the Carry trade[34] *becomes arbitrary small.*

Proposition 8 suggests that our model provides a good approximation to the the observed empirical patterns. As in the data, positive interest rate differentials predict expected exchange rate appreciations — a pattern of the *UIP deviations* known as the *Forward Premium puzzle* (Fama 1984). This result follows directly from (41), as a $\psi_t > 0$ shock results both in a positive interest rate differential and an expected appreciation of the home currency.

At the same time, the predictive ability of the interest rate differentials for future devaluations is very weak in the data (see e.g. Valchev 2016), and our model captures this with a vanishingly small R^2 in the Fama regression as the ψ_t shocks become more persistent. Recall from Proposition 4 that in this case the unexpected changes dominate the dynamics of the exchange rate, while the expected changes play a vanishingly small role. Under our baseline parameterization, the R^2 in the Fama regression is below 0.05, in line with the data. In addition, the interest rate differentials, unlike exchange rate changes, are very smooth and persistent in the data — another disconnect pattern captured by our model. For example, under the baseline parametrization, the model produces interest rate differentials that are less than one-tenth as volatile as the exchange rate changes (see Appendix Table A1). Lastly, the UIP shock in our model does not result in counterfactually large returns on the Carry trade. In Appendix Figure A5, we illustrate the small-sample properties of the Fama regression and the carry trade returns, and show that the associated Sharpe ratio varies between 0.15 and 0.3, in line with the empirical patterns.[35]

To summarize the results of Section 3, our baseline model with a simple transmission mechanism and a single financial exchange rate shock is consistent, both qualitatively and quantitatively, with a rich set of moments describing the dynamic comovement between exchange rates and macro variables. Many of these moments correspond to the long-standing puzzles from the point of view of the conventional international-macro models, including the PPP puzzle and the Backus-Smith puzzle. The financial shock ψ_t admits a number of micro-foundations, yet is not directly observable in the data. When ψ_t is assumed to follow a persistent AR(1) process with small innovations, the model reproduces both the empirical Meese-Rogoff exchange rate dynamics and the comovement properties between exchange rates and interest rates, including the Forward Premium (UIP) puzzle.

[34] A carry trade is a zero-capital investment strategy, which shorts the low interest rate currency and longs the high interest rate currency. For concreteness, following Lustig and Verdelhan (2011), we focus on a strategy with an intensity (size of the positions taken, x_t) proportional to the expected return, i.e. $x_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t$ (see Appendix A.5).

[35] The unconditional Sharpe ratio of the carry trade in the data is about 0.5, but at least half of it comes from the cross-sectional country fixed effects not modeled in our framework, which instead focuses on the time-series properties. Our empirical target for the Sharpe ratio of 0.2 corresponds to the “forward premium trade” in Hassan and Mano (2014).

4 Extensions

This section considers two main extensions to our analysis. Section 4.1 generalizes the baseline model to a full-fledged monetary model with nominal stickiness and a Taylor rule. In the context of this extension, we discuss the properties of the model with multiple shocks, as well as the Mussa puzzle associated with the switch between monetary regimes. Section 4.2 extends the baseline model to feature an explicit financial sector with risk-averse intermediaries and noise traders to shed light on a number of issues, including the recent Engel (2016) puzzle. Additional extensions, including a model with capital and a full international busyness cycle calibration, are considered in Appendix A.11.

4.1 A monetary model with nominal rigidities

We now consider the robustness of our findings in Sections 2 and 3 in a fully specified monetary model with nominal rigidities, arguably a salient feature of the real world. First, we demonstrate that nominal shocks *per se* cannot reproduce the empirical exchange rate behavior, as suggested by Proposition 1. Second, we show that the financial shock ψ_t has similar quantitative properties in the monetary model, as in our baseline model of Section 3, despite a different transmission mechanism for the interest rates. Third, we study a calibrated multi-shock model to quantify the contribution of monetary and productivity shocks to the exchange rate volatility. Lastly, we discuss the robustness of the results to various policy rules and the associated Mussa puzzle.

We introduce nominal rigidities as in the standard New Keynesian model (see e.g. Woodford 2003), while leaving the structure of international financial markets as in the baseline model. We focus on a cashless-limit economy and abstract from ZLB, commitment problems and multiplicity of equilibria. In particular, the nominal interest rate is set by a central bank according to a conventional Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m)\, \delta_\pi \pi_t + \varepsilon_t^m, \tag{43}$$

where ρ_m and δ_π are parameters, π_t is the CPI inflation rate and ε_t^m is an exogenous monetary shock.

Firms are subject to a Calvo price-setting friction with the probability of price adjustment equal to $1 - \lambda_p$. We maintain the assumption that the desired price depends on both own marginal cost and competitor prices, as in (12) with the exogenous markup shock shut down. We further assume that exporters set prices in the local currency (LCP), as for example in Chari, Kehoe, and McGrattan (2002) and Devereux and Engel (2002). Households set wages and are also subject to the Calvo friction with the probability of wage adjustment equal to $1 - \lambda_w$, as described in Galí (2008). Appendix A.9 provides a full description of the model with the characterization of its solution, as well as several extensions with PCP price stickiness and alternative Taylor rules.

To calibrate the model, we keep the same values of the parameters as in the baseline case discussed in Section 3. The prices are assumed to adjust on average once a year, i.e. $\lambda = 0.75$ (Nakamura and Steinsson 2008). For wages, we set $\lambda_w = 0.85$ corresponding to a longer expected wage duration equal to 1.5 years. For the inflation response elasticity in the Taylor rule we use the estimates from Clarida, Galí, and Gertler (2000), namely $\delta_\pi = 2.15$, which satisfies the Taylor principle. Following the literature, we set the interest rate smoothness parameter $\rho_m = 0.95$ to match the empirical persistence of the interest rate, and we assume that the monetary shocks ε_t^m follow an iid process.

We summarize the results of our analysis in Table 2, where we contrast the moments in the data and in various versions of the model. Panel A considers various single-shock models. We first report the moments in two versions of the model with the financial shock ψ_t alone. Specifically, column 1 reports the moments from a model without nominal rigidities ($\lambda_p = \lambda_w = 0$), but subject to the Taylor rule (43), which is the only departure from our benchmark model of Section 3. Column 1 reconfirms our findings that the model captures well the quantitative behavior of the exchange rates, both nominal and real, including their persistence, volatility relative to other macro variables, as well as the direction of comovement with consumption and interest rates.

The model in column 2 of Table 2 features additionally wage and price stickiness, as discussed above, while maintaining ψ_t as the only shock. We see that the introduction of nominal rigidities has very little effect on the quantitative properties of the model, even despite the differences in the transmission mechanism. Indeed, in the baseline model, the interest rate settles down to clear the markets, while in the monetary model the path of the interest rate is chosen by the monetary authority according to the Taylor rule (43). In both models, a ψ_t shock results in a sharp nominal depreciation, which in turn leads to a mild home inflation as the prices of the foreign goods increase. In a monetary model, the central bank responds by raising the interest rate, and the households respond by cutting their current consumption expenditures, thus enabling the model to reproduce both the Backus-Smith puzzle and the UIP puzzle. Furthermore, the sluggish price adjustment in the model with nominal rigidities increases the volatility of the real exchange rate and reduces the volatility of consumption and interest rates relative to the volatility of the nominal exchange rate, improving somewhat the fit of the model.[36]

Finally, in column 3 and 4 of Table 2, we shut down the financial shock and instead consider two conventional international macro models — a NOEM model with nominal rigidities subject to monetary (Taylor rule) shocks ε_t^m and also for comparison an IRBC model without nominal frictions subject to productivity shocks a_t.[37] These models fail on a number of moments and reproduce the familiar exchange rate puzzles. In particular, both of these models cannot reproduce the direction of the comovement between interest rates and exchange rate (the UIP puzzle) and consumption and exchange rate (the Backus-Smith puzzle). Furthermore, the NOEM model is challenged to reproduce the persistence of the real exchange rate, yielding a half life of less than a year (the PPP puzzle), while for the IRBC model matching the volatility of the nominal exchange rate requires incredibly large volatility of the relative TFP shocks across countries, resulting in excessively volatile real exchange rate, consumption and interest rates.

[36] On the other hand, as pointed out by Obstfeld and Rogoff (2000), models with LCP imply a counterfactual negative correlation between the exchange rate and the terms-of-trade. Matching the empirical positive, yet imperfect, correlation between these variables requires a model with a mixture of price setting patterns (flexible, PCP, LCP), which also allows for DCP (dollar/dominant currency pricing), as emphasized recently by Casas, Díez, Gopinath, and Gourinchas (2016).

[37] With our focus on exchange rate moments, we only need to specify the processes for the relative shocks, i.e. $a_t - a_t^*$ in case of the productivity shock, which we assume follows an AR(1) process with persistence $\rho_a = 0.97$.

Table 2: Quantitative properties and comparisons across models

Moment	Data	A: Single-shock models				B: Multi-shock models		
		Fin. shock ψ		NOEM	IRBC	NOEM	IRBC	Financial
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\rho(\Delta e)$	0.00	-0.02 (0.09)	-0.03 (0.09)	-0.05 (0.09)	0.00 (0.09)	-0.03 (0.09)	-0.02 (0.09)	-0.01 (0.09)
$\rho(q)$	0.95	0.93 (0.04)	0.91 (0.05)	0.84 (0.05)	0.93 (0.04)	0.93 (0.04)	0.93 (0.04)	0.93 (0.04)
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.79	0.97	0.97	1.64	0.98	0.94	0.76
$\text{corr}(\Delta q, \Delta e)$	0.98	1	1	0.99	0.99	1.00	0.97	0.94
$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$	0.20	0.31	0.12	0.52	0.64	0.20	0.30	0.31
$\text{corr}(\Delta c - \Delta c^*, \Delta q)$	-0.20	-1	-0.95	1	1	-0.20 (0.09)	-0.20 (0.09)	-0.22 (0.09)
$\sigma(\Delta nx)/\sigma(\Delta q)$	0.10	0.26	0.17	0.08	0.14	0.32	0.30	0.10
$\text{corr}(\Delta nx, \Delta q)$	≈ 0	1	0.99	1	1	-0.00 (0.09)	-0.00 (0.09)	-0.02 (0.09)
$\sigma(\Delta s)/\sigma(\Delta e)$	0.35	0.23	0.80	0.82	0.49	0.80	0.28	0.23
$\text{corr}(\Delta s, \Delta e)$	0.60	1	-0.93	-0.96	0.99	-0.93	0.97	0.94
Fama β	$\lesssim 0$	-2.4 (1.7)	-3.4 (2.6)	1.2 (0.7)	1.4 (0.5)	-0.6 (1.4)	-0.7 (1.3)	-2.8 (3.5)
Fama R^2	0.02	0.03 (0.02)	0.03 (0.02)	0.03 (0.03)	0.09 (0.02)	0.00 (0.01)	0.00 (0.01)	0.01 (0.02)
$\sigma(i - i^*)/\sigma(\Delta e)$	0.06	0.07 (0.02)	0.05 (0.02)	0.14 (0.02)	0.21 (0.06)	0.06 (0.02)	0.08 (0.02)	0.03 (0.01)
$\rho(i - i^*)$	0.90	0.93 (0.04)	0.98 (0.01)	0.84 (0.05)	0.93 (0.04)	0.91 (0.04)	0.93 (0.04)	0.90 (0.04)
Carry SR	0.20	0.21 (0.04)	0.20 (0.04)	0	0	0.17 (0.06)	0.19 (0.06)	0.12 (0.07)

Note: The table reports moments in the data (see details in the Data Appendix A.12) and in the simulated data for various specification of the model, as described in the text. $\rho(\cdot)$ denotes autocorrelation, $\sigma(\cdot)$ denotes standard deviation. The numbers in brackets report standard deviation across 10,000 simulations with 120 quarters (30 years) each, only for the moments that vary considerably across simulations. In all specifications, model parameters are as in Table 1, and columns 2, 3 and 5 additionally feature sticky prices ($\lambda_p = 0.75$) and wages ($\lambda_w = 0.85$), while columns 1, 4 and 6–7 have flexible prices ($\lambda_p = \lambda_w = 0$). Monetary policy in columns 1–6 follows the Taylor rule (43) with parameters given in the text, while in column 7 it fully stabilizes the wage inflation ($W_t = W_t^* = 0$), as in the baseline model of Section 3.

Multiple shocks and variance decomposition A natural deficiency of any one-shock model is that it can only speak to the relative volatilities of variables, while implying counterfactual perfect correlations between them. We consider now two models with multiple shocks to study whether they can successfully reproduce the imperfect, and in general weak, empirical correlations between exchange rates and macro variables. In particular, we extend the NOEM and IRBC models from columns 3 and 4 of Table 2 to feature two additional shocks each — the financial shock ψ_t and a demand shock for foreign goods ξ_t.[38] Leaving unchanged all parameter values, we only need to additionally calibrate the

[38]We have also experimented with a LOP-deviation shock η_t and a nearly-equivalent iceberg trade cost shock, but find them largely redundant as long as ξ_t is included, which has superior quantitative properties for the moments we have chosen.

Table 3: Sources of exchange rate variation

Shocks		NOEM		IRBC	
		$\text{var}(\Delta e_t)$	$\text{var}(\Delta q_t)$	$\text{var}(\Delta e_t)$	$\text{var}(\Delta q_t)$
Monetary (Taylor rule)	ε_t^m	10%	10%	—	—
Productivity	a_t	—	—	3%	9%
Foreign-good demand	ξ_t	19%	20%	23%	39%
Financial	ψ_t	71%	70%	74%	52%

Note: The table reports the variance decomposition for nominal and real exchange rate changes into the contributions of shocks in the two multi-shock models, corresponding to columns 5 and 6 of Table 2. The relative volatilities of the shocks are calibrated to match $\text{corr}(\Delta c - \Delta c^*, \Delta q) = -0.2$ and $\text{corr}(\Delta nx, \Delta q) = 0$, with other parameters as in Table 1.

relative variances of the shocks, with the overall level of variance kept to match the volatility of the nominal exchange rate. We set the relative volatilities of the shock to match the correlations of the real exchange rate with consumption and net exports.[39]

From columns 5 and 6 of Table 2, we see that these extended NOEM and IRBC models are indeed capable at perfectly matching the weak negative correlation between the real exchange rate and consumption and a close-to-zero correlation between the real exchange rate and net exports, without compromising the fit of the other moments relative to the baseline monetary model in column 2. We conclude that these parsimonious, and rather conventional, models provide a quantitative resolution to a broad class of international macro puzzles assembled under the exchange rate disconnect umbrella. The key to this success is the presence of the financial shock, as we now show more formally.

In the context of these two multi-shock models, we carry out variance decompositions in order to assess the relative importance of productivity, monetary and financial shocks for the exchange rate dynamics. This decomposition, reported in Table 3, reveals a clear pattern, which echoes our theoretical predictions in Propositions 1 and 2. First, the financial shock ψ_t plays the dominant role, explaining over 70% of the nominal exchange rate variation in both models. Second, the international shock in the goods market ξ_t also plays an important role, contributing about 20% to the nominal exchange rate volatility, to partly balance out the comovement between exchange rate and macro variables induced by the ψ_t shock. Third, the contribution of productivity and monetary shocks is minimal, never exceeding 10% for either nominal or real exchange rate. Indeed, Proposition 1 suggests that both of these shocks, if too important in shaping the exchange rate dynamics, result in conventional exchange rate puzzles.

To be clear, the conventional productivity and monetary shocks are still central for the dynamics of the macro variables such as consumption, employment, output and prices levels. For these variables our model replicates the standard international business cycle patterns emphasized by e.g. Backus, Kehoe, and Kydland (1992, 1994), as we show in Appendix Table A4, which reports a formal BKK-style calibration of a version of our model with capital accumulation. Furthermore, our model of disconnect does

[39] In the NOEM model, $\sigma_m/\sigma_\varepsilon = 0.315$, where $\sigma_m = \sigma(\varepsilon_t^m - \varepsilon_t^{m*})$ for the Taylor rule shock in (43) and $\sigma_\varepsilon = \sigma(\varepsilon_t)$ for the innovation to ψ_t in (23). In the IRBC model, $\sigma_a/\sigma_\varepsilon = 2.1$, where relative productivity $(1-\rho_a L)(a_t - a_t^*) \sim \mathcal{N}(0, \sigma_a^2)$ with persistence $\rho_a = \rho = 0.97$. We do not combine productivity and monetary shocks together in any one specification because their relative roles cannot be identified from the set of exchange rate moments that we focus on. Instead, both NOEM and IRBC models include a foreign good demand shock: $(1-\rho_\xi L)(\xi_t - \xi_t^*) \sim \mathcal{N}(0, \sigma_\xi)$ with $\rho_\xi = 0.97$ and $\gamma\sigma_\xi/\sigma_\varepsilon$ set at 2.7 and 2.4 in the two specifications respectively.

not fundamentally change the way in which these conventional shocks affect exchange rates. Instead, it puts an upper bound on how important these shocks can be in shaping the overall *unconditional* exchange rate dynamics, which we argue must be largely driven by ψ_t-like shocks in financial markets.

Mussa puzzle Mussa (1986) puzzle refers to the striking discontinuous difference in the behavior of the real exchange rate when the monetary authority switches between a pegged and a floating nominal exchange rate regimes. Indeed, in conventional models the real and nominal exchange rates are shaped by different forces, at least once prices have adjusted, and therefore a change in the monetary regime should not alter the behavior of the real exchange rate in such a fundamental way. After briefly reviewing a larger set of stylized facts, to which we collectively refer as the Mussa puzzle, we turn to our model of exchange rate disconnect to study whether it reproduces these empirical patterns in response to a switch in the monetary regime.

Comparing empirical moments for several developed countries before and after the end of the Bretton Woods system of fixed exchange rates, the literature has emphasized the following stylized facts:

1. The volatility of the *real* exchange rate increased almost as much as the volatility of the nominal exchange rate (Mussa 1986). More precisely, the volatility of the nominal exchange rate was about 8 times smaller under the peg, while for the real exchange rate it was about 4 times smaller, and the correlation between the two variables was 0.66 under the peg relative to a nearly perfect correlation under the float (Monacelli 2004).
2. At the same time, there was almost no difference in the output or consumption volatilities across the two periods (Baxter and Stockman 1989, Flood and Rose 1995).
3. The Backus-Smith risk-sharing condition and the UIP condition both held better in the data during the Bretton Woods period. Specifically, the data from the peg period exhibits the theoretically-predicted positive signs of the respective correlations (Colacito and Croce 2013).[40]

We summarize these facts in the Data column of Table 4, which also reports the respective moments in our simulated model. Our goal here is not to simply match the moments under the peg, but rather to check whether our quantitative exchange rate disconnect model can simultaneously account for the broad patterns of the Mussa puzzle. To this end, we adopt the extended multi-shock NOEM model from column 5 of Table 2, and study two alternative nominal peg scenarios, keeping all other parameters unchanged.

By definition, a nominal peg regime requires a change from the Taylor rule (43) to a monetary policy rule that directly stabilizes the nominal exchange rate. We focus on the following policy rule:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\,\delta_e(e_t - \bar{e}) + \varepsilon_t^m, \tag{44}$$

where $\bar{e}$ is the level of the peg and δ_e is the strength of the peg. We adopt this policy rule in both scenarios, calibrating δ_e in each case to exactly match the empirical volatility of the nominal exchange rate during the peg regimes (the first moment in Table 4).

[40]Similarly, Devereux and Hnatkovska (2014) document that Backus-Smith condition holds better across regions within countries, in contrast with its cross-country violations. Another pattern emphasized by Berka, Devereux, and Engel (2012) is

Table 4: Mussa puzzle: moments under nominal peg relative to float

Moment	Data	Model (1)	Model (2)
$\text{std}(\Delta e_t)$	0.13	0.13	0.13
$\text{std}(\Delta q_t)$	0.26	0.18	0.16
$\text{corr}(\Delta q_t, \Delta e_t)$	0.66	0.79	0.84
$\text{std}(\Delta c_t - \Delta c_t^*)$	$\approx$1	2.63	1.33
$\text{corr}(\Delta c_t - \Delta c_t^*, \Delta q_t)$	$>$0	$-$0.63	0.13
Fama β	$>$0	$-$0.1	1.1

Note: The first four lines report moments under a peg relative to their values under a float, e.g. 0.13 for $\text{std}(\Delta e_t)$ means that nominal exchange rate is about 8 ($\approx 0.13^{-1}$) times less volatile under the peg. In contrast, the last two lines simply report the moments for the peg regime. The model in both columns corresponds to the multi-shock NOEM model from column 5 of Table 2, with the Taylor rule changed from (43) to a peg (44); column 2 additionally shuts down the ψ_t shock under the peg.

Another important difference under the peg is a substantially lower volatility in the UIP deviations (as, for example, documented in Kollmann 2005), which corresponds to a lower variance of the financial shock ψ_t. Jeanne and Rose (2002) suggest a theoretical limits-to-arbitrage mechanism, in which the variance of ψ_t endogenously declines when the monetary authority commits to a peg, reducing the risk associated with carry trades and making arbitrageurs less averse to large currency positions, as we discuss in Section 4.2.[41] To capture this, we consider two extreme scenarios: in the first scenario in column 1 we keep the variance of all shocks $(\psi_t, \xi_t, \varepsilon_t^m)$ unchanged, while in the second scenario in column 2 we fully shut down the dispersion of ψ_t, leaving the other two shocks unchanged.

From Table 4, we see that under both scenarios, the model captures a sharp reduction in the volatility of the real exchange rate under a nominal peg (in fact, overstating it), as well as a sizable reduction in the correlation between the nominal and the real exchange rates from a nearly perfect correlation under the float. The model under the first scenario (i.e., no reduction in ψ_t shocks) fails, however, on the remaining moments, predicting an increase in the volatility of consumption and no reversal in the signs of the Backus-Smith correlation and the Fama coefficient. In turn, the second scenario, with the financial shock shut down, makes the model consistent with these three moments: in this case, the volatility of consumption changes little across the regime switch and the signs of the Backus-Smith correlation and the Fama coefficient turn to positive under the peg.

We conclude that our model of exchange rate disconnect captures, at least qualitatively, the additional set of empirical patterns associated with a switch to a nominal peg regime, which we collectively labeled as the Mussa puzzle. This, however, requires that a pegged nominal exchange rate endogenously reduces the volatility of the UIP deviation shocks. We now turn to an explicit model of the financial sector, which in particular sheds light on this mechanism.

a substantially greater role of the non-tradable (Balassa-Samuelson) component in the RER variation under a nominal peg.

[41] Other structural interpretations of the ψ_t shock may suggest further patters of comovement between ψ_t and the primitive shocks of the model.

4.2 A model with a financial sector

In this section we study an extension of the baseline model from Section 3 with an explicit financial sector, which microfounds the upward-sloping supply in the financial market and the financial shock ψ_t. The model embeds in general equilibrium the noise trader and limits-to-arbitrage model of De Long, Shleifer, Summers, and Waldmann (1990) and its adaptation to the exchange rate market by Jeanne and Rose (2002). This extension achieves the following goals. First, it allows us to analyze a symmetric world economy, in which each country offers a bond in its own currency, and all international transactions are intermediated by a financial sector, which is averse to large risky positions. This leads to an upward-sloping supply of international bonds, and as a result the dispersion of the UIP deviation shock and the volatility of the nominal exchange rate are determined endogenously in equilibrium. Second, unlike our baseline, the model with a financial sector is stationary, featuring a unique long-run equilibrium and long-run mean reversion in the real exchange rate. Nonetheless, we show that the small-sample quantitative properties of the two models are nearly indistinguishable. Lastly, in the context of this extension we address an additional set of facts on the comovement between interest rates and exchange rates, emphasized recently by Engel (2016) and Valchev (2016).

We start by briefly describing the extended model, relegating the details to Appendix A.10, and then proceed to study its qualitative and quantitative properties. We consider an international financial market with three types of agents trading assets. First, there are home and foreign households that trade their local-currency bond only, taking net foreign positions B_{t+1} and B^*_{t+1} respectively.[42] Second, there are n noise traders that take a zero-capital position long N^*_{t+1} in foreign-currency bond and short $N_{t+1} = -N^*_{t+1}\mathcal{E}_t$ in home-currency bond, and vice versa when $N^*_{t+1} < 0$. We assume

$$N^*_{t+1} = n\left(e^{\psi_t} - 1\right), \tag{45}$$

where ψ_t is the noise-trader demand shock for foreign currency, which follows an exogenous AR(1) process (23) with zero mean.

Third, intermediation in the financial market is done by a measure m of competitive arbitrageurs that collectively take a zero-capital position D^*_{t+1} in foreign-currency bond and short $D_{t+1} = -D^*_{t+1}\mathcal{E}_t$ home-currency bonds, again allowing for $D^*_{t+1} < 0$. We denote the return on a one-dollar D^*_{t+1} position by $\tilde{R}^*_{t+1} \equiv R^*_t - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$. Each arbitrageur chooses his individual position d^*_{t+1} to maximize a mean-variance utility of returns, $\mathbb{E}_t\tilde{R}_{t+1} \cdot d^*_{t+1} - \frac{\omega}{2}\mathrm{var}_t(\tilde{R}_{t+1}) \cdot d^{*2}_{t+1}$, where ω is the risk aversion parameter. The resulting demand for foreign currency bonds by the financial intermediary sector is then:

$$D^*_{t+1} = m\frac{\mathbb{E}_t\tilde{R}_{t+1}}{\omega\ \mathrm{var}_t(\tilde{R}_{t+1})}. \tag{46}$$

The financial market clears when the interest rates R_t and R^*_t are such that $B_{t+1}+N_{t+1}+D_{t+1} = 0$ and $B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0$, which in particular implies that in equilibrium net foreign asset position

[42] For convenience, we now have B_{t+1} denote the value of home-currency bonds purchased at t and paying out R_tB_{t+1} units of home currency at $t+1$. Furthermore, B^*_{t+1} now refers to the position of the foreign households (as home household no longer hold foreign-currency bonds), and it pays $R^*_tB^*_{t+1}$ units of foreign currency at $t+1$.

of home equals net foreign liabilities of foreign, $B_{t+1} = -B^*_{t+1}\mathcal{E}_t$. Lastly, we assume for concreteness that the profits and losses of the arbitrageurs and noise traders are transferred to the foreign households, and thus their budget constraint becomes:

$$B^*_{t+1} - R^*_{t-1}B^*_t = NX^*_t + \tilde{R}^*_t(D^*_t + N^*_t),$$

while the budget constraint of the home is $B_{t+1} - R_{t-1}B_t = NX_t$, where $NX_t = -NX^*_t\mathcal{E}_t$. The rest of the model is unchanged. We again solve the model by log-linearization around a non-stochastic equilibrium with $B = B^* = N^* = D^* = \tilde{R}^* = 0$ and $\mathcal{E} = 1$. In light of this approximate solution approach, the strong assumptions made above — namely, the quadratic utility over dollar returns and the transfer of profits to foreign households — are without loss of generality.[43]

The only equation in the linearized equilibrium system that changes in this extension with a financial sector is the UIP condition (22), which now becomes (see Appendix A.10):

$$i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1} = \chi_1\psi_t - \chi_2 b_{t+1} \qquad \text{with} \qquad \chi_1 \equiv \frac{n/\beta}{m/(\omega\sigma_e^2)} \quad \text{and} \quad \chi_2 \equiv \frac{\bar{Y}}{m/(\omega\sigma_e^2)}, \tag{47}$$

where $\sigma_e^2 \equiv \text{var}_t(\Delta e_{t+1})$ is the variance of the innovation to the nominal exchange rate, which is determined endogenously in equilibrium, yet taken as given by the competitive financial sector. The generalized UIP condition (47) derives from the international bond market clearing condition combined with (45) and (46). It characterizes the excess return on the home bond, which ensures that the arbitrageurs are willing to satisfy the relative demand for foreign bonds by both noise traders (ψ_t) and households (b_{t+1}). The net foreign asset position of the home households, $b_{t+1} \equiv RB_{t+1}/\bar{Y}$, reflects the demand for home-currency bond as a savings vehicle, and hence increases the price of the home bond, or equivalently reduces its relative interest rate.

