Infrequent Rebalancing, Return Autocorrelation, and Seasonality*

Vincent Bogousslavsky†

First Draft: May 26, 2013
This Draft: October 15, 2015

Abstract

A model of infrequent rebalancing can explain specific predictability patterns in the time-series and cross-section of stock returns. First, infrequent rebalancing produces return autocorrelations that are consistent with empirical evidence from intraday returns and new evidence from daily returns. Autocorrelations can switch sign and become positive at the rebalancing horizon. Second, variations in the degree of infrequent rebalancing across periods increase the cross-sectional variance in expected returns in the period during which more traders rebalance. This effect generates seasonality in the cross-section of stock returns, which can help explain the empirical evidence.

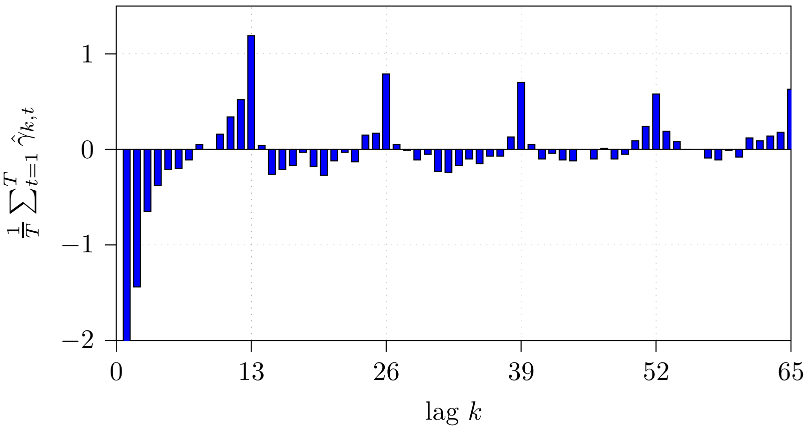
*I would like to thank Pierre Collin-Dufresne for his advice and many helpful discussions. I would also like to thank Daniel Andrei, Jérôme Detemple (discussant), Darrell Duffie, Thierry Foucault (discussant), Amit Goyal, Erwan Morellec, Ilaria Piatti, Jeffrey Pontiff, Omar Rachedi, Kenneth Singleton, and seminar participants at the 2013 Swiss Doctoral Workshop in Finance, EFA 2014 Doctoral Tutorial, and CQA 2014 Fall Conference for their comments. Financial support from the National Centre of Competence in Research "Financial Valuation and Risk Management" of the Swiss National Science Foundation is gratefully acknowledged. A previous version of this paper was titled "Infrequent Rebalancing and Short-Term Return Predictability."

†Swiss Finance Institute and EPFL. E-mail: vincent.bogousslavsky@epfl.ch

1 Introduction

Heston, Korajczyk, and Sadka (2010) document a striking pattern of periodicity in intraday returns that persists over several days. Reproducing their main finding, Figure 1 shows that the estimate from a cross-sectional regression of current half-hour returns on lagged half-hour returns spikes at intervals of one trading day for several days. The estimate can be interpreted as the return on a momentum strategy; a high or low return on a stock in a given half-hour interval today can help predict the return on the stock at the same time tomorrow and in the next days.

Figure 1: Time-series averages of cross-sectional regression estimates. The following cross-sectional regression is estimated using NYSE half-hour simple returns over 2001-2005: $r_{i,t} = \alpha_{k,t} + \gamma_{k,t} r_{i,t-k} + u_{i,t}$ for $k = 1, \ldots, 65$. The cross-sectional regressions are overlapping and run for every half-hour return. The data are reproduced from Heston et al. (2010) and scaled so that the units are percentages.



In addition, changes in trading volume exhibit a similar periodicity but do not fully explain the return periodicity. Heston et al. (2010) conjecture that systematic trading and institutional fund flows lead to predictable patterns in trading volume. They argue, however, that "if these patterns are fully anticipated, then they should not cause predictability in stock returns."

Motivated by this evidence, this paper contributes new theoretical and empirical results to the literature on return autocorrelation and seasonality by highlighting the role of infrequent rebalancing for asset price dynamics. I study a dynamic model in which a subset of agents only trade infrequently.[1] Indeed, the literature on slow-moving capital documents that many market participants are only active intermittently (Duffie, 2010).

The model shows that infrequent rebalancing generates specific return autocorrelation patterns. When traders absorb a liquidity shock in an asset, they end up holding an excess position in the asset relative to its normal weight in their portfolio. At a

[1] The setup builds on the model of Duffie (2010) and relates to the finance literature on models with overlapping generations. Related papers include Spiegel (1998); Watanabe (2008); Biais, Bossaerts, and Spatt (2010); Banerjee (2011); and Albagli (2014).

rebalancing date, traders with an excess position in the asset unload part of their position in the market. This unloading is equivalent to another liquidity shock. Infrequent rebalancing can then result in positive return autocorrelation by propagating liquidity shocks across periods. This effect also modifies the dynamics of trading volume. A large liquidity shock results in high volume during the current period. One rebalancing period later, infrequent traders adjust their abnormal positions by trading with market makers, which generates high volume again.

Unless liquidity shocks are highly persistent, autocorrelations are negative at any horizon in the economy without infrequent traders. More importantly, all autocorrelations have the same sign. With infrequent rebalancing, autocorrelations can switch sign around traders' rebalancing horizon and become positive. Momentum at the rebalancing date is key in matching the empirical evidence. Similarly, variations in trading volume are negatively autocorrelated at any horizon without infrequent rebalancing. The results appear robust to having infrequent traders with heterogeneous rebalancing horizons. This suggests that the model can simultaneously apply to different frequencies.

The infrequent rebalancing mechanism stressed by the theory can help understand the previous empirical evidence on intraday returns. Assuming that a fraction of agents trade only once a day, the model can reproduce the periodicity documented by Heston et al. (2010).[2] In the model, systematic trading generates predictable patterns in returns despite being perfectly anticipated. The model can also explain other recent evidence on intraday index returns. Gao, Han, Li, and Zhou (2014) find that the first half-hour return on the SPDR S&P 500 ETF predicts the last half-hour return. This result is in line with a fraction of agents adjusting their portfolios at the open and close of the market.[3]

Empirically, I provide new evidence on the impact of infrequent rebalancing on daily U.S. stock returns from 1983 to 2012.[4] Cross-sectional regressions in the spirit of Jegadeesh (1990) reveal patterns in return serial correlations that are consistent with a significant fraction of investors rebalancing at a weekly frequency.[5] The model can fit the short-term autocorrelation pattern. Neglected stocks do not drive the result since high turnover stocks display more pronounced patterns than low turnover stocks. This is in line with the theory, in which infrequent rebalancing is distinct from

[2] Heston et al. (2010) discuss why funds flows and trading algorithms may lead to periodicity in trading volume and order imbalances.

[3] As anecdotal evidence, *The Wall Street Journal* (September 10, 2010) reports the story of a proprietary-trading firm that is mostly active at the open and close of the market ("The Traders Who Skip Most of the Day").

[4] Papers that are closest to this one include the studies of Jegadeesh (1990) on the profitability of monthly contrarian strategies, and Lehmann (1990) on weekly return reversal in individual securities. Nagel (2012) provides a more recent analysis on the profitability of reversal strategies.

[5] Rakowski and Wang (2009) find a day-of-the-week effect in mutual fund flows. Besides, the rebalancing methodology documentation of several investment products suggests that weekly reviews may take place on specific days of the week.

thin trading. Daily volume change autocorrelations are broadly consistent with the theoretical predictions.

Both return autocorrelation and changes in the cross-sectional variance of average returns across calendar periods can drive the periodicity in Figure 1 (see Section 2). Intraday and monthly returns cross-sectional regressions show persistent seasonality patterns that go beyond autocorrelation effects. I extend the model to allow for variations in the proportion of infrequent traders across calendar periods. For instance, the well-known intraday U-shaped pattern in trading volume suggests that many market participants concentrate their trading at specific hours (Admati and Pfleiderer, 1988).[6] I show that this extended model can generate persistent seasonality patterns in line with the empirical evidence from intraday and monthly returns.[7]

In this extension, price impact varies across calendar periods. Traders require a larger risk premium to hold an asset when they anticipate price impact to be higher next period. More precisely, variations in the proportion of infrequent traders across calendar periods generate a seasonality in the market risk premium. If assets have different exposures to the market, then this mechanism amplifies the cross-sectional variance in expected returns in the period during which more traders rebalance. This effect generates seasonality in the cross-section of stock returns.

Several papers examine the impact of infrequent rebalancing on asset prices. Duffie (2010) surveys the literature on slow-moving capital and studies the conditional price response to a large liquidity shock. He does not discuss unconditional return properties and trading volume. Bacchetta and van Wincoop (2010) study the role of infrequent portfolio adjustments for the forward discount puzzle. Their setup is tailored to the foreign exchange market. In particular, liquidity shocks do not matter for predictability in their economy, while they play a key role in mine. Chien, Cole, and Lustig (2012) show that intermittent rebalancing increases the volatility of the market price of risk in a standard incomplete markets economy. Rinne and Suominen (2012) also investigate short-term return reversals. Their paper, however, focuses on liquidity and does not obtain the key prediction emphasized in this paper; namely, that infrequent rebalancing generates shifts in return autocorrelations. In contemporaneous research, Hendershott, Li, Menkveld, and Seasholes (2014) test a modified version of Duffie's model to shed light on deviations from efficient prices at different frequencies. Their analysis uses impulse response functions and does not overlap my approach and results. None of these papers examines return seasonality.

More broadly, this paper relates to the literature on heterogeneous investment

[6]Similarly, the fraction of agents who adjust their portfolios is likely not constant over a trading week or year (Dellavigna and Pollet, 2009; Hong and Yu, 2009).

[7]Investors' inertia has been shown to affect asset properties at longer horizons. Lou (2012) shows that the high persistence in mutual fund flows can explain part of the medium and long-term predictability in stock returns. Vayanos and Woolley (2013) provide a theory of momentum and reversal based on investment flows in a setup with rational agents.

horizons and trading frequencies. For instance, Corsi (2009) motivates a cascade model of realized volatility with heterogeneity in market participants' trading frequencies. Beber, Driessen, and Tuijp (2012) use heterogeneous investment horizons to study the pricing of liquidity risk. More recently, Kamara, Korajczyk, Lou, and Sadka (2015) empirically highlight the role of investors' different rebalancing horizons in determining risk premia.

The paper is organized as follows. Section 2 decomposes the cross-sectional regressions used in Figure 1 and in rest of the paper. Section 3 introduces a dynamic model with infrequent rebalancing. Section 4 studies return autocorrelation. Section 5 studies return seasonality. Section 6 examines trading volume. Section 7 concludes. All the proofs are in Appendix A.

2 Patterns in the Cross-Section of Stock Returns

Heston et al. (2010) estimate the following regression to obtain Figure 1:

$$r_{i,t} = \alpha_{k,t} + \gamma_{k,t} r_{i,t-k} + u_{i,t}, \tag{1}$$

where $r_{i,t}$ is the return on stock i in half-hour interval t. The regression coefficients are first estimated cross-sectionally at each date and then averaged over time (Fama and MacBeth, 1973). The cross-sectional regression methodology avoids several shortcomings of time-series estimates of serial correlation (Jegadeesh, 1990; Lehmann, 1990). As explained below, the cross-sectional regression estimates are, however, not exactly equivalent to autocorrelations.

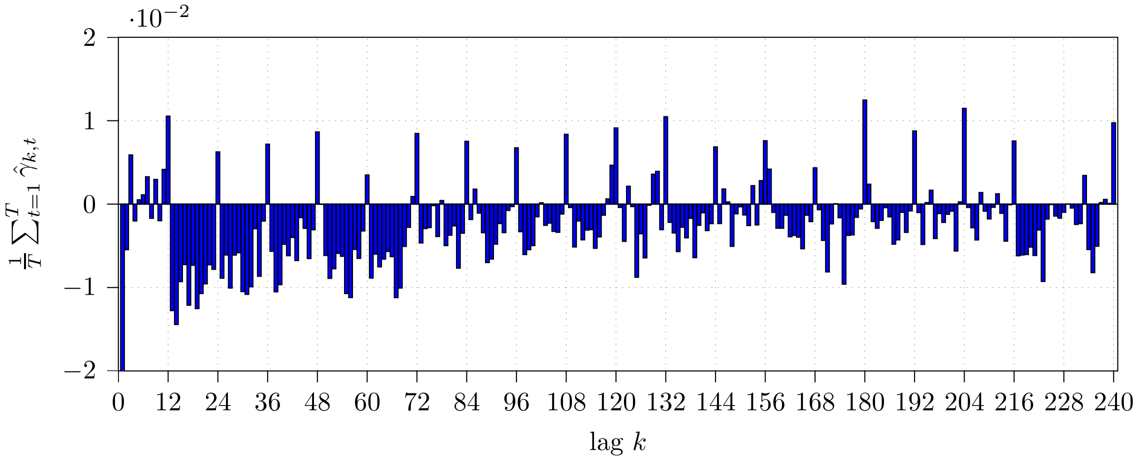
Heston and Sadka (2008) estimate the same cross-sectional regression on monthly returns. They document a striking seasonality pattern. I replicate their analysis in Figure 2. The average regression coefficient spikes every twelve lag. Most of the spikes are significant. Contrary to the intraday evidence, the seasonal spikes in the coefficients do not show any decay with the horizon.[8] Using similar strategies, Keloharju, Linnainmaa, and Nyberg (2014) provide substantial evidence about the pervasiveness of seasonalities across asset classes and markets.

In Section 4.3, I provide empirical evidence with daily returns using a similar methodology. Daily returns also exhibit specific patterns in their regression coefficients (Figure 5). The coefficients spike at intervals of five trading days.

In summary, periodicity patterns in stock returns exist at different frequencies. To better understand the sources of such patterns, one can decompose the average cross-sectional regression coefficient. Let $\bar{r}_t = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$. The slope coefficient estimate is

[8]In addition, an investment strategy that builds on this result earns an economically significant average return over the sample period (Heston and Sadka, 2008). This strategy shows a strong January seasonality, but the returns remain significant in other months and are not restricted to small stocks.

Figure 2: Time-series averages of cross-sectional regression estimates. The following cross-sectional regression is estimated each month: $r_{i,t} = \alpha_{k,t} + \gamma_{k,t} r_{i,t-k} + u_{i,t}$ for $k = 1, \ldots, 240$. The sample consists in U.S. common stock returns over the period 1964 to 2013 for the dependent variable. The right-hand side series starts in 1944. Stocks with a price lower than \$1 are excluded from the regressions.



given by

$$\hat{\gamma}_{k,t} = \frac{1}{\frac{1}{N}\sum_{i=1}^{N}\left(r_{i,t-k} - \bar{r}_{t-k}\right)^2} \underbrace{\sum_{i=1}^{N} r_{i,t} \frac{1}{N}\left(r_{i,t-k} - \bar{r}_{t-k}\right)}_{\equiv \pi_t(k)}. \tag{2}$$

The estimate closely relates to the profit of a relative strength strategy, denoted as $\pi_t(k)$. This zero-investment strategy is long past winners and short past losers based on their return in period $t-k$. Define the calendar function $c(t)$ that gives the calendar period for each date t (for instance, the day of the week). The expected return on the strategy in calendar period $c(t)$ is

$$\begin{aligned}\mathbb{E}[\pi_t(k)|c(t)] =& \frac{1}{N}\sum_{i=1}^{N}\mathrm{Cov}[r_{i,t}, r_{i,t-k}|c(t)] - \mathrm{Cov}[\bar{r}_t, \bar{r}_{t-k}|c(t)] \\ &+ \frac{1}{N}\sum_{i=1}^{N}\left(\mu_{i,c(t)} - \mu_{c(t)}\right)\left(\mu_{i,c(t-k)} - \mu_{c(t-k)}\right), \end{aligned} \tag{3}$$

where $\mu_{i,c(t)} \equiv \mathbb{E}[r_{i,t}|c(t)]$, and $\mu_{c(t)} \equiv \mathbb{E}[\bar{r}_t|c(t)]$. As a result, the average $\gamma_{k,t}$ coefficient in Equation (2) reflects three components: return autocorrelation, return cross-autocorrelation, and cross-sectional variation in average returns (Lo and MacKinlay, 1990).[9] Separating these different components is important because the periodicity patterns may not reflect the same components at different frequencies.

[9] Many papers investigate the source of momentum profits using a similar decomposition (see, for instance, Conrad and Kaul, 1998; Jegadeesh and Titman, 2002). Jegadeesh and Titman (1995) point out, however, that applying this decomposition empirically may not correctly distinguish between the autocovariance and cross-autocovariance components.

Since changes in trading volume also exhibit marked periodicity patterns, investors' trading seems a natural candidate to explain the patterns.[10] In this paper, I explore how infrequent rebalancing can help explain the empirical evidence from the cross-sectional regressions previously shown. First, infrequent rebalancing generates specific return autocorrelation patterns linked to the rebalancing horizon of traders (first component in Equation (3)). Second, infrequent rebalancing can generate persistent seasonality patterns. Indeed, the last component in Equation (3) does not decay with the lag. Persistent seasonality patterns in the average $\gamma_{k,t}$ can therefore arise when expected returns vary across calendar periods. I show that infrequent rebalancing can generate variations in expected returns across calendar periods. The next section presents a model to formalize this intuition.

3 A Dynamic Model with Infrequent Rebalancing

To better understand the impact of investors' trading on return and volume predictability patterns, I study a model in which some traders readjust their portfolio infrequently in an otherwise standard economy. The setup of the model builds on that of Duffie (2010). In particular, I extend the model to multiple assets to study the cross-sectional evidence of Section 2.

In addition, as suggested by extant empirical evidence on trading volume, the fraction of agents who adjust their portfolios is likely not constant over a trading day, week, or year. In this respect, Heston et al. (2010) find that their pattern is stronger in the first and last half-hour of trading. Following this evidence, I further extend the model to allow for a fixed but non-constant proportion of infrequent traders across periods. Theoretically, Admati and Pfleiderer (1988) demonstrate that traders may optimally cluster their orders at given periods.

3.1 The Economy

Time is discrete and goes from zero to infinity. At each date, N risky assets pay dividends. The $N \times 1$ vector of dividends follows a simple autoregressive process:

$$D_{t+1} = a_D D_t + \epsilon^D_{t+1}, \tag{4}$$

where $0 \leq a_D \leq 1$ represents the common dividend persistence. I assume that $\epsilon^D_{t+1} \sim \mathcal{N}(0, \Sigma_D)$, where Σ_D denotes the $N \times N$ variance-covariance matrix of dividend shocks. The mean dividend does not matter for return autocorrelation and seasonality and is

[10] Using changes in turnover instead of returns to estimate Regression (1) produces patterns that are similar to the return patterns (Heston and Sadka, 2008; Heston et al., 2010). In Section 6, I provide new empirical evidence using daily changes in turnover.

assumed to be zero. In addition, a risk-free asset with gross return $R > 1$ is available in perfectly elastic supply.

Two types of agents with exponential utility over terminal wealth trade in the economy. Frequent traders (also referred to as market makers) are present in the market at every date. A frequent trader of age j maximizes the value of her terminal wealth in $h-j$ periods. At the end of her trading cycle, the agent starts investing again with an horizon h. I assume a constant fraction of frequent traders across investment horizons. Given this assumption, at each date the following groups of frequent traders are active in the market: a fraction $\frac{1}{h}$ of frequent traders with horizon h, a fraction $\frac{1}{h}$ of frequent traders with horizon $h-1$, and so on.

Long horizons frequent traders are a natural extension to evaluate the robustness of multi-period return predictability patterns. Furthermore, investment horizons can have large effects on asset prices, as illustrated by Albagli (2014). Let $h-j$ be the remaining horizon of a frequent trader ($0 \leq j \leq h-1$). Her optimization problem is then given by

$$\max_{X^F_{t,j}} \mathbb{E}_t \left[-e^{-\gamma_F W^F_{t+h-j}} \right], \tag{5}$$

$$\text{s.t.} \quad W^F_{t+1} = (X^F_{t,j})' \left(P_{t+1} + D_{t+1} - RP_t \right) + RW^F_t,$$

where P_t is the vector of asset prices, and W^F_t is the initial wealth. The expectation is taken with respect to an information set that is common to all traders and includes the current and past levels of all state variables (defined below), as well as the current calendar period.

Infrequent traders—the second group of agents—trade to maximize the value of their terminal wealth and then leave the market for k period. The inattention period k is taken as exogenous. Bacchetta and van Wincoop (2010), Duffie (2010), and Chien et al. (2012) make a similar assumption. The tractability offered by this assumption allows one to draw clear predictions from the model. Solving for endogenous participation or inattention in general equilibrium settings is challenging.[11] It is unlikely that a fixed fraction of infrequent traders participate in the market each period; some investors may enter into the market when they perceive that profit opportunities outweigh their participation cost, which is a state-dependent trading rule, as opposed to the time-dependent rule implied by the exogenous k. In a partial equilibrium setting, Abel, Eberly, and Panageas (2007) find that a constant rebalancing interval is optimal when agents are subject to observation costs. In further research, Abel, Eberly, and Panageas (2013) show that in the presence of both information costs

[11] Orosel (1998) studies an overlapping generations economy with endogenous participation arising from a fixed cost of participation, but his setup does not include liquidity shocks. Taking another modeling approach, Peng and Xiong (2006) define an agent's attention to a particular stock as the precision of the signal he receives about the stock's future dividend. In this case, the agent is always active in the market but allocates his limited attention across different stocks.

and transactions costs, a time-dependent rule survives if the fixed component of the transactions costs is small enough. Sections 4 and 5 show that the model's implications are consistent with empirical evidence; hence, a simple approximation of investors' trading policies may help understand asset return properties.

The infrequent traders who are rebalancing at date t select their vector of asset demands X_t^I to maximize their expected utility:

$$\max_{X_t^I} \mathbb{E}_t\left[-e^{-\gamma_I W^I_{t+k+1}}\right], \tag{6}$$

$$\text{s.t.} \quad W^I_{t+k+1} = (X_t^I)' \underbrace{\left(P_{t+k+1} + \sum_{j=1}^{k+1} R^{k+1-j} D_{t+j} - R^{k+1} P_t\right)}_{\text{cumulative payoff from } t \text{ (ex-dividend) to } t+k+1} + R^{k+1} W_t^I,$$

where W_t^I is the initial wealth. Infrequent traders adjust their portfolio and do not trade for the rest of their investment horizon. The dividends paid while the agent is out of the market are reinvested at the risk-free rate.

The model requires an additional element to generate trade. Here, liquidity traders supply inelastic quantities of assets every period. Equivalently, a fraction of market makers could receive state-contingent endowment shocks as in the setup of Biais et al. (2010). Liquidity traders' supplies are given by the following zero-mean $N \times 1$ process:

$$\theta_{t+1} = a_\theta \theta_t + \epsilon^\theta_{t+1}, \tag{7}$$

where $0 \leq a_\theta \leq 1$ represents liquidity trading persistence. I assume that $\epsilon^\theta_{t+1} \sim \mathcal{N}(0, \Sigma_\theta)$, where Σ_θ denotes the $N \times N$ variance-covariance matrix of liquidity shocks.

The autocorrelation effect highlighted in Section 4 requires that a shock affecting traders' positions reverses over time. The model allows this shock to be asset-specific or common to many assets. Importantly, the infrequent rebalancing mechanism does not require any persistence in the shock to generate specific return predictability patterns ($a_\theta = 0$). To focus on the simplest possible setting, I use an autoregressive process of order one. This assumption also makes the setup comparable to previous literature.

3.2 Equilibrium

Infrequent and frequent traders are present in proportion q and $1 - q$ in the economy, respectively. I consider two cases. First, the mass of rebalancing infrequent traders at each date is constant over time. Second, the mass of rebalancing infrequent traders varies with the calendar period and equals $q_{c(t)}$, where $c(t)$ indicates the calendar period at date t. With C calendar periods, $\sum_{j=1}^{C} q_j = q$. In this general case, market clearing requires

$$q_{c(t)} X_t^I + \frac{1-q}{h} \sum_{j=0}^{h-1} X_{j,t}^F = \bar{S} + \theta_t - \sum_{i=1}^{k} q_{c(t-i)} X_{t-i}^I, \tag{8}$$

where $\bar{S}$ is the $N \times 1$ vector of share supplies.[12] The lagged demands of infrequent traders reduce the number of shares available in the market today.

The following three conditions define a linear rational expectations equilibrium (REE). (i) Prices and demands are linear functions of the state variables. (ii) Agents optimize Problems (5) and (6). (iii) Markets clear according to (8).

I first focus on the case in which the mass of rebalancing infrequent traders is constant every period. This provides a benchmark model to focus on return autocorrelation. I study the general model with a varying mass of infrequent traders in Section 5.

3.3 Constant Proportion of Infrequent Traders

A constant and identical proportion of infrequent traders readjust their portfolio every period, such that $q_{c(t)} = \frac{q}{k+1}$.

Proposition 1. *In a linear stationary REE, if it exists, the vector of asset prices is given by*

$$P_t = \bar{P} + P_\theta \theta_t + \frac{a_D}{R - a_D} D_t + \sum_{i=1}^{k} P_{X_i} X_{t-i}^I, \tag{9}$$

where the matrices of coefficients are solutions to a system of nonlinear equations given in the Appendix.

The lagged demands of infrequent traders are state variables in equilibrium. The matrices P_{X_i} determine how lagged demands affect current prices. The matrix P_θ reflects the price impact of liquidity shocks. The price vector includes the present value of expected future dividends discounted at the risk-free rate; indeed, $\mathbb{E}_t\left[\sum_{j=1}^{\infty} R^{-j} D_{t+j}\right] = \frac{a_D}{R-a_D} D_t$.

Polar cases of the economy help gain intuition since the equilibrium coefficients have to be solved for numerically.[13] When $q = 0$ (or $k = 0$), only frequent traders are active in the market, and therefore lagged demands are not state variables anymore.

[12] If $\bar{S} = 0_{N\times 1}$, then the unconditional expected excess return is zero for all assets. Thus, to study expected returns I assume that all the assets are in positive supply. Some securities can be in zero net supply as long as they are correlated with securities in positive supply.

[13] This results from having multiple groups of traders. Watanabe (2008), Biais et al. (2010), and Banerjee (2011) also resort to numerical solutions.

The price vector is then given by

$$P_t = \bar{P} + P_\theta \theta_t + \frac{a_D}{R - a_D} D_t. \tag{10}$$

The price vector (10) has the same form whether $h = 1$ or $h > 1$, but an analytical solution for P_θ is only available when $h = 1$ because of the nonlinear hedging demands.[14] I refer to this economy as the *frictionless economy*.

The following corollary solves for the equilibrium coefficients when the economy contains only infrequent traders with inattention period k (*infrequent rebalancing economy*).

Corollary 1. *Infrequent rebalancing economy. Assume that $q = 1$. In a linear stationary REE, the lagged demands price coefficients in Equation (9) are given by*

$$P_{X_1} = P_{X_2} = \ldots = P_{X_k} = -\frac{1}{k+1}\left(\frac{R^{k+1} - a_\theta^{k+1}}{R^{k+1} - a_\theta^k}\right) P_\theta, \tag{11}$$

where P_θ solves a quadratic matrix equation given in the Appendix.

When $q = 1$, Equation (11) shows that P_{X_i} and P_θ are proportional to each other.[15] Since agents only trade on liquidity shocks, lagged demands directly reflect past liquidity shocks. To gain intuition, assume for example that liquidity traders sell a large quantity of the asset. The price drops to give agents an incentive to hold the additional asset supply. The traders who accommodate the liquidity shock now hold the asset in excess of their steady-state optimal position. As a result, these traders want to liquidate their abnormal holdings when they rebalance their portfolio in $k + 1$ periods. At that future date, the rebalancing trades create a selling pressure proportional to the initial liquidity shock. This mechanism has specific implications for return autocorrelation, which I explain in Section 4.1.

3.3.1 Equilibrium Multiplicity and Existence

The infrequent rebalancing economy solves the same problem as the frictionless economy with adjusted fundamental parameters. Thus, the results of Watanabe (2008) for the frictionless economy apply. In particular, he shows that if liquidity and dividend shocks volatilities and correlations are the same for all assets, then only four "symmetric" equilibria exist (i.e., equilibria in which price and demand coefficients are equal across assets). A "low volatility" equilibrium coexist with three "high volatility" equilibria. This multiplicity stems from the infinite horizon of the economy and the

[14] The case $h = 1$ is similar to the model of Spiegel (1998). Corollary 2 in the Appendix provides the analytical solution for this case.

[15] The negative sign in Equation (11) comes from the market clearing condition (8); if θ_t represents demand shocks instead of supply shocks, then P_θ and P_{X_1} have the same sign.

finite lives of agents. The low volatility equilibrium is the unique equilibrium of the finite horizon frictionless economy (Banerjee, 2011). Moreover, as agents lives' goes to infinity in the frictionless economy (with intermediate consumption), a unique linear equilibrium always exists (Albagli, 2014). Albagli's analysis further suggests that the low volatility equilibrium converges to this unique equilibrium. I show in the online Appendix that the low volatility equilibrium is the only "stable" equilibrium when $q = 0$ or $q = 1$.[16]

When $0 < q < 1$, I found multiple equilibria in all my numerical calibrations. Assuming that fundamental parameters are the same for all assets, I always found four symmetric equilibria that converged to the analytical polar cases as $q \to 0$ or $q \to 1$. For the previous reasons, I focus my analysis on the low volatility equilibrium. Importantly, the paper's main results also hold in the high volatility equilibria. This is because the analysis does not rely on comparative statics, for which different equilibria typically give opposite results (see, for instance, Banerjee, 2011).

Concerning existence, the effect of fundamental parameters is intuitive in both polar economies; more volatile and persistent sources of risk shrink the existence region. Increasing the persistence of liquidity trading a_θ may, however, widen the existence region when $q = 1$, as explained in the online Appendix. The exact equilibrium existence conditions in the polar economies are given in the online Appendix. When $0 < q < 1$, numerical experiments indicate that a small q helps obtain an equilibrium. Increasing a_D, σ_θ, or σ_D directly increases volatility. High volatility leads to non-existence. More precisely, a risk-averse agent with a finite horizon requires a price discount to absorb a liquidity shock. This price discount increases price volatility. Increased volatility leads the agent to require an even larger discount. An equilibrium fails to exist if the loop does not converge. Since a_θ may have an opposite effect on the existence region when $q = 0$ and $q = 1$, a_θ can have an ambiguous effect on the existence region when $0 < q < 1$. Increasing h helps find an equilibrium, in line with the results of Albagli (2014).

4 Return Autocorrelation

This section examines return autocorrelation in a dynamic equilibrium model in which some traders adjust their portfolios infrequently.

[16]Multiple equilibria arise because agents have self-fulfilling beliefs about the volatility of future prices. Following Bacchetta and Van Wincoop (2006), stability requires an equilibrium to be robust to a small deviation in next period's belief regarding volatility. The online Appendix is available at `https://documents.epfl.ch/users/b/bo/bogoussl/www/IR_onlineAppendix.pdf`.

4.1 Theory

In the frictionless economy, the vector of (dollar) excess returns between time $t+s-1$ and $t+s$ is given by

$$
\begin{aligned}
Q_{t+s} \equiv & P_{t+s} + D_{t+s} - RP_{t+s-1} \\
= & P_{\theta}\epsilon^{\theta}_{t+s} + \frac{R}{R-a_D}\epsilon^{D}_{t+s} + (a_{\theta} - R)P_{\theta}\theta_{t+s-1}.
\end{aligned}
$$

A dividend shock affects prices but does not modify expected returns (Wang, 1994). Return autocovariances are then given by

$$
\mathrm{Cov}[Q_{t+s}, Q_t] = (a_{\theta}R - 1)a_{\theta}^{s-1}\frac{R-a_{\theta}}{1-a_{\theta}^2}P_{\theta}\Sigma_{\theta}P_{\theta}', \quad a_{\theta} < 1, s \geq 1. \tag{12}
$$

Dividend persistence does not affect the sign of excess return autocovariances.[17] Since the price vector (10) takes the same form when $h > 1$, Equation (12) also shows that long horizons affect neither the sign of the autocovariances nor their rate of decay. The model requires $a_{\theta}R < 1$ to produce short-term return reversal, which is widely documented by previous research (see, for instance, Jegadeesh, 1990) and confirmed by the empirical analysis on daily returns in Section 4.3.

When $0 < a_{\theta}R < 1$, the frictionless model predicts that all return autocovariances are negative at any horizon and decay exponentially. The negative autocorrelation of price changes stems from the reversal of transitory order flows and the risk-aversion of market makers (Grossman and Miller, 1988). Makarov and Rytchkov (2012) demonstrate that a version of Equation (12) holds for the more general case of asymmetrically informed traders. They show that asymmetric information alone cannot generate price momentum in the standard *stationary* setting in which liquidity trading follows a first order autoregressive process. This implication contrasts with the finite horizon model of Cespa and Vives (2012), in which autocorrelations are positive if information quality increases sufficiently across periods and liquidity trading is persistent enough (see also Holden and Subrahmanyam, 2002).