The crucial difference of the new UIP condition (47) from (22) is that now it features two sources of UIP deviations — an exogenous shock ψ_t as before and an endogenous feedback via the state variable b_{t+1}. The elasticities χ_1 and χ_2 of respectively exogenous and endogenous sources of UIP deviations decrease in the absorption (risk-bearing) capacity of the financial sector $m/(\omega\sigma_e^2)$, which in turn depends on the size of this sector m, its risk aversion ω and the endogenous volatility of the nominal exchange rate σ_e^2 (*cf.* Gabaix and Maggiori 2015). As the financial sector becomes larger or less risk averse, with $\frac{\bar{Y}}{m/\omega}, \frac{n}{m/\omega} \to 0$, the model features an undistorted UIP condition in the limit (with $\chi_1 = \chi_2 = 0$). If, instead, $\chi_1 \propto \frac{n}{m/\omega}$ remains positive in the limit, the model admits the baseline UIP condition (22) as a special case.

The other equilibrium relationships of the extended model, apart from the UIP condition (47), remain unchanged. This includes the proportional relationship between the nominal and the real exchange rates (32), as well as the relationships between exchange rates and respectively terms of trade (35), consumption (38) and interest rates (41). As a result, Propositions 5–7 still hold in the model with a financial sector. The equilibrium dynamics of the exchange rate (Proposition 3) differs, however, as we now characterize:

[43]Note, in particular, that the profits and losses of the noise traders and arbitrageurs, $\tilde{R}^*_t(D^*_t + N^*_t)$, is a second order term.

Proposition 9 (Exchange rate process redux) *In the model with a financial sector and a noise-trader shock $\psi_t \sim AR(1)$ with persistence ρ and innovation ε_t, the equilibrium nominal exchange rate e_t follows an ARMA(2,1) process with AR roots ρ and $\zeta_1 < 1$ (with $\zeta_1 \to 1$ iff $\chi_2 \to 0$) and an MA root $1/\beta$:*

$$(1-\rho L)(1-\zeta_1 L)e_t = \frac{1}{1+\gamma\lambda_1}\frac{\beta\zeta_1\chi_1}{1-\beta\zeta_1\rho}\big(1-\beta^{-1}L\big)\varepsilon_t. \tag{48}$$

Provided that $\frac{n\sigma_\varepsilon}{m/\omega}$ is large enough, there exists a solution with $\chi_1 > 0$ and $\sigma_e^2 = \mathrm{var}_t(\Delta e_{t+1}) > 0$, such that σ_e^2 increases in $\frac{n\sigma_\varepsilon}{m/\omega}$. There always exists another solution with $\sigma_e^2 = \chi_1 = 0$.

A formal proof of this proposition is contained in Appendix A.10, which also defines the cutoff value for $\frac{n\sigma_\varepsilon}{m/\omega}$ and shows that it tends to zero as $\beta\rho \to 1$. In the absence of other shocks, there always exists a zero-variance equilibrium, in which the arbitrageurs coordinate to fully offset the noise-trader shock, as this involves no risk as a matter of a self-fulfilling prophecy. However, for a large enough noise-trader shock $n\sigma_\varepsilon$, there also exists a positive-variance equilibrium, in which noise-trader shocks result in a volatile exchange rate. Furthermore, any fundamental shock to current account (e.g., productivity shock) shifts the economy to a positive-variance equilibrium. The government, however, has the ability to commit to peg the exchange rate, providing a coordination device for the financial sector to fully absorb the noise-trader shocks. This justifies why in Section 4.1 we considered the case in which a monetary peg was associated with a reduction in the variance of the UIP deviation shock.

The equilibrium exchange rate process (48) is stationary, unlike (28) in the baseline model. However, the two become indistinguishable as $\chi_2 \to 0$ and consequently $\zeta_1 \to 1$. We discuss below that quantitatively, under our calibration with $\chi_2 > 0$, the two processes are nearly identical in finite samples, even though one is stationary and one is integrated in the long run. The model with a financial sector is stationary around a unique steady state with $B = B^* = \tilde{R}^* = 0$.[44] The stationarity of the model emerges from the fact that the financial intermediaries are averse to holding positions that expose them to exchange rate risk and require a premium. As a result, a country-borrower faces a higher interest rate relative to $1/\beta$, which provides an incentive to gradually close its NFA position. This mechanism offers a microfoundation for the state-dependent borrowing rate often adopted to close small open economy models (see Schmitt-Grohé and Uribe 2003).

Quantitative properties We now turn to a multi-shock version of the model with a financial sector to study its quantitative properties. As in the earlier IRBC model (in column 6 of Table 2), we add a productivity shock a_t and a foreign-good-demand shock ξ_t, however, for simplicity instead of a Taylor rule we maintain the assumption that monetary policy stabilizes the nominal value of the wage rate, as in the baseline model. We keep all parameters unchanged as in Table 1, and only need to calibrate the new parameter χ_2 and the relative volatilities of the three shocks. As before, we choose the relative volatilities to match the consumption and net export correlation with the real exchange rate.[45] We set $\chi_2 = 0.01$ to match the impulse response of the UIP deviations in Figure 3a, as we discuss below.

[44]We show in the appendix that the NFA position b_{t+1} follows a stationary AR(2) process with the same roots ρ and ζ_1.

[45]The exogenous shock to the UIP is now $\chi_1\psi_t$ with the coefficient χ_1 not separately identified from the volatility of ψ_t. Thus, to match the correlation moments, we now set $\sigma_a/(\chi_1\sigma_\varepsilon) = 4$ and $\gamma\sigma_\xi/(\chi_1\sigma_\varepsilon) = 2.6$ (*cf.* footnote 39).

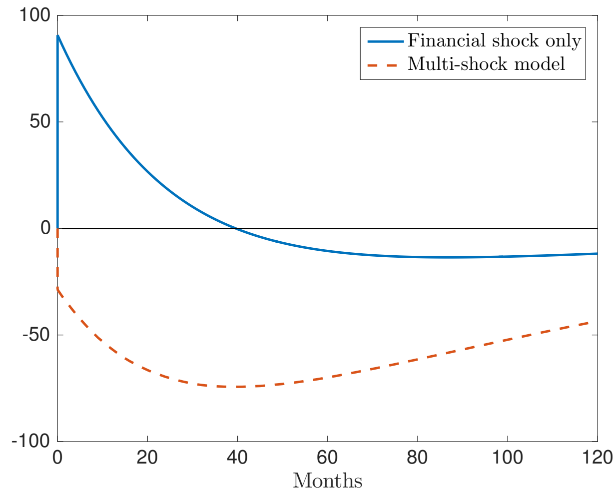

Figure 2: The response of exchange rate e_{t+j} to innovation in $i_t - i_t^*$

Note: The figure plots two alternative shapes of the exchange rate impulse response to an innovation in the interest rate differential. The construction of the impulse response is explained in Appendix A.10. The blue solid line is from the single-ψ_t-shock model and the dashed red line is from the multi-shock model, both featuring the financial sector and calibrated as discussed in Section 4.2.

We report the model-generated moments resulting from this calibration in the last column of Table 2. This column shows that the model with a financial sector is as successful at matching the empirical moments as the other multi-shock models (NOEM and IRBC) discussed above, yet as we discuss next it matches an additional set of moments. In particular, the model still reproduces a near-random-walk exchange rate process, with nominal and real exchange rates nearly perfectly correlated, while consumption remains about six times less volatile than the real exchange rate and weakly negatively correlated with it.[46] The similarity of the fit is, perhaps, not surprising, as the only difference of the model with a financial sector sector is the presence of the endogenous feedback $-\chi_2 b_{t+1}$ in the UIP condition (47). Despite the implied qualitative difference resulting in stationarity of the exchange rate, the finite sample properties of the model are almost unchanged. One difference that emerges in the model with a financial sector is that non-financial shocks such as a_t and ξ_t now result in endogenous UIP violations through their effect on the state variable b_{t+1}, and hence contribute more to the exchange rate volatility. In particular, our calibration of the model with a financial sector attributes a larger role to the good-market shock ξ_t in comparison with the multi-shock IRBC model.

Engel puzzle We now turn to the Engel (2016) puzzle, which concerns the shape of the exchange rate response to the movements in the interest rate differential. The Fama (1984) regression suggests that a positive interest rate differential predicts an exchange rate appreciation, that is $\mathrm{cov}(i_t - i_t^*, \Delta e_{t+1}) < 0$. Engel (2016) argues that most models of the forward premium achieve this with an exchange rate process that depreciates on impact at t and then gradually appreciates starting at $t+1$. This is indeed the case in our model with a single financial shock ψ_t following an AR(1) process, as we depict with

[46]The only noticeable difference in the fit of this model from the multi-shock IRBC model is the lower volatility of the real exchange rate, which is due to the difference in the monetary policy rule.

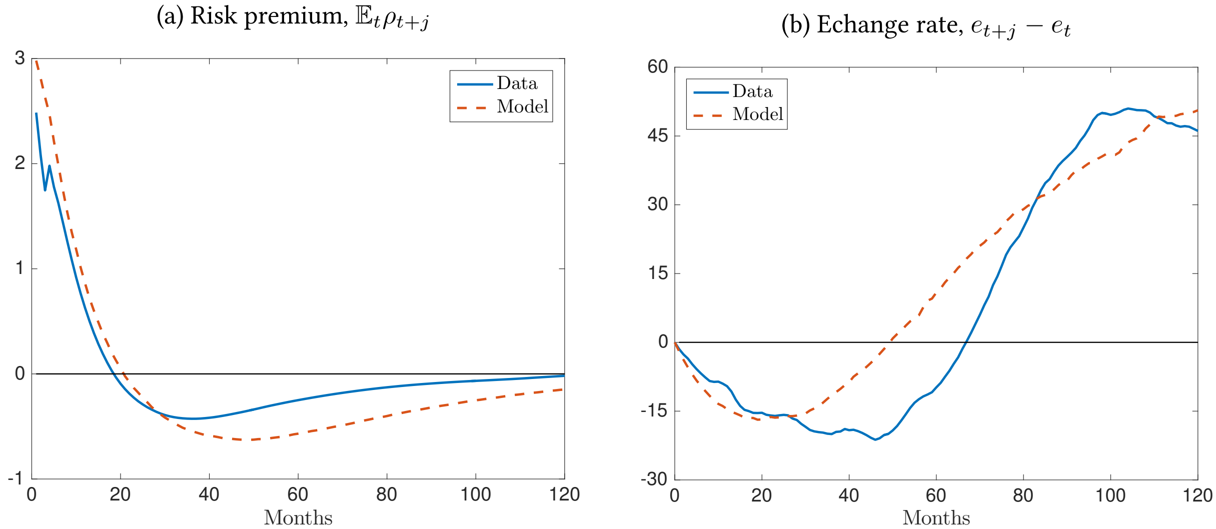

Figure 3: Data vs Model: projection coefficients on $i_t - i_t^*$

Note: Panel (a) plots the projection coefficients of $\mathbb{E}_t\rho_{t+j}$ on $i_t - i_t^*$ for $j \geq 1$, where risk premium $\rho_t = i_{t-1} - i_{t-1}^* - \Delta e_t$ is the one-period return on future carry trades, and the expectation $\mathbb{E}_t\rho_{t+j}$ is constructed using a VEC model, as in Engel (2016). Panel (b) plots the projection of $e_{t+j} - e_t$ on $i_t - i_t^*$ for $j \geq 0$. The blue lines in the figures correspond to the empirical estimates in: (a) Figure 2 in Engel (2016) and (b) Figure 2 in Valchev (2016), both reconstructed using data provided by Engel (2016). The dashed red lines plot corresponding estimates in the data simulated from the calibrated multi-shock model with a financial sector. The details of estimation are provided in Appendix A.10.

a solid blue line in Figure 2. Engel (2016), however, shows that in the data the exchange rate response appears to be different, with an appreciation on impact at t followed by further appreciation over some period and an eventual reversal into depreciation (see also Valchev 2016).[47] We illustrate this alternative impulse response with a dashed red line in Figure 2.

We show here that the calibrated multi-shock version of the model with a financial sector matches the empirical impulse responses of the risk premium in Engel (2016) and of the exchange rate in Valchev (2016). The results are reported in Figure 3, in which solid blue lines correspond to the empirical impulse responses and the dashed red lines plot the corresponding responses calculated using simulated data from the model. The calibration of the model is the same that yields moments for column 7 of Table 2. In our choice of $\chi_2 = 0.01$, which parameterizes the endogenous feedback elasticity in the UIP condition (47), we targeted the duration till risk premium becomes negative in the impulse response in Figure 3a. Recall that all other parameters were kept unchanged from our calibration of the baseline model in Section 3.

Our calibrated model captures well that an increase in the interest rate differential today predicts an increase in the risk premium $\mathbb{E}_t\rho_{t+j}$ on impact, where $\rho_{t+j} \equiv i_{t+j-1} - i_{t+j-1}^* - \Delta e_{t+j}$, which then gradually decreases and turns negative 20 months out (Figure 3a). Similarly, it captures an exchange rate appreciation on impact, followed by further appreciation over the next 20 months, which then

[47] The main empirical results in Engel (2016) are reported in terms of the impulse response of the UIP deviations, as we reproduce in Figure 3a, yet his paper also shows that $\text{cov}(i_t - i_t^*, e_t) < 0$ and $\text{cov}(\Delta i_t - \Delta i_t^*, \Delta e_t) < 0$, consistent with the shape of the dashed red impulse response in Figure 2, but not with the solid blue one. We report these correlations in the data and for different version of the model in Appendix Table A2.

reverts into an expected depreciation (Figure 3b).[48] Indeed, these impulse responses capture the subtle departures of the exchange rate process from a random walk, which are present both in the data, as well as in our model.

The mechanism by which our model reproduces these empirical patterns relies crucially on the endogenous state variable b_{t+1} in the UIP condition (47) in the presence of multiple shocks. In particular, the short run dynamics of the UIP deviations is dominated by the noise-trader shock ψ_t. The other shocks do not have a direct impact on the UIP deviations, but shape them indirectly through their effect on the net foreign assets, which build up gradually over time, resulting in a non-monotonic impulse response in Figure 2. Non-monotonicity is not guaranteed in general, but does emerge from the combinations of ψ_t and ξ_t shocks, as they move risk premium in opposite directions for a given direction of the interest rate change. We, thus, find that the same mix of shocks that results in the broad exchange rate disconnect properties summarized in Table 2 also reproduces the dynamic comovement between interest rate differentials and exchange rate changes documented by Engel (2016) and Valchev (2016).

5 Conclusion

We propose a parsimonious general equilibrium model of exchange rate determination, which is simultaneously consistent with a wide array of exchange-rate-related moments, offering a unifying resolution to the main exchange rate puzzles in international macroeconomics to which we collectively refer as the exchange rate disconnect. The model is analytically tractable, allowing for a complete closed-form characterization, essential for a transparent exploration of the underlying mechanisms. Beyond reproducing the qualitative patterns in the data, the model also matches quantitatively a rich set of moments describing the comovement between exchange rates and macro variables.

With this general equilibrium model, one can reconsider the conclusions of the vast international macro literature plagued by the exchange rate puzzles. In particular, the model reproduces exchange rate disconnect without altering the international transmission mechanism for monetary and productivity shocks, including international spillovers from monetary policy (see e.g. Corsetti, Dedola, and Leduc 2010). This is the case because, *conditional* on a shock, the model relies on a transmission mechanism with conventional properties. What we emphasize instead is that conventional productivity and monetary shocks cannot be the main drivers of the *unconditional* behavior of exchange rates.[49]

In contrast, the model likely invalidates the conventional normative analysis in open economies, and in particular the studies of the optimal exchange rate regimes and capital controls. The model emphasizes the role of the shocks in the financial markets, as opposed to monetary and goods market shocks, in accounting for the bulk of the *unconditional* exchange rate variation. Therefore, a normative analysis must allow for financial shocks, which introduce new tradeoffs for alternative policy options. For example, an exchange rate peg may simultaneously reduce monetary policy flexibility, yet improve

[48] Here the model is somewhat off on the timing of the reversal, yet the uncertainty bounds around the exact shapes of these impulse responses are wide enough that the two are not statistically different.

[49] Therefore, while we emphasize the same empirical patterns as Alvarez, Atkeson, and Kehoe (2007), our conclusions are not as far reaching as theirs (summarized in their title), suggesting in particular that the unconditional *disconnect* behavior of the nominal exchange rate may be consistent with the conventional transmission of monetary shocks.

international risk-sharing by offsetting the noise-trader risk (*cf.* Devereux and Engel 2003). Furthermore, a microfoundation for the financial shock is essential, as it may endogenously interact with the policy.

In addition, our framework can be used as a theoretical foundation for the vast empirical literature, which relies on exchange rate variation for identification (see e.g. Burstein and Gopinath 2012). Similarly, it can serve as a point of departure for the equilibrium analysis of the *international price system* (Gopinath 2016, Mukhin 2017) and the *global financial cycle* (Rey 2013). Our model also offers a simple general equilibrium framework for nesting the financial sector in an open economy environment. In particular, it summarizes the macroeconomic relationships in a few simple log-linear equations, which can be combined with richer models of the financial sector (as e.g. Evans and Lyons 2002). We see this as a particularly promising next step in exploring further the nature of the financial shocks and disciplining them with additional moments on the comovement between exchange rates and financial variables.

A Appendix

A.1 Additional Tables and Figures referenced in the text

Table A1: Quantitative properties of the baseline model and robustness

		Data	Baseline	Robustness				
				$\theta = 2.5$	$\alpha = 0$	$\gamma = .15$	$\rho = 0.9$	$\sigma = 1$
1.	$\rho(\Delta e)$	0.00	−0.02 (0.09)				−0.05	
2.	$\rho(q)$	0.95	0.93 (0.04)				0.87	
	$HL(q)$	12.0	9.9 (6.4)				4.9	
	$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.75			0.54		
3.	$\sigma(\Delta s)/\sigma(\Delta q)$	0.35	0.30		1.16	0.46		
	$\sigma(\Delta q^P)/\sigma(\Delta q)$	0.98	1.10		1.16	1.26		
4.	$\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q)$	0.20	0.31	0.42	0.42	0.81		0.48
	$\sigma(\Delta gdp - \Delta gdp^*)/\sigma(\Delta q)$	0.25	0.19	0.44		0.49		0.07
	$\sigma(\Delta nx)/\sigma(\Delta q)$	0.10	0.25			0.65		
5.	Fama β_F	$\lesssim 0$	−8.1 (4.7)					
	Fama R^2	0.02	0.04 (0.02)				0.07	
	$\sigma(i - i^*)/\sigma(\Delta e)$	0.06	0.03 (0.01)					
	Carry SR	0.20	0.21 (0.04)				0.29	

Notes: The baseline model corresponds to the model of Section 3 with parameters as in Table 1. The table reports robustness analysis with respect to deviations of various parameters from their baseline values. The moments are robust to changes in ν, ϕ, μ and β (not reported for brevity). The robustness panel of the table shows only the moments that are sensitive to the change in the parameter values. Data moments and notation as in Table 2, and $HL(\cdot)$ corresponds to the half-life estimate. The dynamic moments are calculated as the median of the in-sample estimates across 10,000 simulations with 30 years (120 quarters) each and the standard deviation across simulations are reported in brackets. The asymptotic values of the estimates are similar to the medians except for $\rho(q) \to 1$, $HL(q) \to \infty$ and $\beta_F \to -4.6$. The five sections in the table correspond to the five puzzles defined in Section 1 and addressed in Section 3.1–3.5 respectively. See Appendix A.5.2 for further discussion.

Table A2: Unconditional correlations between exchange rate and interest rates

	Data	Models			
		NOEM	IRBC	Financial	
		(1)	(2)	(3)	(4)
$\text{corr}(e_t, i_t - i_t^*)$	−0.09 (0.09)	0.40 (0.34)	0.25 (0.33)	0.95 (0.05)	−0.19 (0.28)
$\text{corr}(\Delta e_t, \Delta i_t - \Delta i_t^*)$	−0.18 (0.05)	0.38 (0.08)	−0.20 (0.09)	1.00 (0.00)	−0.12 (0.05)

Note: Models: (1) multi-shock NOEM (from column 5 of Table 2); (2) multi-shock IRBC (column 6 of Table 2); (3) single-ψ_t-shock model with a financial sector; (4) multi-shock model with a financial sector (column 7 of Table 2). See Appendix A.10. The numbers in the brackets give standard errors calculated as Newey-West standard errors with 12 lags (months) for the data moments and as standard deviations across 10,000 simulations for the model-simulated moments.

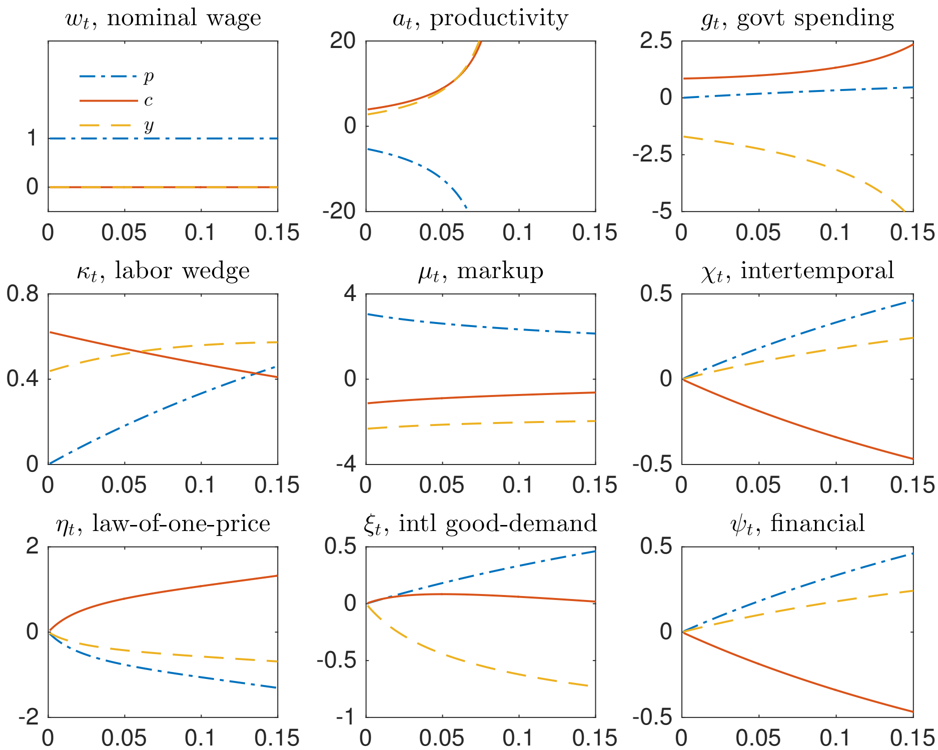

Figure A1: Relative impulse responses to shocks as a function of γ

Note: The figure plots $\frac{\mathrm{d}z_t}{\mathrm{d}e_t} \equiv \frac{\partial z_t/\partial \varepsilon_t}{\partial e_t/\partial \varepsilon_t}$ for three variables $z_t \in \{p_t - p_t^*, c_t - c_t^*, y_t - y_t^*\}$ (relative price level, relative consumption and relative output respectively) and shocks $\varepsilon_t \in \Omega_t = \{w_t, a_t, g_t, \kappa_t, \mu_t, \chi_t, \eta_t, \xi_t, \psi_t\}$ across models with different home bias parameter $\gamma \in [0, 0.15]$ and the other parameters as in Table 2. For three shocks (η_t, ξ_t, ψ_t), the impulse responses for all three x_t are negligible relative to e_t in the autarky limit ($\gamma \to 0$), and tend to monotonically depart away from zero with $\gamma > 0$. For the other five shocks $(w_t, a_t, g_t, \kappa_t, \mu_t)$, the impulse response for at least one z_t is of the same order of magnitude as that for e_t, even near $\gamma = 0$. The χ_t shock is equivalent to ψ_t shock in terms of its effect on prices and quantities, but they differ in their effect on interest rates (not shown). See discussion in Section 2.2 and Propositions 1–2.

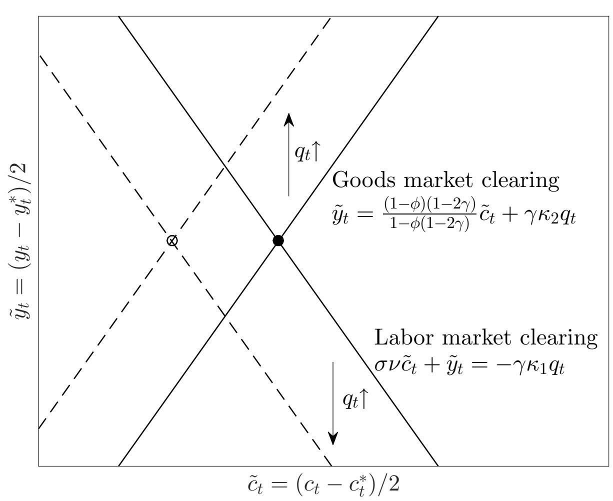

Figure A2: Illustration: Consumption-RER relationship

Note: Illustration for Section 3.4. A RER depreciation ($q_t \uparrow$) has two effects corresponding to equations (36) and (37) in the text. The latter is the expenditure switching towards domestic goods, which increases $\tilde{y}_t$ given $\tilde{c}_t$. The former is the reduction in the home real wage, which reduces the supply of labor and home goods $\tilde{y}_t$ given $\tilde{c}_t$. The joint equilibrium in the goods and labor market, hence, requires a reduction in $\tilde{c}_t$, with in general ambiguous effect on output $\tilde{y}_t$.

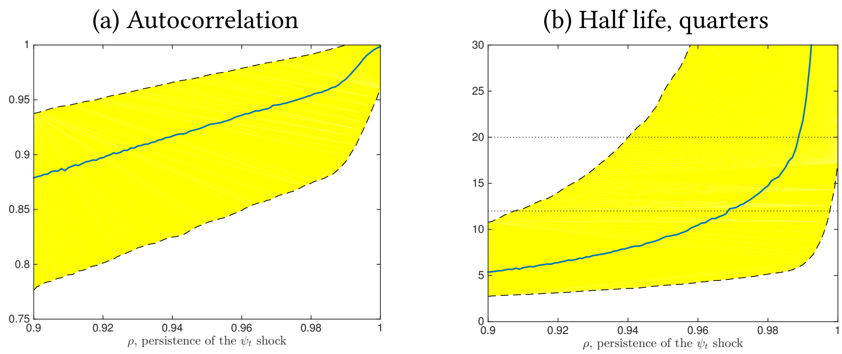

Figure A3: Persistence of the real exchange rate q_t

Note: Left panel: OLS estimates $\hat{\rho}_q$ from projection $q_t = \rho_q q_{t-1} + \epsilon_t^q$. Right panel: corresponding half-life estimates calculated according to $HL_q = \frac{\log 0.5}{\log \hat{\rho}_q}$. Based on 10,000 simulations with 120 quarters (30 years) each, where the solid lines plot the median estimates and the areas are the 90% bootstrap sets. The dotted lines in the right panel indicate the conventional 3–5 year half life estimates in the data (Rogoff 1996).