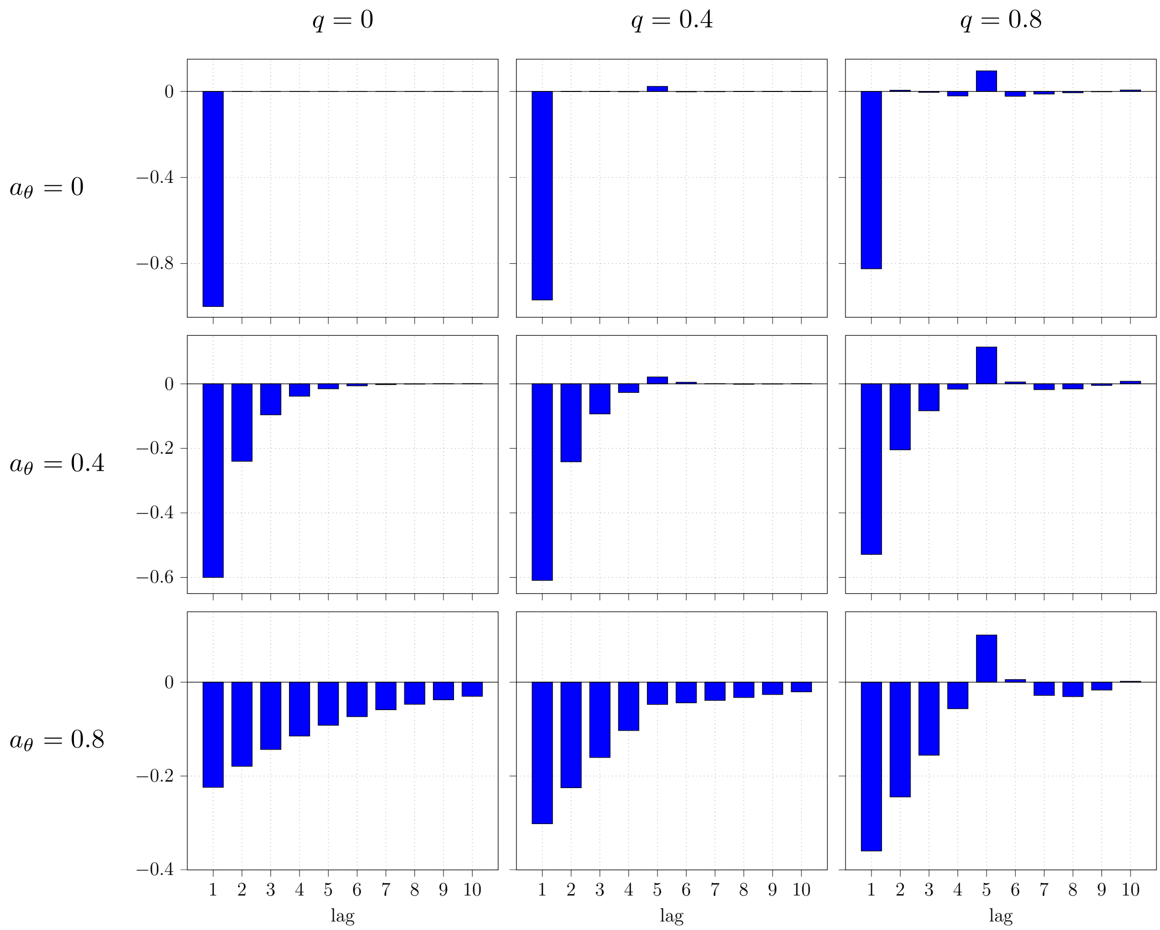
In a stationary setup, liquidity shocks determine autocovariance dynamics because of the market clearing condition. When $q = 0$ (and $h = 1$), the market clearing condition is $\gamma_F\Sigma\left(\theta_t + \bar{S}\right) = \mathbb{E}_t[Q_{t+1}]$, where $\Sigma \equiv \mathrm{Var}_t[P_{t+1} + D_{t+1}]$ is a constant matrix. This implies that $\mathrm{Cov}[Q_{t+1}, Q_t] = \gamma_F\Sigma\mathrm{Cov}[\theta_t, Q_t]$. Since $\mathrm{Cov}\left[\epsilon^D_t, \epsilon^{\theta}_t\right] = 0$, signals about future dividends are not informative about future liquidity shocks and

[17] Campbell, Grossman, and Wang (1993) derive a similar equation in a single-asset setup with myopic agents and exogenously time-varying risk aversion instead of liquidity shocks. Following their paper and the related literature, I focus my analysis on dollar returns Q_t to highlight the economic intuition. Percentage returns are not well-defined with normally distributed prices and do not have analytical expressions. Numerical experiments indicate that the main qualitative results hold with percentage returns.

cannot help generate positive return autocorrelation alone.[18]

According to the model, infrequent rebalancing can have a large impact on return autocorrelation. Figure 3 displays the first ten autocorrelations generated by the model for different degrees of infrequent rebalancing and persistence parameters of liquidity trading. The patterns are robust to variations in the other parameters. The calibration is detailed Appendix B and assumes that infrequent traders readjust their portfolios every five periods. To focus solely on the patterns generated by infrequent rebalancing, I scale the autocorrelations so that their absolute values sum up to one for the first ten lags.

Figure 3: Autocorrelations (scaled) for different persistence levels of liquidity trading (a_θ) and degree of infrequent rebalancing (q). The figure plots the scaled first element of the matrix $\text{Cov}[Q_{t+s}, Q_t]$ for $s = 1, \ldots, 10$. The autocorrelations are scaled so that their absolute values sum up to one for the first ten lags. The calibration is shown in Table 1 (left column).



[18]The previous result holds in the model of Biais et al. (2010), which uses endowment shocks. Asymmetric information can increase return autocorrelation but cannot make it positive unless $a_\theta R > 1$. In a stationary setup, Albuquerque and Miao (2014) obtain positive autocorrelation with a signal about future dividends. The main trading mechanism of their model is, however, the existence of a non-traded investment opportunity as in the model of Wang (1994). The hedging motive relies on a non-zero correlation between dividend shocks and private investment shocks, which is why the signal affects return autocorrelation.

The left column shows autocorrelations in the frictionless economy. These autocorrelations are always negative and decay proportionally to the persistence of liquidity trading. As shown in the middle and right columns, infrequent rebalancing shifts the autocorrelations around the rebalancing horizon. In particular, autocorrelations can switch sign and become positive regardless of the persistence of liquidity trading. Even in a similar non-stationary setting, returns reverse when the liquidity trading is transient. In the model of Cespa and Vives (2012), return autocorrelations are always negative when $a_\theta = 0$, in spite of the non-stationary variance dynamics associated with the gradual revelation of information.

To understand the underlying mechanism, consider the single-asset case and assume that a large liquidity shock takes place at date t. The price drops so that agents who are present in the market accommodate the shock; hence, Q_t is low. Infrequent traders partially absorb the liquidity shock, and X_t^I is larger than its steady-state level. At time $t+k+1$, the infrequent traders come back to the market. Since liquidity trading is transient, these traders now hold an abnormal position in the asset relative to the current asset supply. Thus, they liquidate part of their excess holdings. The resulting order flow is equivalent to a liquidity shock; the price drops, and Q_{t+k+1} is low. This effect increases $\mathrm{Cov}[Q_{t+k+1}, Q_t]$. Infrequent rebalancing is akin to serially correlated liquidity shocks, which is why autocorrelations can become positive despite the result of Makarov and Rytchkov (2012). A liquidity shock today transmits to the future date when agents rebalance their holdings.

More formally, consider a single-asset economy with $k = 1$ and $a_\theta = 0$. In this case, all autocovariances beyond the first lag are zero in the frictionless economy. This provides a clean benchmark. The next proposition formalizes the intuition developed previously.

Proposition 2. *Let $a_\theta = 0$, $k = 1$, and $h = 1$. In the single-asset economy with $0 < q < 1$, if $P_\theta < 0$ and $P_X > 0$, then $\mathrm{Cov}[Q_t, Q_{t+1}] < 0$ and $\mathrm{Cov}[Q_t, Q_{t+2}] > 0$.*

The conditions $P_\theta < 0$ and $P_X > 0$ are intuitive and hold in the polar economies. First, a liquidity shock should have a negative price impact. Second, a positive lagged demand should increase the price of the asset since it restricts the current asset supply. Under these conditions, infrequent traders absorb part of the liquidity shocks and therefore provide liquidity when $0 < q < 1$.[19]

Proposition 2 formally shows that infrequent rebalancing generates positive return autocorrelation when liquidity trading is transient and that autocorrelations can switch sign. As indicated by Figure 3, a similar effect applies when $k > 1$. In summary, the theory produces the following implication.

[19] See Lemma 4 in the Appendix. These conditions always held in the four symmetric equilibria that I found numerically. Assuming that $k = 1$ and $h = 1$ is made for convenience and does not appear to affect the result.

Implication. With infrequent rebalancing, return autocorrelations are subject to shifts linked to traders' rebalancing horizon and can switch sign. Without infrequent rebalancing, all return autocorrelations have the same sign and decay exponentially.

This mechanism seems specific to the model. The online Appendix presents a model in which liquidity trading occurs at low and high frequencies. That is, a fraction of liquidity traders trade infrequently. I show that autocorrelations are negative unless liquidity trading is highly persistent and cannot switch sign if the first autocorrelation is negative. The key difference is that infrequent traders provide liquidity (Lemma 4 in the Appendix). Thus, when they liquidate their abnormal positions, they trade in the same direction as the initial liquidity shock that they absorbed. The same is not true for low-frequency liquidity shocks since they revert over time.

Positive return autocorrelation can be obtained by mechanically adjusting the liquidity trading process (7). Assuming that $\theta_t = \epsilon_t^\theta + \beta\epsilon_{t-k}^\theta$ leads to a price function of the form $P_t = P_\theta\epsilon_t^\theta + \sum_{i=1}^k P_{\theta,i}\epsilon_{t-i}^\theta$. Economically, this specification of liquidity trading can be broadly interpreted as a form of order-splitting strategy. If $\beta > 0$, this setup produces positive autocorrelation between the excess return today and the excess return in k periods. For instance, with $k = 2$, $\text{Cov}[Q_t, Q_{t+2}] = \beta\Sigma\Sigma_\theta\Sigma$. This result illustrates that infrequent rebalancing propagates liquidity shocks across periods.

4.1.1 Heterogeneous Rebalancing Horizons

In the online Appendix, I extend the benchmark model to allow for infrequent traders with heterogeneous rebalancing horizons. More precisely, I consider an economy with two groups of infrequent traders (in addition to frequent traders). Group i has a mass q_i and an inattention period k_i. Though analytical solutions are again not available, the rebalancing mechanism seems robust to having multiple groups of infrequent traders. Namely, the autocorrelation pattern is subject to shifts at *both* rebalancing horizons, $k_1 + 1$ and $k_2 + 1$. In particular, both autocorrelations can switch sign. This suggests that the model can simultaneously explain predictability patterns at different frequencies.

4.2 Empirical Evidence: Intraday Returns

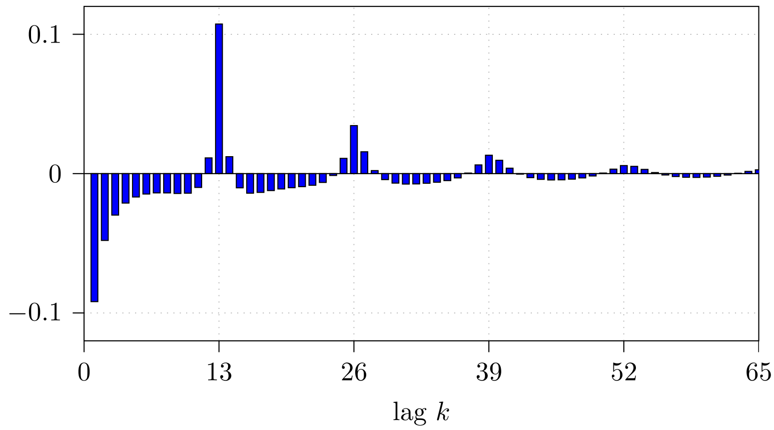
Figure 3 suggests that a model in which a fraction of traders only adjust their portfolio once a day can help explain the predictability pattern documented by Heston et al. (2010) and reproduced in Figure 1. The multi-asset settings allows for an exact replication of the regressions using simulated returns from a calibrated version of the model.[20]

[20] Solving the model for a large number of assets is numerically challenging with high k and correlated assets. To ease the procedure, one can assume that the variance-covariance matrices of dividends and liquidity shocks commute and use an eigenvalue decomposition. The method only requires to solve for $(2k + 2)$ eigenvalues independently of the number of assets.

Since the current model only relies on the autocorrelation component of Equation (3), the regression estimates are almost identical to autocorrelations in the model. For clarity, I report autocorrelations. This paper does not aim to provide an exact quantitative match to the data. The parameters are therefore chosen to broadly match the patterns observed in the data while keeping the calibration as simple and transparent as possible. Appendix B details the calibrations used in the paper.

Figure 4 plots the autocorrelations obtained from the model. The results are in line with the empirical evidence; as expected, the regression coefficient spikes at horizons that are multiples of one trading day (since a trading day is composed of 13 half-hour intervals, traders' inattention period is set to $k = 12$). Infrequent rebalancing produces a persistent pattern of return predictability despite being perfectly anticipated by frequent traders.

Figure 4: Autocorrelations predicted by the model for intraday returns. The calibration is shown in Table 1.



In Figure 4, the proportion of infrequent traders must be set to a high level (i.e., $q = 0.99$) for the pattern to persist over several days. A small fraction of frequent traders is consistent with the calibrations of related papers.[21] The model also abstracts from transaction costs; these costs limit the arbitrage activity of frequent traders and could therefore partially explain the persistence of the pattern in the data (Heston et al., 2010). The decay in the coefficients is consistent with a repeated shock explanation. But the persistence of the pattern at higher lags points towards cross-sectional variations in average returns that differ across calendar periods (see Section 2). Section 5 investigates this effect, which generates persistent seasonality patterns.

Heston et al. (2010) report that changes in trading volume exhibit similar periodic daily patterns. The model also predicts this relationship. A large liquidity shock

[21] Chien et al. (2012) assume 5% of active traders, 45% of intermittent traders and 50% of non-participants in their economy. Bacchetta and van Wincoop (2010) study a foreign exchange market setup only populated by infrequent traders. The results are robust to variations in the other parameters; for instance, liquidity shocks volatility can be adjusted to calibrate the magnitudes of the coefficients.

results in high volume during the current period. One day later, infrequent traders reduce their abnormal positions and generate high volume again. I examine trading volume in Section 6.

4.3 Empirical Evidence: Daily Returns

This section examines whether daily returns exhibit predictability consistent with infrequent rebalancing. I use daily returns on NYSE and Amex common stocks from CRSP over the period January 1983 to December 2012. The data are cleaned as follows: CRSP share code is equal to 10 or 11; penny stocks (average price less than one dollar) are eliminated; returns above 400% are winsorized; each stock is required to have at least 250 days of data. This procedure leaves an average of 2000 stocks each period in the data set. I focus on the last thirty years of data because structural shifts in investors' rebalancing frequencies are likely to be an issue over longer samples.

Intuitively, conjecture that some traders rebalance at a weekly frequency (i.e., every five consecutive trading days). For instance, Rakowski and Wang (2009) find a day-of-the-week effect in mutual fund flows. Alternatively, it could be that investment products are rebalanced on specific days.[22] To test this intuition, I use the methodology of Jegadeesh (1990) and estimate a multiple cross-sectional regression of current returns on lagged returns at each date.

As explained in Section 2, cross-sectional variations in average returns across calendar periods can generate persistent seasonality patterns that are picked up by the regression coefficients. This is likely to be a concern here since prior research documents that average stock returns are not equal across days of the week (French, 1980; Gibbons and Hess, 1981). The infrequent rebalancing model developed in Section 3.3 provides a repeated shock explanation for return predictability, although variations in unconditional expected returns across days of the week could arise from variations in the degree of infrequent trading throughout the week (see Section 5). To focus on the repeated shock mechanism, I estimate the following cross-sectional regression at each date:

$$r_{i,t} = \alpha_t + \gamma_{1,t} r_{i,t-1} + \ldots + \gamma_{l,t} r_{i,t-l} + \gamma_{\mu,t} \mu_{i,t} + u_{i,t}, \tag{13}$$

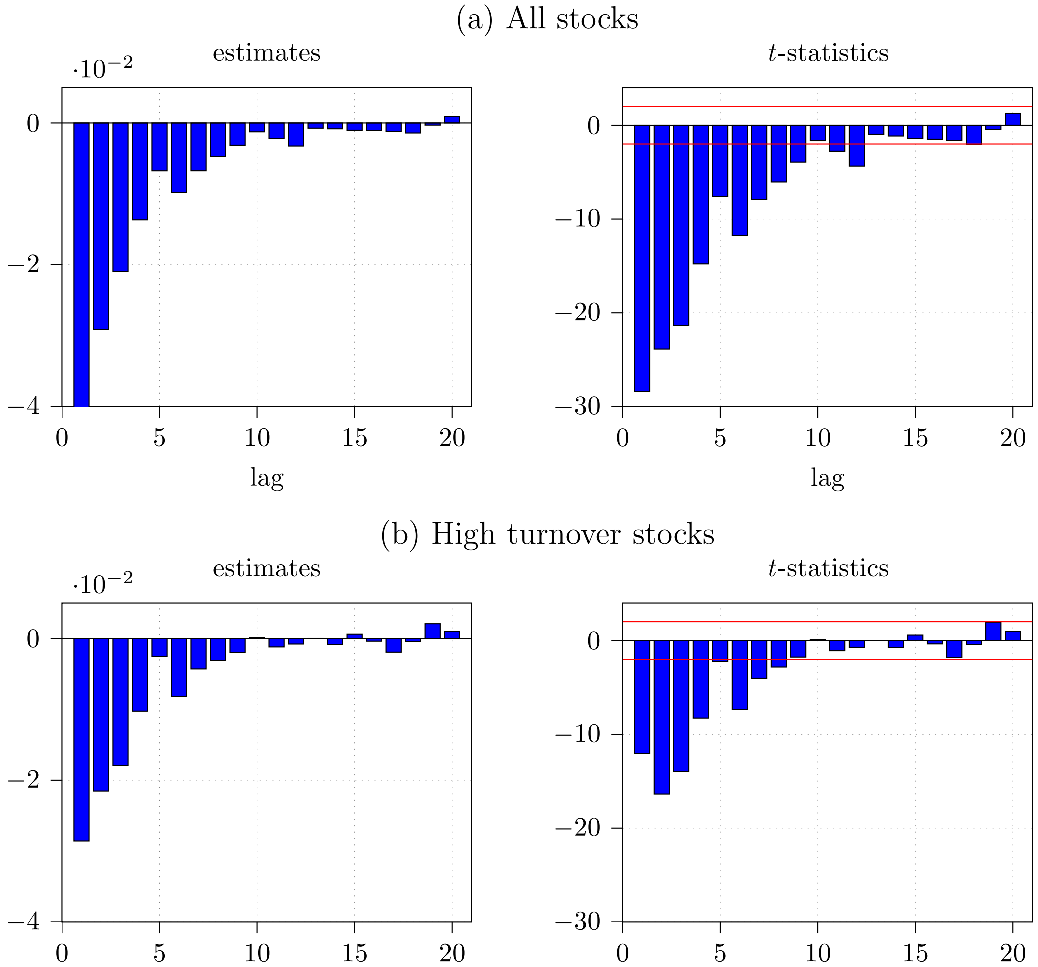
where μ_{it} is the average *same-day* (as day t) return on stock i over the previous year (excluding the past l returns). Here, μ_{it} controls for variation in expected returns across days of the week, which is similar to a day-of-the-week fixed effect. Regression (13) follows the methodology of Keloharju et al. (2014) but uses a multiple regression. Multiple regressions provide a cleaner picture of autocorrelation patterns

[22] An example is the S&P Leveraged Loan Index for which each weekly review "typically occurs on Friday." Source:
http://us.spindices.com/indices/fixed-income/sp-lsta-us-leveraged-loan-100-index.

than univariate regressions (1).

The upper panel of Figure 5 plots the time-series averages of the cross-sectional regression estimates with $l = 20$ and their associated Newey and West (1987) t-statistics. The results are not sensitive to the precise number of lags. At short horizons, the co-

Figure 5: Time-series averages of cross-sectional regression estimates. The following cross-sectional regression is estimated each day: $r_{i,t} = \alpha_t + \gamma_{1,t} r_{i,t-1} + \ldots + \gamma_{20,t} r_{i,t-20} + \gamma_{\mu,t}\mu_{i,t} + u_{i,t}$, where μ_{it} is the average same-day (as day t) return on stock i over the previous year excluding the past 20 returns. The sample consists in NYSE/Amex common stock returns over the period 1983 to 2012. The t-statistics are computed using a Newey-West correction with twenty lags. Significance bounds at the level of 5% are shown in red. Panel (a): all stocks. Panel (b): one-third of stocks with highest average turnover over 250 days at date $t - 20$.



efficients are all negative and significant. The first estimate is large in absolute value because of the bid-ask bounce (-0.09, truncated in the figure). The decaying pattern in slope coefficients is consistent with the $q = 0$ model. But the fifth and tenth estimates appear abnormally high relative to the other estimates. More formally, the frictionless model predicts that all autocorrelations decay exponentially. This implies the following null hypothesis:

Hypothesis 1. $|\hat{\gamma}_5| \geq |\hat{\gamma}_6|$

This hypothesis is rejected at 1% with a t-statistic of 2.93, which is inconsistent with the frictionless model and may indicate infrequent rebalancing every five trading days as illustrated in Figure 3. Hypothesis 1 can only invalidate the frictionless model and does not constitute direct evidence of infrequent rebalancing. Still, infrequent rebalancing offers a plausible explanation that seems difficult to obtain with other theories. Furthermore, variations in average returns across days of the week do not generate the results, though $\hat{\gamma}$ is strongly significant. Using simple regressions or demeaning returns in the cross-section before estimating $\gamma_{\mu,t}$ does not affect this result.

To evaluate the role of trading volume, I split stocks into three portfolios at each date based on their average turnover during the past 250 days. The cross-sectional regression (13) is then estimated on the one-third of stocks that are in the high turnover portfolio at date $t-20$. Panel (b) of Figure 5 shows that the shift at lag five is markedly stronger for high turnover stocks. Hypothesis 1 is rejected at the level of 1% with a t-statistic of 3.61. Neglected stocks do not drive the results; the shift at lag five is weak for low turnover stocks (not reported). Moreover, the γ_k coefficients tend to be lower in absolute value for high turnover stocks, indicating smaller reversal for these stocks.

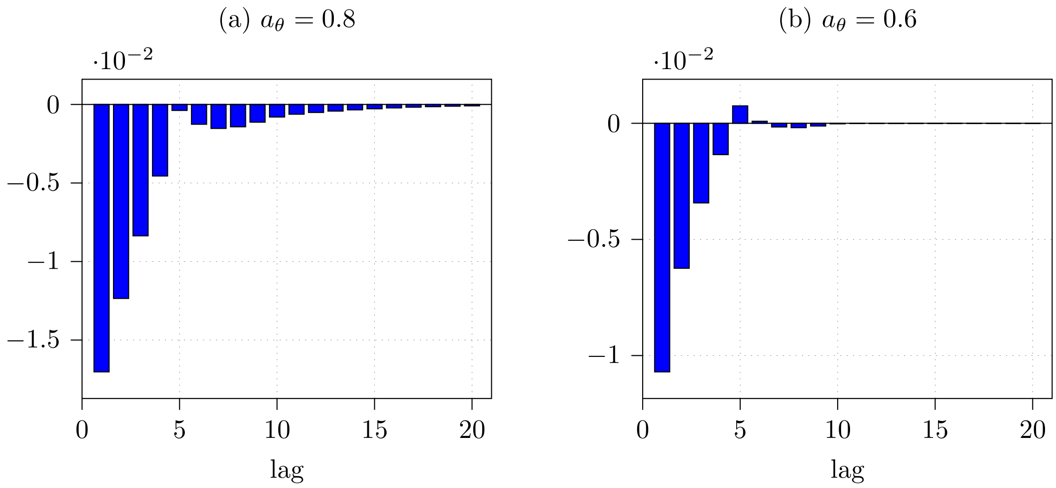
The model can match the predictability patterns in daily returns. As for intraday returns, I compare the regression estimates to the partial autocorrelations predicted by the model since they are almost identical.[23] Panel (a) of Figure 6 reports the model's partial autocorrelations. The model seems to fit the short-term dependence in stock returns in Figure 5. Infrequent rebalancing generates a shift in the autocorrelation pattern at the rebalancing horizon.

The turnover results in Figure 5 are also potentially consistent with the model. A decrease in the persistence of liquidity trading a_θ increases turnover and decreases return autocorrelation (in absolute value). Nevertheless, a_θ has an ambiguous role on equilibrium price coefficients with infrequent rebalancing. Section C in the online Appendix explains why this is the case. Numerically, I find that, when a_θ is large, the pattern becomes more pronounced as a_θ decreases, consistent with the evidence. As an illustrative example, Panel (b) of Figure 6 shows that the infrequent rebalancing pattern is more pronounced for a lower value of a_θ. In particular, the autocorrelation becomes positive.

The previous results are robust to using midquote returns, controlling for firm size, and over different subsamples. The online Appendix reports the detailed resulted. In addition, the results do not appear to be driven by a quarterly measure of institutional ownership after controlling for turnover. The coefficients are, however, insignificant over an older sample that runs from 1963 to 1993.

[23]The regression coefficients cannot be directly compared to partial autocorrelations. Nevertheless, adjusting the volatility of dividends or liquidity shocks can fit the magnitudes of the autocorrelations while preserving the shape of the autocorrelation pattern. The calibration is discussed in detail in Appendix B.

Figure 6: Partial autocorrelations predicted by the model (with 20 lags) for daily returns with different levels of liquidity trading persistence (a_θ). The calibration is shown in Table 1.



4.4 Additional Empirical Evidence

The model can potentially shed light on additional recent evidence from intraday returns. Gao et al. (2014) find that the first half-hour return on the SPDR S&P 500 ETF predicts the last half-hour return of the trading day. The infrequent rebalancing model is consistent with this evidence assuming that some infrequent traders adjust their portfolios at the open and close of the market. This assumption is economically intuitive. The U-shaped pattern in trading volume across the trading day suggests that many market participants concentrate their trading at market open and close. Increasing the fraction of traders adjusting their portfolios in a given calendar period increases trading volume and strengthens the autocorrelation pattern in this period. Thus, the model can provide a simple explanation for the results of Gao et al. (2014). Furthermore, these results come from time-series regressions and therefore only reflect autocorrelations.

5 Return Seasonality

The persistence of the coefficients in Figures 1 and 2 strongly suggests that the cross-sectional variance in average returns is not constant across half-hour intervals of a trading day and months of the year. The benchmark infrequent rebalancing model of Section 3.3 focuses on the autocovariance component and abstracts from cross-sectional variation in expected returns. In what follows, I show that variations in the proportion of infrequent traders across calendar periods can generate persistent seasonality patterns.

In the general setup of Section 3, the following proposition holds:

Proposition 3. *In a linear stationary rational expectations equilibrium, if it exists, the vector of asset prices is given by*

$$P_t = \bar{P}_{c(t)} + \frac{a_D}{R - a_D} D_t + P_{\theta,c(t)}\theta_t + \sum_{i=1}^{k} P_{X_i,c(t)} X^I_{t-i}, \tag{14}$$

where the matrices of coefficients are solutions to a system of nonlinear equations given in the Appendix.

The main insights developed using the simpler model of Section 3.3 hold, but here the equilibrium price coefficients vary with the calendar period $c(t)$ at date t. Expected returns now differ across calendar periods.[24]

To convey the main intuition in the simplest possible way, I focus on the case with two different calendar periods and let $k = 1$. The mass of frequent traders is fixed and equals $1 - q$, where $q = q_1 + q_2$. Further, let $h = 1$ for ease of exposition. Using the market clearing condition (8), the expected return in a given calendar period is

$$\mathbb{E}[Q_{t+1}|c(t)] = \frac{\gamma_F}{(1-q)} \text{Var}[Q_{t+1}|c(t)] \left(\bar{S} - q_{c(t)}\mathbb{E}\left[X^I_t|c(t)\right] - q_{c(t-1)}\mathbb{E}\left[X^I_{t-1}|c(t)\right]\right), \tag{15}$$

where I used the fact that $\text{Var}_t[Q_{t+1}] = P_{\theta,c(t+1)}\Sigma_\theta P'_{\theta,c(t+1)} + \left(\frac{R}{R-a_D}\right)^2 \Sigma_D$ is constant for a given calendar period. The term in the parenthesis in Equation (15) is independent of the calendar period. Thus, when $q_1 \neq q_2$, differences in the conditional variance across calendar periods solely generate differences in expected return across calendar periods. When $q_1 > q_2$, a larger mass of rational traders is present in the market in period 1, which reduces the price impact of liquidity shocks. This remark implies that $|P_{\theta,i,2}| > |P_{\theta,i,1}|$ for asset i; hence, expected returns are larger in period 1 than in period 2. In summary, traders require a higher premium to hold an asset when they anticipate price impact to be higher next period. The next proposition formalizes this reasoning using the same intuitive conditions as Proposition 2.

Proposition 4. *Consider a single-asset economy with two calendar periods, and assume that $k = 1$ and $h = 1$. Infrequent traders rebalance their portfolio only in the first calendar period. If $P_{\theta,c} < 0$ and $P_{X,c} > 0$ $(c = 1, 2)$, then the expected excess return on the asset is larger in the first calendar period than in the second calendar period.*

The previous result is specific to the infrequent rebalancing setup. As a point of comparison, consider a frictionless economy ($q = 0$) in which the mass of traders—or equivalently the risk aversion—varies deterministically from one calendar period to the next. In this economy, the opposite result holds.

[24] Let date t be the beginning of a calendar period. The vector of expected returns in calendar period j is then given by $\mathbb{E}[P_{t+1} + D_{t+1} - RP_t|c(t) = j]$. This definition ensures that increasing traders' risk aversion in a calendar period increases expected returns in the same calendar period.

Proposition 5. *Consider a single-asset economy with two calendar periods and only frequent traders with $h = 1$. The expected excess return on the asset is largest in the period when less traders are in the market.*

A smaller mass of traders requires a larger expected return to absorb liquidity shocks. This effect dominates the price impact effect described above. In the infrequent rebalancing economy, the average asset supply that frequent traders must absorb is the same in both calendar period, as shown in Equation (15).

Expected returns are larger in the period in which more traders rebalance. This effect also leads to a larger *spread* in expected returns between assets in the rebalancing period. Intuitively, high risk assets are disproportionately affected relative to low risk assets—the extreme case being a riskless asset, which is not affected. To see this, note that a conditional form of the CAPM holds. The expected excess return on asset i in a given calendar period is

$$\mathbb{E}[Q_{i,t+1}|c(t)] = \frac{\mathrm{Cov}[Q_{i,t+1}, Q_{m,t+1}|c(t)]}{\mathrm{Var}[Q_{m,t+1}|c(t)]}\mathbb{E}[Q_{m,t+1}|c(t)], \tag{16}$$

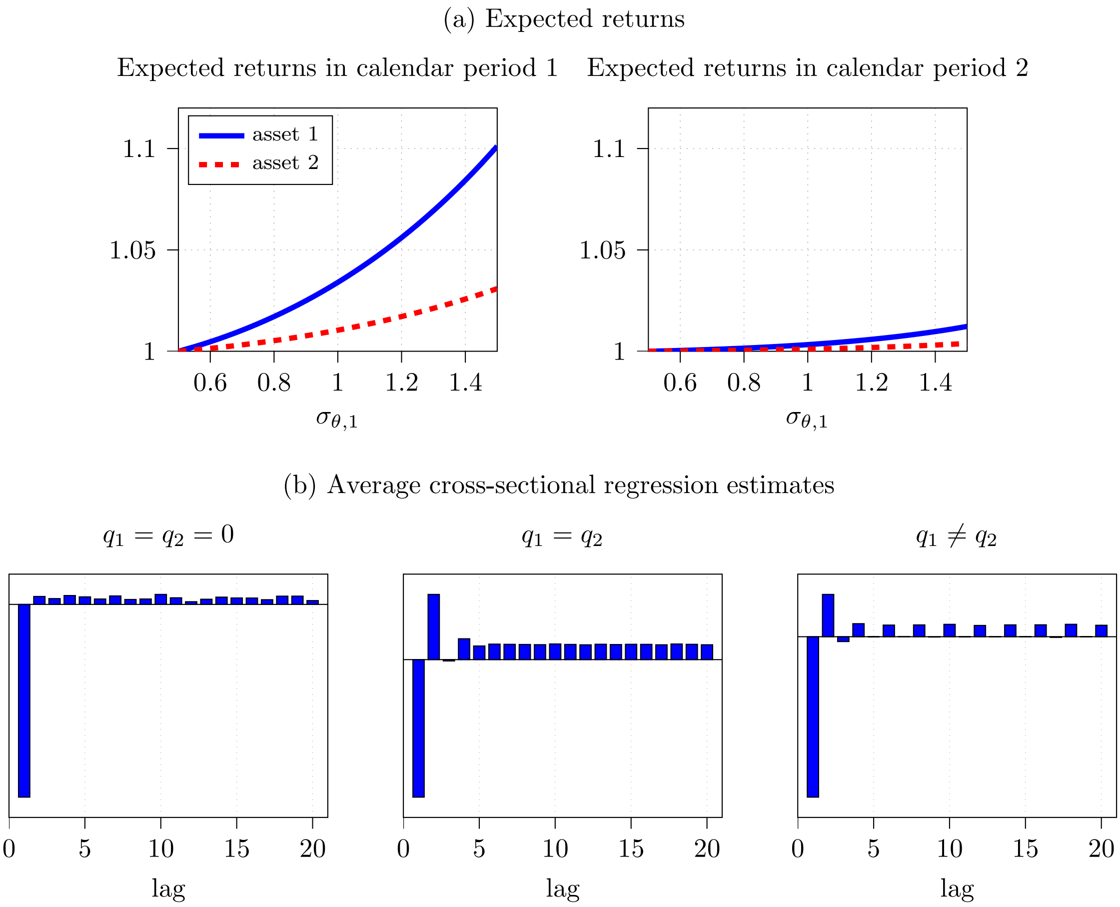
where $Q_{m,t+1}$ is the market excess return.[25] Variations in the degree of infrequent rebalancing generate a seasonality in the market risk premium. If assets have different exposures to market risk, then the model generates a seasonality in the cross-section of asset returns.