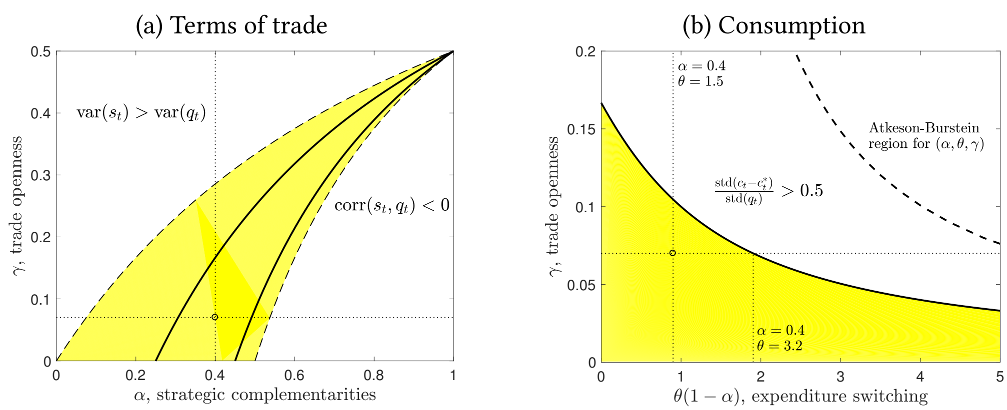

Figure A4: Volatility of prices and quantities relative to real exchange rate

Note: The figures plot the regions in the model parameter space resulting in the empirically-relevant volatilities of prices (terms of trade s_t, left panel) and quantities (relative consumption $c_t - c_t^*$, right panel) relative to the volatility of the real exchange rate q_t. The left panel is in the (α, γ)-space and the yellow region within dashed lines corresponds to the parameter combinations that result simultaneously in $\text{var}(\Delta s_t) < \text{var}(\Delta q_t)$ and $\text{corr}(\Delta s_t, \Delta q_t) > 0$, while within the solid lines $\frac{\text{std}(\Delta s_t)}{\text{std}(\Delta q_t)} < 0.5$ (refer to Proposition 6). The right panel is in the $(\theta(1-\alpha), \gamma)$-space with the yellow region under the solid line resulting in $\frac{\text{std}(\Delta c_t - \Delta c_t^*)}{\text{std}(\Delta q_t)} < 0.5$ (refer to Proposition 7). The region above the dashed line corresponds to the possible parameter combinations for oligopolistic competition under CES demand (as in Atkeson and Burstein 2008), which does not jointly allow for such θ and α that result in a quantitatively moderate response of consumption.

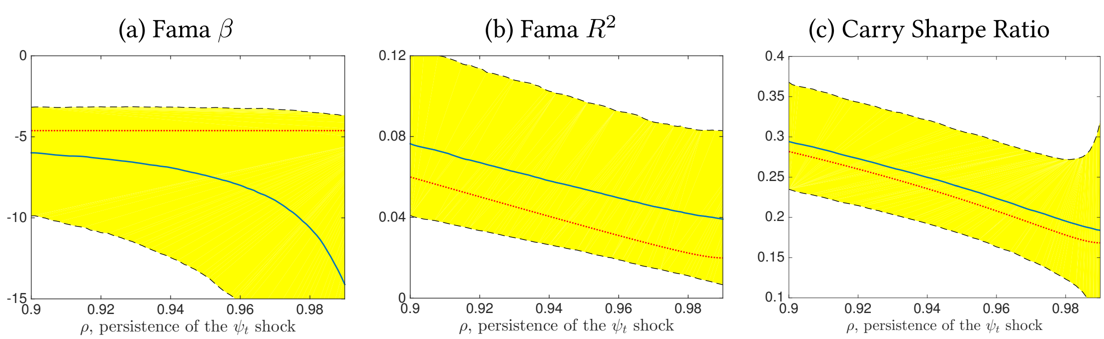

Figure A5: Fama regression, UIP deviations and Carry trade returns

Note: Monte Carlo study of the baseline model (with parameters from Table 1) based on 10,000 simulations of the model with 120 quarters (30 years). The solid lines plot the median estimates across simulations, the areas represent 90% bootstrap sets, and the red dotted lines are the asymptotic values. Panel (a) plots the β coefficient from the Fama regression of Δe_{t+1} on $(i_t - i_t^*)$, while panel (b) plots the R^2 from this regression. Panel (c) plots the unconditional within-sample Sharpe ratio calculated as the coefficient of variation for the carry return $r_{t+1}^C = \psi_t \cdot (i_t - i_t^* - \Delta e_{t+1})$, as defined in (A71).

A.2 Demand structure

Consider the general separable homothetic (Kimball 1995) demand aggregator C_t defined implicitly by:

$$\Omega_{Ht}\, g\left(\frac{C_{Ht}}{\Omega_{Ht}C_t}\right) + \Omega_{Ft}\, g\left(\frac{C_{Ft}}{\Omega_{Ft}C_t}\right) = 1, \tag{A1}$$

where $g' > 0$, $g'' < 0$ and $g(1) = g'(1) = 1$ (a normalization), and

$$\Omega_{Ht} \equiv (1-\gamma)e^{-\gamma\xi_t} \qquad \text{and} \qquad \Omega_{Ft} \equiv \gamma e^{(1-\gamma)\xi_t}$$

are the weights which satisfy the required properties. Then expenditure minimization results in the following demand schedules:

$$C_{Ht} = (1-\gamma)e^{-\gamma\xi_t} h\left(\frac{P_{Ht}}{D_t}\right) C_t \qquad \text{and} \qquad C_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{D_t}\right) C_t, \tag{A2}$$

where $h(x) \equiv g'^{-1}(x)$, which implies $h' < 0$ and $h(1) = 1$, and where D_t is the Lagrange multiplier on the aggregator (A1). We can solve for D_t by substituting demand (A2) into the aggregator (A1):

$$(1-\gamma)e^{-\gamma\xi_t}\, g\left(h\left(\frac{P_{Ht}}{D_t}\right)\right) + \gamma e^{(1-\gamma)\xi_t}\, g\left(h\left(\frac{P_{Ft}}{D_t}\right)\right) = 1, \tag{A3}$$

and also define the price index in this economy from the expenditure per unit of C_t:

$$P_t = \frac{P_{Ht}C_{Ht} + P_{Ft}C_{Ft}}{C_t} = (1-\gamma)e^{-\gamma\xi_t} P_{Ht} h\left(\frac{P_{Ht}}{D_t}\right) + \gamma e^{(1-\gamma)\xi_t} P_{Ft} h\left(\frac{P_{Ft}}{D_t}\right). \tag{A4}$$

Proof of Lemma 1 It is immediate to check from (A3)–(A4), using $g(1) = h(1) = 1$, that when $\xi_t = 0$ and $P_{Ht} = P_{Ft}$, we have $P_t = D_t = P_{Ht} = P_{Ft}$, which corresponds to the symmetric steady state. Using demand (A2), we immediately have that the symmetric steady state foreign share is:

$$\left.\frac{P_{Ft}C_{Ft}}{P_{Ht}C_{Ht} + P_{Ft}C_{Ft}}\right|_{\xi_t=0,\ P_{Ht}=P_{Ft}} = \frac{\gamma C_t}{(1-\gamma)C_t + \gamma C_t} = \gamma,$$

where we again use $h(1) = 1$. We next use (A3)–(A4) to obtain the log-linear approximation for P_t and D_t around the symmetric steady state:[50]

$$p_t = d_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}, \tag{A5}$$

where p_t and d_t are the log deviations from the steady state values. Note that the taste shock $\xi_t \neq 0$ does not affect the first order approximation to the prices index (due to the way it enters the weights Ω_{Ht} and Ω_{Ft}). Finally, log-linearizing (A2), we have:

$$c_{Ht} = -\gamma\xi_t - \theta(p_{Ht} - d_t) + c_t \qquad \text{and} \qquad c_{Ft} = (1-\gamma)\xi_t - \theta(p_{Ft} - d_t) + c_t,$$

[50] In the CES case, which obtains with $g(z) = \frac{1}{\theta-1}\big(\theta z^{\frac{\theta-1}{\theta}} - 1\big)$, we have $P_t = D_t$, while for a more general demand P_t and D_t different by a second order term around a symmetric steady state. Since our analysis relies on the first order approximation to the equilibrium system, we replace D_t with P_t in the demand equations (5) in the text.

where $\theta \equiv -\frac{h'(x)x}{h(x)}\big|_{x=1} = -\frac{\partial \log h(x)}{\partial x}\big|_{x=1}$. Together with (A5), these expressions results in (7). Subtracting, we have $c_{Ft} - c_{Ht} = \xi_t - \theta(p_{Ft} - p_{Ht})$, which implies that the elasticity of substitution is indeed θ. ■

Monopolistic competition and price setting Consider now a unit continuum of symmetric domestic firms with marginal cost MC_t and a unit continuum of symmetric foreign firms with marginal cost $\mathcal{E}_t MC_t^*$ monopolistically competing in the domestic market. We generalize the consumption aggregator C_t to be defined in the following way:

$$\int_0^1 \Omega_{Ht}\, g\left(\frac{C_{Ht}(i)}{\Omega_{Ht}C_t}\right) \mathrm{d}i + \int_0^1 \Omega_{Ft}\, g\left(\frac{C_{Ft}(i)}{\Omega_{Ft}C_t}\right) \mathrm{d}i = 1, \tag{A6}$$

with taste shocks $(\Omega_{Ht}, \Omega_{Ft})$ determined as above by a common home bias parameter γ and a common demand shifter ξ_t for all varieties $i \in [0,1]$. The households choose $\{C_{Ht}(i), C_{Ft}(i)\}$ to maximize C_t given prices and total expenditure:

$$E_t = P_t C_t = \int_0^1 P_{Ht}(i) C_{Ht}(i) \mathrm{d}i + \int_0^1 P_{Ft}(i) C_{Ft}(i) \mathrm{d}i. \tag{A7}$$

This expenditure minimization results in individual firm demand as in (A2). A representative home firm takes (C_t, P_t, D_t) as given and sets its price to maximize profits from serving the domestic market:

$$P_{Ht}(i) = \arg\max_{P_{Ht}(i)} \left\{ (P_{Ht}(i) - MC_t)(1-\gamma)e^{-\gamma\xi_t} h\left(\frac{P_{Ht}(i)}{D_t}\right) C_t \right\},$$

which results in the standard markup pricing rule, with the markup $\mathcal{M}_t$ determined by the elasticity of the demand curve $h(\cdot)$. Since all domestic firms are symmetric, we have $C_{Ht} = C_{Ht}(i)$ and $P_{Ht} = P_{Ht}(i)$ for all $i \in [0,1]$. Similar price setting rule is used by symmetric foreign firms with marginal costs $\mathcal{E}_t MC_t^*$, and we also have $C_{Ft} = C_{Ft}(i)$ and $P_{Ft} = P_{Ft}(i)$ for all $i \in [0,1]$. Following the proof of Lemma 1, the elasticity of demand in a symmetric steady state equals θ, and therefore the steady state markup is given by $\mathcal{M} = \theta/(\theta-1)$ for both home and foreign firms.

We next take a log-linear approximation to the optimal price P_{Ht} around the symmetric steady state:

$$p_{Ht} = -\Gamma(p_{Ht} - p_t) + mc_t,$$

where we use the approximation $d_t = p_t$ and Γ denotes the elasticity of the markup $\mathcal{M}_t$ with respect to the relative price of the firm, evaluated at the symmetric steady state. Note that this equation is the counterpart to (12) in the text with $\alpha \equiv \frac{\Gamma}{1+\Gamma}$ and $\mu_t = 0$.

Lastly, we provide further details about the primitive determinants of θ and α (see Amiti, Itskhoki, and Konings 2016, for a more indepth exposition). Define the demand elasticity function $\tilde{\theta}(x) \equiv -\frac{\partial \log h(x)}{\partial \log x}$, so that $\theta \equiv \tilde{\theta}(1)$. Then the markup function is given by $\tilde{\mathcal{M}}(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x)-1}$, and the elasticity of the markup is given by $\tilde{\Gamma}(x) \equiv -\frac{\partial \log \tilde{\mathcal{M}}(x)}{\partial x}$, with $\mathcal{M} = \tilde{\mathcal{M}}(1)$ and $\Gamma = \tilde{\Gamma}(1)$. Manipulating these definitions, we can represent

$$\tilde{\Gamma}(x) = \frac{\tilde{\epsilon}(x)}{\tilde{\theta}(x) - 1}, \qquad \text{where} \qquad \tilde{\varepsilon}(x) \equiv \frac{\partial \log \tilde{\theta}(x)}{\partial \log x}$$

is the elasticity of elasticity (or super-elasticity) of demand. Therefore, $\Gamma = \frac{\epsilon}{\theta - 1}$, where $\epsilon = \tilde{\epsilon}(1)$, and we further have:

$$\alpha = \frac{\Gamma}{1 + \Gamma} = \frac{\epsilon}{\epsilon + \theta - 1}.$$

To the extent ϵ and θ are controlled by independent parameters, we can decouple the elasticity of substitution θ from the strategic complementarity elasticity α. Indeed, θ is a characteristic of the slope (the first derivative) of demand h', while ϵ is a characteristic of the curvature (the second derivative) of demand h''. Formally, we have:

$$\theta = -\frac{h'(x)x}{h(x)}\Big|_{x=1} \qquad \text{and} \qquad \epsilon = \frac{\partial \log \tilde{\theta}(x)}{\partial \log x} = \left[1 - \frac{h'(x)x}{h(x)} + \frac{h''(x)x}{h'(x)}\right]\Bigg|_{x=1} = 1 + \theta + \frac{h''(x)x}{h'(x)}\Big|_{x=1}.$$

We assume that the demand schedule $h(\cdot)$ is log-concave, that is $\epsilon \geq 0$, and therefore $\alpha \in [0, 1)$, since $\theta > 1$ is the second order requirement for price setting optimality. An appropriate choice of ϵ produces any required value of α for any given value of θ. A suitable parametric example can be found in Klenow and Willis (2006) and Gopinath and Itskhoki (2010), where $h(x) = [1 - \epsilon \log(x)]^{\theta/\epsilon}$ for some elasticity parameter $\theta > 1$ and super-elasticity parameter $\epsilon > 0$.

A.3 Equilibrium system

We summarize here the equilibrium system of the general model from Section 2.1 by breaking it into blocks:

1. Labor supply (3) and its exact foreign counterpart.

2. Labor demand in (10), used together with the definition of the marginal cost (9), and its exact foreign counterpart.[51]

3. Demand for home and foreign goods:

$$Y_t = Y_{Ht} + Y^*_{Ht} \qquad \text{and} \qquad Y^*_t = Y_{Ft} + Y^*_{Ft}, \tag{A8}$$

where the sources of demand for home good are given in (17) and (18), and the counterpart sources of demand for foreign good are given by:

$$Y_{Ft} = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}}{P_t}\right)[C_t + X_t + e^{g_t}], \tag{A9}$$

$$Y^*_{Ft} = (1-\gamma) e^{-\gamma \xi^*_t} h\left(\frac{P^*_{Ft}}{P^*_t}\right)\left[C^*_t + X^*_t + e^{g^*_t}\right], \tag{A10}$$

where X_t and X^*_t satisfy the intermediate good demand in (10) and its foreign counterpart.

[51]Note that the input demand equations (10) together with the marginal cost (9) imply the production function equation (8).

4. Supply of goods: given price setting (12)–(13) and their foreign counterparts given by:

$$P_{Ft} = e^{\mu_t^* + \eta_t^*} (MC_t^* \mathcal{E}_t)^{1-\alpha} P_t^{\alpha}, \tag{A11}$$

$$P_{Ft}^* = e^{\mu_t^*} MC_t^{*1-\alpha} P_t^{*\alpha}, \tag{A12}$$

output produced is determined by the demand equation (A8).

Given prices $(P_{Ht}, P_{Ht}^*, P_{Ft}, P_{Ft}^*)$, equation (6) defines the price level P_t as a log-linear approximation, and a similar equation defines P_t^*.[52]

5. Asset demand by home and foreign households (4) and (15), which can be rewritten as an international risk sharing condition and a no-arbitrage condition:

$$\mathbb{E}_t \left\{ e^{\psi_t} \Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} - \Theta_{t+1}^* \right\} = 0, \tag{A13}$$

$$\mathbb{E}_t \left\{ \Theta_{t+1} \left[e^{\psi_t} R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} - R_t \right] \right\} = 0, \tag{A14}$$

with the stochastic discount factors Θ_{t+1} and Θ_{t+1}^* defined in the text.

6. Home-country flow budget constraint (19), with its foreign counterpart redundant by Walras Law.

A.3.1 Symmetric steady state

In a symmetric steady state, $B^* = B^{*F} = 0$, and the shocks (defined in Table 1) take the following values:

$$\psi = \xi = \xi^* = \eta = \eta^* = \chi = \chi^* = 0,$$

and we normalize $W = W^* = 1$ (corresponding to $w = w^* = 0$). We let the remaining shocks take arbitrary (zero or non-zero) symmetric values:

$$a = a^*, \qquad g = g^*, \qquad \kappa = \kappa^* \qquad \text{and} \qquad \mu = \mu^*.$$

We start with the equations for prices. In a symmetric steady state, exchange rates and terms of trade are equal to 1:

$$\mathcal{E} = \mathcal{Q} = \mathcal{S} = 1, \tag{A15}$$

and therefore we can evaluate the prices using the equilibrium conditions described above:

$$P = P^* = P_H = P_F^* = P_H^* = P_F = \left[\frac{e^{\frac{\mu}{1-\alpha} - a}}{(1-\phi)^{1-\phi} \phi^{\phi}} \right]^{\frac{1}{1-\phi}}, \tag{A16}$$

[52] Log-linear expression for p_t in (6) can be replaced with two non-linear expressions (A3)–(A4) defining (P_t, D_t), and P_t should be replaced with D_t in demand equations (17)–(18) and (A9)–(A10). The rest of the equilibrium system stays unchanged. However, these adjustments do not have first order consequences, as P_t and D_t are the same up to second order terms, and therefore the log-linearized system in Appendix A.3.2 is unchanged.

with the marginal costs given by $MC = MC^* = \frac{e^{-a}P\phi}{(1-\phi)^{1-\phi}\phi^{\phi}} = \left[\frac{e^{\frac{\phi\mu}{1-\alpha}-a}}{(1-\phi)^{1-\phi}\phi^{\phi}}\right]^{\frac{1}{1-\phi}}$.

Next we use these expressions together with production function, labor demand and labor supply to obtain two relationships for (C, Y, L):

$$L = e^{\frac{\phi}{1-\phi}\frac{\mu}{1-\alpha}-\frac{a}{1-\phi}}\phi^{-\frac{\phi}{1-\phi}}Y, \tag{A17}$$

$$C^{\sigma}L^{1/\nu} = \frac{e^{-\kappa}}{P} = e^{\frac{a}{1-\phi}-\frac{\mu}{(1-\alpha)(1-\phi)}-\kappa}(1-\phi)\phi^{\frac{\phi}{1-\phi}}. \tag{A18}$$

Substituting prices (and using $h(1) = 1$) and intermediate good demand $X = \phi\frac{MC}{P}Y = e^{\frac{-\mu}{1-\alpha}}\phi Y$ into the goods market clearing, we obtain an additional relationship between C and Y:

$$C + e^{g} = \left[1 - e^{-\frac{\mu}{1-\alpha}}\phi\right]Y. \tag{A19}$$

We further have $Y = Y^*$, and $Y_H = Y_F^* = (1-\gamma)Y$ and $Y_H^* = Y_F = \gamma Y$.

The asset demand conditions imply that $R = R^* = 1/\beta$.

Lastly, we define the following useful ratios:

$$\zeta \equiv \frac{\text{GDP}}{\text{Output}} = \frac{P(C+G)}{P_H Y} = 1 - e^{-\frac{\mu}{1-\alpha}}\phi, \tag{A20}$$

$$\gamma \equiv \frac{\text{Import}}{\text{Expenditure}} = \frac{P_F Y_F}{P_H Y_H + P_F Y_F} = \frac{P_F Y_F}{P_H Y} = \gamma, \tag{A21}$$

$$\frac{\text{Import+Export}}{\text{GDP}} = \frac{\mathcal{E}P_H^* Y_H^* + P_F Y_F}{P(C+G)} = \frac{2\gamma}{\zeta}. \tag{A22}$$

A.3.2 Log-linearized system

We log-linearize the equilibrium system (summarized above in Appendix A.3) around the symmetric steady state (described in Appendix A.3.1). We split the equilibrium system into three blocks — prices, quantities and dynamic equations — and solve them sequentially, as the equilibrium system is block-recursive.

Exchange rates and prices The price block contains the definitions of the price index (6) and its foreign counterpart:

$$p_t^* = \gamma p_{Ht}^* + (1-\gamma)p_{Ft}^*, \tag{A23}$$

as well as the price setting equations (12)–(13) and (A11)–(A12), in which we substitute the marginal cost (9) and its foreign counterpart and log-linearize to obtain:

$$p_{Ht} = \mu_t - (1-\alpha)a_t + (1-\alpha)(1-\phi)(w_t - p_t) + p_t, \tag{A24}$$

$$p_{Ht}^* = \mu_t + \eta_t - (1-\alpha)a_t + (1-\alpha)[(1-\phi)(w_t - p_t) + p_t - e_t] + \alpha p_t^*, \tag{A25}$$

$$p_{Ft}^* = \mu_t^* - (1-\alpha)a_t^* + (1-\alpha)(1-\phi)(w_t^* - p_t^*) + p_t^*, \tag{A26}$$

$$p_{Ft} = \mu_t^* + \eta_t^* - (1-\alpha)a_t^* + (1-\alpha)[(1-\phi)(w_t^* - p_t^*) + p_t^* + e_t] + \alpha p_t. \tag{A27}$$

In addition, we use the logs of the definitions of the real exchange rate and terms of trade (14) and (20):

$$q_t = p_t^* + e_t - p_t, \tag{A28}$$
$$s_t = p_{Ft} - p_{Ht}^* - e_t. \tag{A29}$$

First, it is useful to define the log LOP deviations (as in equation (14) and in its foreign counterpart):

$$q_{Ht} \equiv p_{Ht}^* + e_t - p_{Ht} = \eta_t + \alpha q_t, \tag{A30}$$
$$q_{Ft} \equiv p_{Ft}^* + e_t - p_{Ft} = -\eta_t^* + \alpha q_t, \tag{A31}$$

where the expression on the right-hand side are obtained by using (A24)–(A27) together with (A28). Then, we combine (A28)–(A29) together with these expressions, to obtain:

$$s_t = q_t^P - 2\tilde{\eta}_t - 2\alpha q_t, \tag{A32}$$
$$q_t = (1-\gamma)q_t^P - \gamma s_t, \tag{A33}$$

where $q_t^P = p_{Ft}^* + e_t - p_{Ht}$ is the producer-price-based real exchange rate and we use the tilde notation $\tilde{x}_t \equiv (x_t - x_t^*)/2$ for any pair of variables (x_t, x_t^*). Lastly, we solve for q_t^P and s_t as function of q_t:

$$q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma} q_t - \frac{2\gamma}{1-2\gamma}\tilde{\eta}_t, \tag{A34}$$
$$s_t = \frac{1-2\alpha(1-\gamma)}{1-2\gamma} q_t - \frac{2(1-\gamma)}{1-2\gamma}\tilde{\eta}_t. \tag{A35}$$

Next, we use these solutions together with the expressions for price indexes (6) and (A23), to solve for:[53]

$$p_{Ht} - p_t = -\frac{\gamma}{1-\gamma}(p_{Ft} - p_t) = \gamma(p_{Ht} - p_{Ft}) = -\frac{(1-\alpha)\gamma}{1-2\gamma} q_t + \frac{\gamma^2\eta_t - \gamma(1-\gamma)\eta_t^*}{1-2\gamma}, \tag{A36}$$
$$p_{Ft}^* - p_t^* = -\frac{\gamma}{1-\gamma}(p_{Ht}^* - p_t^*) = \gamma(p_{Ft}^* - p_{Ht}^*) = \frac{(1-\alpha)\gamma}{1-2\gamma} q_t + \frac{\gamma^2\eta_t^* - \gamma(1-\gamma)\eta_t}{1-2\gamma}. \tag{A37}$$

Combining these expression with (A24) and (A26), we can solve for the price levels:

$$p_t = w_t + \frac{1}{1-\phi}\left[\frac{\mu_t - \frac{\gamma^2\eta_t - \gamma(1-\gamma)\eta_t^*}{1-2\gamma}}{1-\alpha} - a_t + \frac{\gamma}{1-2\gamma} q_t\right], \tag{A38}$$
$$p_t^* = w_t^* + \frac{1}{1-\phi}\left[\frac{\mu_t^* - \frac{\gamma^2\eta_t^* - \gamma(1-\gamma)\eta_t}{1-2\gamma}}{1-\alpha} - a_t^* - \frac{\gamma}{1-2\gamma} q_t\right], \tag{A39}$$

[53] Note from (6) that $p_{Ht} - p_t = \gamma(p_{Ht} - p_{Ft})$, and we use the following steps to solve for:

$$p_{Ht} - p_{Ft} = -(p_{Ft} - p_{Ht}^* - e_t) - (p_{Ht}^* + e_t - p_{Ht}) = -(s_t + q_{Ht}) = -(s_t + \alpha q_t + \eta_t),$$

in which we then substitute (A35) to solve out s_t. Similarly, we solve for $p_{Ft}^* - p_t^*$.

which together allow to solve for the relationship between q_t and nominal exchange rate e_t:

$$\left[(1-\phi)+\frac{2\gamma}{1-2\gamma}\right]q_t=(1-\phi)e_t-(1-\phi)2\tilde{w}_t+2\tilde{a}_t-\frac{2\tilde{\mu}_t}{1-\alpha}+\frac{2\gamma}{1-2\gamma}\frac{\tilde{\eta}_t}{1-\alpha}. \tag{A40}$$

Real exchange rate and quantities The supply side is the combination of labor supply (3) and labor demand (10) (together with marginal cost (9)), which we log-linearize as:

$$\kappa_t+\sigma c_t+\tfrac{1}{\nu}\ell_t=w_t-p_t, \tag{A41}$$
$$\ell_t=-a_t-\phi(w_t-p_t)+y_t. \tag{A42}$$

Combining the two to solve out ℓ_t, and using (A38) to solve out (w_t-p_t), we obtain:[54]

$$\nu\sigma c_t+y_t=\frac{1+\nu}{1-\phi}a_t-\frac{\nu+\phi}{1-\phi}\left[\frac{\mu_t-\frac{\gamma^2\eta_t-\gamma(1-\gamma)\eta_t^*}{1-2\gamma}}{1-\alpha}+\frac{\gamma}{1-2\gamma}q_t\right]-\nu\kappa_t. \tag{A43}$$

Symmetrically, the same expression for foreign is:

$$\nu\sigma c_t^*+y_t^*=\frac{1+\nu}{1-\phi}a_t^*-\frac{\nu+\phi}{1-\phi}\left[\frac{\mu_t^*-\frac{\gamma^2\eta_t^*-\gamma(1-\gamma)\eta_t}{1-2\gamma}}{1-\alpha}-\frac{\gamma}{1-2\gamma}q_t\right]-\nu\kappa_t^*.$$

Adding and subtracting the two we obtain:

$$\nu\sigma\bar{c}_t+\bar{y}_t=\frac{1+\nu}{1-\phi}\bar{a}_t-\frac{\nu+\phi}{1-\phi}\frac{\bar{\mu}_t+\gamma\bar{\eta}_t}{1-\alpha}-\nu\bar{\kappa}_t, \tag{A44}$$
$$\nu\sigma\tilde{c}_t+\tilde{y}_t=\frac{1+\nu}{1-\phi}\tilde{a}_t-\frac{\nu+\phi}{1-\phi}\left[\frac{\tilde{\mu}_t-\frac{\gamma}{1-2\gamma}\tilde{\eta}_t}{1-\alpha}+\frac{\gamma}{1-2\gamma}q_t\right]-\nu\tilde{\kappa}_t, \tag{A45}$$

where $\bar{x}_t\equiv(x_t+x_t^*)/2$ and $\tilde{x}_t\equiv(x_t-x_t^*)/2$ for any pair of variables (x_t,x_t^*).