As an example, consider two assets that are identical but for their liquidity shocks volatility. Panel (a) of Figure 7 plots the expected excess return for each asset in both calendar periods as a function of the first asset's liquidity shocks volatility. Since $q_1 > q_2$ in this example, the cross-sectional variation in expected returns is larger in calendar period one than in calendar period two. This effect comes from anticipated price impact; the conditional variance is more sensitive to variations in the mass of traders for the riskier asset than for the safer asset. In addition, expected returns are larger in the period in which more traders rebalance, in line with Proposition 4 (not shown in the figure since returns are normalized).

The previous mechanism generates persistent return seasonalities. Panel (b) of Figure 7 plots the average coefficients in Regression (1) estimated from simulated returns with different proportions of infrequent traders. Return autocorrelation mainly determine the coefficients at lower lags; with infrequent rebalancing, the repeated shock mechanism produces a large positive autocorrelation in the second period (middle and right panels). At higher lags, the coefficients are positive because of cross-sectional variation in mean returns. When $q_1 \neq q_2$, these coefficients shift from period to period since the cross-sectional variance in mean returns differs across calendar periods.

[25]The market return is computed using the expected number of shares available in the market. More precisely, $Q_{m,t+1} = \sum_{i=1}^{N} s_i Q_{i,t+1}$, where s_i is the i^{th} element of the vector $\bar{S} - \sum_{i=1}^{k} q_{c(t-i)}\mathbb{E}\left[X_{t-i}^{I}|c(t)\right]$.

Figure 7: Panel (a) shows the expected excess return for each stock in each calendar period as a function of the first stock's liquidity shocks volatility ($\sigma_{\theta,1}$). The expected returns are normalized to one for $\sigma_{\theta,1} = \sigma_{\theta,2} = 0.5$. Panel (b) shows cross-sectional regression estimates from $Q_{i,t} = \alpha_{k,t} + \gamma_{k,t} Q_{i,t-k} + u_{i,t}$ based on averages of 500 simulations of a 20-stock economy over 500 periods. The calibration assumes $q_1 = 0.65$, $q_2 = 0.05$, $a_\theta = 0$, $a_D = 0$, $\sigma_D = 0.2$, $\rho_D = 0.3$, $R = 1.05$, $h = 2$, and $\bar{S} = 10$.



Therefore, variations in the degree of infrequent rebalancing can potentially explain the evidence presented by Heston et al. (2010) and other persistent seasonality patterns in cross-sectional regression estimates. At low lags the cross-sectional regressions pick up a repeated shock mechanism, while at high lags they only reflect cross-sectional variation in mean returns.

Market risk is the single risk factor in the model.With additional sources of risk, variations in the proportion of rebalancing traders may generate seasonality in multiple risk premia. Return seasonalities could then persist even after sorting assets on specific characteristics or factors. This effect may shed light on the evidence presented by Keloharju et al. (2014) about seasonality strategies across asset classes. Furthermore, a seasonality strategy is exposed to systematic risk in the model, consistent with their findings. The model is also consistent with the evidence on similar seasonalities in trading volume (Section 2).

In the online Appendix, I show that a model with a seasonality in mean liquidity trading can also generate persistent seasonality patterns. Buying or selling pressures on some stocks at the open and close could generate the seasonality in mean liquidity trading and explain the persistence of the pattern in Figure 1. This model cannot, however, explain the decaying pattern in the coefficients (from lag 13 to 26 and so on), the daily return empirical evidence in Section 4.3, and any predictability evidence based on time series regressions (Section 4), which are all consistent with an autocorrelation effect from infrequent rebalancing.

Moreover, the seasonal mean model does not generate any calendar pattern in return volatility. In this model, it is the price of risk that varies with the calendar period. This model may therefore better apply to seasonalities at lower frequencies, such as the January effect. First, the monthly seasonality in Figure 2 is much stronger when estimating the regression on January returns only. Second, volatility does not appear to be larger in January. The shifts in mean liquidity trading could arise from tax-loss selling and rebalancing in January (Ritter, 1988).

6 Trading Volume

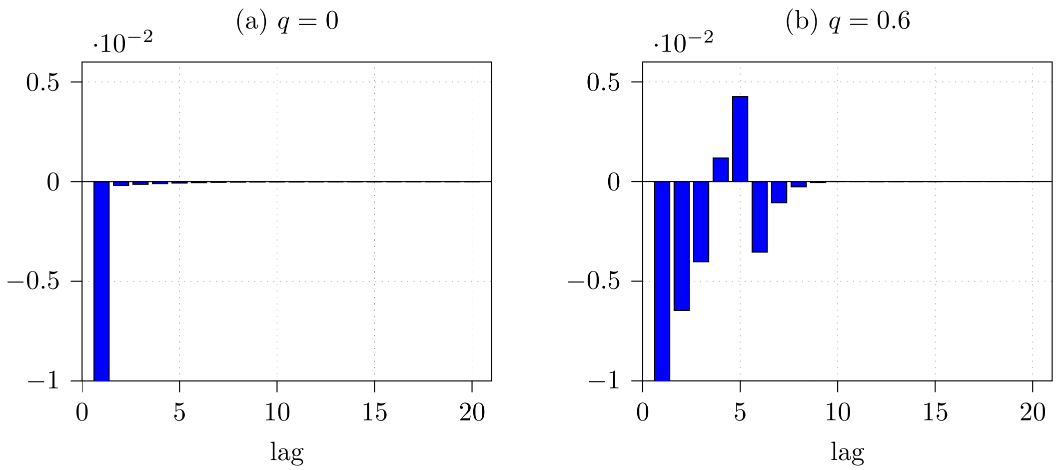
Infrequent rebalancing generates specific volume autocorrelation patterns. When only one group of agents trades in the market, the dynamics of volume are exogenously given by the dynamics of liquidity trading. Banerjee and Kremer (2010) discuss this feature of standard RE model, which makes the study of volume uninformative.

Proposition 6. *When $q = 0$ or $q = 1$, and $0 < a_\theta < 1$, changes in trading volume are negatively autocorrelated. That is,* $\text{Corr}\left[\Delta V_t, \Delta V_{t+j}\right] < 0$*,* $j \geq 1$*.*

In the model, the multiple groups of agents can generate specific volume dynamics. First, the rebalancing of infrequent traders directly modifies trading volume dynamics. A large liquidity shock today reverberates in $k+1$ periods when traders readjust their portfolios. These rebalancing trades increase the autocorrelation between changes in trading volume; hence, $\text{Corr}\left[\Delta V_t, \Delta V_{t+k+1}\right]$ can be positive. Proposition 6 shows that this is impossible in the frictionless economy. Second, market makers anticipate liquidity shocks by trading with the rebalancing infrequent traders.

Figure 8 plots $\text{Corr}\left[\Delta V_t, \Delta V_{t+j}\right]$ for the $q = 0$ economy and the $q = 0.6$ economy using the baseline calibration (the online Appendix explains how to compute volume autocorrelations when $0 < q < 1$). For both cases, $\text{Corr}\left[\Delta V_t, \Delta V_{t+1}\right]$ is large and negative (≈ -0.49). The autocorrelations are negligible beyond the first lag in the $q = 0$ economy. When $0 < q < 1$, the autocorrelations are still small but many times larger than in the frictionless economy. Patterns linked to infrequent rebalancing appear. Interestingly, $\text{Corr}\left[\Delta V_t, \Delta V_{t+4}\right] > 0$. This effect—not discernible in the case of returns—reflects the trading of market makers. For instance, when market makers expect a positive liquidity shock tomorrow, they sell the asset; their counterparty is the group of infrequent traders currently in the market, who are not affected by a liquidity shock tomorrow.

Figure 8: Volume changes autocorrelations ($\text{Corr}\left[\Delta V_t, \Delta V_{t+j}\right]$) predicted by the model. The calibration is shown in Table 1.



The setup can potentially explain why Heston et al. (2010) find that the half-hour volume periodicity does not fully account for the return periodicity. When $q = 1$, liquidity trading determines trading volume (Proposition 6), but infrequent rebalancing still generates a return periodicity pattern. Therefore, the volume pattern cannot explain the return pattern in this polar case. When $q < 1$ volume remains partly determined by liquidity trading and therefore cannot fully explain return periodicity.

To test the model's predictions, I estimate the following regression on the daily data set:

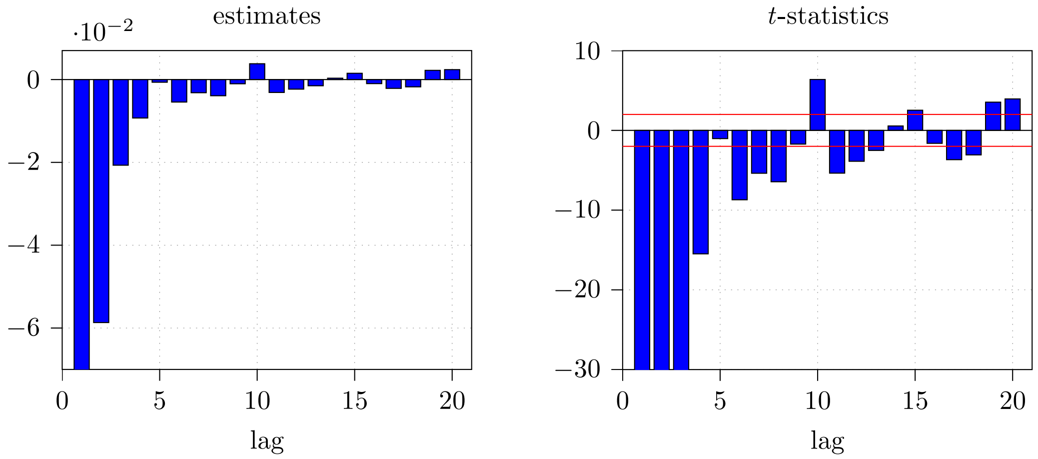
$$v_{it} = \alpha_t + \gamma_k v_{i,t-k} + \gamma_{\nu,t}\nu_{i,t} + u_{i,t}, \tag{17}$$

where $v_{it} = \ln\left(\frac{\text{Turnover}_{i,t}}{\text{Turnover}_{i,t-1}}\right)$, and $\nu_{i,t}$ is the average same-day (as day t) change in turnover over the past year.[26] Figure 9 plots the average γ_k coefficient and their t-statistics. The first coefficient γ_1 (truncated in the figure) is large and negative (-0.39). The regression reveals shifts in the autocorrelation at the fifth and tenth lags that are qualitatively consistent with an infrequent rebalancing mechanism. Similar to the daily return evidence, the fixed effect coefficient $\hat{\gamma}_\nu$ is positive and highly significant but does not explain the shifts in the coefficients. Clearly, specific predictability patterns also exist for daily changes in trading volume.

The model overestimates the magnitude of the fifth coefficient and does not produce a large positive tenth lag coefficient. Moreover, the fourth lag coefficient does not exhibit any shift, which seems to indicate that either traders do not anticipate the

[26] To estimate Regression (17), I exclude all stocks that have zero volume on one day from the sample. This procedure leaves an average of roughly 950 observations per period. The results of Section 4.3 are unaffected.

Figure 9: Time-series averages of cross-sectional regression estimates. The following cross-sectional regression is estimated each day: $v_{it} = \alpha_t + \gamma_k v_{i,t-k} + \gamma_{\nu,t}\nu_{i,t} + u_{i,t}$. The sample consists in NYSE/Amex common stock turnover series over the period 1983 to 2012. The t-statistics are computed using a Newey-West correction with twenty lags. Significance bounds at the level of 5% are shown in red.



repeated liquidity shocks on average, or that they are not able to reliably trade on them. In summary, the model provides new insights to understand the short-term dynamics of trading volume, though additional elements are needed to realistically model these dynamics.

7 Conclusion

This paper shows that infrequent rebalancing can have an important impact on asset return autocorrelation and seasonality. In the model, return autocorrelations exhibit specific patterns and can even switch sign, consistent with empirical evidence from intraday returns and new evidence from daily returns. Despite being perfectly anticipated, the lagged demands of infrequent traders affect return dynamics. The model also makes specific predictions about trading volume, for which I find support in the data.

A variable proportion of infrequent traders across calendar periods can generate return seasonality. The spread in expected returns between assets with different exposures to the market widens when more traders rebalance in the market. As a result, the cross-sectional variance in mean returns differs across calendar periods. This variation generates persistent seasonality patterns in cross-sectional regression estimates of current returns on lagged returns. More work remains to be done to better understand the fundamental driving factors behind seasonalities in stock returns. This is important because seasonalities have a large impact on the cross-section of stock returns.[27]

[27] For instance, monthly returns on many well-known anomalies are subject to strong seasonalities

References

[1] Abel, A. B., J. C. Eberly, and S. Panageas (2007). Optimal inattention to the stock market. *American Economic Review, Papers and Proceedings 97*(2), 244–249. [8]

[2] Abel, A. B., J. C. Eberly, and S. Panageas (2013). Optimal inattention to the stock market with information costs and transactions costs. *Econometrica 81*(4), 1455–1481. [8]

[3] Admati, A. R. and P. Pfleiderer (1988). A theory of intraday patterns: Volume and price variability. *Review of Financial Studies 1*(1), 3–40. [4, 7]

[4] Albagli, E. (2014). Investment horizons and asset prices under asymmetric information. Working paper. [2, 8, 12]

[5] Albuquerque, R. and J. Miao (2014). Advance information and asset prices. *Journal of Economic Theory 149*, 236–275. [14]

[6] Bacchetta, P. and E. Van Wincoop (2006). Can information heterogeneity explain the exchange rate determination puzzle? *The American Economic Review 96*(3), 552–576. [12]

[7] Bacchetta, P. and E. van Wincoop (2010). Infrequent portfolio decisions: A solution to the forward discount puzzle. *American Economic Review 100*(3), 870–904. [4, 8, 17]

[8] Banerjee, S. (2011). Learning from prices and the dispersion in beliefs. *Review of Financial Studies 24*(9), 3025–3068. [2, 10, 12]

[9] Banerjee, S. and I. Kremer (2010). Disagreement and learning: Dynamic patterns of trade. *The Journal of Finance 65*(4), 1269–1302. [25]

[10] Beber, A., J. Driessen, and P. Tuijp (2012). Pricing Liquidity Risk with Heterogeneous Investment. Working paper. [5]

[11] Biais, B., P. Bossaerts, and C. Spatt (2010). Equilibrium asset pricing and portfolio choice under asymmetric information. *Review of Financial Studies 23*(4), 1503–1543. [2, 9, 10, 14]

[12] Bogousslavsky, V. (2015). Seasonalities in anomalies. Working paper. [28]

[13] Campbell, J. Y., S. J. Grossman, and J. Wang (1993). Trading volume and serial correlation in stock returns. *Quarterly Journal of Economics 108*(4), 905–939. [13, 41]

[14] Cespa, G. and X. Vives (2012). Dynamic trading and asset prices: Keynes vs. Hayek. *Review of Economic Studies 79*(2), 539–580. [13, 15]

[15] Chien, Y., H. Cole, and H. Lustig (2012). Is the volatility of the market price of risk due to intermittent portfolio rebalancing? *American Economic Review 102*(6), 2859–96. [4, 8, 17]

[16] Conrad, J. and G. Kaul (1998). An anatomy of trading strategies. *Review of Financial Studies 11*(3), 489–519. [6]

[17] Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics 7*(2), 174–196. [5]

[18] Dellavigna, S. and J. M. Pollet (2009). Investor Inattention and Friday Earnings Announcements. *Journal of Finance 64*(2), 709–749. [4]

[19] Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance 65*(4), 1237–1267. [2, 4, 7, 8, 31]

[20] Fama, E. F. and J. D. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy 81*(3), 607–36. [5]

(see Bogousslavsky, 2015, for an overview).

[21] French, K. R. (1980). Stock returns and the weekend effect. *Journal of Financial Economics 8*(1), 55–69. [18]

[22] Gao, L., Y. Han, S. Z. Li, and G. Zhou (2014). Intraday Momentum: The First Half-Hour Return Predicts the Last Half-Hour Return. Working paper. [3, 21]

[23] Gibbons, M. R. and P. Hess (1981). Day of the week effects and asset returns. *The Journal of Business 54*(4), 579–96. [18]

[24] Grossman, S. J. and M. H. Miller (1988). Liquidity and market structure. *Journal of Finance 43*(3), 617–633. [13]

[25] He, H. and J. Wang (1995). Differential informational and dynamic behavior of stock trading volume. *Review of Financial Studies 8*(4), 919–972. [32]

[26] Hendershott, T., S. X. Li, A. J. Menkveld, and M. S. Seasholes (2014, January). Asset price dynamics with limited attention. Working paper. [4]

[27] Heston, S. L., R. A. Korajczyk, and R. Sadka (2010). Intraday patterns in the cross-section of stock returns. *Journal of Finance 65*(4), 1369–1407. [2, 3, 5, 7, 16, 17, 24, 26]

[28] Heston, S. L. and R. Sadka (2008). Seasonality in the cross-section of stock returns. *Journal of Financial Economics 87*(2), 418–445. [5, 7]

[29] Holden, C. W. and A. Subrahmanyam (2002). News events, information acquisition, and serial correlation. *Journal of Business 75*(1), 1–32. [13]

[30] Hong, H. and J. Yu (2009, November). Gone fishin: Seasonality in trading activity and asset prices. *Journal of Financial Markets 12*(4), 672–702. [4]

[31] Jegadeesh, N. (1990). Evidence of predictable behavior of security returns. *Journal of Finance 45*(3), 881–898. [3, 5, 13, 18]

[32] Jegadeesh, N. and S. Titman (1995). Overreaction, Delayed Reaction, and Contrarian Profits. *Review of Financial Studies 8*(4), 973–993. [6]

[33] Jegadeesh, N. and S. Titman (2002). Cross-Sectional and Time-Series Determinants of Momentum Returns. *Review of Financial Studies 15*(1), 143–157. [6]

[34] Kamara, A., R. A. Korajczyk, X. Lou, and R. Sadka (2015). Horizon pricing. Journal of Financial and Quantitative Analysis, Forthcoming. [5]

[35] Keloharju, M., J. T. Linnainmaa, and P. Nyberg (2014). Common factors in return seasonalities. NBER Working paper 20815. [5, 18, 24]

[36] Lehmann, B. N. (1990). Fads, martingales, and market efficiency. *Quarterly Journal of Economics 105*(1), 1–28. [3, 5]

[37] Lo, A. and A. MacKinlay (1990). When are contrarian profits due to stock market overreaction? *Review of Financial Studies 3*(2), 175–205. [6]

[38] Lou, D. (2012). A flow-based explanation for return predictability. *Review of financial studies 25*(12), 3457–3489. [4]

[39] Makarov, I. and O. Rytchkov (2012). Forecasting the forecasts of others: Implications for asset pricing. *Journal of Economic Theory 147*(3), 941–966. [13, 15]

[40] Nagel, S. (2012). Evaporating liquidity. *Review of Financial Studies 25*(7), 2005–2039. [3]

[41] Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–08. [19]

[42] Orosel, G. O. (1998). Participation costs, trend chasing, and volatility of stock prices. *Review of Financial Studies 11*(3), 521–557. [8]

[43] Peng, L. and W. Xiong (2006). Investor attention, overconfidence and category learning. *Journal of Financial Economics 80*(3), 563–602. [8]

[44] Rakowski, D. and X. Wang (2009). The dynamics of short-term mutual fund flows and returns: A time-series and cross-sectional investigation. *Journal of Banking & Finance 33*(11), 2102–2109. [3, 18]

[45] Rinne, K. and M. Suominen (2012, May). A structural model of short-term reversals. Working paper. [4]

[46] Ritter, J. (1988). The Buying and Selling Behavior of Individual Investors at the Turn of the Year. *Journal of Finance 43*(3), 701–717. [25]

[47] Spiegel, M. (1998). Stock price volatility in a multiple security overlapping generations model. *Review of Financial Studies 11*(2), 419–447. [2, 11, 34]

[48] Vayanos, D. and P. Woolley (2013). An institutional theory of momentum and reversal. *Review of Financial Studies 26*(5), 1087–1145. [4]

[49] Vives, X. (2010). *Information and Learning in Markets: The Impact of Market Microstructure*. Princeton University Press. [32]

[50] Wang, J. (1994). A model of competitive stock trading volume. *Journal of Political Economy 102*(1), 127–68. [13, 14]

[51] Watanabe, M. (2008). Price volatility and investor behavior in an overlapping generations model with information asymmetry. *Journal of Finance 63*(1), 229–272. [2, 10, 11, 34]

A Appendix: Proofs

To derive Proposition 1, I first conjecture that asset prices and infrequent traders' demands are linear in the state variables (defined below). Using this conjecture I derive frequent traders' demands (Lemma 1) and infrequents traders' demands (Lemma 3). Finally, I verify the initial conjectures by plugging the demands into the market clearing condition (Proposition 1). The dividend and liquidity trading mean vectors are given by $\bar{D}$ and $\bar{\theta}$ in the proofs and are set to $0_{N\times 1}$ in the analysis.

Derivation of the state variables process. I follow Duffie (2010) and focus on linear equilibria. Conjecture that the price and infrequent traders' demand vectors are given by

$$P_t = AY_t, \qquad\qquad X_t^I = BY_t, \tag{A.1}$$

where A and B are constant parameter matrices of dimensions $N \times 1 + (2+k)N$, and Y_t is the following $(1+(2+k)N)$-dimensional vector of state variables:

$$Y_t \equiv \begin{pmatrix} 1 & \theta_t' & D_t' & X_{t-1}^{I\prime} & \dots & X_{t-k}^{I\prime} \end{pmatrix}'. \tag{A.2}$$

The lagged demands of infrequent traders X_{t-i}^I $(1 \le i \le k)$ are state variables in equilibrium. Given the vector of state variables (A.2) it follows that

$$Y_{t+1} = A_Y Y_t + B_Y \epsilon_{t+1}, \tag{A.3}$$

where $\epsilon_t \equiv \begin{pmatrix}\epsilon_t^\theta & \epsilon_t^D\end{pmatrix}' \sim \mathcal{N}(0, \Sigma_Y)$ is the vector of innovations and the matrices A_Y, B_Y, and Σ_Y are defined below. First,

$$A_Y = \left[\begin{array}{cccccccc}
1 & 0_{1\times N} & \cdots & & & & & 0_{1\times N} \\
(1-a_\theta)\bar{\theta} & a_\theta I_N & 0_N & \cdots & & & & 0_N \\
(1-a_D)\bar{D} & 0_N & a_D I_N & 0_N & \cdots & & & 0_N \\
\multicolumn{8}{c}{B} \\
0_{N\times 1} & 0_N & 0_N & I_N & 0_N & \cdots & 0_N & 0_N \\
\vdots & \vdots & \vdots & 0_N & \ddots & & \vdots & \vdots \\
 & & & \vdots & & & 0_N & \\
0_{N\times 1} & 0_N & 0_N & 0_N & \dots & 0_N & I_N & 0_N
\end{array}\right],$$

where I_N denotes the identity matrix of dimension $N \times N$, and B is the $N \times (1+(2+k)N)$ matrix of conjectured equilibrium demand coefficients from (A.1). Similarly,

$$B_Y = \begin{bmatrix} 0_{1\times N} & 0_{1\times N} \\ I_N & 0_N \\ 0_N & I_N \\ 0_{kN\times N} & 0_{kN\times N} \end{bmatrix}.$$

The variance-covariance matrix of innovations is

$$\Sigma_Y = \begin{bmatrix} \Sigma_\theta & 0_N \\ 0_N & \Sigma_D \end{bmatrix}.$$

The dynamics of Y_t imply that

$$Y_{t+j} = A_Y^j Y_t + \sum_{i=1}^{j} A_Y^{j-i} B_Y \epsilon_{t+i}, \qquad j \ge 1. \tag{A.4}$$

To simplify notation, let $A_Y^0 = I_N$. I also introduce the following matrices for convenience:

φ_D, φ_θ, and φ_x, which are defined such that $\theta_t = \varphi_\theta Y_t$, $D_t = \varphi_D Y_t$, $\left(X^I_{t-1} \dots X^I_{t-k}\right)' = \varphi_X Y_t$, and $\varphi_{\bar{S}} Y_t = \bar{S}$.

Define $Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t$, the vector of excess dollar returns. It follows that

$$Q_{t+1} = A_Q Y_t + B_Q \epsilon_{t+1}, \tag{A.5}$$

where $A_Q \equiv (A + \varphi_D)A_Y - RA$, and $B_Q \equiv (A + \varphi_D)B_Y$.

Finally, denote the cumulative payoff from t (ex-dividend) to $t + k + 1$ as

$$T_{t,t+k+1} \equiv P_{t+k+1} + \sum_{j=1}^{k+1} R^{k+1-j} D_{t+j}. \tag{A.6}$$

Lemma 1. *Given the initial conjectures (A.1), the asset demands of frequent traders' with remaining horizon $h - j$ $(0 \leq j < h)$ at date t are given by*

$$X^F_{t,j} = \frac{1}{\alpha_{j+1}} F_{j+1} Y_t, \tag{A.7}$$

where

$$\begin{aligned}
F_{j+1} &= (B_Q \Xi_{j+1} B_Q')^{-1}(A_Q - B_Q \Xi_{j+1} B_Y' U_{j+1}' A_Y), \\
\alpha_j &= R\alpha_{j+1}.
\end{aligned}$$

The coefficients are solved for recursively starting from the conditions $\alpha_h = \gamma_F$ and $U_h = 0_{1+2N+kN}$. Ξ_{j+1} and U_{j+1} $(0 \leq j < h)$ are constant matrices defined below.

Proof. The proof parallels the derivations of He and Wang (1995) in a non-stationary setup. Let j be the age of the investor $(0 \leq j < h)$ and $J(W_t, Y_t, j)$ be the value function. The Bellman optimization problem for an investor aged j at date t is

$$J(W_t, Y_t, j) = \max_{X_{t,j}} \mathbb{E}_t[J(W_{t+1}, Y_{t+1}, j + 1)]$$

such that $W_{t+1} = X_{t,j}' Q_{t+1} + RW_t$ and $J(W_t, Y_t, h) = -e^{-\gamma_F W_t}$. Conjecture that $J(W_{t+1}, Y_{t+1}, j + 1) = -e^{-\alpha_{j+1} W_{t+1} - \frac{1}{2} Y_{t+1}' U_{j+1} Y_{t+1}}$. It then follows that

$$\begin{aligned}
\mathbb{E}_t[J(W_{t+1}, Y_{t+1}, j + 1)] &= - e^{-\alpha_{j+1}(RW_t + X_{t,j}' A_Q Y_t)} \mathbb{E}_t\left[e^{-\alpha_{j+1} X_{t,j}' B_Q \epsilon_{t+1} - \frac{1}{2} Y_{t+1}' U_{j+1} Y_{t+1}}\right] \\
&= - e^{-\alpha_{j+1}(RW_t + X_{t,j}' A_Q Y_t) - \frac{1}{2} Y_t' A_Y' U_{j+1} A_Y Y_t} \\
&\quad \mathbb{E}_t\left[e^{(-\alpha_{j+1} X_{t,j}' B_Q - Y_t' A_Y' U_{j+1} B_Y)\epsilon_{t+1} - \frac{1}{2} \epsilon_{t+1}' B_Y' U_{j+1} B_Y \epsilon_{t+1}}\right].
\end{aligned}$$

Using the multivariate normality of ϵ (see for instance Vives, 2010, sect. 10.2.4) gives

$$\begin{aligned}
\mathbb{E}_t\left[e^{(-\alpha_{j+1} X_{t,j}' B_Q - Y_t' A_Y' U_{j+1} B_Y)\epsilon_{t+1} - \frac{1}{2} \epsilon_{t+1}' B_Y' U_{j+1} B_Y \epsilon_{t+1}}\right] &= \left|I + \Sigma_Y B_Y' U_{j+1} B_Y\right|^{-\frac{1}{2}} \\
& e^{\frac{1}{2}(-\alpha_{j+1} X_{t,j}' B_Q - Y_t' A_Y' U_{j+1} B_Y)(I + \Sigma_Y B_Y' U_{j+1} B_Y)^{-1} \Sigma_Y (-\alpha_{j+1} B_Q' X_{t,j} - B_Y' U_{j+1}' A_Y Y_t)}.
\end{aligned}$$

Define

$$\begin{aligned}
\rho_{j+1} &\equiv \left|I + \Sigma_Y B_Y' U_{j+1} B_Y\right|^{-\frac{1}{2}}, \\
\Xi_{j+1} &\equiv (\Sigma_Y^{-1} + B_Y' U_{j+1} B_Y)^{-1}.
\end{aligned}$$

Using the previous results the first-order conditions are

$$A_Q Y_t - \alpha_{j+1} B_Q \Xi_{j+1} B_Q' X_{t,j} - B_Q \Xi_{j+1} B_Y' U_{j+1}' A_Y Y_t = 0.$$

Thus, the vector of demands is given by

$$X_{t,j} = \frac{1}{\alpha_{j+1}} F_{j+1} Y_t,$$

where $F_{j+1} \equiv (B_Q \Xi_{j+1} B_Q')^{-1}(A_Q - B_Q \Xi_{j+1} B_Y' U_{j+1}' A_Y)$. Last, I verify the conjecture for the value function. Plugging the optimal demand expression in the optimization problem gives

$$\mathbb{E}_t[J(W_{t+1}, Y_{t+1}, j+1)] = -\rho_{j+1} e^{-\alpha_{j+1} R W_t - \frac{1}{2} Y_t' M_{j+1} Y_t}$$

where $M_{j+1} \equiv A_Y' U_{j+1} A_Y + F_{j+1}' B_Q \Xi_{j+1} B_Q' F_{j+1} - A_Y' U_{j+1} B_Y \Xi_{j+1} B_Y' U_{j+1}' A_Y$. Matching terms with the conjectured valued function yields

$$\alpha_j = R\alpha_{j+1}, \qquad U_j = M_{j+1} - 2\ln(\rho_{j+1}) I_{11},$$

where I_{11} is a matrix whose first element is 1 and all others are zero (i.e., $Y_t' I_{11} Y_t = 1$). The terminal condition gives $\alpha_h = \gamma_F$ and $U_h = 0_{1+2N+kN}$. All the coefficients can then be solved for recursively.■

The following lemma is needed to derive infrequent traders' demands:

Lemma 2. *Given the initial conjectures (A.1), the equilibrium stationary j-period payoff variance $Var_t(T_{t,t+j})$ is a constant matrix Σ_j given by ($j \geq 1$)*

$$\sum_{i=1}^{j} \left(A A_Y^{j-i} + \frac{R^{j-i+1} - a_D^{j-i+1}}{R - a_D} \varphi_D \right) B_Y \Sigma_Y B_Y' \left(A A_Y^{j-i} + \frac{R^{j-i+1} - a_D^{j-i+1}}{R - a_D} \varphi_D \right)'. \quad \text{(A.8)}$$

Proof. Since $T_{t,t+j} = AY_{t+j} + \sum_{i=1}^{j} R^{j-i} D_{t+i}$, it follows that (using (A.4))

$$\text{Var}_t[T_{t,t+j}] = \text{Var}\left[A \sum_{i=1}^{j} A_Y^{j-i} B_Y \epsilon_{t+i} + \sum_{i=1}^{j} R^{j-i} \left(\sum_{s=1}^{i-1} a_D^s \epsilon_{t+i-s}^D + \epsilon_{t+i}^D \right) \right]. \quad \text{(A.9)}$$

The matrix $\text{Var}_t[T_{t,t+j}]$ only depends on independent shocks and is therefore constant. To compute $\text{Var}_t[T_{t,t+j}]$, note that

$$\sum_{i=1}^{j} R^{j-i} \left(\sum_{s=1}^{i-1} a_D^s \epsilon_{t+i-s}^D + \epsilon_{t+i}^D \right) = \sum_{i=1}^{j} g(R, a_D, j-i) \varphi_D B_Y \epsilon_{t+i}, \quad \text{(A.10)}$$

where the function $g(R, a_D, j-i)$ is defined recursively by

$$g(R, a_D, i) = g(R, a_D, i-1) R + a_D^i, \quad i \geq 1,$$

and $g(R, a_D, 0) = 1$. It is direct to prove by induction that $g(R, a_D, i) = \frac{R^{i+1} - a_D^{i+1}}{R - a_D}$. Plugging this function in the conditional variance expression (A.9) gives

$$\Sigma_j = \text{Var}\left[\sum_{i=1}^{j} \left(A A_Y^{j-i} + \frac{R^{j-i+1} - a_D^{j-i+1}}{R - a_D} \varphi_D \right) B_Y \epsilon_{t+i} \right]. \quad \text{(A.11)}$$

Since the error terms ϵ_{t+i} in (A.11) are independent of each other, the lemma follows.■

Lemma 3. *Given the initial conjectures (A.1), infrequent traders' demands are given by*

$$X_t^I = \frac{1}{\gamma_I} \Sigma_{k+1}^{-1} \sum_{j=0}^{k} R^{k-j} A_Q A_Y^j Y_t, \quad \text{(A.12)}$$

where $\Sigma_{k+1} \equiv \mathrm{Var}_t[T_{t,t+k+1}]$ is the equilibrium stationary $(k+1)$-period payoff variance and is shown to be constant in Lemma 2.