The demand side is the goods market clearing (A8) together with (17)–(18), which we log-linearize as:

$$\begin{aligned}
y_t&=(1-\gamma)y_{Ht}+\gamma y_{Ht}^*,\\
y_{Ht}&=-\gamma\xi_t-\theta(p_{Ht}-p_t)+\zeta[\varsigma c_t+(1-\varsigma)g_t]+(1-\zeta)\big[(1-\phi)(w_t-p_t)-a_t+y_t\big],\\
y_{Ht}^*&=(1-\gamma)\xi_t^*-\theta(p_{Ht}^*-p_t^*)+\zeta[\varsigma c_t^*+(1-\varsigma)g_t^*]+(1-\zeta)\big[(1-\phi)(w_t^*-p_t^*)-a_t^*+y_t^*\big],
\end{aligned}$$

where $\varsigma\equiv C/(C+G)$, $\zeta\equiv P(C+G)/(P_HY)$, and we used expression (10) and (9) to substitute for X_t (and correspondingly for X_t^*). Combining together, we derive:

$$y_t-(1-\zeta)[y_t-2\gamma\tilde{y}_t^*]-\zeta\varsigma[c_t-2\gamma\tilde{c}_t]=\gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma}-(1-\zeta)\right]q_t \tag{A46}$$
$$+\zeta(1-\varsigma)[g_t-2\gamma\tilde{g}_t]-\frac{1-\zeta}{1-\alpha}[\mu_t-2\gamma\tilde{\mu}_t]+\frac{\theta\gamma(1-\gamma)}{1-2\gamma}\eta_t+\left(\frac{\theta\gamma(1-\gamma)}{1-2\gamma}-\frac{1-\zeta}{1-\alpha}\gamma\right)\eta_t^*-2\gamma(1-\gamma)\tilde{\xi}_t,$$

where we have slowed out (w_t-p_t) and $(w_t^*-p_t^*)$ using (A38)–(A39) and solved out $(p_{Ht}-p_t)$ and

[54] A useful interim step is: $\nu\sigma c_t+y_t=(\nu+\phi)(w_t-p_t)+a_t-\nu\kappa_t$.

$(p^*_{Ht} - p^*_t)$ using (A36)–(A37). Adding and subtracting the foreign counterpart, we obtain:[55]

$$\bar{y}_t = \varsigma\bar{c}_t + (1-\varsigma)\bar{g}_t - \frac{1}{\zeta}\frac{1-\zeta}{1-\alpha}\bar{\mu}_t + \frac{1}{\zeta}\left(\frac{2\theta\gamma(1-\gamma)}{1-2\gamma} - \frac{1-\zeta}{1-\alpha}\gamma\right)\bar{\eta}_t, \tag{A47}$$

$$\begin{aligned}[1-(1-2\gamma)(1-\zeta)]\tilde{y}_t = (1-2\gamma)\zeta\varsigma\tilde{c}_t + (1-2\gamma)\left[\zeta(1-\varsigma)\tilde{g}_t - \frac{1-\zeta}{1-\alpha}\tilde{\mu}_t\right] \\ + \gamma\frac{1-\zeta}{1-\alpha}\tilde{\eta}_t - 2\gamma(1-\gamma)\tilde{\xi}_t + \gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} - (1-\zeta)\right]q_t.\end{aligned} \tag{A48}$$

An immediate implication of (A44) and (A47) is that $(\bar{y}_t, \bar{c}_t)$ depends only on $(\bar{a}_t, \bar{g}_t, \bar{\kappa}_t, \bar{\mu}_t, \bar{\nu}_t)$ and does not depend on the real exchange rate q_t. In particular, if $\bar{a}_t = \bar{g}_t = \bar{\kappa}_t = \bar{\mu}_t = \bar{\nu}_t = 0$, then $\bar{y}_t = \bar{c}_t = 0$. This is the case we focus on throughout the paper, since as we see below the variation in $(\bar{a}_t, \bar{g}_t, \bar{\kappa}_t, \bar{\mu}_t, \bar{\nu}_t)$ does not affect q_t. Combining (A45) and (A48) we can solve for $\tilde{y}_t$ and $\tilde{c}_t$. For example, the expression for $\tilde{c}_t$ is:

$$\begin{aligned}\Big[(1-2\gamma)\zeta(\nu\sigma+\varsigma) + 2\gamma\nu\sigma\Big]\tilde{c}_t = [(1-2\gamma)\zeta + 2\gamma]\left[\frac{1+\nu}{1-\phi}\tilde{a}_t - \frac{\nu+\phi}{1-\phi}\frac{\tilde{\mu}_t}{1-\alpha} - \nu\tilde{\kappa}_t\right] - (1-2\gamma)\zeta(1-\varsigma)\tilde{g}_t \\ + (1-2\gamma)\frac{1-\zeta}{1-\alpha}\tilde{\mu}_t - \gamma\left[\frac{1-\zeta}{1-\alpha} - \frac{\nu+\phi}{1-\phi}\frac{1-(1-\zeta)(1-2\gamma)}{1-\alpha}\right]\tilde{\eta}_t + 2\gamma(1-\gamma)\tilde{\xi}_t \\ - \gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} + \frac{\nu+\phi}{1-\phi}\frac{1}{1-2\gamma} - \frac{1+\nu}{1-\phi}(1-\zeta)\right]q_t.\end{aligned} \tag{A49}$$

Lastly, we provide the linearized expression for net exports:

$$nx_t = \gamma\Big(y^*_{Ht} - y_{Ft} - s_t\Big),$$

where $nx_t = \frac{1}{P_H Y}NX_t$ is linear deviation of net exports from steady state $NX = 0$ relative to the total value of output. Substituting in the expressions for s_t, y^*_{Ht} and y_{Ft}, we obtain:

$$\begin{aligned}nx_t = \gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} + \alpha - \frac{1-\alpha}{1-2\gamma} + \frac{2\gamma(1-\zeta)}{1-2\gamma}\right]q_t - 2\gamma\Big[\zeta[\varsigma\tilde{c}_t + (1-\varsigma)\tilde{g}_t] + (1-\zeta)\tilde{y}_t\Big] \\ + 2\gamma\left[\frac{1-\zeta}{1-\alpha}\tilde{\mu}_t - (1-\gamma)\tilde{\xi}_t\right] - 2\gamma\left[\theta(1-\gamma) + \frac{1-\gamma}{1-2\gamma} - \frac{1-\zeta}{1-\alpha}\right]\tilde{\eta}_t.\end{aligned} \tag{A50}$$

The log-linear approximation to the flow budget constraint (19) is given then by:

$$\beta b^*_{t+1} - b^*_t = nx_t, \tag{A51}$$

where $b^*_t = \frac{\mathcal{E}}{P_H Y}B^*_t$ is the linear deviation of the net foreign asset (NFA) position from its steady state value of $B^* = 0$ relative to the total value of output (both in foreign currency, using steady state exchange rate of $\mathcal{E} = 1$). Note that the dynamics of $\mathcal{E}_t$ and R^*_t has only second order effects on

[55]The foreign counterpart is obtained from combining together and rearranging:

$$\begin{aligned} y^*_t &= (1-\gamma)y^*_{Ft} + \gamma y_{Ft}, \\ y^*_{Ft} &= -\gamma\xi^*_t - \theta(p^*_{Ft} - p^*_t) + \zeta[\varsigma c^*_t + (1-\varsigma)g^*_t] + (1-\zeta)\big[(1-\phi)(w^*_t - p^*_t) - a^*_t + y^*_t\big], \\ y_{Ft} &= (1-\gamma)\xi_t - \theta(p_{Ft} - p_t) + \zeta[\varsigma c_t + (1-\varsigma)g_t] + (1-\zeta)\big[(1-\phi)(w_t - p_t) - a_t + y_t\big].\end{aligned}$$

the returns on NFA (and hence drops out from the linearized system), as we approximate around a symmetric steady state with zero NFA position. Equations (A51) is part of the dynamic block.

Exchange rate and interest rates It only remains now to log-linearize the asset demand conditions (4) and (15), which pins down the equilibrium interest rates, as well as provides an international risk sharing condition:

$$\begin{aligned} i_t &= \mathbb{E}_t \left\{ \sigma \Delta c_{t+1} + \Delta p_{t+1} - \Delta \chi_{t+1} \right\}, \\ i_t^* &= \mathbb{E}_t \left\{ \sigma \Delta c_{t+1} + \Delta p_{t+1} - \Delta e_{t+1} - \Delta \chi_{t+1} \right\} - \psi_t, \\ i_t^* &= \mathbb{E}_t \left\{ \sigma \Delta c_{t+1}^* + \Delta p_{t+1}^* - \Delta \chi_{t+1}^* \right\}, \end{aligned}$$

where $i_t \equiv \log R_t - \log R$ and $i_t^* \equiv \log R_t^* - \log R^*$. We combine the first two to obtain a no-arbitrage (UIP) condition, the last two to obtain a risk-sharing (Backus-Smith) condition, and the first with the third to solve for the interest rate differential:

$$i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \psi_t, \tag{A52}$$

$$\mathbb{E}_t \left\{ \sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} \right\} = \psi_t + \mathbb{E}_t \left\{ 2\Delta \tilde{\chi}_{t+1} \right\}, \tag{A53}$$

$$\tilde{i}_t \equiv \tfrac{1}{2}(i_t - i_t^*) = \mathbb{E}_t \left\{ \sigma \Delta \tilde{c}_{t+1} + \Delta \tilde{p}_{t+1} - \Delta \tilde{\chi}_{t+1} \right\}. \tag{A54}$$

Substituting out $\Delta c_{t+1} - \Delta c_{t+1}^* = 2\Delta \tilde{c}_{t+1}$ in (A53) using (A49), we obtain an equation characterizing the expected real depreciation $\mathbb{E}_t \Delta q_{t+1}$ as a function of exogenous shocks. Together with (A51), in which we substitute (A50), it forms a system of two dynamic equations that describe the equilibrium dynamics of the real exchange rate given the exogenous dynamic processes for the shocks.

A.4 Autarky Limit and Proofs for Section 2.2

Proof of Propositions 1 The strategy of the proof is to evaluate the log deviations of the macro variables $z_t \equiv (w_t, p_t, c_t, \ell_t, y_t, i_t)$ from the deterministic steady state (described in Appendix A.3.1) in response to a shock $\varepsilon_t = \mathbf{V}'\Omega_t \neq 0$.[56] In particular, we explore under which circumstances $\lim_{\gamma \to 0} z_t = 0$. It is sufficient to consider the log-linearized equilibrium conditions described in Appendix A.3.2, as providing a counterexample is sufficient for the prove (hence, the focus on the small log deviations is without loss of generality). Furthermore, the proof does not rely on the international risk sharing conditions, and hence does not depend on the assumptions about the (in)completeness of the international asset markets.

To prove the propositions, consider any shock ε_t with the restriction that

$$\eta_t = \eta_t^* = \xi_t = \xi_t^* = \psi_t \equiv 0. \tag{A55}$$

We now go through the list of requirements imposed by the first part of the condition (21):

[56] We do not impose any restrictions on the process for shocks in Ω_t, with the exception of the mild requirement that any innovation in Ω_t has some contemporaneous effect on the value of shocks in Ω_t, i.e. we rule out pure news shocks. We discuss examples with specific time series processes for the shocks in the end of this subsection.

1. No wage response $\lim_{\gamma\to 0} w_t = 0$ implies $w_t = 0$, i.e. the unit of account shocks cannot lead to the exchange rate disconnect in the limit.

2. No price level response implies, using (A38) and (A55):

$$\lim_{\gamma\to 0} p_t = w_t + \frac{1}{1-\phi}\left[\frac{\mu_t}{1-\alpha} - a_t\right] = 0,$$

   which in light of $w_t = 0$ requires $\mu_t = (1-\alpha)a_t$, i.e. the markup shocks must offset the productivity shocks to avoid variation in the price level.

   When the same requirements are imposed for foreign, it ensures $\lim_{\gamma\to 0}\{q_t - e_t\} = 0$, as immediartely follows from the the definition of the real exchange rate $q_t = p_t^* + e_t - p_t$ (see also (A40)).

3. From the labor supply and labor demand conditions (A41)–(A42), no consumption, employment and output response require:

$$\lim_{\gamma\to 0}\left\{\sigma c_t + \tfrac{1}{\nu}\ell_t + p_t\right\} = w_t - \kappa_t = 0,$$
$$\lim_{\gamma\to 0}\left\{y_t - \ell_t + \phi p_t\right\} = a_t + \phi w_t = 0,$$

   which then implies $a_t = \kappa_t = w_t \equiv 0$ and by consequence $\mu_t \equiv 0$ from the result above. That is, there cannot be productivity, markup or labor wedge shocks, if the price level, consumption, output and employment are not to respond in the autarky limit.

4. Rearranging the goods market clearing in the home market (A46), we have:

$$\lim_{\gamma\to 0}\left\{\zeta y_t - \zeta\varsigma c_t\right\} = \zeta(1-\varsigma)g_t - \frac{1-\zeta}{1-\alpha}\mu_t = 0,$$

   which in light of the above results requires $g_t \equiv 0$.

5. Lastly, the home bond demand requires:

$$\lim_{\gamma\to 0}\left\{\sigma\Delta\mathbb{E}_t c_{t+1} + \Delta\mathbb{E}_t p_{t+1} - i_t\right\} = \mathbb{E}_t\Delta\chi_{t+1} = 0,$$

   therefore there cannot be predictable changes in χ_t and unpredictable changes in χ_t do not affect allocations in a one-period bond economies, hence without loss of generality we impose $\chi_t \equiv 0$.

To summarize, the first condition in (21) (combined with the absence of η_t, ξ_t and ψ_t shocks) implies:

$$w_t = \chi_t = \kappa_t = a_t = \mu_t = g_t \equiv 0,$$

i.e. no other shock can be consistent with $\lim_{\gamma\to 0} z_{t+j} = 0$ for all $j \geq 0$, however in the absence of shocks $\lim_{\gamma\to 0} e_{t+j} = 0$, violating the second condition in (21). A symmetric argument for foreign rules out the foreign counterparts of these shocks. This completes the proof. ■

Proof of Proposition 2 For the proof, we consider the equilibrium system in the autarky limit by only keeping the lowest order terms in γ for each shock or variable.[57] Throughout the proof we impose $w_t = \chi_t = \kappa_t = a_t = \mu_t = g_t \equiv 0$, as well as for their foreign counterparts.

First, we consider our three moments of interest when ψ_t is the only shock, that is we set $\eta_t = \xi_t \equiv 0$. For this purpose, it is sufficient to consider the static equilibrium conditions only, as the effect of the ψ_t shock on the macro variables is exclusively indirect through q_t. Specifically:

1. Consider the near-autarky comovement between the terms of trade and the real exchange rate from (A35):
$$\lim_{\gamma\to 0} \frac{\operatorname{cov}(\Delta s_t, \Delta q_t)}{\operatorname{var}(\Delta q_t)} = (1-2\alpha) > 0 \quad \text{iff} \quad \alpha < \frac{1}{2},$$
since we have $\tilde{\eta}_t = 0$. $\alpha < 1/2$ is a necessary parameter requirement for this result, which is borne out in the data, as we discuss in Section 3.

2. Consider the near-autarky comovement between the relative consumption and the real exchange rate from (A49), which in the absence of all shocks but ψ_t simplifies to:
$$\Big[(1-2\gamma)\zeta(\nu\sigma+\varsigma)+2\gamma\nu\sigma\Big]\tilde{c}_t = -\gamma\left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma}+\frac{\nu+\phi}{1-\phi}\frac{1}{1-2\gamma}-\frac{1+\nu}{1-\phi}(1-\zeta)\right]q_t.$$
Hence, we have:
$$\lim_{\gamma\to 0}\frac{1}{\gamma}\frac{\operatorname{cov}\left(\Delta c_t-\Delta c_t^*, \Delta q_t\right)}{\operatorname{var}(\Delta q_t)} = -\frac{2}{\zeta(\nu\sigma+\varsigma)}\left(2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta)\right) < 0,$$
which is negative for all parameter values since
$$\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta) = \frac{\zeta\nu+\zeta-(1-\phi)}{1-\phi} > 0$$
as from (A20) $\zeta = 1-e^{-\mu/(1-\alpha)}\phi > 1-\phi$.

3. Consider the near-autarky comovement between the nominal exchange rate and the nominal interest rate differential (the Fama coefficient) by using (A54), which we write in the limit as:
$$i_t - i_t^* = \mathbb{E}_t\{2\sigma\Delta\tilde{c}_{t+1}+2\Delta\tilde{p}_{t+1}\} = -\frac{2\gamma\sigma}{\zeta(\nu\sigma+\varsigma)}\left[2\theta(1-\alpha)-1+\frac{\zeta(1-\varsigma/\sigma)}{1-\phi}\right]\mathbb{E}_t\Delta q_{t+1}.$$
where we used expression (A49) for $\tilde{c}_t$ and expressions (A38)–(A39) for p_t and p_t^*. Furthermore, (A40) and (A53) imply $\mathbb{E}_t\Delta e_{t+1} = \mathbb{E}_t\Delta q_{t+1} = -\psi_t$ in the limit and with ψ_t shocks only. There-

[57] For example, consider equation (A40), which we now rewrite as:
$$q_t - e_t = 2\left[\frac{1}{1-\phi}\left(\tilde{a}_t - \frac{\tilde{\mu}_t}{1-\alpha}\right)-\tilde{w}_t\right]+2\gamma\frac{1}{1-\phi}\frac{\tilde{\eta}_t}{1-\alpha}.$$
Note that the gap between q_t and e_t is zero-order in γ for shocks $(\tilde{a}_t, \tilde{\mu}_t, \tilde{w}_t)$ and first-order in γ for shock $\tilde{\eta}_t$.

fore, the Fama regression coefficient in the limit is:[58]

$$\lim_{\gamma\to 0}\gamma\frac{\operatorname{cov}\left(\mathbb{E}_t\Delta e_{t+1}, i_t - i_t^*\right)}{\operatorname{var}\left(i_t - i_t^*\right)} = -\frac{\zeta(\nu\sigma+\varsigma)}{2\sigma}\frac{1}{2\theta(1-\alpha)-1+\frac{\zeta(1-\varsigma/\sigma)}{1-\phi}} < 0,$$

which is always negative under a mild additional requirement that $\theta > 1$ and $\sigma > 1$ (since $\varsigma \leq 1$ and $\alpha < 1/2$), with a necessary condition being substantially weaker.[59]

This proves the first claim of the proposition that ψ_t robustly and simultaneously produces all three empirical regularities in the autarky limit.[60]

Second, recall that the uncovered interest rate parity (A52) implies that the Fama regression coefficient:

$$\beta_F \equiv \frac{\operatorname{cov}(\Delta e_{t+1}, i_t - i_t^*)}{\operatorname{var}(i_t - i_t^*)} = 1 \qquad \text{whenever} \qquad \psi_t \equiv 0.$$

Therefore, $(\eta_t, \eta_t^*, \xi_t, \xi_t^*)$ shocks that follow any joint process cannot resolve the forward premium puzzle. This is sufficient for the second claim of the proposition that the remaining shocks cannot deliver the empirical comovement for all three moments. Nonetheless, we explore the remaining two moments as well.

Third, in the remainder of the proof, we focus on the ξ_t and η_t shocks (setting all other shocks including ψ_t to zero), and impose specific time series process for these two types of shocks, which can be viewed as providing counterexamples sufficient for our argument in Proposition 2. Specifically, we focus on AR(1) processes for relative shocks:

$$\tilde{\xi}_t = \rho_\xi\tilde{\xi}_{t-1} + \sigma_\xi\varepsilon_t^\xi,$$
$$\tilde{\eta}_t = \rho_\eta\tilde{\eta}_{t-1} + \sigma_\eta\varepsilon_t^\eta,$$

with $\rho_\xi, \rho_\eta \in [0,1]$ and where $\varepsilon_t^\xi, \varepsilon_t^\eta \sim iid(0,1)$. We focus on the zero-order component of the exchange rate dynamics in γ, as this component is non-trivial for both $\tilde{\xi}_t$ and $\tilde{\eta}_t$ shocks. Therefore, we drop the first and higher order components in γ, so that we have $e_t = q_t$ from (A40) and $\mathbb{E}_t\Delta q_{t+1} = \mathbb{E}_t\Delta e_{t+1} = 0$ from (A53) together with (A49). Hence, the dynamics of the exchange rates is a random walk with jumps that satisfy the intertemporal budget constraints. The flow budget constraint (A51) (with net exports (A50), in which we substitute the solutions for $\tilde{c}_t$ and $\tilde{y}_t$ from (A45) and (A48)) up to first order terms in γ is given by:

$$\beta b_{t+1}^* - b_t^* = 2\gamma\big[\vartheta q_t - \tilde{\xi}_t - (\theta-1)\tilde{\eta}_t\big],$$

where $\vartheta \equiv \theta(1-\alpha) - \frac{1-2\alpha}{2}$. Solving this equation forward and imposing $\lim_{T\to\infty}\beta^T b_{T+1}^* = 0$, we

[58]We make use of the fact that $\operatorname{cov}\left(\Delta e_{t+1}, i_t - i_t^*\right) = \operatorname{cov}\left(\mathbb{E}_t\Delta e_{t+1}, i_t - i_t^*\right)$ since $i_t - i_t^*$ is known at t.

[59]The Fama coefficient for the real interest rates is always negative without any further parameter restrictions, as it is proportional to the expression for the Backus-Smith correlation, since the real interest rate $r_t \equiv i_t - \mathbb{E}_t\Delta p_{t+1} = \sigma\mathbb{E}_t\Delta c_{t+1}$ in the absence of χ_t shocks.

[60]It is also easy to verify that the dispersion of the (real and nominal) exchange rate is separated from zero in response to a ψ_t shock since from (A53) $\mathbb{E}_t\Delta q_{t+1} = -\psi_t$ and q_t needs to adjust in response to ψ_t to ensure intertemporal budget constraint with net exports following (A50). We show this formally in Appendix A.5 for ψ_t following an AR(1) process.

obtain the solution for the equilibrium exchange rate:[61]

$$\Delta q_{t+1} = \frac{1}{\vartheta}\frac{1-\beta}{1-\beta\rho_\xi}\sigma_\xi\varepsilon_{t+1}^{\xi} + \frac{\theta-1}{\vartheta}\frac{1-\beta}{1-\beta\rho_\eta}\sigma_\eta\varepsilon_{t+1}^{\eta}.$$

We can now calculate the moments using static equilibrium conditions (A35) for s_t and (A49) for $\tilde{c}_t$:[62]

$$\lim_{\gamma\to 0}\frac{\mathrm{cov}(\Delta s_t,\Delta q_t)}{\mathrm{var}(\Delta q_t)} = \left[\begin{array}{ll} (1-2\alpha)>0, & \text{for } \xi_t \text{ shock} \\ \begin{array}{l} (1-2\alpha)-2\frac{\mathrm{cov}(\Delta\tilde{\eta}_t,\Delta q_t)}{\mathrm{var}(\Delta q_t)} \\ =(1-2\alpha)\left[1-\frac{\frac{2(1-\alpha)}{1-2\alpha}\theta-1}{\theta-1}\frac{1-\beta\rho_\eta}{1-\beta}\right]<0, \end{array} & \text{for } \eta_t \text{ shock,} \end{array}\right.,$$

$$\lim_{\gamma\to 0}\frac{1}{\gamma}\frac{\mathrm{cov}\left(\Delta\tilde{c}_t,\Delta q_t\right)}{\mathrm{var}(\Delta q_t)} = \left[\begin{array}{ll} \frac{2\theta(1-\alpha)-(1-2\alpha)}{\zeta(\nu\sigma+\varsigma)}\left[\frac{\beta(1-\rho_\xi)}{1-\beta}-\frac{(1-2\alpha)+\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta)}{2\theta(1-\alpha)-(1-2\alpha)}\right] \gtrless 0, & \text{for } \xi_t, \\ \frac{1}{\zeta(\nu\sigma+\varsigma)}\frac{2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}}{\frac{\theta-1}{\theta-\frac{1-2\alpha}{2(1-\alpha)}}\frac{1-\beta}{1-\beta\rho_\eta}}\left[1-\frac{\theta-1}{\theta-\frac{1-2\alpha}{2(1-\alpha)}}\frac{1-\beta}{1-\beta\rho_\eta}\frac{2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}-(1-\zeta)}{2\theta(1-\alpha)+\zeta\frac{\nu+\phi}{1-\phi}}\right] > 0, & \text{for } \eta_t, \end{array}\right.$$

where for the first moment we maintain the assumption that $\alpha < 1/2$ and to sign the second moment we use the fact that $\zeta > 1-\phi$. To see that the Backus-Smith correlation under ξ_t shocks can take both signs, it is sufficient to consider the case with $\rho_\xi = 1$ (when the correlation is negative) and the case with $\rho_\xi = 0$ and $\beta \approx 1$ (when the correlation is positive). If $\beta \geq \rho_\xi$, under our parameterization it is sufficient to have the quarterly discount factor $\beta > 0.75$ for the sign to be positive (with the calibrated value of $\beta = 0.99$). This shows that the $\tilde{\eta}_t$ shock robustly generates counterfactual comovement with all three macro variables, while the $\tilde{\xi}_t$ shock does not robustly deliver empirically relevant comovement between exchange rates on one hand and interest rates and relative consumption on the other hand. ■

A.5 The Baseline Model of Section 3 with ψ_t Shock

Consider the log-linearized equilibrium system from Appendix A.3.2, in which we set $w_t = \mu_t = \eta_t = \xi_t = g_t = a_t = \kappa_t = \chi_t = 0$ at Home, and equivalently in Foreign, and also specialize to $\varsigma = 1$ and $\zeta = 1-\phi$ (corresponding to $G = 0$ and $\bar{\mu} = 0$ respectively). The equilibrium system is block recursive, and we solve it in turn for prices, quantities and equilibrium dynamics, followed by the discussion of interest rates.

[61] We describe a rigorous solution method in Appendix A.5, while here we offer a heuristic argument: the net present value (using β as a discount factor) of any innovation to the right hand side of the flow budget constraint needs to be zero for intertemporal budget balance. Denote $\varepsilon_t \equiv \Delta q_t$ the (random walk) innovation of the exchange rate. The net present value of the innovation to the flow budget constraint is therefore $\sum_{j=0}^{\infty}\beta^j\left[\vartheta\varepsilon_t - \rho_\xi^j\sigma_\xi\varepsilon_t^\xi - (\theta-1)\rho_\eta^j\sigma_\eta\varepsilon_t^\eta\right] = 0$, and solving for ε_t from this equation we obtain the expression in the proof.