Proof. From the optimization problem (6) and given that prices are normally distributed under the conjecture (A.1), infrequent traders' demands are as follows:

$$
\begin{aligned}
X_t^I =& \frac{1}{\gamma_I}\Sigma_{k+1}^{-1}\left(\mathbb{E}_t\left[P_{t+k+1}+\sum_{j=1}^{k+1}R^{k+1-j}D_{t+j}\right]-R^{k+1}P_t\right) \\
=& \frac{1}{\gamma_I}\Sigma_{k+1}^{-1}\sum_{j=0}^{k}R^{k-j}\mathbb{E}_t[Q_{t+j+1}]. \qquad \text{(A.13)}
\end{aligned}
$$

Using (A.4) and (A.5), infrequent traders' demands (A.13) reduce to (A.12). The vector of demands is linear in the state variables, as conjectured initially.■

Proof of Proposition 1. Replacing the demands (A.12) and (A.7) in the market clearing condition (8) with $q_{c(t)} = \frac{q}{k+1}$ and rearranging terms yields the following system of fixed point equations:

$$
\frac{q/\gamma_I}{k+1}\Sigma_{k+1}^{-1}\left(\sum_{j=0}^{k}R^{k-j}A_QA_Y^j\right)+\frac{1-q}{h}\left(\sum_{j=0}^{h-1}\frac{1}{\alpha_{j+1}}F_{j+1}\right)-\varphi_\theta-\varphi_{\bar{S}}+\frac{q}{k+1}\varphi_X=0, \qquad \text{(A.14)}
$$

$$
\frac{1}{\gamma_I}\Sigma_{k+1}^{-1}\left(\sum_{j=0}^{k}R^{k-j}A_QA_Y^j\right)-B=0. \qquad \text{(A.15)}
$$

A linear REE exists if this system of equations admits a solution.

Using the expressions for A_Q and A_Y, the dividend coefficient matrix of (A.15) can be rewritten as

$$
\left(a_D\left(P_D+I_N\right)-RP_D\right)\left(\sum_{j=1}^{k}R^{k-j}a_D^j+R^k\right)=0_N,
$$

where the equality follows from the fact that agents do no trade on dividends (no-trade theorem). Hence, $P_D=\frac{a_D}{R-a_D}I_N$. The result can also be directly deduced from (A.14) and (A.15).■

Corollary 2. *Frictionless economy. Assume that $q=0$ and $h=1$. In a linear stationary REE, the price vector is given by*

$$
P_t=\bar{P}+P_\theta\theta_t+\frac{a_D}{R-a_D}D_t.
$$

P_θ solves the following quadratic matrix equation:

$$
P_\theta\Sigma_\theta P_\theta'+\frac{R-a_\theta}{\gamma_F}P_\theta+\left(\frac{R}{R-a_D}\right)^2\Sigma_D=0_N. \qquad \text{(A.16)}
$$

This equation has 2^N solutions if $\frac{1}{4}\left(\frac{R-a_\theta}{\gamma_F}\right)^2I_N-\left(\frac{R}{R-a_D}\right)^2\Sigma_\theta^{\frac{1}{2}}\Sigma_D\Sigma_\theta^{\frac{1}{2}}$ is positive definite.

Proof. Spiegel (1998) and Watanabe (2008) provide similar derivations. Conjecture that $P_t=\bar{P}+P_\theta\theta_t+P_DD_t$. The demand of frequent traders is $X_t^F=\frac{1}{\gamma_F}\Sigma_1^{-1}\mathbb{E}_t[Q_{t+1}]$, where

$\Sigma_1 \equiv \mathrm{Var}_t[Q_{t+1}] = P_\theta \Sigma_\theta P_\theta' + (P_D + I_N)\Sigma_D(P_D + I_N)'$ is a constant matrix under the price conjecture. The market clearing condition is $\gamma_F \Sigma_1 \left(\theta_t + \bar{S}\right) = \mathbb{E}_t[Q_{t+1}]$, where

$$\mathbb{E}_t[Q_{t+1}] = \bar{P} + (1 - a_\theta)P_\theta\bar{\theta} + (1 - a_D)(P_D + I_N)\bar{D} + a_\theta P_\theta \theta_t + a_D(P_D + I_N)D_t - RP_t.$$

Matching terms with the price conjecture gives

$$\begin{aligned}
P_D &= \frac{a_D}{R - a_D} I_N, \\
P_\theta \Sigma_\theta P_\theta' &+ \frac{R - a_\theta}{\gamma_F} P_\theta + \left(\frac{R}{R - a_D}\right)^2 \Sigma_D = 0_N, \text{ and} \\
\bar{P} &= \frac{1}{R - 1}\left((R - a_\theta)P_\theta \bar{S} + (1 - a_\theta)P_\theta\bar{\theta} + \frac{(1 - a_D)R}{R - a_D}\bar{D}\right).
\end{aligned} \tag{A.17}$$

The last equation uses the fact that $\gamma_F \Sigma_1 = -(R - a_\theta)P_\theta$ from the second equation. The price impact matrix P_θ solves the quadratic matrix equation (A.17) and must be symmetric. Assuming that Σ_θ is positive definite, multiply both sides of (A.17) by $\Sigma_\theta^{\frac{1}{2}}$ (the unique positive definite square root of Σ_θ) and reorganize the terms to obtain

$$\left(\Sigma_\theta^{\frac{1}{2}} P_\theta \Sigma_\theta^{\frac{1}{2}} + \frac{R - a_\theta}{2\gamma_F} I_N\right)^2 = \frac{1}{4}\left(\frac{R - a_\theta}{\gamma_F}\right)^2 I_N - \left(\frac{R}{R - a_D}\right)^2 \Sigma_\theta^{\frac{1}{2}} \Sigma_D \Sigma_\theta^{\frac{1}{2}}. \tag{A.18}$$

If $\frac{1}{4}\left(\frac{R-a_\theta}{\gamma_F}\right)^2 \Sigma_\theta^{-2} - \left(\frac{R}{R-a_D}\right)^2 \Sigma_\theta^{-\frac{1}{2}} \Sigma_D \Sigma_\theta^{-\frac{1}{2}}$ is positive definite, then its spectral decomposition is given by $\Gamma\Lambda\Gamma'$, where Λ is a diagonal matrix of eigenvalues λ_i $(i = 1, \ldots, N)$ and Γ is an orthonormal matrix with eigenvectors as columns. Thus,

$$P_\theta = -\frac{1}{2}\left(\frac{R - a_\theta}{\gamma_F}\right)\Sigma_\theta^{-1} + \Gamma\Lambda^{\frac{1}{2}}\Gamma'. \tag{A.19}$$

Since each diagonal element of $\Lambda^{\frac{1}{2}}$ can take values $\pm\sqrt{\lambda_i}$ to satisfy (A.18), P_θ admits 2^N solutions.■

Proof of Corollary 1. For simplicity, let $\bar{P}_\theta = 0$, $\bar{D} = 0$, and $\bar{S} = 0$. This implies that $\bar{P} = 0$. When $q = 1$, the market clearing condition becomes

$$\frac{1}{k + 1} X_t^I = \theta_t - \frac{1}{k + 1}\sum_{i=1}^{k} X_{t-i}^I. \tag{A.20}$$

This gives the B coefficients in (A.1). Plugging infrequent traders' demands (A.13) in the market clearing condition yields

$$R^{k+1}P_t = \mathbb{E}_t[T_{t,t+k+1}] - \gamma_I(k + 1)\Sigma_{k+1}\theta_t + \gamma_I\Sigma_{k+1}\sum_{i=1}^{k} X_{t-i}^I. \tag{A.21}$$

The cumulative payoff from date t to date $t + k + 1$ is given by

$$T_{t,t+k+1} = P_\theta\theta_{t+k+1} + (P_D + I_N)D_{t+k+1} + \sum_{i=1}^{k} R^{k+1-i}D_{t+i} + \sum_{i=1}^{k} P_{X_i} X_{t+k+1-i}^I. \tag{A.22}$$

Using (4) and matching terms for the dividends in (A.21) gives

$$R^{k+1}P_D = a_D^{k+1}(P_D + I_N) + \sum_{i=1}^{k} R^{k+1-i} a_D^i I_N.$$

This condition simplifies to $P_D = \frac{a_D}{R-a_D} I_N$.

From (7), $\theta_{t+k+1} = a_\theta^{k+1}\theta_t + \sum_{i=0}^{k-1} a_\theta^{k-i}\epsilon_{t+1+i}^\theta + \epsilon_{t+k+1}^\theta$. Now the sum of future lagged demands $\sum_{i=1}^{k} P_{X_i} X_{t+k+1-i}^I$ has to be expressed in terms of current lagged demands. The following equality holds:

$$\begin{bmatrix} X_{t+k}^I \\ \vdots \\ X_{t+1}^I \end{bmatrix} = \begin{bmatrix} X_{t-1}^I \\ \vdots \\ X_{t-k}^I \end{bmatrix} + (k+1) \begin{bmatrix} \theta_{t+k} - \theta_{t+k-1} \\ \theta_{t+k-1} - \theta_{t+k-2} \\ \vdots \\ \theta_{t+1} - \theta_t \end{bmatrix}.$$

This equation follows from the market clearing condition (A.20) and since agents trade only every $k+1$ periods. The $(t+k)$-demand of an infrequent trader equals her $(t-1)$-demand plus the additional liquidity trading that takes place between $t+k-1$ and $t+k$. Using this result and (7) again, it follows that

$$\begin{aligned} \mathbb{E}_t\left[X_{t+i}^I\right] &= X_{t+i-(k+1)}^I + (k+1)\mathbb{E}_t[\theta_{t+i} - \theta_{t+i-1}] \\ &= \begin{cases} X_{t+i-(k+1)}^I - (k+1)a_\theta^{i-1}(1-a_\theta)\theta_t, & 1 < i < k+1. \\ X_{t-k}^I - (k+1)(1-a_\theta)\theta_t, & i = 1. \end{cases} \end{aligned}$$

Finally, using the previous results and matching terms for the liquidity shocks and lagged demands in (A.21) gives

$$R^{k+1}P_\theta = a_\theta^{k+1}P_\theta - \gamma_I(k+1)\Sigma_{k+1} - (k+1)(1-a_\theta)\left(\sum_{i=1}^{k-1} a_\theta^{k-i}P_{X_i} + P_{X_k}\right), \text{ and} \tag{A.23}$$

$$R^{k+1}P_{X_i} = P_{X_i} + \gamma_I \Sigma_{k+1}, \qquad i = 1, \ldots, k. \tag{A.24}$$

Equation (11) follows from (A.23), (A.24), and $(1-a_\theta)\left(\sum_{i=1}^{k-1} a_\theta^{k-i} + 1\right) = 1 - a_\theta^k$.

To prove the second part of the corollary I show that $\Sigma_{k+1} = \mathrm{Var}_t[T_{t,t+k+1}]$ defines a system of quadratic matrix equations that admits 2^N solutions under some parametric condition. Since $P_{X_1} = P_{X_2} = \ldots = P_{X_k} \equiv P_X$, it follows that

$$\begin{aligned} \sum_{i=1}^{k} P_{X_i} X_{t+k+1-i}^I &= P_X\left(\sum_{i=1}^{k} X_{t-i}^I + (k+1)\sum_{j=0}^{k-1}(\theta_{t+k-j} - \theta_{t+k-1-j})\right) \\ &= P_X\left(\sum_{i=1}^{k} X_{t-i}^I + (k+1)(\theta_{t+k} - \theta_t)\right). \end{aligned}$$

Plugging the last formula in the expression for $T_{t,t+k+1}$ (A.22) and using (11) to replace P_X with P_θ gives

$$\Sigma_{k+1} = \mathrm{Var}_t\left[\frac{a_D}{R-a_D}D_{t+k+1} + \sum_{i=1}^{k+1} R^{k+1-i}D_{t+i} + P_\theta \epsilon_{t+k+1}^\theta - \frac{R^{k+1}(1-a_\theta)}{R^{k+1} - a_\theta^k}P_\theta \theta_{t+k}\right].$$

Since dividends and liquidity shocks are uncorrelated, I can focus on both terms separately. First, consider dividend shocks. Tedious computations show that

$$\mathrm{Var}_t\left[\frac{a_D}{R-a_D}D_{t+k+1} + \sum_{i=1}^{k+1} R^{k+1-i}D_{t+i}\right] = \left(\frac{R}{R-a_D}\right)^2 \left(\sum_{i=0}^{k} R^{2i}\right)\Sigma_D. \tag{A.25}$$

Lemma 2 provides more details about similar computations.

Second, consider liquidity shocks. Again, tedious computations show that

$$\mathrm{Var}_t\left[P_\theta \epsilon^\theta_{t+k+1} - \frac{R^{k+1}(1-a_\theta)}{R^{k+1}-a^k_\theta} P_\theta \theta_{t+k}\right] = \left(1 + \left(\frac{R^{k+1}(1-a^k_\theta)}{R^{k+1}-a^k_\theta}\right)^2\right) P_\theta \Sigma_\theta P'_\theta. \tag{A.26}$$

This last expression implies that Σ_{k+1} defines a quadratic matrix equation for P_θ. Finally, use (A.23), (A.24), and simplify terms to obtain

$$\Sigma_{k+1} + \frac{(R^{k+1}-1)(R^{k+1}-a^{k+1}_\theta)}{\gamma_I(k+1)(R^{k+1}-a^k_\theta)} P_\theta = 0. \tag{A.27}$$

Replacing Σ_{k+1} with (A.25) and (A.26) gives the following quadratic matrix equation for P_θ:

$$\begin{aligned} &\left(1 + \left(\frac{R^{k+1}(1-a^k_\theta)}{R^{k+1}-a^k_\theta}\right)^2\right) P_\theta \Sigma_\theta P'_\theta + \frac{(R^{k+1}-1)(R^{k+1}-a^{k+1}_\theta)}{\gamma_I(k+1)(R^{k+1}-a^k_\theta)} P_\theta \\ &+ \left(\frac{R}{R-a_D}\right)^2 \left(\sum_{i=0}^{k} R^{2i}\right) \Sigma_D = 0. \end{aligned} \tag{A.28}$$

The rest of the proof is similar to Corollary 2's. The quadratic matrix equation admits 2^N solutions if

$$\begin{aligned} &\frac{1}{4}\left(\frac{(R^{k+1}-1)(R^{k+1}-a^{k+1}_\theta)}{\gamma_I(k+1)(R^{k+1}-a^k_\theta)}\right)^2 I_N \\ &- \left(\frac{R}{R-a_D}\right)^2 \left(1 + \sum_{i=1}^{k} R^{2i}\right) \left(1 + \left(\frac{R^{k+1}(1-a^k_\theta)}{R^{k+1}-a^k_\theta}\right)^2\right) \Sigma_\theta^{\frac{1}{2}} \Sigma_D \Sigma_\theta^{\frac{1}{2}} \end{aligned} \tag{A.29}$$

is positive definite.■

To prove Proposition 2, I use the following lemma:

Lemma 4. *Let $k = 1$ and $h = 1$. In the single-asset economy with $0 < q < 1$, if $P_\theta < 0$ and $P_X > 0$, then infrequent traders absorb part of the liquidity shocks.*

Proof. Infrequent traders' demand at time t is linear in the state variables and can be written as $X^I_t = \bar{X}_I + X_\theta \theta_t + X_X X^I_{t-1}$. If $X_\theta > 0$, then infrequent traders absorb part of the liquidity shocks. When $k = 1$, $h = 1$, and $N = 1$, the following four equations hold:

$$\frac{q}{2} X_\theta + (1-q)\gamma_F^{-1}\Sigma_1^{-1}\left((a_\theta - R)P_\theta + P_X X_\theta\right) = 1, \tag{A.30}$$

$$\frac{q}{2} X_X + (1-q)\gamma_F^{-1}\Sigma_1^{-1} P_X \left(X_X - R\right) = -\frac{q}{2}, \tag{A.31}$$

$$X_\theta = \gamma_I^{-1}\Sigma_2^{-1}\left(a_\theta\left(a_\theta P_\theta + P_X X_\theta\right) + P_X X_X X_\theta - R^2 P_\theta\right), \tag{A.32}$$

$$X_X = \gamma_I^{-1}\Sigma_2^{-1}\left(P_X X_X^2 - R^2 P_X\right). \tag{A.33}$$

Equations (A.30) and (A.31) are obtained from the market clearing condition. Equations (A.32) and (A.33) follow from the optimization problem of infrequent traders. Since I assume that $P_\theta < 0$ and $P_X > 0$, Equation (A.31) implies that $X_X < R$. But then Equation (A.33) requires $-R < X_X < 0$.