[62] In our calculations, we use the interim results that in response to η_t shocks:

$$\mathrm{var}(\Delta q_{t+1}) = \left(\frac{\theta-1}{\theta(1-\alpha)-\frac{1-2\alpha}{2}}\frac{1-\beta}{1-\beta\rho_\eta}\right)^2\sigma_\eta^2 \qquad \text{and} \qquad \mathrm{cov}(\Delta\eta_{t+1},\Delta q_{t+1}) = \frac{\theta-1}{\theta(1-\alpha)-\frac{1-2\alpha}{2}}\frac{1-\beta}{1-\beta\rho_\eta}\sigma_\eta^2,$$

and similarly for the ξ_t shock.

Price block We rewrite the solution from Appendix A.3.2 for this special case as follows:[63]

$$s_t = \frac{1-2\alpha(1-\gamma)}{1-2\gamma}q_t \qquad \text{and} \qquad q_t^P = \frac{1-2\alpha\gamma}{1-2\gamma}q_t,$$
$$p_t = \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t \qquad \text{and} \qquad p_t^* = -\frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}q_t, \tag{A56}$$

and

$$q_t = \frac{1}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}e_t. \tag{A57}$$

Quantity block Again, rewriting the general solution (A45) and (A48) for this special case, we have:

$$\nu\sigma\tilde{c}_t + \tilde{y}_t = -\frac{\nu+\phi}{1-\phi}\frac{\gamma}{1-2\gamma}q_t, \tag{A58}$$
$$[1-(1-2\gamma)(1-\zeta)]\tilde{y}_t - (1-2\gamma)\zeta\tilde{c}_t = \gamma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - (1-\zeta)\right]q_t, \tag{A59}$$

where we used the fact that now $\varsigma = \frac{C}{C+G} = 1$, and in what follows we also use $\zeta = 1-e^{-\frac{\bar{\mu}}{1-\alpha}}\phi = 1-\phi$. (Recall that $\tilde{c}_t \equiv \frac{1}{2}(c_t - c_t^*)$.)

We solve (A58)–(A59) for consumption and output explicitly:

$$c_t - c_t^* = -\gamma\kappa_q^c q_t, \qquad \kappa_q^c \equiv \frac{2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \nu + \frac{\nu+\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}}{1+\sigma\nu\big(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\big)}\frac{1}{1-\phi}\frac{2}{1-2\gamma} > 0, \tag{A60}$$
$$y_t - y_t^* = \gamma\kappa_q^y q_t, \qquad \kappa_q^y \equiv \sigma\nu\kappa_q^c - \frac{\nu+\phi}{1-\phi}\frac{2}{1-2\gamma} = \frac{\sigma\nu\big(2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \phi\big) - (\nu+\phi)}{1+\sigma\nu\big(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\big)}\frac{1}{1-\phi}\frac{2}{1-2\gamma}.$$

Note that in the text in (38) we simply use κ_q for κ_q^c.

Using (A50), we can rewrite net exports in this case as:[64]

$$nx_t = \gamma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + \alpha - \frac{1-\alpha}{1-2\gamma} + \frac{2\gamma}{1-2\gamma}\phi\right]q_t - 2\gamma((1-\phi)\tilde{c}_t + \phi\tilde{y}_t)$$
$$= \gamma\kappa_q^{nx}q_t, \qquad \kappa_q^{nx} \equiv \left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} + 2(1-\gamma)\alpha - 1 - \gamma\kappa_q^y\right]\frac{1}{1-2\gamma}. \tag{A61}$$

Note that $\kappa_q^y < \kappa_q^c$ and may be negative. Furthermore, $\kappa_q^{nx} > 0$ iff (after using κ_q^y and simplifying):

$$2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}\frac{1+\sigma\nu}{1+\sigma\nu\big(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\big)} > 1-2(1-\gamma)\alpha - \frac{\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\big[\nu+(1+\sigma\nu)\phi\big]}{1+\sigma\nu\big(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\big)}. \tag{A62}$$

Finally, note that we scaled the coefficient κ_q^c, κ_q^y and κ_q^{nx} so that they are zero-order in γ, that is the limits of these coefficients as $\gamma \to 0$ are separated from both 0 and ∞.

[63] We also have $p_{Ht} - p_t = -\frac{\gamma}{1-\gamma}(p_{Ft} - p_t) = -\frac{(1-\alpha)\gamma}{1-2\gamma}q_t$, and symmetrically in the Foreign market.

[64] Note that $\phi\tilde{y}_t + (1-\phi)\tilde{c}_t = \frac{1}{1-2\gamma}\tilde{y}_t - \Big[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \phi\Big]\frac{\gamma}{1-2\gamma}q_t$.

Dynamic block We combine together the international risk-sharing condition (A53) and the flow budget constrain (A51), and use the solution (A60)–(A61) above, to obtain:

$$\psi_t = \mathbb{E}_t\left\{\sigma\Delta(c_{t+1}-c^*_{t+1})-\Delta q_{t+1}\right\} = -\left(1+\gamma\sigma\kappa^c_q\right)\mathbb{E}_t\Delta q_{t+1}, \tag{A63}$$

$$\beta b^*_{t+1}-b^*_t = nx_t = \gamma\kappa^{nx}_q q_t. \tag{A64}$$

Given the proportional relationship between real and nominal exchange rates (A57), we can equivalently rewrite the dynamic system above in terms of the nominal exchange rate e_t, corresponding to (24)–(25) in the text, with:

$$\lambda_1 = \frac{\sigma\kappa^c_q - \frac{1}{1-\phi}\frac{2}{1-2\gamma}}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}} = \frac{\frac{1}{1-\phi}\frac{2}{1-2\gamma}}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}\frac{\sigma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}+\frac{\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}\right]-1}{1+\sigma\nu\left(1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right)}, \tag{A65}$$

$$\lambda_2 \equiv \frac{\kappa^{nx}_q}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}. \tag{A66}$$

Note that (A62) ensures $\kappa^{nx}_q > 0$, which also implies $\lambda_2 > 0$. In turn, $\lambda_1 > 0$ iff:

$$\sigma\left[2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma}+\frac{\phi}{1-\phi}\frac{2\gamma}{1-2\gamma}\right] > 1. \tag{A67}$$

We discuss sufficient conditions for (A62) and (A67) in Appendix A.5.1. Note, however, that the signs of λ_1 and λ_2 are inconsequential for the proofs of the main results.

Equations (A63)–(A64) define a dynamic system in (b_t, q_t), where we assume the exogenous shock ψ_t follows an AR(1) process:

$$\psi_t = \rho\psi_{t-1}+\varepsilon_t. \tag{A68}$$

This system can be solved for the equilibrium dynamics of q_t and b_t using the Blanchard and Kahn (1980) method.[65] We prove:

Lemma A2 *The unique non-explosive solution to the dynamic system* (A63)–(A64) *with* (A68) *is given by:*

$$\Delta q_{t+1} = \rho\Delta q_t + \frac{1}{1+\gamma\sigma\kappa^c_q}\frac{\beta}{1-\beta\rho}\left(\varepsilon_{t+1}-\frac{1}{\beta}\varepsilon_t\right), \tag{A69}$$

$$\Delta b^*_{t+1} = \rho\Delta b^*_t + \frac{\gamma\kappa^{nx}_q}{1+\gamma\sigma\kappa^c_q}\frac{1}{1-\beta\rho}\varepsilon_t. \tag{A70}$$

Proof: We rewrite the dynamic system (A63)–(A64) in matrix form as:

$$\mathbb{E}_t\begin{pmatrix} q_{t+1} \\ \hat{b}_{t+1}\end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 1/\beta & 1/\beta\end{pmatrix}}_{\equiv A}\begin{pmatrix} q_t \\ \hat{b}_t\end{pmatrix} - \begin{pmatrix} \hat{\psi}_t \\ 0\end{pmatrix},$$

[65] Alternatively, this system can be solved by the method of undetermined coefficients. Rewriting (A63) as $\mathbb{E}_t q_{t+1} = q_t - \hat{\psi}_t$, it must be that $q_t = \frac{1}{1-\rho}\hat{\psi}_t + m_t$, where m_t is a martingale with the innovation given by the only fundamental shock ε_t. That is, $m_{t+1} = m_t + \vartheta\varepsilon_{t+1}$, where ϑ is the undetermined coefficient. We then use (A64) to find the unique value of ϑ that results in a non-explosive path for b^*_t.

where we use the rescaled variables $\hat{b}_t = \frac{b_t}{\gamma\kappa_q^{nx}}$ and $\hat{\psi}_t = \frac{\psi_t}{1+\gamma\sigma\kappa_q^c}$ for convenience. Matrix A has two eigenvalues: 1 and $1/\beta > 1$. The left eigenvector of matrix A associated with eigenvalue $1/\beta$ is $v = (1,\ 1-\beta)$. Premultiplying the system by v from the left, we have:

$$\mathbb{E}_t z_{t+1} = \frac{1}{\beta} z_t - \hat{\psi}_t, \qquad \text{where} \qquad z_t \equiv v \begin{pmatrix} q_t \\ \hat{b}_t \end{pmatrix} = q_t + (1-\beta)\hat{b}_t.$$

The unique non-explosive (forward) solution of this dynamic equation is:[66]

$$z_t = \beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{\psi}_{t+j} = \frac{\beta}{1-\beta\rho}\hat{\psi}_t.$$

This implies a cointegration relationship for the endogenous variables, $q_t + (1-\beta)\hat{b}_t = \frac{\beta}{1-\beta\rho}\hat{\psi}_t$, which can be used to solve out q_t in (A64), yielding:

$$\Delta\hat{b}_{t+1} = \frac{1}{1-\beta\rho}\hat{\psi}_t,$$

which implies (A70) given the definitions of $\hat{b}_t$ and $\hat{\psi}_t$, and using the fact that $(1-\rho L)\psi_t = \varepsilon_t$, where the lag operator $Lx_t = x_{t-1}$ for an arbitrary variable x_t.

Next we take the first difference of (A64):

$$\Delta q_t = \beta\Delta\hat{b}_{t+1} - \Delta\hat{b}_t = \frac{\beta}{1-\beta\rho}\hat{\psi}_t - \frac{1}{1-\beta\rho}\hat{\psi}_{t-1},$$

where the second equality substitutes in the solution for $\Delta\hat{b}_{t+1}$. Applying the $(1-\rho L)$ operator on both sides, we obtain equation (A69), since $(1-\rho L)\hat{\psi}_t = \hat{\psi}_t - \rho\hat{\psi}_{t-1} = \frac{1}{1+\gamma\sigma\kappa_q^c}\varepsilon_t$. ■

Proof of Proposition 3 Lemma A2 implies that the unique (NPG-admissible) solution of the dynamic system results in an ARIMA(1,1,1) process for the exchange rate and an ARIMA(1,1,0) process of the NFA position of the country. Equivalently, the change in the exchange rate Δq_t follows an ARMA(1,1) with the AR root ρ and the MA root $1/\beta$, while the change in the NFA Δb^*_{t+1} follows an AR(1) process with root ρ. Also note that the smaller is γ, the larger is the response of q_t and the smaller is the response of b^*_{t+1} to the innovation ε_t of the financial shock ψ_t. In light of (A57) and (A65), Lemma A2 implies Proposition 3. ■

Proofs for Proposition 4 and 5 Next we discuss the properties of the equilibrium exchange rate dynamics, which in light of (A57) apply equally to both the nominal and the real exchange rate. Given the solution (A69), we can now characterize the statistical properties of the exchange rate process:

1. Unconditional variance of Δq_t can be calculated from (A69) as follows:

$$\sigma^2_{\Delta q} = \rho^2\sigma^2_{\Delta q} + \frac{1}{(1+\gamma\sigma\kappa_q^c)^2}\frac{1+\beta^2}{(1-\beta\rho)^2}\sigma^2_\varepsilon - \frac{2\beta\rho}{(1+\gamma\sigma\kappa_q^c)^2}\frac{1}{(1-\beta\rho)^2}\sigma^2_\varepsilon,$$

[66] The remaining explosive solutions feature $\lim_{j\to\infty}\beta^j\mathbb{E}_t z_{t+j+1} = \infty$, violating the No Ponzi Game Condition for b^*_t.

where $\sigma^2_{\Delta q} = \text{var}(\Delta q_{t+1})$. Solving for $\sigma^2_{\Delta q}$, and for $\sigma^2_{\Delta e}$ using (A57) and (A65), we have:

$$\sigma^2_{\Delta q} = \frac{1}{(1+\gamma\sigma\kappa^c_q)^2}\frac{1-2\beta\rho+\beta^2}{(1-\beta\rho)^2}\frac{\sigma^2_\varepsilon}{1-\rho^2} \qquad \text{and} \qquad \sigma^2_{\Delta e} = \frac{1}{(1+\gamma\lambda_1)^2}\frac{1-2\beta\rho+\beta^2}{(1-\beta\rho)^2}\frac{\sigma^2_\varepsilon}{1-\rho^2},$$

since we have $1+\gamma\lambda_1 = \frac{1+\gamma\sigma\kappa^q_c}{1+\frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}}$.

Noting that $\text{var}(\psi_{t+1}) = \sigma^2_\varepsilon/(1-\rho^2)$, we further have:

$$\frac{\text{var}(\Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1+\gamma\lambda_1)^2}\frac{1-2\beta\rho+\beta^2}{(1-\beta\rho)^2} \xrightarrow[\beta\to1]{} \frac{1}{(1+\gamma\lambda_1)^2}\frac{2}{1-\rho},$$

which increases without bound as $\rho \to 1$.

2. Auto-covariance of Δq_t can be similarly calculated using (A69) as:

$$\text{cov}(\Delta q_{t+1}, \Delta q_t) = \rho\sigma^2_{\Delta q} - \frac{1}{(1+\gamma\sigma\kappa^c_q)^2}\frac{\beta}{(1-\beta\rho)^2}\sigma^2_\varepsilon = -\frac{(\beta-\rho)(1-\beta\rho)}{1-2\beta\rho+\beta^2}\sigma^2_{\Delta q}.$$

The autocorrelation of the exchange rate changes (both nominal and real, in light of (A57)) is:

$$\rho_{\Delta e} = \rho_{\Delta q} = \frac{\text{cov}(\Delta q_{t+1}, \Delta q_t)}{\sigma^2_{\Delta q}} = -\frac{(\beta-\rho)(1-\beta\rho)}{1-2\beta\rho+\beta^2} \xrightarrow[\beta\to1]{} -\frac{1-\rho}{2},$$

confirming claim 1 in Propositions 4.

3. The variance of innovation of Δq_{t+1} is:

$$\text{var}_t(\Delta q_{t+1}) = \text{var}(\Delta q_{t+1} - \mathbb{E}_t \Delta q_{t+1}) = \left(\frac{\beta}{1-\beta\rho}\right)^2 \frac{\sigma^2_\varepsilon}{(1+\gamma\sigma\kappa^c_q)^2},$$

where we assume that the information set at time t includes $\{q_t, q_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots\}$. Since Δe_t equals Δq_t scaled by a constant, we further have:

$$\frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = \frac{\text{var}(\Delta q_{t+1} - \mathbb{E}_t\Delta q_{t+1})}{\text{var}(\Delta q_{t+1})} = \frac{\beta^2(1-\rho^2)}{1-2\beta\rho+\beta^2} \xrightarrow[\beta\to1]{} \frac{1+\rho}{2},$$

confirming claim 2 in Propositions 4.

Combining with the result in point 1 above, we have:

$$\frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t\Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1+\gamma\lambda_1)^2}\frac{\beta^2(1-\rho^2)}{(1-\beta\rho)^2} \xrightarrow[\beta\to1]{} \frac{1}{(1+\gamma\lambda_1)^2}\frac{1+\rho}{1-\rho},$$

which tends to infinity with $\rho \to 1$, confirming claim 3 in Propositions 4. ■

4. We now calculate the finite-sample autocorrelation of the real exchange rate in levels, that is the coefficient from a regression of q_t on q_{t-1} (with a constant) in a sample with $T+1$ observations. Even though the second moments are not well-defined in population, this finite sample

correlation is well-defined. We have:

$$\hat{\rho}_q(T) = \frac{\frac{1}{T}\sum_{t=1}^{T}(q_t - \bar{q})(q_{t-1} - \bar{q})}{\frac{1}{T}\sum_{t=1}^{T}(q_{t-1} - \bar{q})^2} = 1 + \frac{\frac{1}{T}\sum_{t=1}^{T}\Delta q_t q_{t-1}}{\frac{1}{T}\sum_{t=1}^{T}(q_{t-1} - \bar{q})^2}.$$

Note that the denominator is positive and finite for any finite T, but diverges as $T \to \infty$, since q_t is an integrated process. The numerator, however, has a finite limit (assuming $\beta, \rho < 1$, which ensures stationarity of Δq_t, and conditioning on the given initial value of the process q_0):

$$\begin{aligned} p\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\Delta q_t q_{t-1} &= \mathrm{cov}(\Delta q_t, q_{t-1}) = \sum_{j=1}^{\infty}\mathrm{cov}(\Delta q_t, \Delta q_{t-j}) \\ &= \frac{\mathrm{cov}(\Delta q_t, \Delta q_{t-1})}{1-\rho} = -\frac{\beta - \rho}{(1-\rho)(1-\beta\rho)}\frac{\sigma_\psi^2}{(1+\gamma\sigma\kappa_q^c)^2}, \end{aligned}$$

where we used the fact that for process (A69) $\mathrm{cov}(\Delta q_t, \Delta q_{t-j}) = \rho^{j-1}\mathrm{cov}(\Delta q_{t-j+1}, \Delta q_{t-j})$ for $j \geq 1$ and the expression for $\mathrm{cov}(\Delta q_t, \Delta q_{t-1})$ obtained above.

To summarize, this analysis implies that the finite sample autocorrelation of q_t: (a) tends to 1 asymptotically as samples size increases; and (b) is smaller than 1 in large but finite samples, provided that $\rho < \beta$. This confirms the claims in Proposition 5. ■

Interest rates and Carry trades Finally, we turn to the properties of the interest rates, which are linked to consumption and prices by (A54). Substituting in the solution for consumption (A60) and prices (A56), we arrive at:

$$i_t - i_t^* = -\left(\sigma\gamma\kappa_q^c - \tfrac{1}{1-\phi}\tfrac{2\gamma}{1-2\gamma}\right)\mathbb{E}_t\Delta q_{t+1} = -\gamma\lambda_1\mathbb{E}_t\Delta e_{t+1},$$

corresponding to (41) and with λ_1 defined in (A65). In this analysis we assume that the parameter restriction (A67) is satisfied, and $\lambda_1 > 0$.[67] Combining the expression for interest rate differential with the UIP condition (A52), we obtain expression (42) in the text.

Lastly, we define a Carry trade. Consider a trade strategy that invests $x_t \equiv i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1}$ in the home bond and sells short x_t units of foreign bond, including the case when $x_t < 0$ (i.e., shorting the home bond and investing in foreign bond in this case). We refer to this strategy as a *Carry trade.* Note that this trade requires zero capital at t and the intensity (exposure) of the trade is proportional to its expected return, which from (A52) equals $x_t = \psi_t$. The return on this trade and the corresponding (unconditional) Sharpe ratio are given by:

$$r_{t+1}^C = x_t(i_t - i_t^* - \Delta e_{t+1}) \qquad \text{and} \qquad SR^C = \frac{\mathbb{E}r_{t+1}^C}{\mathrm{std}(r_{t+1}^C)}. \tag{A71}$$

[67] There exists a parallel relationship in real terms (where $r_t \equiv i_t - \mathbb{E}_t\Delta p_{t+1}$ denotes the real interest rate):

$$r_t - r_t^* = \sigma\mathbb{E}_t\{\Delta c_{t+1} - \Delta c_{t+1}^*\} = -\gamma\sigma\kappa_q^c\,\mathbb{E}_t\Delta q_{t+1},$$

with the negative sign independently of the parameter values.

Proof of Proposition 8 First, consider the Fama regression of Δe_{t+1} on $i_t - i_t^*$. From (26), it follows:

$$\Delta e_{t+1} = \mathbb{E}_t \Delta e_{t+1} + \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\varepsilon_{t+1},$$

since ε_{t+1} is the only innovation relative to the information set of time t. Furthermore, from our derivations above we have $i_t - i_t^* = -\gamma\lambda_1 \mathbb{E}_t \Delta e_{t+1}$, and therefore, we can write the regression as:

$$\Delta e_{t+1} = -\frac{1}{\gamma\lambda_1}(i_t - i_t^*) + \frac{1}{1+\gamma\lambda_1}\frac{\beta}{1-\beta\rho}\varepsilon_{t+1}.$$

Since ε_{t+1} is a regression residual (i.e., $\mathbb{E}\{\varepsilon_{t+1}|i_t - i_t^*\} = 0$), the Fama regression coefficient is given by $\beta_F \equiv -\frac{1}{\gamma\lambda_1} < 0$. The R^2 in this regression is given by the share of the predictable variation in Δe_{t+1}, since $i_t - i_t^*$ absorbes the entire predictable component $\mathbb{E}_t \Delta e_{t+1}$ (see Proposition 4 and its proof for derivation of the variance share of the unpredictable component):

$$R^2 = \frac{\text{var}(\mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = 1 - \frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = 1 - \frac{\beta^2(1-\rho^2)}{1-2\beta\rho+\beta^2} = \frac{(1-\beta\rho)^2}{1-2\beta\rho+\beta^2}.$$

Note that $\lim_{\beta\to 1} R^2 = \frac{1-\rho}{2}$, which tends to zero with $\rho \to 1$. This proves claim (i).

Using the same arguments, we prove claim (ii):

$$\frac{\text{var}(\Delta e_{t+1})}{\text{var}(i_t - i_t^*)} = \frac{1}{(\gamma\lambda_1)^2}\frac{\text{var}(\Delta e_{t+1})}{\text{var}(\mathbb{E}_t \Delta e_{t+1})} = \frac{1}{(\gamma\lambda_1)^2}\frac{1}{R^2}.$$

Since $\gamma\lambda_1$ is separated from zero when $\gamma > 0$ and does not depend on β and ρ, the asymptotics of this relative variances is the same as that of $1/R^2$, which goes to infinity as $\beta, \rho \to 1$.

Claim (iii) follows form the fact that $\rho_{\Delta e} = \text{corr}(\Delta e_{t+1}, \Delta e_t) \to 0$ as $\beta, \rho \to 1$ (see Proposition 4), while the persistence of $i_t - i_t^* = \frac{\gamma\lambda_1}{1+\gamma\lambda_1}\psi_t$ equals $\rho \to 1$.

Lastly, we make use of the definition of the Carry trade return r_{t+1}^C and its Sharpe ratio SR^C in (A71), to prove claim (iv). In particular, we calculate the expected return and the variance of the returns (using the fact that from (A52) we have $i_t - i_t^* - \Delta e_{t+1} = \psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})$):

$$\begin{aligned}
\mathbb{E} r_{t+1}^C &= \mathbb{E}\big\{\psi_t \mathbb{E}_t\{i_t - i_t^* - \Delta e_{t+1}\}\big\} = \mathbb{E}\psi_t^2 = \text{var}(\psi_t) = \sigma_\psi^2, \\
\text{var}(r_{t+1}^C) &= \mathbb{E}(r_{t+1}^C)^2 - (\mathbb{E} r_{t+1}^C)^2 = \mathbb{E}\big\{\psi_t^2[\psi_t - (\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})]^2\big\} - \sigma_\psi^4 \\
&= \mathbb{E}\psi_t^4 + \mathbb{E}\left\{\psi_t^2 \text{var}_t(\Delta e_{t+1})\right\} - \sigma_\psi^4 = 2\sigma_\psi^4 + \text{var}_t(\Delta e_{t+1})\sigma_\psi^2,
\end{aligned}$$

where the last line uses the fact that $\text{var}_t(\Delta e_{t+1}) = \mathbb{E}_t\{\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1}\}^2$ depends only on the parameters and does not depend on ψ_t (i.e., the unexpected component of Δe_{t+1} is homoskedastic; see the proof of Proposition 4), the fact that $\mathbb{E}\{\psi_t^3(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})\} = \mathbb{E}\{\psi_t^3 \mathbb{E}_t\{\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1}\}\} = 0$, and lastly that $\mathbb{E}\psi_t^4 = 3(\mathbb{E}\psi_t^2)^2 = 3\sigma_\psi^4$ under the additional assumption that ε_t is normally distribution (in which case ψ_t is also normal). With this, we calculate:

$$SR^C = \frac{\sigma_\psi^2}{\sqrt{2\sigma_\psi^4 + \text{var}_t(\Delta e_{t+1})\sigma_\psi^2}} = \left(2 + \frac{1}{(1+\gamma\lambda_1)^2}\frac{\beta^2(1-\rho^2)}{(1-\beta\rho)^2}\right)^{-1/2},$$

where we use the expression for $\text{var}_t(\Delta e_{t+1})/\sigma_\psi^2$ from the proof of Proposition 4. Note that $SR^C \to 0$ as $\beta, \rho \to 1$, as $\text{var}_t(\Delta e_{t+1})/\sigma_\psi^2 \to \infty$. ■

A.5.1 Parameter restrictions

The primitive parameters (defined in Table 1) can take the following values:

- $\beta \in (0,1)$ and $\rho \in [0,1]$, while the model admits solution as long as $\beta\rho < 1$
- $\sigma, \theta > 0$ and $\nu \geq 0$
- $\gamma \in [0, 1/2]$ with $\gamma = 0$ corresponding to autarky and $\gamma = 1/2$ corresponding to no home bias
- $\alpha \in [0,1)$ with $\alpha = 0$ corresponding to no strategic complementarities and complete pass-through
- $\phi \in [0,1)$ with $\phi = 0$ corresponding to no intermediate inputs.

In addition, we impose the following sufficient parameter restrictions needed for certain results:

Assumption A1: $\alpha < 1/2$.

A1 ensures positive correlation between RER and ToT (Proposition 6). A1 is consistent with the range of empirical estimates for the elasticity of strategic complementarities.

Assumption A2: $\theta > 1/\sigma$.

Together with A1, A2 implies $2\sigma\theta(1-\alpha) > 1$, which is a sufficient condition for (A67), ensuring that $\lambda_1 > 0$, i.e. that the nominal interest rate falls with expected deprecation (see (41); needed for Proposition 8, and useful but not necessary in discussion of Proposition 3).[68] Note that here $1/\sigma$ plays the role of IES (elasticity of intertemporal substitution), rather than the income effect in the labor supply (and therefore does not exclude GHH preferences). Empirically, $\theta > 1$ and $1/\sigma \in (1/2, 1)$, so this sufficient condition is met with ease, and the necessary condition (A67) is even further lax.

Assumption A3: $\theta > \frac{1}{2} + \frac{1}{1-\phi}\frac{\gamma}{1-2\gamma}$.

A3 is a sufficient condition for (A62), a variant of the Marshall-Lerner condition in our general equilibrium model, which ensuring that $\lambda_2 > 0$ and $\kappa_q^{nx} > 0$, i.e. that net export improves in response to a devaluation (see (25); a useful but not necessary condition for discussion of Proposition 3). An alternative necessary condition for (A62) can be written as $\theta > \frac{1}{2}\frac{1}{1-\gamma}\big[1 + 2\gamma\frac{\phi}{1-\phi}\big]$. A necessary condition (A62) is noticeably weaker, and in particular is relaxed when $\alpha > 0$. In the limit of autarky ($\gamma \approx 0$), $\theta > 1/2$ is both necessary and sufficient, corresponding to the classical Marshall-Lerner condition. Since empirically $\theta > 1$ and $\phi \approx 1/2$, A3 would be easily satisfied even for countries that are a number of times more open than the United States.

[68] Condition A2 effectively ensures that nominal and real interest rates move in the same direction, i.e. the expected inflation response does not more than offset the movement in the real interest rate.

A.5.2 Quantitative properties and robustness

We now explore the robustness of our quantitative findings in the baseline model (with ψ_t shocks only), away from both limites $\gamma \to 0$ and $\beta\rho \to 1$, and with respect to departures from our baseline parameterization summarized in Table 1. The results are reported in Table A1, along with the benchmark empirical moments. The robustness columns report only the moments that are sensitive to the changes in the parameter values.

The robustness analysis suggests that the model requires a high ρ in order to capture the dynamic properties of the exchange rates, as a lower ρ results in less persistent exchange rates and in more predictable exchange rate changes. The quantitative success of the model also relies on home bias (a low γ), as when γ is doubled (corresponding to a 60% trade to GDP ratio), the model predicts a considerably more volatile response of the real variables to the real exchange rate, in contrast with the data. Having moderately low θ and high α is also important for the fit of the model, while it is more robust with respect to variation in other parameters, including risk aversion σ and Frisch elasticity ν (not reported in the table). Similarly, we check robustness with respect to $\varsigma = C/(C+G)$ and $\bar{\mu}$ (average markup).

Notes: (a) In response to ψ_t shock, c_t and c_t^* are perfectly negatively correlated so that $\Delta c_t - \Delta c_t^* = 2\Delta c_t$. Therefore, while $\Delta c_t - \Delta c_t^*$ is about three times less volatile than Δq_t, Δc_t alone is about six times less volatile than Δq_t. (b) The in-sample Fama regression coefficient is -8 with huge variation covering zero within two standard deviations. When ψ_t is combined with other shocks, the median coefficient becomes closer to zero, as in the data (see Table 2).

A.6 Market (in)completeness

We consider here the generalization of our framework to a version of a complete market environment to assess the importance of a single internationally-traded bond assumption in the baseline model.

We make the following assumptions: Only the foreign-currency assets are traded internationally and are the only types of assets held by foreign households. This assumption is for convenience of exposition and is without loss of generality if there exists a full set of state-contingent foreign-currency assets. We make the following assumptions about SDFs:

1. Foreign nominal SDF for foreign-currency assets: $\Theta^*_{t+1} = \beta \exp\{-(\sigma\Delta c^*_{t+1} + \Delta p^*_{t+1})\}$.
2. Home nominal SDF for home-currency assets: $\Theta_{t+1} = \beta \exp\{-(\sigma\Delta c_{t+1} + \Delta p_{t+1})\}$.
3. Home nominal SDF for foreign-currency assets:

$$\Theta_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}e^{\Delta\zeta_{t+1}} = \beta\exp\{\Delta\zeta_{t+1} - \sigma\Delta c_{t+1} - \Delta p_{t+1} + \Delta e_{t+1}\}.$$

Hence, ζ_t is an exogenous home preference shock for holding foreign currency assets at t, or alternatively a shock to international risk sharing. It can also be viewed as a type of deviation from the joint assumption of CRRA utility and complete markets (see e.g. Lustig and Verdelhan 2016).

With a full set of state contingent foreign-currency assets, the risk-sharing condition becomes:

$$\Theta^*_{t+1} = \Theta_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}e^{\Delta\zeta_{t+1}} \qquad \Leftrightarrow \qquad \sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) = \Delta q_{t+1} + \Delta\zeta_{t+1},$$

which is equivalent to a static condition (in log deviations):

$$\sigma(c_t - c^*_t) = q_t + \zeta_t. \tag{A72}$$

Therefore, ζ_t is the shock to/wedge in the Backus-Smith condition. We take equation (A72) as the reduced-form characterization of this environment, which can be consistent with both complete markets, as well as specific types of deviations from complete markets.

In what follows, we allow for productivity shock a_t and risk sharing shock ζ_t, shutting down all other shocks for simplicity. We also maintain the assumption of flexible wages and prices. Under complete markets, the budget constraint is satisfied as a side-equations (due to state-contingent payoffs) and does not affect the equilibrium dynamics. The equilibrium characterization simply combines the risk-sharing condition (A72) with the goods and labor market clearing relationship (40), which remains unchanged.[69] Together, they result in the solution for the real exchange rate:

$$q_t = \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}(a_t - a^*_t) - \frac{1}{1+\gamma\sigma\kappa_q}\zeta_t. \tag{A73}$$

Therefore, in this complete market environment, the real exchange rate is driven directly by the contemporaneous shocks to productivity and to international risk sharing, and the persistence in these shocks directly translates into persistence in q_t.

Given the solution for equilibrium real exchange rate in (A73), the equilibrium nominal exchange rate is characterized by (31), which also stays unchanged, and we reproduce it here as:

$$e_t = \left[1 + \frac{1}{1-\phi}\frac{2\gamma}{1-2\gamma}\right]q_t + \left((w_t - w^*_t) - \frac{1}{1-\phi}(a_t - a^*_t)\right),$$

and where the path of the nominal wage $w_t - w^*_t$ is determined by the monetary policy.

The conclusion from this analysis is that the exchange rate disconnect properties of the baseline one-bond model can be also obtained in this version of a complete-market environment, provided that productivity $(a_t - a^*_t)$ and monetary $(w_t - w^*_t)$ shocks are small relative to a persistent risk-sharing shock ζ_t. Indeed, when ζ_t is the only shock and it follows a persistent AR(1) process, the real and nominal exchange rates will be perfectly correlated and will follow the same persistent process. Furthermore, there will be analogous limiting disconnect properties for the macro variables. Consider for example consumption, which we solve for by combining (A72) with (A73):

$$\sigma(c_t - c^*_t) = \frac{\sigma\kappa_a}{1+\gamma\sigma\kappa_q}(a_t - a^*_t) + \frac{\gamma\sigma\kappa_q}{1+\gamma\sigma\kappa_q}\zeta_t.$$

[69] Recall that in the absence of markup shocks, this condition implicitly assumes flexible prices and wages, an assumption we maintain throughout this appendix. With nominal rigidities, the equilibrium real exchange rate would be additionally influenced by the implied markup shocks arising from sticky prices and wages.

We see that ζ_t shock generates the Backus-Smith negative correlation between $c_t - c_t^*$ and q_t, and the volatility of consumption is arbitrary small relative to the real exchange rate when the economy is closed to international trade ($\gamma \to 0$). Therefore, we can prove equivalents of Propositions 1–2 and 5–8 in the complete market environment as well. To summarize, market incompleteness in the specific form of a single internationally-traded bond is not necessary for our exchange rate disconnect results, however, a shock ζ_t to the risk-sharing condition is, and it can emerge due to a variety of reasons, such as risk premia, financial frictions, limits to arbitrage, or market incompleteness.

Relationship to the baseline model Under incomplete markets with a single risk-free foreign nominal bond traded internationally, the optimal risk sharing results in the (log-linearized) UIP condition:[70]

$$i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \psi_t, \qquad \text{where} \qquad \psi_t \equiv \mathbb{E}_t \Delta \zeta_{t+1}.$$

Therefore, the shock ψ_t in our baseline model is equivalent to the expect change in the SDF shock ζ_t, as the martingale disturbances to the SDF (with $\mathbb{E}_t \Delta \zeta_{t+1} = 0$) wash out in the linearized model.

We consider now two special cases:

1. ζ_t follows an AR(1): $\zeta_t = \rho \zeta_{t-1} + \varepsilon_t$. Then $\psi_t = \mathbb{E}_t \Delta \zeta_t = -(1-\rho)\zeta_t$ also follows an AR(1) with persistence ρ, and ψ_t and ζ_t are negatively correlated. In the limit of $\rho \to 1$, ψ_t becomes both more persistent and less volatile, with $\mathrm{var}(\psi_t) \to 0$ holding $\mathrm{var}(\varepsilon_t)$ constant.
2. $\Delta\zeta_t$ follows an AR(1): $\Delta\zeta_t = \rho \Delta\zeta_{t-1} + \varepsilon_t$. Then: $\psi_t = \mathbb{E}_t \Delta \zeta_{t+1} = \rho \Delta \zeta_t$ also follows an AR(1) with persistence ρ, yet now ψ_t and ζ_t are positively correlated. The variance of the ψ_t process goes to zero with $\rho \to 0$.

We consider the first case as our benchmark, with $\rho < 1$ but in the neighborhood of 1. Note that a persistent ψ_t requires a persistent ζ_t, which however does not need to be integrated.

In the baseline model, equilibrium real exchange rate follows a dynamic process (A69) in Lemma A2, which differs markedly from the simple static relationship of q_t to shocks in (A73). The reason is that the RER in a single-bond economy not only needs to clear markets statically, but also ensures the dynamic intertemporal budget constraint, which is not relevant for equilibrium dynamics in a complete market environments.

A.7 Relationship to Engel and West (2005)

Consider a simple monetary extension to our baseline model, with an interest-elastic money demand:

$$m_t - p_t = \sigma c_t - \chi i_t, \tag{A74}$$

and an exogenous stochastic money supply process m_t, resulting in an endogenous path for nominal wages w_t instead of $w_t \equiv 1$ in the baseline model. Since prices and wages are flexible, this change has no effect on the equilibrium path of real variables, including the real exchange rate, but can be consequential for the path of nominal variables, including the nominal exchange rate.

[70]This derives from a log linear approximation to $i_{t+1}^* = -\log \mathbb{E}_t \Theta_{t+1}^* = -\log \mathbb{E}_t \big\{ \Theta_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} e^{\Delta \zeta_{t+1}} \big\}$.

This model extension corresponds to one of the special cases considered in Engel and West (2005; henceforth, EW), and their other special case with a Taylor rule admits a similar characterization (omitted here for brevity). For concreteness, we focus on three types of shocks: productivity shock a_t, financial (UIP) shock ψ_t, and monetary shock m_t.

Engel and West (2005) solution. EW combine the UIP condition (22) with money demand (A74) in both countries and the definition of the real exchange rate (29) to obtain the equilibrium dynamic expression for the nominal exchange rate as a function of fundamentals:

$$e_t = \delta \mathbb{E}_t e_{t+1} + \delta \psi_t + (1-\delta) f_t, \tag{A75}$$

where $f_t \equiv q_t + (m_t - m_t^*) - \sigma(c_t - c_t^*)$ and $\delta \equiv \chi/(1+\chi)$. Under two alternative assumptions on the equilibrium dynamic processes for ψ_t and f_t, EW prove that the equilibrium process for nominal exchange rate e_t that satisfies (A75) must converge to a random walk as $\delta \to 1$. Specifically, the conditions are that either (a) $\psi_t \equiv 0$ and $f_t \sim I(1)$, or (b) that $\psi_t \sim I(1)$ and $f_t \sim I(0)$ or $I(1)$, and then $\text{cov}(\Delta e_t, \Delta e_{t+k}) \to 0$ for all $k \neq 0$ as $\delta \to 1$. Interestingly, what matters for the equilibrium exchange rate process here is the elasticity of money demand χ (which determines δ), but not the discount factor β, which we emphasize in Proposition 3.

Equilibrium cointegration. In general, other equilibrium conditions of the model impose cointegration between fundamentals (ψ_t, f_t) featured in (A75). In particular, the international risk-sharing condition (39) can be rewritten as:

$$\mathbb{E}_t \Delta \tilde{f}_{t+1} = -\psi_t, \qquad \text{where} \qquad \tilde{f}_t \equiv q_t - \sigma(c_t - c_t^*), \tag{A76}$$

so that $f_t \equiv \tilde{f}_t + (m_t - m_t^*)$ and $\tilde{f}_t$ isolates the endogenous fundamentals in f_t. Condition (A76) is an equilibrium cointegration relationship between exogenous shock ψ_t and endogenous variable $\tilde{f}_t$. We can use it to solve out ψ_t in (A75) and rewrite it as:

$$(1 - \delta L^{-1}) e_t = (1 - \delta L^{-1}) \tilde{f}_t + (1-\delta)(m_t - m_t^*).$$

This makes it clear that due to cointegration the endogenous fundamentals also have a forward looking root δ, just like nominal exchange rate, and therefore it cancels out, resulting in the following solution:

$$e_t = \tilde{f}_t + (1-\delta) \sum\nolimits_{j=0}^{\infty} \delta^j \mathbb{E}_t \{ m_{t+j} - m_{t+j}^* \}.$$

Furthermore, consider the case when $m_t = m_t^* \equiv 0$, which results in:

$$e_t = \tilde{f}_t = (1 + \gamma\sigma\kappa_q) q_t - \sigma\kappa_a (a_t - a_t^*),$$

where we used the solution for consumption (40) to express $\tilde{f}_t$ as a function of endogenous q_t and exogenous $(a_t - a_t^*)$. Note that in this case the root δ disappears altogether from the equilibrium characterization of $\{e_t\}$. More concretely, assume $\psi_t \sim AR(1)$ is the only source of shocks, as in the baseline model, and then $e_t = (1+\gamma\sigma\kappa_q)q_t \sim ARIMA(1,1,1)$, as characterized in Lemma A2.[71] In this case, the process for e_t does not depend on δ and convergence to a random walk when $\beta\rho \to 1$, in seeming contrast with the predictions of EW.

Given that our model can be formulated as a special case of the EW representation (A75), a natural question is how can it be that our solution does not satisfy their theorem? The following three special cases explain the source of this discrepancy:

1. A model with monetary shocks (m_t, m_t^*) only. This case features $\psi_t = \tilde{f}_t \equiv 0$ and $f_t = m_t - m_t^*$, and the EW theorem applies. The same would be true under an alternative formulation with a Taylor rule and Taylor rule shocks.

2. A model with productivity shocks (a_t, a_t^*) only. This case features $e_t = f_t = \tilde{f}_t$ and $\psi_t \equiv 0$, which implies $\mathbb{E}_t\Delta e_{t+1} = \mathbb{E}_t\Delta f_{t+1} = 0$, that is e_t follows an exact random walk independently of the value of δ or β, and hence the EW theorem holds trivially in this case. This case extends to all fundamental shocks as long as $m_t = m_t^* = \psi_t \equiv 0$, as in this case $i_t - i_t^* = 0$, and the UIP condition immediately implies $\mathbb{E}_t\Delta e_{t+1} = 0$, independently of the shock processes and parameters of the model.

3. A model with financial (UIP) shocks ψ_t, and possibly all other shocks. This case features cointegration relationship $\mathbb{E}_t\Delta\tilde{f}_{t+1} = -\psi_t$. Therefore, if $\psi_t \sim I(k)$ for some integer $k \geq 0$, then f_t is at least $I(k+1)$, and therefore the conditions of the EW theorem are never satisfied in this case.

To summarize, we find that in the presence of the UIP shocks ψ_t in our model, the equilibrium cointegration relationship between endogenous and exogenous fundamentals violates the conditions of the EW theorem, explaining why we find that an equilibrium exchange rate process has different properties, and in particular does not depend on δ, but instead depends on β.

A.8 Productivity shocks and the Backus-Smith puzzle

We consider here alternative mechanisms, which can resolve the Backus-Smith puzzle in a bond-only economy subject exclusively to productivity shocks, following much of the literature (in particular, Corsetti, Dedola, and Leduc 2008). Using the static equilibrium relationship (40), we can express the Backus-Smith correlation as:

$$\frac{\mathrm{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t)}{\mathrm{var}(\Delta q_t)} = \kappa_a \varrho_{a,q} \frac{\mathrm{std}(\Delta a_t - \Delta a_t^*)}{\mathrm{std}(\Delta q_t)} - \gamma\kappa_q,$$

where $\varrho_{a,q} \equiv \mathrm{corr}(\Delta a_t - \Delta a_t^*, \Delta q_t)$. The puzzle persists as the typical calibrations imply $\varrho_{a,q} > 0$ and $\gamma \approx 0$, resulting in the counterfactually positive correlation between relative consumption growth and real exchange rate depreciation. The two mechanism we discuss below either make $\varrho_{a,q} < 0$, or increase $\mathrm{std}(\Delta q_t)/\mathrm{std}(\Delta a_t - \Delta a_t^*)$ to obtain the empirical negative correlation.

In addition to (40), the two dynamic equilibrium conditions are the risk sharing condition (39) and the budget constraint (analog of (25)), which we reproduce here altogether as the special case of the

[71] Note the difference between $e_t = (1+\gamma\sigma\kappa_q)q_t$ and (32), which arises due to the fact that under monetary policy rule $m_t \equiv 0$, $w_t \neq 0$ in general. Nonetheless, the path of q_t is the same independently of the monetary policy rule (as prices and wages are flexible). Furthermore, e_t and q_t are still proportional to each other, albeit with a different factor of proportionality.

equilibrium system in Appendix A.3.2 with productivity shocks only:

$$c_t - c_t^* = \kappa_a(a_t - a_t^*) - \gamma\kappa_q q_t, \tag{40}$$

$$\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}\} = 0, \tag{39$'$}$$

$$\beta b_{t+1}^* - b_t^* = nx_t = \gamma\left[\lambda_q q_t - \lambda_a(a_t - a_t^*)\right], \tag{25$'$}$$

Further, for simplicity, we consider the case with $\phi = 0, \zeta = \varsigma = 1$ and $\nu = 0$, and the results generalize immediately beyond this special case. In this case, the coefficients in the system above are given by:

$$\kappa_a = \lambda_a = \frac{1}{1-2\gamma}, \qquad \kappa_q = \frac{4\theta(1-\alpha)(1-\gamma)}{(1-2\gamma)^2}, \qquad \lambda_q = \frac{\kappa_q}{2} - \frac{1-2(1-\gamma)\alpha}{1-2\gamma}.$$

Two noteworthy features of this system are:

1. Both coefficients $\gamma\lambda_a$ and $\gamma\lambda_q$ tend towards zero with $\gamma \to 0$, while this is the case only for $\gamma\kappa_q$, and not for κ_a. This suggests that the direct effect of productivity on consumption will tend to dominate the expenditure switching effect, when γ is small (that is, economy is sufficiently closed). This constitutes the key challenge for the productivity-based models in obtaining the empirical Backus-Smith correlation.

2. Coefficients $\kappa_a, \kappa_q, \lambda_a > 0$, while $\lambda_q > 0$ iff a version of the Marshal-Lerner condition holds:

$$\theta > \frac{1}{2}\frac{1-2\gamma}{1-\gamma}\frac{1-2(1-\gamma)\alpha}{1-\alpha},$$

   for which Assumption A3 in Appendix A.5.1 is a sufficient condition.

The fact that λ_q can flip sign (when θ is sufficiently low) is one path towards a resolution of the Backus-Smith puzzle. The other possibility relies on reducing the volatility of innovation to productivity (making $\kappa_a(a_t - a_t^*)$ small in (40)), while simultaneously increasing its persistence (to increase the response of q_t and hence the term $\gamma\kappa_q q_t$ in (40)). We consider these two possibilities in turn.

Low elasticity of substitution For simplicity we focus here on the case of a random walk process for productivity, and results generalize outside this case. With $\Delta(a_t - a_t^*) = \varepsilon_t^a$, the combination of (39$'$) and (40) results in $\mathbb{E}_t\Delta q_{t+1} = 0$. Intuitively, in response to a permanent shift in productivity, the real exchange rate also shifts permanently. Given a random walk path for both q_t and $(a_t - a_t^*)$, the intertemporal budget constraint holds only if $\lambda_q q_t - \lambda_a(a_t - a_t^*) \equiv 0$. Therefore, the real exchange rate depreciates with a positive productivity shock iff the Marshal-Lerner condition is satisfied, but it appreciates otherwise. Intuitively, an increase in consumption and import demand from a productivity shock must be offset by an increase in exports, which requires a depreciation iff the Marshal-Lerner condition holds, and vice versa. Combining this relationship between q_t and $(a_t - a_t^*)$ with the solution for consumption, we arrive at:

$$c_t - c_t^* = \left\{\tfrac{1}{1-2\gamma}\left[\tfrac{2\theta(1-\alpha)(1-\gamma)}{1-2\gamma} - [1-2(1-\gamma)\alpha]\right] - \gamma\kappa_q\right\} q_t.$$

Therefore, the violation of the Marshal-Lerner condition is sufficient for the negative correlation between $c_t - c_t^*$ and q_t, but the necessary condition is weaker and is given by:

$$\theta < \frac{1}{2}\frac{1}{1-\gamma}\frac{1-2(1-\gamma)\alpha}{1-\alpha}.$$

With $\gamma = 0.28$ (four times that of the US) and $\alpha = 0$, this requires $\theta < 0.7$. For smaller γ and larger α, this requirement becomes considerably more strict (e.g., for our baseline values of α and γ, $\theta < 0.25$).

Persistent productivity shocks Here we assume that the Marshal-Lerner condition is satisfied and $\lambda_q > 0$, and instead consider a case with a persistent process for relative productivity growth rates:

$$\Delta\tilde{a}_t = \rho_a \Delta\tilde{a}_{t-1} + \varepsilon_t^{\varepsilon},$$

where $\tilde{a}_t \equiv (a_t - a_t^*)/2$ and $\rho_a \in [0,1)$.[72] Combining (39′) and (40), we have in this case:

$$\mathbb{E}_t\Delta q_{t+1} = \frac{2\sigma\kappa_a}{1+\gamma\sigma\kappa_q}\mathbb{E}_t\Delta\tilde{a}_{t+1} = \frac{2\sigma\kappa_a\rho_a}{1+\gamma\sigma\kappa_q}\Delta\tilde{a}_t.$$

Therefore, a positive productivity growth shock results in an expected appreciation. Combining this with the flow budget constraint, we have a system of dynamic equations, which we solve again using the Blanchard-Kahn method (as in Lemma A2):[73]

$$\Delta q_{t+1} = \frac{\hat{\lambda} - \beta\rho_a\hat{\kappa}}{1-\beta\rho_a}\left(\Delta\tilde{a}_{t+1} - \frac{(\hat{\lambda}-\hat{\kappa})\rho_a}{\hat{\lambda}-\beta\rho_a\hat{\kappa}}\Delta\tilde{a}_t\right),$$

where $\hat{\kappa} \equiv \frac{2\sigma\kappa_a}{1+\gamma\sigma\kappa_q}$ and $\hat{\lambda} \equiv 2\lambda_a/\lambda_q$. With this solution, we can calculate the Backus-Smith covariance (making use of (40)):

$$\begin{aligned}\frac{\mathrm{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t)}{\mathrm{var}(\Delta q_t)} &= 2\kappa_a\frac{\mathrm{cov}(\Delta\tilde{a}_t, \Delta q_t)}{\mathrm{var}(\Delta q_t)} - \gamma\kappa_q \\ &= 2\kappa_a\frac{(1-\beta\rho_a)[\hat{\lambda}(1-\rho_a^2) - \hat{\kappa}\rho_a(\beta-\rho_a)]}{[\hat{\lambda}(1-\rho_a)+\hat{\kappa}\rho_a(1-\beta)]^2 + \rho_a(2-\rho_a)(\hat{\lambda}-\hat{\kappa})(\hat{\lambda}-\hat{\kappa}\beta\rho_a)} - \gamma\kappa_q.\end{aligned}$$

Around $\rho_a \approx 0$ (random walk), the Marshal-Lerner condition is sufficient to ensure that this expression is positive (corresponding to the case we considered earlier). However, as ρ_a increases, this equation switches sign to negative, as in the limit $\beta\rho_a \to 1$, $\mathrm{var}(\Delta q_t)/\mathrm{var}(\Delta\tilde{a}_t) \to \infty$. This is an intuitive result: as productivity growth shocks become very persistent, the contemporaneous improvement in productivity is small relative to the cumulative expected improvement, and the RER responds to the

[72] The results in the stationary AR(1) case are qualitatively similar to the random walk limit with ρ_a.

[73] To apply the Blanchard-Kahn method, we rewrite the system in matrix form as:

$$\mathbb{E}_t\begin{pmatrix} q_{t+1} \\ \hat{b}_{t+1}\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\beta\end{pmatrix}\begin{pmatrix} q_t \\ \hat{b}_t\end{pmatrix} + \begin{pmatrix}\hat{\kappa}\rho_a\Delta\tilde{a}_t \\ -\hat{\lambda}\tilde{a}_t\end{pmatrix},$$

where $\hat{b}_t \equiv \beta b_t^*/(\gamma\lambda_q)$, and look for the unique stationary solution associated with the explosive root $1/\beta$. Note that the coefficients do not depend on ρ_a, which facilitates taking the limits below. Note that, given the productivity process, the RER equilibrium process is still an ARIMA(1,1,1), as in Lemma A2, yet with a different MA root.

cumulative expectation. Therefore, tiny shocks to current productivity act like news shocks about future productivity, and trigger large RER movements. Under these circumstances, indirect expenditure switching effect of RER on consumption can dominate the direct contemporaneous productivity effect in (40). Long-run risk shocks in Colacito and Croce (2013) operate in a similar way, yet have an additional risk premia effects in (39). Lastly, we point out that persistent growth rate shocks are not necessary per se, as persistent effects to output growth can be obtained from endogenous amplification, such as capital accumulation in Corsetti, Dedola, and Leduc (2008).

A.9 Monetary model with nominal rigidities (Section 4.1)

We outline the details of the monetary model, adopting a general enough setup to nest several extensions as special cases. In particular, we allow for both nominal wage and price rigidities. As before, we focus on Home and symmetric relationships hold in Foreign.

Households Consider a standard New Keynesian two country model in a cashless limit, as described in Galí (2008). In particular, the aggregate labor input is a CES aggregate of individual varieties with elasticity of substitution ϵ, which results in labor demand:

$$L_{it} = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon} L_t, \quad \text{where} \quad L_t = \left(\int L_{it}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{and} \quad W_t = \left(\int W_{it}^{1-\epsilon} \mathrm{d}i\right)^{\frac{1}{1-\epsilon}},$$

and the rest of the model production structure is unchanged. The first order conditions of the household optimization result in the New Keynesian IS curve and the UIP condition:

$$\mathbb{E}_t \left\{\sigma \Delta \tilde{c}_{t+1} + \Delta \tilde{p}_{t+1}\right\} = \tilde{i}_t, \tag{A77}$$

$$\mathbb{E}_t \Delta \tilde{e}_{t+1} = 2\tilde{i}_t - \psi_t, \tag{A78}$$

where as before we use notation $\tilde{x}_t = \frac{1}{2}\left(x_t - x_t^*\right)$.

Households set wages a la Calvo and supply as much labor as demanded at a given wage rate. The probability of changing wage in the next period is $1 - \lambda_w$. The first order condition for wage setting is:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \lambda_w)^{s-t} \frac{C_s^{-\sigma}}{P_s} W_s^{\epsilon} L_s \left(\bar{W}_t^{1+\epsilon/\nu} - \frac{\kappa \epsilon}{\epsilon - 1} P_s C_s^{\sigma} L_s^{1/\nu} W_s^{\epsilon/\nu}\right) = 0.$$

Substituting in labor demand and log-linearizing, we obtain:

$$\hat{w}_t = \frac{1 - \beta \lambda_w}{1 + \epsilon/\nu} \left(\sigma c_t + \frac{1}{\nu}\ell_t + p_t + \frac{\epsilon}{\nu} w_t\right) + \beta \lambda_w \mathbb{E}_t \hat{w}_{t+1},$$

where $\hat{w}_t$ denotes log deviation from the steady state of the wage rate reset at t. Note that the wage inflation can be expressed as $\pi_t^w \equiv \Delta w_t = (1 - \lambda_w)(\hat{w}_t - w_{t-1})$. Aggregate wages using these equalities and express the wage process in terms of cross-country differences to obtain the NKPC for wages:

$$[1 + \beta + k_w]\, \tilde{w}_t - \beta \mathbb{E}_t \tilde{w}_{t+1} - \tilde{w}_{t-1} = k_w \left[\sigma \tilde{c}_t + \frac{1}{\nu}\tilde{\ell}_t + \tilde{p}_t\right], \tag{A79}$$

where $k_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)}$.

Firms Assume that firms set prices a la Calvo with probability of changing price next period equal $1-\lambda_p$. There are two Phillips curves, one for domestic sales $\tilde{p}_{Ht}$ and one for export $\tilde{p}^*_{Ht}$. The first order conditions for reset prices in log-linearized form are

$$\hat{p}_{Ht} = (1-\beta\lambda_p)\,\mathbb{E}_t\sum_{j=t}^{\infty}(\beta\lambda_p)^{j-t}\left[(1-\alpha)\left(-a_j+(1-\phi)\,w_j+\phi p_j\right)+\alpha p_j\right],$$
$$\hat{p}^*_{Ht} = (1-\beta\lambda_p)\,\mathbb{E}_t\sum_{j=t}^{\infty}(\beta\lambda_p)^{j-t}\left[(1-\alpha)\left(-a_j+(1-\phi)\,w_j+\phi p_j-e_j\right)+\alpha p^*_j\right].$$

The law of motion for home prices and the resulting NKPC are then:

$$\pi_{Ht} = (1-\lambda_p)\left(\hat{p}_{Ht}-p_{Ht-1}\right) = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}_{Ht}-p_{Ht}\right),$$
$$\begin{aligned}\big[1+\beta+&k_p\left(\gamma+(1-\alpha)\,(1-\gamma)\,(1-\phi)\right)\big]\tilde{p}_{Ht}-\beta\mathbb{E}_t\tilde{p}_{Ht+1}-\tilde{p}_{Ht-1}\\ &=k_p\,(1-\alpha)\left[-\tilde{a}_t+(1-\phi)\,\tilde{w}_t\right]-k_p\gamma\left[1-(1-\alpha)\,(1-\phi)\right]\tilde{p}^*_{Ht},\end{aligned}$$

where $k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}$. On the other hand, the law of motion for export prices depends on currency of invoicing. Assuming LCP one obtains

$$\pi^*_{Ht} = (1-\lambda_p)\left(\hat{p}^*_{Ht}-p^*_{Ht-1}\right) = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}^*_{Ht}-p^*_{Ht}\right),$$
$$\begin{aligned}\big[1+\beta+&k_p\left(1-\alpha\gamma+\gamma\phi\,(1-\alpha)\right)\big]\tilde{p}^*_{Ht}-\beta\mathbb{E}_t\tilde{p}^*_{Ht+1}-\tilde{p}^*_{Ht-1}\\ &=k_p\,(1-\alpha)\left[-\tilde{a}_t+(1-\phi)\,\tilde{w}_t-e_t\right]-k_p\,(1-\gamma)\left[\alpha-(1-\alpha)\,\phi\right]\tilde{p}_{Ht},\end{aligned}$$

In case of PCP the law of motion of price index and NKPC are

$$\pi^*_{Ht} = (1-\lambda_p)\left(\hat{p}^*_{Ht}-p^*_{Ht-1}\right)-\lambda_p\Delta e_t = \frac{1-\lambda_p}{\lambda_p}\left(\hat{p}^*_{Ht}-p^*_{Ht}\right)-\Delta e_t,$$
$$\begin{aligned}\big[1+\beta+&k_p\left(1-\alpha\gamma+\gamma\phi\,(1-\alpha)\right)\big]\left(\tilde{p}^*_{Ht}+e_t\right)-\beta\mathbb{E}_t\{\tilde{p}^*_{Ht+1}+e_{t+1}\}-\left(\tilde{p}^*_{Ht-1}+e_{t-1}\right)\\ &=k_p\,(1-\alpha)\left[-\tilde{a}_t+(1-\phi)\,\tilde{w}_t\right]+k_p\left[\alpha(1-\gamma)+\gamma\phi\,(1-\alpha)\right]e_t-k_p\,(1-\gamma)\left[\alpha-(1-\alpha)\,\phi\right]\tilde{p}_{Ht}.\end{aligned}$$

Government policy and shocks We assume that Central Bank conducts active monetary policy, while the government chooses the fiscal policy (taxes) passively to balance the budget. The monetary policy is represented by a standard Taylor rule:

$$i_t = \rho_m i_{t-1} + (1-\rho_m)\left[\delta_\pi\pi_t+\delta_y y_t\right]+\epsilon^m_t, \tag{A80}$$

where δ_y is a coefficient on the output gap, which in the baseline case was absent (i.e., $\delta_y = 0$). The Taylor rules are symmetric in both countries. In the case with exchange rate peg (Table 4), we check robustness with asymmetric Taylor rules, where one country follows a conventional Taylor rule (43), while the other pegs its exchange rate according to (44).

We allow persistence of the interest rate to be different from the autocorrelation of other shocks, which in addition to the ψ_t shock in (23) also include foreign-good demand and productivity shocks:

$$\tilde{\xi}_t = \rho\tilde{\xi}_{t-1} + \epsilon_t^{\xi}, \tag{A81}$$

$$\tilde{a}_t = \rho\tilde{a}_{t-1} + \epsilon_t^{a}. \tag{A82}$$

Market clearing The last dynamic equation is the country's budget constraint:

$$\beta b_{t+1}^* = b_t^* + 2\left(\tilde{p}_{Ht}^* + \tilde{y}_{Ht}^*\right) + e_t, \tag{A83}$$

where b_t^* is the net foreign asset position of the Home country. The static part of the model is represented by labor demand and goods market equilibrium conditions:

$$\tilde{\ell}_t = \tilde{y}_t - \tilde{a}_t + \phi\left(\left(1-\gamma\right)\tilde{p}_{Ht} - \gamma\tilde{p}_{Ht}^* - \tilde{w}_t\right) \tag{A84}$$

$$\tilde{y}_{Ht} = -\gamma\tilde{\xi}_t - \theta\gamma\left(\tilde{p}_{Ht} + \tilde{p}_{Ht}^*\right) + \left(1-\phi\right)\tilde{c}_t + \phi\left(\left(1-\phi\right)\left(\tilde{w}_t - \tilde{p}_t\right) - \tilde{a}_t + \tilde{y}_t\right) \tag{A85}$$

$$\tilde{y}_{Ht}^* = -\tilde{y}_{Ht} - \tilde{\xi}_t - \theta\left(\tilde{p}_{Ht} + \tilde{p}_{Ht}^*\right) \tag{A86}$$

$$\tilde{y}_t = \left(1-\gamma\right)\tilde{y}_{Ht} + \gamma\tilde{y}_{Ht}^* \tag{A87}$$

The numbered equations above define the system that describes the equilibrium dynamics of the model.

Robustness All baseline parameters are as described in the main text and we set the elasticity of substitution between different types of labor to $\epsilon = 4$. Table A3 presents the results from alternative monetary models with multiple shocks, which can be compared to the baseline multi-shock monetary model in column 5 of Table 2. In particular, we consider the following alternative specifications, adjusting one feature of the model at a time relative to the baseline:

1. Flexible wages ($\lambda_w = 0$): no noticeable difference
2. Flexible prices ($\lambda_p = 0$): the volatility and correlation of terms of trade and real exchange rate relative to the nominal exchange rate deteriorate somewhat, volatility of consumption goes up, and the role of monetary shocks is larger.
3. Lower persistence in the Taylor rule ($\rho_m = 0.8$): Fama coefficient becomes positive, interest rates become more volatile and less persistent.
4. Expected inflation ($\mathbb{E}_t\pi_{t+1}$) instead of π_t in the Taylor rule (43): no noticeable differences.
5. Positive weight on output gap in the Taylor rule ($\delta_y = 0.2$): no noticeable differences.
6. PCP stickiness instead of LCP (with the same $\lambda_p = 0.75$): correlation between RER and ToT becomes approximately $+1$ instead of -1, with few other differences.

In each case, we recalibrate the relative volatilities of the shocks (reported in the last two lines of Table A3) to still match the correlations between consumption and net exports and the real exchange rate.

Table A3: Monetary model: robustness

	$\lambda_w = 0$ (1)	$\lambda_p = 0$ (2)	$\rho_m = 0.8$ (3)	$\mathbb{E}_t\pi_{t+1}$ (4)	$\delta_y = 0.2$ (5)	PCP (6)
$\rho\,(\Delta e)$	-0.03 (0.09)	-0.02 (0.09)	-0.04 (0.09)	-0.03 (0.09)	-0.02 (0.09)	-0.02 (0.09)
$\rho\,(q)$	0.92 (0.04)	0.94 (0.04)	0.93 (0.04)	0.92 (0.04)	0.92 (0.04)	0.94 (0.04)
$\frac{\sigma(\Delta q)}{\sigma(\Delta e)}$	0.99 (0.01)	0.75 (0.00)	0.98 (0.00)	0.98 (0.00)	0.97 (0.01)	0.85 (0.01)
$\mathrm{corr}\,(\Delta e, \Delta q)$	1.00	1.00	1.00	1.00	1.00	1.00
$\frac{\sigma(\Delta c-\Delta c^*)}{\sigma(\Delta q)}$	0.16 (0.01)	0.38 (0.03)	0.30 (0.03)	0.17 (0.02)	0.23 (0.02)	0.31 (0.03)
$\mathrm{corr}\,(\Delta c - \Delta c^*, \Delta q)$	-0.20 (0.09)	-0.20 (0.09)	-0.20 (0.09)	-0.20 (0.09)	-0.20 (0.09)	-0.20 (0.09)
$\frac{\sigma(\Delta nx)}{\sigma(\Delta q)}$	0.28 (0.03)	0.42 (0.04)	0.31 (0.03)	0.31 (0.03)	0.23 (0.02)	0.42 (0.04)
$\mathrm{corr}\,(\Delta nx, \Delta q)$	-0.00 (0.09)	0.00 (0.09)	-0.00 (0.09)	-0.00 (0.09)	-0.00 (0.09)	-0.00 (0.09)
$\frac{\sigma(\Delta s)}{\sigma(\Delta e)}$	0.79 (0.01)	0.22 (0.00)	0.81 (0.01)	0.80 (0.01)	0.80 (0.01)	0.85 (0.01)
$\mathrm{corr}\,(\Delta s, \Delta e)$	-0.92 (0.02)	1.00 (0.00)	-0.93 (0.02)	-0.93 (0.02)	-0.92 (0.02)	0.97 (0.01)
Fama β	-0.6 (1.5)	-0.0 (1.1)	0.4 (0.5)	-0.8 (1.6)	-2.4 (2.7)	-0.1 (1.2)
Fama R^2	0.00 (0.01)	0.00 (0.01)	0.01 (0.02)	0.00 (0.01)	0.01 (0.02)	0.00 (0.01)
$\frac{\sigma(i-i^*)}{\sigma(\Delta e)}$	0.06 (0.02)	0.08 (0.02)	0.16 (0.03)	0.06 (0.02)	0.05 (0.02)	0.08 (0.02)
$\rho\,(i - i^*)$	0.91 (0.05)	0.88 (0.05)	0.81 (0.06)	0.92 (0.04)	0.96 (0.02)	0.90 (0.05)
Sharpe Ratio	0.17 (0.06)	0.16 (0.06)	0.19 (0.06)	0.17 (0.06)	0.19 (0.06)	0.16 (0.07)
Decomposition of $\mathrm{var}(\Delta e_{t+1})$:						
Monetary shock, ε_t^m	8%	29%	19%	8%	1%	21%
Foreign-good shocks, ξ_t	23%	20%	21%	19%	39%	24%
Financial shocks, ψ_t	69%	51%	60%	73%	60%	55%
Calibrated variances of the shocks:						
$\sigma_m/\sigma_\varepsilon$	0.31	0.64	1.07	0.27	0.1	0.51
$\gamma\sigma_\xi/\sigma_\varepsilon$	2.6	3.1	2.3	2.7	1.8	3.5

Note: The table reports moments as in Table 2 for six alternative specifications of the multi-shock monetary model, as explained in the text. The lower panels report the variances decomposition for the nominal exchange rate into the contribution of the shocks (as in Table 3) and the calibrated relative volatilities of the shocks, which are adjusted to match $\mathrm{corr}(\Delta c - \Delta c^*, \Delta q) = -0.20$ and $\mathrm{corr}(\Delta nx, \Delta q) = 0.00$ (see footnote 39).

A.10 A model with a financial sector (Section 4.2)

This appendix provides the details for the model of Section 4.2. We discuss here the equations that change relative to the baseline model summarized in Appendix A.3. This concerns only the blocks 5 and 6, namely the Euler equations and the budget constraints, and additionally it involves the equilibrium (market clearing) conditions for the financial intermediation sector.

We start with the home, which now has the following consolidated budget constraint:

$$B_{t+1} - R_{t-1}B_t = NX_t, \quad \text{where} \quad NX_t = \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - P_{Ft}Y_{Ft} = P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht}Y^*_{Ht} - P_tC_t.$$

Since home households can trade only the home currency bond, their intertemporal optimization is characterized by a single Euler equation:

$$R_t\mathbb{E}_t\Theta_{t+1} = 1, \qquad \text{where} \qquad \Theta_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}.$$

The log linearization of these two conditions results in:

$$\beta b_{t+1} - b_t = nx_t, \tag{A88}$$
$$i_t = \mathbb{E}_t\{\sigma\Delta c_{t+1} - \Delta p_{t+1}\},$$

both equations exactly as before.[74] Given that the static relationship in the model are unchanged, we still have as in (25) that $nx_t = \gamma\lambda_2 e_t$ with λ_2 defined in (A66).

The foreign households differ only in that the profits and losses of the noise traders and arbitrageurs are transferred to them, but that constitutes a second order term, which vanishes in the log linearization. Specifically, we have the two parallel equations for foreign:

$$B^*_{t+1} - R^*_{t-1}B^*_t = NX^*_t + \tilde{R}_t(N^*_t + D^*_t), \qquad \tilde{R}_t \equiv R^*_{t-1} - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t},$$
$$R^*_t\mathbb{E}_t\Theta^*_{t+1} = 1, \qquad \Theta^*_{t+1} = \beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\frac{P^*_t}{P^*_{t+1}}.$$

The log-linearization of the former results in $\beta b^*_{t+1} - b^*_t = nx^*_t$, since in steady state both $\tilde{R} = 0$ and $N^* = D^* = 0$, and hence the transfer term is second order. Also note that this log-linearized equation is equivalent to (A88), since by definition $NX^*_t = -\mathcal{E}_t NX_t$ and from market clearing $B^*_{t+1} = -\mathcal{E}_t B_{t+1}$ (see below), and hence we drop it from the equilibrium system (Walras law). The log linearization of the second conditions is, as before:

$$i^*_t = \mathbb{E}_t\{\Delta c^*_{t+1} - \Delta p^*_{t+1}\}.$$

Finally, we turn to the financial market clearing $B_{t+1}+N_{t+1}+D_{t+1} = 0$ and $B^*_{t+1} + N^*_{t+1} + D^*_{t+1} = 0$. These conditions imply that, given that noise traders and arbitrageurs both hold zero-capital positions ($N_{t+1} = -\mathcal{E}_t N^*_{t+1}$ and $D_{t+1} = -\mathcal{E}_t D^*_{t+1}$), the net foreign assets of foreign equal the net foreign liabil-

[74] There is a slight change in notation to $b_{t+1} = RB_{t+1}/\bar{Y}$ from $b^*_{t+1} = B^*_{t+1}/\bar{Y}$, since previously the NFA of home was in foreign-currency bonds (while now it is in home-currency bonds), and B^*_{t+1} used to denote the nominal value of the bond.

ities of home: $\mathcal{E}_t B^*_{t+1} = -B_{t+1}$. Therefore, in light of the exogenous demand of noise traders (45) and optimal demand of the arbitrageurs (46), we have the following market clearing condition:

$$\frac{B_{t+1}}{\mathcal{E}_t} = n\left(e^{\psi_t} - 1\right) + m\frac{\mathbb{E}_t\tilde{R}_{t+1}}{\omega\ \mathrm{var}_t(\tilde{R}_{t+1})}.$$

Using the fact that

$$\tilde{R}_{t+1} = R^*_t\left(1 - \frac{R_t}{R^*_t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = R^*_t\left(1 - e^{i_t - i^*_t - \Delta e_{t+1}}\right),$$

we rewrite this condition as:

$$\frac{R^*_t B_{t+1}}{\mathcal{E}_t} = R^*_t n\left(e^{\tilde{\psi}_t} - 1\right) + m\frac{\mathbb{E}_t\big\{1 - e^{i_t - i^*_t - \Delta e_{t+1}}\big\}}{\omega\ \mathrm{var}_t\big(1 - e^{i_t - i^*_t - \Delta e_{t+1}}\big)}.$$

Using the facts that in steady state $R^* = 1/\beta$, $\mathcal{E} = 1$, $B = 0$, $\psi = 0$ and $i - i^* - \Delta e = 0$, we obtain the approximation:[75]

$$\bar{Y}b_{t+1} = \frac{n}{\beta}\psi_t - \frac{m}{\omega\sigma_e^2}(i_t - i^*_t - \mathbb{E}_t e_{t+1}),$$

where $b_{t+1} \equiv \frac{R^* B_{t+1}}{\bar{Y}} = \frac{B_{t+1}}{\beta\bar{Y}}$ and $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$, which after rearranging results in the UIP condition (47) in the text, reproduced here as:

$$i_t - i^*_t - \mathbb{E}_t\Delta e_{t+1} = \chi_1\psi_t - \chi_2 b_{t+1}, \tag{A89}$$

where

$$\chi_1 \equiv \frac{n/\beta}{m/(\omega\sigma_e^2)} \qquad \text{and} \qquad \chi_2 \equiv \frac{\bar{Y}}{m/(\omega\sigma_e^2)}.$$

Now combing (A88) and (A89) together with the static equilibrium relationships of net exports and nominal interests with the exchange rate (which are unchanged, and as derived in Appendix A.3.2), we obtain the dynamic equilibrium system:

$$\mathbb{E}_t\Delta e_{t+1} = -\frac{\chi_1}{1+\gamma\lambda_1}\psi_t + \frac{\chi_2}{1+\gamma\lambda_1}b_{t+1}, \tag{A90}$$

$$\beta b_{t+1} - b_t = \gamma\lambda_2 e_t, \tag{A91}$$

where we again assume that the exogenous shock follows an AR(1):

$$\psi_t = \rho\psi_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim iid(0, \sigma_\varepsilon^2). \tag{A92}$$

Note that this system takes the one in the baseline model as a special case as $\chi_2 \to 0$, and it generalizes it by allowing for $\chi_2 > 0$. Note also the additional complication that the χ_1, χ_2 coefficients depend on the equilibrium volatility of the nominal exchange rate innovations, which needs to be taken into account in the solution.

We now prove a generalization of Lemma A2 in Appendix A.5 for the model's extension with a

[75] This can be viewed as a log-linear approximation under an asymptotics where σ_e^2 is first order in the size of the shocks, as for example can be the case when the number of arbitrageurs m decreases with the volatility of the shocks.

financial sector:

Lemma A3 (a) *The unique non-explosive solution to the dynamic system* (A90)–(A92) *is given by:*

$$(1-\zeta_1 L)e_{t+1} = \frac{1}{1+\gamma\lambda_1}\frac{\beta\zeta_1}{1-\beta\zeta_1\rho}\left(1-\beta^{-1}L\right)\chi_1\psi_{t+1}, \tag{A93}$$

$$(1-\zeta_1 L)b_{t+1} = \frac{\gamma\lambda_2}{1+\gamma\lambda_1}\frac{\zeta_1}{1-\beta\zeta_1\rho}\chi_1\psi_t, \tag{A94}$$

where $\zeta_1 \in (0,1]$ *and* $\zeta_1 \to 1$ *as* $\chi_2 \to 0$.

(b) *There exists a cutoff* $\hat{d} > 0$*, such that for* $\frac{1}{\beta(1+\gamma\lambda_1)}\frac{n\omega\sigma_\varepsilon}{m} < \hat{d}$*, the only equilibrium has* $\sigma_e^2 = \chi_1 = \chi_2 = 0$*, while for* $\frac{1}{\beta(1+\gamma\lambda_1)}\frac{n\omega\sigma_\varepsilon}{m} > \hat{d}$ *there also exists an equilibrium with* $\sigma_e > 0$ *and* $\frac{\partial\sigma_e}{\partial(m\omega\sigma_\varepsilon/m)} > 0$.

Proof: We define the following normalized variables: $\hat{\psi}_t \equiv \frac{\chi_1\psi_t}{1+\gamma\lambda_1}$ and $\hat{b}_{t+1} = \frac{\beta b_{t+1}}{\gamma\lambda_2}$. Then we can rewrite (A90)–(A91) in the matrix form as:

$$\begin{pmatrix} \mathbb{E}_t e_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = A \begin{pmatrix} e_t \\ \hat{b}_t \end{pmatrix} + \begin{pmatrix} -\hat{\psi}_t \\ 0 \end{pmatrix}, \qquad A \equiv \begin{pmatrix} 1+\kappa & \kappa/\beta \\ 1 & 1/\beta \end{pmatrix},$$

where $\kappa \equiv \frac{\gamma\lambda_2\chi_2/\beta}{1+\gamma\lambda_1} \geq 0$ with $\kappa \to 0$ iff $\chi_2 \to 0$. The two eigenvalues of A are the solutions of $(1+\kappa-\zeta_i)(1/\beta-\zeta_i) - \kappa/\beta = 0$, and are given by:

$$\zeta_{1,2} = \frac{\left(1+\kappa+\frac{1}{\beta}\right) \mp \sqrt{\left(1+\kappa+\frac{1}{\beta}\right)^2 - \frac{4}{\beta}}}{2}$$

with the property that $0 < \zeta_1 \leq 1$ and $\frac{1}{\beta} \leq \zeta_2 < \infty$, with the two equalities obtaining iff $\kappa \to 0$ (i.e., $\chi_2 \to 0$). Furthermore, the Vieta's formulas imply $\zeta_1\zeta_2 = 1/\beta$ and $\zeta_1 + \zeta_2 = 1+\kappa+1/\beta$, which we conveniently use below.

The left eigenvector of A associated with $\zeta_2 > 1$ is $v_2 = \left[\left(\zeta_2 - \frac{1}{\beta}\right), \ \frac{\kappa}{\beta}\right]$. Therefore, the cointegration relationship between the variables is (see proof of Lemma A2):

$$e_t + \frac{\kappa/\beta}{\zeta_2 - \frac{1}{\beta}}\hat{b}_t = \frac{1}{\zeta_2 - \rho}\hat{\psi}_t,$$

or equivalently using the Vieta's formulas:

$$e_t + \frac{1}{\beta}(1-\beta\zeta_1)\hat{b}_t = \frac{\beta\zeta_1}{1-\beta\zeta_1\rho}\hat{\psi}_t,$$

Combining with the second equation of the system, this yields the solution for $\hat{b}_{t+1}$:

$$\hat{b}_{t+1} = \frac{1}{\beta}\hat{b}_t + e_t = \zeta_1\hat{b}_t + \frac{\beta\zeta_1}{1-\beta\zeta_1\rho}\hat{\psi}_t.$$

Together with the definitions of $\hat{b}_{t+1}$ and $\hat{\psi}_t$, this yields the solution in Lemma A3. Since ψ_t follows an AR(1), b_{t+1} follows an AR(2) with roots ζ_1 and ρ.

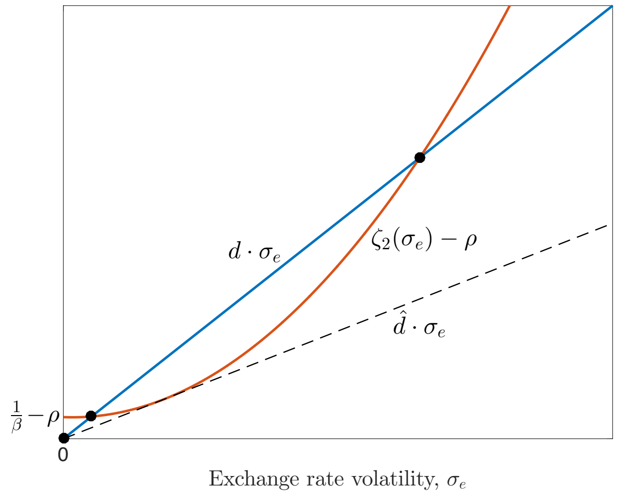

Figure A6: Equilibrium exchange rate volatility

Note: The figure illustrates the three equilibria (black dots), which exist for $d = \frac{1}{\beta(1+\gamma\lambda_1)}\frac{n\omega\sigma_\varepsilon}{m} > \hat{d}$. When $d < \hat{d}$, the only equilibrium is $\sigma_e = 0$. As $\beta\rho \to 1$, the red convex curve $\zeta_2(\sigma_e) - \rho$ starts at the origin, and the two left equilibria coincide.

Next we combine this solution with the cointegration relationship to obtain:

$$(1-\zeta_1 L)e_{t+1} = -\frac{1}{\beta}(1-\beta\zeta_1)\underbrace{(1-\zeta_1 L)\hat{b}_{t+1}}_{=\frac{\beta\zeta_1}{1-\beta\zeta_1\rho}\hat{\psi}_t} + \frac{\beta\zeta_1}{1-\beta\zeta_1\rho}(1-\zeta_1 L)\hat{\psi}_{t+1} = \frac{\beta\zeta_1}{1-\beta\zeta_1\rho}\left(\hat{\psi}_{t+1} - \frac{1}{\beta}\hat{\psi}_t\right),$$

and therefore e_{t+1} follows an ARMA(2,1) with AR roots ζ_1 and ρ and an MA root $1/\beta$.

Note that as $\chi_2 \to 0$, we have $\kappa \to 0$, $\zeta_1 \to 1$ and $\zeta_2 \to 1/\beta$, and therefore the stationary solutions AR(2) and ARMA(2,1) become integrated solutions of Lemma A2, namely ARIMA(1,1,0) and ARIMA(1,1,1).

Lastly, we characterize the equilibrium volatility of the innovation of the nominal exchange rate. We have:

$$\sigma_e^2 = \mathrm{var}_t(\Delta e_{t+1}) = \left(\frac{\beta\zeta_1}{1-\beta\zeta_1\rho}\right)^2 \mathrm{var}_t(\hat{\psi}_{t+1}) = \left(\frac{1}{\zeta_2-\rho}\right)^2\left(\frac{1}{1+\gamma\lambda_1}\frac{n/\beta}{m/(\omega\sigma_e^2)}\right)^2\sigma_\varepsilon^2, \tag{A95}$$

where we used Vieta's formula and the definitions of $\hat{\psi}_t$ as function of the primitive ψ_t with innovation ε_t with variance σ_ε^2. Note that ζ_2 also depends on σ_e^2 through χ_2, which determines κ:

$$\zeta_2 = \frac{(1+\kappa+\frac{1}{\beta}) + \sqrt{(1+\kappa+\frac{1}{\beta})^2 - \frac{4}{\beta}}}{2}, \qquad \kappa = \frac{\gamma\lambda_2\chi_2/\beta}{1+\gamma\lambda_1}, \qquad \chi_2 = \frac{\omega\bar{Y}}{m}\sigma_e^2.$$

Since $\zeta_2 \to 1/\beta$ as $\sigma_e \to 0$, (A95) always has a root $\sigma_e = 0$. Denote

$$d = \frac{1}{\beta(1+\gamma\lambda_1)}\frac{n\omega\sigma_\varepsilon}{m} \geq 0.$$

There exists a $\hat{d} > 0$ such that if $d < \hat{d}$, then $\sigma_e = 0$ is the only solution of (A95). For $d > \hat{d}$, (A95) has two non-zero solution, which satisfy:

$$\zeta_2 - \rho = d \cdot \sigma_e.$$

This is because $\zeta_2 \to 1/\beta$ as $\sigma_e \to 0$ and ζ_2 is convex in σ_e for $\sigma_e > 0$, and $\hat{d}$ is the unique value at which $\hat{d}\sigma_e$ and $\zeta_2 - \rho$ are tangent (see Figure A6). One of these two solutions has the property that $\partial\sigma_e/\partial d > 0$, and we select this solution as economically meaningful. One can rationalize this solution as stable by introducing explicit dynamics of entrepreneur's entry. More importantly, this solution becomes the unique non-zero solution of (A95) in the limit $\beta\rho \to 1$, as in this limit $(\zeta_2 - \rho)|_{\sigma_e=0} \to 0$, hence $\hat{d} \to 0$ and the other non-zero root merges with $\sigma_e = 0$ root. ■

Proof of Proposition 9 follows directly from Lemma A3. ■

Figure 2 plots impulse responses of the nominal exchange rate e_{t+j} to the innovation in $i_t - i_t^*$ for $j \geq 0$ obtained from a model with a financial sector, both its single-ψ_t-shock version and the multi-shock version describe in Section 4.2. For the single shock case, we construct this impulse response as $\frac{\partial e_{t+j}}{\partial \varepsilon_t} \Big/ \frac{\partial (i_t - i_t^*)}{\partial \varepsilon_t}$, where ε_t is the innovation of the ψ_t process (23), the only source of innovations in this version of the model. Given the closed-form solutions for both the nominal exchange rate and the interest rate differential, this impulse response is analytical. In fact, since (42) still holds in this model, we have $\frac{\partial (i_t - i_t^*)}{\partial \varepsilon_t} = \frac{\gamma\lambda_1}{1+\gamma\lambda_1}$, and therefore the impulse response of the exchange rate to the innovation in the interest rate is simply the impulse response of exchange rate to ε_t, as characterized by (48), scalled by $\frac{\gamma\lambda_1}{1+\gamma\lambda_1}$.

Next, consider the multi-shock version. In this case, the innovation to $i_t - i_t^*$ comes from a combination of shocks, and we define the impulse response as follows:

$$IRF_{i_t - i_t^*}^{e_{t+j}} = \sum\nolimits_{z \in \{\psi, a, \xi\}} \frac{\partial e_{t+j}}{\partial \varepsilon_t^z} \sigma_z \Big/ \sum\nolimits_{z \in \{\psi, a, \xi\}} \frac{\partial (i_t - i_t^*)}{\partial \varepsilon_t^z} \sigma_z,$$

where z indexes the shocks (ψ_t, a_t, ξ_t), ε_t^z is the innovation of respective shock, and σ_z^2 is its variance. Therefore, the standard-deviation-weighted response of $i_t - i_t^*$ to the innovations in the model. We calculate this impulse response numerically, by simulating $(i_t - i_t^*)$ and the time path $\{e_{t+j}\}_{j\geq 0}$ 10,000 times for random draws of $\{\varepsilon_t^z\}_{z \in \{\psi, a, \xi\}}$ with all other innovations set to zero, and taking $\text{median}\left(\frac{e_{t+j}}{i_t - i_t^*}\right)$ across this simulations for each $j \geq 0$.

Figure 3 The empirical impulse response functions in Figure 3 are calculated following closely Engel (2016) and Valchev (2016). The data used is for US vs trade-weighted average of Canada, France, Germany, Italy, Japan and UK, using monthly data from 1979:06 to 2009:10 provided by Engel (2016). As a result, the empirical impulse response in Figure 3a reproduces exactly that in Figure 2 of Engel (2016), while the empirical impulse response in Figure 3b differs slightly from that in Figure 2 in Valchev (2016) due to the difference in the dataset, yet the results are consistent qualitatively. The same procedures are applied to calculating impulse response in the model-generated data.

The impulse response in Figure 3a plots δ_j for $j \geq 1$, which are obtained as coefficients from the

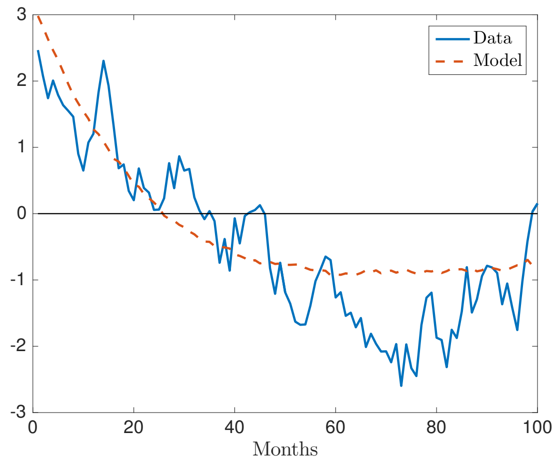

Figure A7: Impulse response of ρ_{t+j} to $i_t - i_t^*$

Note: this Figure complements Figure 3a, and plots the impulse response for the ex post realized UIP deviations $\rho_{t+j} = i_{t+j-1} - i^*_{t+j-1} - \Delta e_{t+j}$, instead of $\mathbb{E}_t\rho_{t+j}$, reproducing Figure 4 in Engel (2016), as described below.

regression:

$$\mathbb{E}_t\rho_{t+j} = \zeta_j + \delta_j(i_t - i_t^*) + u_{t+j},$$

where $\rho_{t+j} = i_{t+j-1} - i^*_{t+j-1} - \Delta e_{t+j}$ is the ex post one-period UIP deviation (risk premium) at $t+j$ and its conditional expectation $\mathbb{E}_t\rho_{t+j}$ is constructed using a VEC model for nominal exchange rate, price differential and nominal interest rate differential between countries, as described in detail in Engel (2016). In Figure A7 we plot a similar impulse for realized UIP deviations ρ_{t+j}, that is obtained from the following regression:

$$\rho_{t+j} = \zeta_j + \delta_j(i_t - i_t^*) + u_{t+j},$$

as in Figure 4 in Engel (2016). The impulse response in Figure 3b plots δ_j for $j \geq 0$ from:

$$e_{t+j} - e_t = \zeta_j + \delta_j(i_t - i_t^*) + u_{t+j},$$

where $\delta_0 = 0$ by construction.

Additional moments Table A2 reports two correlations — $\text{corr}(e_t, i_t - i_t^*)$ and $\text{corr}(\Delta e_t, \Delta i_t - \Delta i_t^*)$ — calculated both in the data (provided in Engel 2016, as descried above) and for different model specifications. In the data, both correlations are mildly *negative*. We consider four model specifications:

(1) multi-shock NOEM (corresponding to column 5 of Table 2) — both correlations are positive;

(2) multi-shock IRBC (column 6 of Table 2) — first correlation is positive, second is negative;

(3) single-ψ_t-shock model with a financial sector — both correlations are strongly positive;

(4) multi-shock model with a financial sector (column 7 of Table 2) — both correlations are mildly negative, as in the data.

Thus, Table A2 shows how a multi-shock model with a financial sector reproduces the empirical unconditional correlation moments in addition to the projection coefficients reported in Figure 3.

A.11 Additional extensions

We consider four additional extensions:

1. **Decreasing returns to scale**. For tractability, the baseline model assumes constant returns to scale in production, which allows to solve for prices as a function of exchange rate and exogenous shocks, independently from quantities. We relax this assumption and show that qualitatively decreasing returns to scale act similarly to a higher Frisch elasticity ν, and quantitatively both have only very mild effects on the properties of the model (see Appendix A.5.2). The detailed results are available from the authors upon request.

2. **GHH preferences**. The baseline model adopts the separable constant-elasticity utility in consumption and labor, for which the parameter σ acts simultaneously as the inverse intertemporal elasticity of substitution in dynamic decisions and the income effect elasticity in labor supply. We consider instead the GHH preference specification with no income effect on labor supply to explore robustness of our qualitative and quantitative results to this feature of the transmission mechanism. We show that the results remain robust qualitatively, and the only quantitative differences result in higher volatilities of the interest rates, consumption and output, slightly deteriorating the quantitative performance of the model. The detailed results are available from the authors upon request.

3. **A model with capital**. For simplicity, the baseline model abstracts from capital and dynamic investment decisions, to reduce the state space to a single net foreign asset variable. Below we show the robustness of our conclusions in Sections 3–4 to the introduction of capital accumulation.

4. **Full international business-cycle calibration** below shows that the ability of our model (with capital and productivity shocks) to match the exchange rate disconnect moments does not compromise its ability to match the conventional BKK-style international business cycle moments.

A.11.1 A model with capital

Setup We assume that firms rent capital from households, who in turn make the investment decisions. Capital is produced from country-specific consumption good with one period lag and potentially subject to capital adjustment costs. We continue to assume that only foreign bond is traded internationally, while domestic bond and capital stocks are traded only by local agents. Below we formulate optimization problems of home agents and derive equilibrium conditions in log-linear form.

Households The problem of Home household now includes the choice of capital investment:

$$\max_{\{C_t, L_t, I_t, K_{t+1}, B_{t+1}, B^*_{t+1}\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{L_t^{1+1/\nu}}{1+1/\nu}\right)$$

$$\text{s.t.}\quad P_t\left(C_t+I_t+\frac{\kappa}{2}\left(\frac{I_t}{K_t}-\delta\right)^2K_t\right)+\frac{B_{t+1}}{R_t}+\frac{B^*_{t+1}\mathcal{E}_t}{e^{\psi_t}R^*_t}\le B_t+B^*_t\mathcal{E}_t+W_tL_t+P_tR^K_tK_t+\Pi_t+T_t,$$

$$K_{t+1}=(1-\delta)K_t+I_t,$$

where κ is adjustment cost parameter and returns on capital R_t^K are in units of the final good. Labor supply and demand for bonds remain the same as in the baseline model and can be written in linearized form as follows:[76]

$$\begin{aligned}
\sigma c_t + \frac{1}{\nu} l_t &= w_t - p_t, \\
\mathbb{E}_t \left\{ \sigma \triangle c_{t+1} + \triangle p_{t+1} \right\} &= r_t, \\
\mathbb{E}_t \left\{ \sigma \triangle c_{t+1} + \triangle p_{t+1} \right\} &= r_t^* + \psi_t.
\end{aligned}$$

In addition, there is now an optimality condition for investment:

$$1+\kappa\left(\frac{I_t}{K_t}-\delta\right) = \beta \mathbb{E}_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left[R_{t+1}^K + (1-\delta)\left(1+\kappa\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\right) + \kappa\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\kappa}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2\right],$$

which equalizes costs of investment with expected future returns and a change in adjustment costs in the future. In steady state, this optimality condition pins down the rate of return on capital: $\beta\left(\bar{R}^K + 1 - \delta\right) = 1$, implying $\bar{R}^K = \frac{1}{\beta} - (1-\delta)$. Log-linearizing the capital law of motion and the optimality condition, and denoting $z_t \equiv \log I_t - \log \bar{I}$, we obtain:

$$\begin{aligned}
k_{t+1} &= (1-\delta)\, k_t + \delta z_t, \\
\kappa\delta\left(z_t - k_t\right) &= -\sigma \mathbb{E}_t \triangle c_{t+1} + \beta \mathbb{E}_t \left[\bar{R}^K r_{t+1}^K + \kappa\delta\left(z_{t+1} - k_{t+1}\right)\right].
\end{aligned}$$

Firms The pricing block of the equilibrium system remains unchanged except for the marginal costs. Assume that production function is Cobb-Douglas with a share ϕ_1 spent on intermediates. Out of the remaining $1-\phi_1$ part, ϕ_2 is the capital share and $1-\phi_2$ is the labor share. We choose steady state productivity level so that marginal costs and prices are equal 1. Log-linear approximation to the pricing block is then:

$$\begin{aligned}
p_t &= (1-\gamma)\, p_{Ht} + \gamma p_{Ft}, \\
p_{Ht} &= (1-\alpha)\, mc_t + \alpha p_t, \\
p_{Ft} &= (1-\alpha)\left(mc_t^* + e_t\right) + \alpha p_t, \\
mc_t &= \phi_1 p_t + (1-\phi_1)\,\phi_2 r_t^K + (1-\phi_1)(1-\phi_2)\, w_t - a_t.
\end{aligned}$$

Market clearing The market clearing conditions are more involved since we now have an additional market for capital:

- demand for labor: $w_t + l_t = y_t + mc_t$
- demand for capital: $r_t^K + p_t + k_t = y_t + mc_t$
- goods market equilibrium now includes investment demand and adjustment costs in addition to consumption and intermediates. For example, home demand for domestic goods is:

$$Y_{Ht} = (1-\gamma)\, e^{-\gamma \xi_t} \left[C_t + X_t + I_t + \frac{\kappa}{2}\left(\frac{I_t}{K_t}-\delta\right)^2 K_t\right].$$

[76] With a slight adjustment in notation, we use r_t in this section to denote the nominal interest rate, with i_t now used for the log-deviation of investment.

Up to the first order approximation, adjustment costs are equal zero, and therefore:

$$\begin{aligned}
y_t &= (1-\gamma)\, y_{Ht} + \gamma y_{Ht}^*, \\
y_{Ht} &= -\gamma\xi_t - \theta\,(p_{Ht} - p_t) + (1-\phi_1)\,(\varsigma c_t + (1-\varsigma)\, i_t) + \phi_1\,(y_t + mc_t - p_t)\,, \\
y_{Ht}^* &= (1-\gamma)\,\xi_t^* - \theta\,(p_{Ht}^* - p_t^*) + (1-\phi_1)\,(\varsigma c_t^* + (1-\varsigma)\, i_t^*) + \phi_1\,(y_t^* + mc_t^* - p_t^*)\,,
\end{aligned}$$

where $\frac{\bar{C}}{\bar{Y}} = (1-\phi_1)\,\varsigma = (1-\phi_1)\left(1 - \frac{\delta\phi_2}{R^K}\right)$. When capital share in production goes to zero, i.e. $\phi_2 = 0$, we get the same market clearing conditions as in the baseline model.

The last dynamic equation is country's budget constraint:

$$\beta b_{t+1}^* = b_t^* + \frac{\gamma}{1-\phi_1} nx_t,$$

where b_t^* is a net foreign asset position of the Home country and $nx_t = p_{Ht}^* + y_{Ht}^* + e_t - p_{Ft} - y_{Ft}$ is its net export. Finally, a log-linear approximation to the GDP is

$$gdp_t = \varsigma c_t + (1-\varsigma)\, z_t + \frac{\gamma}{1-\phi_1} nx_t.$$

Calibration All parameters from the benchmark model take the same values. Following the previous literature, we choose capital share in value added to be equal 0.3 and the quarterly depreciation rate of 0.02, which implies steady state capital-to-GDP ratio of 5. Adjustment cost parameter is calibrated together with relative volatilities of the shocks to match the relative volatility of investment in addition to exchange rate correlation with consumption and net exports.

Effect from capital It is convenient to separate the effect of capital on the economy into static and dynamic components. In the extreme case when adjustment costs go to infinity, the capital stock becomes constant. As a result, dynamic effect of capital vanishes. However, the presence of capital in production function implies that the technology exhibits decreasing returns to scale in labor. As we discuss above, decreasing returns to scale have similar implications as a higher Frisch elasticity of labor supply (with mild consequences for the quantitative performance of the model). In addition, when adjustment costs are finite, there is also a dynamic effect of capital coming from the intertemporal investment choice of households and time-varying stock of capital, a new state variable.

Results The process for exchange rate can be derived following the same steps as in the baseline case. The main difference is that we now have two states (NFA and capital) and two controls (exchange rate and consumption). It can be shown that for economically meaningful parameter values, the system has two eigenvalues greater than one (one of which is $1/\beta$ as in the baseline model), one eigenvalue smaller than one and one unit eigenvalue. It follows that each of the state variables follows ARIMA(2,1,1) processes, in contrast with the ARIMA(1,1,0) process for the NFA position in the baseline model.

In turn, exchange rate, being a linear function of the two state variables and financial shock, follows an ARIMA(2,1,2) process. Thus, the introduction of capital as an endogenous state variable increases the order of the stochastic process for exchange rate in the similar way as additional exogenous shocks in the baseline model. Importantly, the process remains integrated and indistinguishable from a random walk in the finite-sample numerical simulations of the calibrated model.

We further show that the introduction of capital does not affect the qualitative or quantitative properties of the model with respect to the behavior or real exchange rate and terms of trade. In particular, the real exchange rate still follows closely the volatile and persistent nominal exchange rate process, with the capital state variable introducing only a mild wedge in the relative dynamics of the two variables.

What concerns the exchange rate correlations with respectively the relative consumption (Backus-Smith) and the relative interest rates (Fama Forward Premium), the results in the model are no longer analytical, yet we show quantitatively that the calibrated model with realistic adjustment costs (calibrated to match the volatility of investment relative to the real exchange rate) is able to match both correlations (despite a somewhat different transmission mechanism for the interest rates). In addition, the model matches the empirical negative correlation between relative investment and exchange rate changes (similar pattern as with consumption), another moment at odds with both productivity and monetary shocks. Further detailed derivations and quantitative results are available from the authors upon request (also see Appendix Table A4).

A.11.2 International busyness cycle calibration

We now calibrate the model with capital and productivity shock to match the additional international business cycle moments. As a benchmark, we use the Backus, Kehoe, and Kydland (1994; henceforth, BKK) international RBC model, as well as the set of BKK moments on the comovement of macro variables (GDP, consumption, investment z and net exports), in particular across countries. We maintain all baseline parameters as in Table 1, and additionally calibrate the investment adjustment cost parameter κ, as well as the second moments of the shocks. In particular, we study three versions of the model:

1. a model with home and foreign productivity shocks (a_t, a_t^*) only, calibrated analogously to the BKK baseline model. In particular, our baseline calibration is consistent with the BKK's values of the key parameters γ and θ, while $\alpha = 0$ in BKK. The only major difference of the BKK model from ours is the completeness of international asset markets, while other differences (non-separable utility, time-to-build, VAR structure for productivity shocks) are of minor quantitative importance.
2. a full model with four shocks $(a_t, a_t^*, \tilde{\xi}_t, \psi_t)$ which extends the multi-shock IRBC model from Table 2 to allow for imperfectly correlated country-specific productivity shocks, as in BKK, as well as capital accumulation as described above. The goal of this calibration is to check whether an IRBC model with capital accumulation and financial shocks can successfully match the conventional international business cycle (BKK) moments without compromising its fit of the exchange rate disconnect moments.
3. a model with financial shock ψ_t only, for comparison.

In each case we calibrate the adjustment cost parameter to match the relative volatility of investment, which is about three times larger than that of GDP. In the BKK replication and in the full multi-shock model we calibrate the correlation between productivity shocks to match the correlation between GDP growth rates across countries. In the world model we additionally calibrate the relative volatility of the shocks to match the correlation of real exchange rate with consumption and net exports, as we did in Table 2.

The results of these calibrations are reported in Table A4 and we summarize here the main insights:

1. **BKK replication**: Our replication of BKK with incomplete markets allows to bring the cross-country correlations of consumption and DGP closer to the data, while the fit of all other moments is similar to the original BKK.

   The BKK calibration also reproduces the conventional exchange rate puzzles, including Meese-Rogoff, PPP, Backus-Smith, and UIP puzzles.

2. **Multi-shock IRBC**. The multi-shock model is successful at simultaneously matching the international business cycle moments and the exchange rate disconnect moments. In particular, this model does not need to compromise the fit of the disconnect moments we saw in Table 2, in order to do at least as good as the BKK in matching the business cycle moments.

   Furthermore, the model with a financial shock is considerably better on cross-country correlation, matching among other things the consumption correlation puzzle — the fact that consumption is less correlated than output — thanks to the risk-sharing shock ψ_t. The only moment for which the fit deteriorates is the correlation between net exports and GDP, predicted to be mildly positive versus mildly negative in the data.

   All exchange rate disconnect moments are on target as before, with a minor exception of the Fama β coefficient. Now it is about 1, but with a very wide confidence interval, which includes both 0 and -1.

3. **Single financial shock model**. This specification, while good for exchange rate disconnect moments, predicts the wrong signs for most business cycle moments. The reason is that ψ_t shock on its own results in counterfactual correlation for the macro variables, but in a full multi-shock model its role in shaping the macro variables is very modest.

4. **Variance decompositions**. Finally, the bottom panel of Table A4 reports the variance decomposition for nominal exchange rate and consumption (as one example of a macro variable). We see that the financial shock, while being dominant in shaping the exchange rate dynamics (almost 60%), has only a very modest contribution to the dynamics of consumption (16%). At the same time, productivity shock account for 77% of the dynamics of consumption, and for almost nothing in the dynamics of the exchange rate (where instead the international good demand shock $\tilde{\xi}_t$ plays a sizable secondary role). This explains why the multi-shock model can simultaneously reproduce both the exchange rate disconnect moments (due to ψ_t) and international business cycle moments (due to (a_t, a_t^*)).

Table A4: International busyness cycle calibration

	Data	BKK with (a_t, a_t^*) only: Original	BKK with (a_t, a_t^*) only: Replication	Model with ψ_t: Multi-shock	Model with ψ_t: ψ_t only
Panel A: Exchange rate disconnect moments					
$\rho(\triangle e)$	0.00		-0.04 (0.09)	-0.02 (0.09)	-0.01 (0.09)
$\rho(q)$	0.95		0.97 (0.02)	0.93 (0.04)	0.93 (0.04)
$\text{corr}(\triangle e, \triangle q)$	0.98		$-\mathbf{0.96}$ (0.02)	0.99 (0.00)	1
$\sigma(\triangle c - \triangle c^*)/\sigma(\triangle q)$	0.20		**0.81** (0.01)	0.23 (0.02)	0.37
$\text{corr}(\triangle c - \triangle c^*, \triangle q)$	-0.20		**1.00** (0.00)	-0.20 (0.09)	-1
$\text{corr}(\triangle nx, \triangle q)$	≈ 0		$-\mathbf{0.80}$ (0.08)	0.03 (0.09)	1
Fama β	$\lesssim 0$		1.3 (0.6)	1.5 (3.2)	-7.7 (4.4)
Fama R^2	0.02		0.04 (0.02)	0.00 (0.01)	0.04 (0.02)
Panel B: International busyness cycle moments					
$\sigma(\triangle c)/\sigma(\triangle gdp)$	0.49	0.47	0.35 (0.01)	0.53 (0.03)	2.60
$\sigma(\triangle z)/\sigma(\triangle gdp)$	3.15	3.48	3.78 (0.03)	3.15 (0.16)	3.15
$\text{corr}(\triangle c, \triangle gdp)$	0.76	0.88	0.99 (0.00)	0.72 (0.05)	-1
$\text{corr}(\triangle z, \triangle gdp)$	0.90	0.93	0.99 (0.00)	0.83 (0.03)	-1
$\text{corr}(\triangle nx, \triangle gdp)$	-0.22	-0.64	-0.52 (0.07)	**0.26** (0.09)	1
$\text{corr}(\triangle gdp, \triangle gdp^*)$	0.70	0.02	0.31 (0.08)	0.70 (0.05)	-1
$\text{corr}(\triangle c, \triangle c^*)$	0.46	0.77	0.37 (0.08)	0.51 (0.07)	-1
$\text{corr}(\triangle z, \triangle z^*)$	0.33		0.18 (0.09)	0.55 (0.06)	-1
Panel C: Variance decomposition					
Nominal exchange rate, $\text{var}(\Delta e)$:					
Productivity shocks, (a_t, a_t^*)			100%	1%	–
Financial shock, ψ_t			–	59%	100%
Consumption, $\text{var}(\Delta c)$:					
Productivity shocks, (a_t, a_t^*)			100%	77%	–
Financial shock, ψ_t			–	16%	100%

Note: The business cycle data and original model moments are from BKK (1994, as well as 1992). The other three columns are from our calibrations, as discussed in the text. In the multi-shock model, we calibrate $\sigma_a/\sigma_\psi = 5.4$, $\gamma\sigma_\xi/\sigma_\psi = 2.4$ and $\text{corr}(\Delta a, \Delta a^*) = 0.94$. Our BKK replication has $\text{corr}(\Delta a, \Delta a^*) = 0.26$. In the variance decomposition for the multi-shock model, the discrepancy from 100% is the contribution of the international good demand shock $\tilde{\xi}_t$.

A.12 Data appendix

Data sources for moments used in Tables 2, 4 and A1:

1. Persistence and volatility of nominal and real exchange rates: from Meese and Rogoff (1983) and survey by Rogoff (1996).
2. Moments for terms of trade and producer-price real exchange rate: from Table 1 in Atkeson and Burstein (2008), based on manufacturing prices and estimated for annual differences and HP-filtered quarterly data, 1975-2006.
3. Moments for consumption, investment and GDP: estimates by the authors. The data is for France, Germany, Italy and Spain from 1973 to 2000, quarterly.[77] We take first log differences for each series, calculate a weighted average across countries and take the difference with the corresponding series for the U.S. The weights are proportional to the PPP-adjusted GDP averaged across years. We prefer first-differenced moments, but the results are robust to HP-filtering.
4. Moments for net exports: estimates by the authors. Quarterly data for the U.S. versus the rest of the world, 1980 - 2015.
5. Slope coefficient β and R^2 in Fama regression: survey by Engel (1996) and recent estimates by Burnside, Han, Hirshleifer, and Wang (2011, Table 1) and Valchev (2016, Table B.1).
6. Volatility and persistence of the interest rate differential: estimates by the authors. Monthly data for the U.S. versus the U.K., France, Germany and Japan from 1979:06 to 2009:10.

   Note: the interest rates for individual countries have autocorrelation of $0.97 - 0.99$, while the autocorrelation for the interest rate differentials is lower, at $0.85 - 0.90$. In the model of Section 3, ρ corresponds to the persistence of both the level i_t and the differential $(i_t - i_t^*)$. In the multi-shock models of Section 4, we set $\rho = 0.97$ to target $\rho(i_t - i_t^*) = 0.90$.
7. Carry trade Sharpe ratio: the estimates for the forward premium trade from Hassan and Mano (2014, Table 2) .
8. Profits volatility (omitted from the tables for brevity): estimates by the authors. Quarterly data for the U.S., 1973 - 2015. We divide seasonally adjusted corporate profits (before taxes) by the seasonally adjusted nominal GDP, calculate the standard deviation of the first differences of this series and divide it by the standard deviation of changes in exchange rate.

[77] Our data goes through 2015, but we choose the pre-2000 subperiod to be consistent quantitatively with the moments reported in the earlier literature, as in the more recent period the correlation between relative consumption growth and real exchange rate changes became less negative.

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