Next, assume $X_\theta < 0$. Equation (A.30) then implies that $(a_\theta - R)P_\theta + P_X X_\theta > 0$, which is equivalent to $RP_\theta < a_\theta P_\theta + P_X X_\theta < 0$. Moreover, Equation (A.32) implies that $a_\theta\left(a_\theta P_\theta + P_X X_\theta\right) + P_X X_X X_\theta - R^2 P_\theta < 0$. Combining the last two conditions gives $a_\theta R P_\theta + P_X X_X X_\theta - R^2 P_\theta < 0$. This is a contradiction since the middle term is positive. Therefore, if $P_\theta < 0$ and $P_X > 0$, then $X_\theta > 0$ in any equilibrium.■

Proof of Proposition 2. Infrequent traders' demand is linear in the state variables and can be written as $X_t^I = \bar{X}_I + X_\theta \theta_t + X_X X_{t-1}^I$. The first lag autocovariance is then given by

$$\begin{aligned}
\mathrm{Cov}\left[Q_{t+1}, Q_t\right] =& P_\theta^2 \mathrm{Cov}\left[\epsilon_{t+1}^\theta - R\epsilon_t^\theta, \epsilon_t^\theta - R\epsilon_{t-1}^\theta\right] + P_X P_\theta \mathrm{Cov}\left[X_t^I - RX_{t-1}^I, \epsilon_t^\theta - R\epsilon_{t-1}^\theta\right] \\
&+ P_X^2 \mathrm{Cov}\left[X_t^I - RX_{t-1}^I, X_{t-1}^I - RX_{t-2}^I\right] \\
=& - RP_\theta^2 \sigma_\theta^2 + P_X P_\theta X_\theta \left(1 - R\left(X_X - R\right)\right) \sigma_\theta^2 + P_X^2 \left(X_X - R\right)^2 X_X \mathrm{Var}\left[X_t^I\right].
\end{aligned}$$

Lemma 4 implies that $X_\theta > 0$ and $X_X < 0$. Hence, each term is negative.

The second lag autocovariance is given by

$$\begin{aligned}
\mathrm{Cov}\left[Q_{t+2}, Q_t\right] =& P_X^2 \mathrm{Cov}\left[X_{t+1}^I - RX_t^I, X_{t-1}^I - RX_{t-2}^I\right] \\
&+ P_X P_\theta \mathrm{Cov}\left[X_{t+1}^I - RX_t^I, \epsilon_t^\theta - R\epsilon_{t-1}^\theta\right] \\
=& P_X^2 \left(X_X - R\right)^2 X_X^2 \mathrm{Var}\left[X_t^I\right] + P_X^2 \left(X_X - R\right) X_X X_\theta^2 \sigma_\theta^2 \\
&+ P_X P_\theta \left(X_X - R\right)\left(1 - RX_X\right) X_\theta \sigma_\theta^2.
\end{aligned}$$

Since $X_\theta > 0$ and $X_X < 0$, each term is positive.■

Proof of Proposition 3. Since the proof is quite similar to the proof of Proposition 1, I only provide the key steps. Conjecture that $P_t = A_{c(t)} Y_t$ and $X_t^I = B_{c(t)} Y_t$. The dynamics of the state variables and excess returns are then given by

$$Y_{t+1} = A_{Y,c(t)} Y_t + B_Y \epsilon_{t+1} \tag{A.34}$$
$$Q_{t+1} = A_{Q,c(t)} Y_t + B_{Q,c(t+1)} \epsilon_{t+1}, \tag{A.35}$$

where the matrices are defined as in Proposition 1. Using (A.34), it follows that

$$Y_{t+j} = \left(\prod_{i=1}^{j} A_{Y,c(t+j-i)}\right) Y_t + \sum_{i=1}^{j-1} \left(\prod_{s=1}^{i} A_{Y,c(t+j-s)}\right) B_Y \epsilon_{t+j-i} + B_Y \epsilon_{t+j}. \tag{A.36}$$

Infrequent traders demand can then be written as

$$X_t^I = \frac{1}{\gamma_I} \Sigma_{k+1,c(t)}^{-1} \left\{ A_{c(t+k+1)} \left(\prod_{i=1}^{k+1} A_{Y,c(t+k+1-i)}\right) + \left(\sum_{i=1}^{k+1} R^{k+1-i} a_D^i\right) \varphi_D \right\} Y_t, \tag{A.37}$$

where $\Sigma_{k+1,c(t)}$ is a constant matrix. This verifies the conjecture that demands are linear in the state variables.

Consider now the problem of frequent traders. The value function of a frequent trader of age j who trades in calendar period $c(t)$ is

$$J(W_t, Y_t, j, c(t)) = \max_{X_t} \mathbb{E}_t[J(W_{t+1}, Y_{t+1}, j+1, c(t+1))], \tag{A.38}$$

such that $W_{t+1} = X_{t,j}' Q_{t+1} + RW_t$ and $J(W_t, Y_t, h, c(t)) = -e^{-\gamma_F W_t}$. When the agent is one period older, the calendar period is then $c(t+1)$. Conjecture that $J(W_{t+1}, Y_{t+1}, j+1, c(t+1)) = -e^{-\alpha_{j+1} W_{t+1} - \frac{1}{2} Y_{t+1}' U_{j+1,c(t+1)} Y_{t+1}}$.

Using standard arguments, it follows that:

$$X_{t,j} = \frac{1}{\alpha_{j+1}} F_{j+1,c(t+1)} Y_t, \tag{A.39}$$

$$\begin{aligned}
\text{where} \quad F_{j+1,c(t+1)} =& \left(B_{Q,c(t+1)} \Xi_{j+1,c(t+1)} B_{Q,c(t+1)}'\right)^{-1} \\
& \left(A_{Q,c(t)} - B_{Q,c(t+1)} \Xi_{j+1,c(t+1)} B_Y' U_{j+1,c(t+1)}' A_{Y,c(t)}\right).
\end{aligned} \tag{A.40}$$

All the parameter matrices are defined recursively from $\alpha_h = \gamma_F$ and $U_{h,c(t)} = 0_{1+2N+kN}$.

$$\alpha_j = \alpha_{j+1} R, \tag{A.41}$$
$$U_{j,c(t)} = M_{j+1,c(t+1)} - 2 \ln \rho_{j+1,c(t+1)} I_{11}, \tag{A.42}$$
$$\begin{aligned} M_{j+1,c(t+1)} =& A'_{Y,c(t)} U_{j+1,c(t+1)} A_{Y,c(t)} \\ &+ F'_{j+1,c(t+1)} B_{Q,c(t+1)} \Xi_{j+1,c(t+1)} B'_{Q,c(t+1)} F_{j+1,c(t+1)} \\ &- A'_{Y,c(t)} U_{j+1,c(t+1)} B_Y \Xi_{j+1,c(t+1)} B'_Y U'_{j+1,c(t+1)} A_{Y,c(t)}, \end{aligned} \tag{A.43}$$
$$\rho_{j+1,c(t+1)} = |I + \Sigma_Y B'_Y U_{j+1,c(t+1)} B_Y|^{-\frac{1}{2}}, \tag{A.44}$$
$$\Xi_{j+1,c(t+1)} = \left(\Sigma_Y^{-1} + B'_Y U_{j+1,c(t+1)} B_Y \right)^{-1}. \tag{A.45}$$

The market clearing condition is

$$\begin{aligned} q_{c(t)} X_t^I + \frac{1-q}{h} \sum_{j=0}^{h-1} X_{t,j}^F &= \bar{S} + \theta_t - \sum_{i=1}^{k} q_{c(t-i)} X_{t-i}^I \\ &= \left(\varphi_{\bar{S}} + \varphi_\theta - \sum_{i=1}^{k} q_{c(t-i)} \varphi_{X_i} \right) Y_t. \end{aligned} \tag{A.46}$$

Equation (A.46) verifies the conjecture that the price is linear in the state variables. Using Equations (A.37) and (A.39), the market clearing condition determines a system of fixed point equations for the $A_{c(t)}$ coefficients. The demand coefficients $B_{c(t)}$ can be solved for using the fixed point system from Equation (A.37):

$$\frac{1}{\gamma_I} \Sigma_{k+1,c(t)}^{-1} \left\{ A_{c(t+k+1)} \left(\prod_{i=1}^{k+1} A_{Y,c(t+k+1-i)} \right) + \left(\sum_{i=1}^{k+1} R^{k+1-i} a_D^i \right) \varphi_D \right\} - B_{c(t)} = 0. \tag{A.47}$$

This concludes the proof.■

Proof of Proposition 4. Assuming that $k = 1$, infrequent traders' demand vector at time t can be written as $X_t^I = \bar{X}_{I,c(t)} + X_{\theta,c(t)} \theta_t + X_{X,c(t)} X_{t-1}^I$. When $h = 1$, the market clearing condition gives the following equilibrium condition:

$$q_{c(t)} X_{\theta,c(t)} + (1-q) \gamma_F^{-1} \Sigma_{c(t)}^{-1} \left(a_\theta P_{\theta,c(t+1)} - R P_{\theta,c(t)} + P_{X,c(t+1)} X_{\theta,c(t)} \right) = 1, \tag{A.48}$$

where $\Sigma_{c(t)} = P_{\theta,c(t+1)} \Sigma_\theta P'_{\theta,c(t+1)} + \left(\frac{R}{R-a_D} \right)^2 \Sigma_D$.

Consider the case with two calendar periods, and let $q_2 = 0$. Equation (A.48) implies

$$q_1 X_{\theta,1} + (1-q_1) \gamma_F^{-1} \Sigma_1^{-1} \left(a_\theta P_{\theta,2} - R P_{\theta,1} + P_{X,2} X_{\theta,1} \right) = 1, \quad \text{and} \tag{A.49}$$
$$(1-q_1) \gamma_F^{-1} \Sigma_2^{-1} \left(a_\theta P_{\theta,1} - R P_{\theta,2} \right) = 1. \tag{A.50}$$

For simplicity, assume that there is only one asset. Plugging (A.50) in (A.49) gives

$$q_1 X_{\theta,1} + \left(\frac{a_\theta P_{\theta,2} - R P_{\theta,1} + P_{X,2} X_{\theta,1}}{a_\theta P_{\theta,1} - R P_{\theta,2}} \right) \frac{\Sigma_2}{\Sigma_1} = 1. \tag{A.51}$$

Equation (A.50) implies that $a_\theta P_{\theta,1} - R P_{\theta,2} > 0$. Using the methodology of Lemma 4, the conditions $P_{\theta,c(t)} < 0$ and $P_{X,c(t)} > 0$ imply that $X_{\theta,c(t)} > 0$. In that case, if $P_{\theta,1} < P_{\theta,2}$, then $\frac{\Sigma_2}{\Sigma_1} > 1$ and $\frac{a_\theta P_{\theta,2} - R P_{\theta,1} + P_{X,2} X_{\theta,1}}{a_\theta P_{\theta,1} - R P_{\theta,2}} > 1$, which is impossible. As a result, $P_{\theta,1} > P_{\theta,2}$ in any equilibrium. Equivalently, $\Sigma_1 > \Sigma_2$. Equation (15) therefore implies that $\mathbb{E}[Q_{t+1} | c(t) = 1] > \mathbb{E}[Q_{t+1} | c(t) = 2]$.■

Proof of Proposition 5. The risk-aversion (mass) of frequent traders varies with the calendar period and is given by $\gamma_{c(t)}$. In equilibrium, it is direct to show that $a_\theta P_{\theta,c(t+1)} - RP_{\theta,c(t)} = \gamma_{c(t)} \left(P_{\theta,c(t+1)} \Sigma_\theta P'_{\theta,c(t+1)} + \left(\frac{R}{R-a_D}\right)^2 \Sigma_D \right)$. With two calendar periods and one asset, if $\gamma_1 > \gamma_2$, then $P_{\theta,1} < P_{\theta,2}$ in any equilibrium (by contradiction). Since price impact is negative, this implies that $a_\theta P_{\theta,2} - RP_{\theta,1} > a_\theta P_{\theta,1} - RP_{\theta,2}$.

Using the market clearing condition, the expected excess return is

$$\begin{aligned}\mathbb{E}[Q_{t+1}|c(t)] &= \gamma_{c(t)} \left(P^2_{\theta,c(t+1)} \Sigma_\theta + \left(\frac{R}{R-a_D}\right)^2 \Sigma_D \right) \bar{S} \\ &= \left(a_\theta P_{\theta,c(t+1)} - RP_{\theta,c(t)}\right) \bar{S}. \end{aligned} \tag{A.52}$$

The proof follows from applying the previous result in Equation (A.52).■

Proof of Proposition 6. When $q = 0$ (or $q = 1$), trading volume is given by $V_t = |\theta_t - \theta_{t-1}|$. To compute volume autocorrelation, the following standard lemma is useful and stated without proof:

Lemma 5. *Let X and Y be jointly normal r.v. with zero mean, variances σ_X^2 and σ_Y^2, and correlation ρ.*

$$\text{Cov}[|X|, |Y|] = \frac{2}{\pi} \left(\rho \arcsin(\rho) + \sqrt{1-\rho^2} - 1 \right) \sigma_X \sigma_Y.$$

Using the properties of θ_t gives

$$\theta_{t+j} - \theta_{t+j-1} = \text{const} + (a_\theta - 1) a_\theta^j \theta_{t-1} + \epsilon^\theta_{t+j} + (a_\theta - 1) \sum_{i=0}^{j-1} a_\theta^{j-1-i} \epsilon^\theta_{t+i}, \quad j \geq 1.$$

Hence, the autocovariance of $\Delta\theta_{t+j} \equiv \theta_{t+j} - \theta_{t+j-1}$ is given by

$$\text{Cov}[\Delta\theta_t, \Delta\theta_{t+j}] = -\left(\frac{1-a_\theta}{1+a_\theta}\right) a_\theta^{j-1} \sigma_\theta^2, \quad j \geq 1.$$

Therefore, $\rho_{\Delta\theta_t, \Delta\theta_{t+j}} \equiv \rho_{\Delta\theta}(j) = \frac{-\left(\frac{1-a_\theta}{1+a_\theta}\right) a_\theta^{j-1} \sigma_\theta^2}{\frac{2}{1+a_\theta} \sigma_\theta^2} = -\left(\frac{1-a_\theta}{2}\right) a_\theta^{j-1}$, $j \geq 1$. Thus, $\rho_{\Delta\theta}(j) < 0$ and is an increasing concave function of j for $0 < a_\theta < 1$.

Using Lemma 5, one has

$$\text{Cov}[V_t, V_{t+j}] = \frac{2}{\pi} \left(\rho_{\Delta\theta}(j) \arcsin(\rho_{\Delta\theta}(j)) + \sqrt{1 - \rho_{\Delta\theta}(j)^2} - 1 \right) \sigma^2_{\Delta\theta}.$$

Note that $\frac{d\text{Cov}[V_t, V_{t+j}]}{d\rho_{\Delta\theta}(j)} = \frac{2}{\pi} \arcsin(\rho_{\Delta\theta}(j)) \sigma^2_{\Delta\theta}$. Using this fact and the properties of the arcsin function, it is direct to show that $\text{Cov}[V_t, V_{t+j}] > 0$ and is a decreasing convex function of j ($j \geq 1$). Note that, when $a_\theta = 1$, $\text{Cov}[V_t, V_{t+j}] = 0$, $j \geq 1$.

Since $\text{Cov}[\Delta V_t, \Delta V_{t+j}] = 2\text{Cov}[V_t, V_{t+j}] - \text{Cov}[V_t, V_{t+j-1}] - \text{Cov}[V_t, V_{t+j+1}]$, it follows that $\text{Cov}[\Delta V_t, \Delta V_{t+j}] < 0$ by Jensen's inequality.■

B Appendix: Calibration

Parameter		Daily returns	Intraday returns
Proportion of infrequent traders	q	0.6	0.99
Inattention period	k	4	13
Risk aversion	γ_F, γ_I	1	1
Risk-free rate	R	$1.05^{\frac{1}{250}}$	1.0001
Persistence of dividends	a_D	0	0
Persistence of liquidity trading	a_θ	0.8	0.7
Volatility of dividends	σ_D	0.04	0.01
Volatility of liquidity shocks	σ_θ	1	0.6
Correlation of dividend shocks	ρ_D	0.3	0.3
Correlation of liquidity shocks	ρ_θ	0	0
Number of assets	N	2	2
Horizon of frequent traders	h	20	20

Table 1: Model calibrations for daily and intraday returns

Table 1 shows the calibration used to compare the model's predictions to the empirical analysis on intraday returns in Section 4.2 and daily returns in Section 4.3. This paper does not aim to provide an exact quantitative match to the data. The parameters are therefore chosen to broadly match the patterns observed in the data while keeping the calibration as simple and transparent as possible.

Trading frequencies. The calibration of q for intraday returns is discussed in Section 4.2. For daily returns, I assume that 60% of the agents adjust their portfolios once a week ($q = 0.6$, $k = 4$), while the remaining agents trade every period with a monthly horizon ($h = 20$). Frequent traders have long horizons to illustrate that they do not arbitrage away the return predictability pattern.

Dividends. Dividends are *iid* for simplicity since dividend persistence does not affect excess return autocorrelation. Dividend shocks volatility does not affect the qualitative results. I use $\sigma_D = 0.04$ for daily returns and $\sigma_D = 0.01$ for intraday returns. Dividend shocks correlation is set to 0.3.

Liquidity shocks. The persistence of liquidity shocks is the only parameter that can generate persistence in excess return autocorrelation in this type of setup. In this respect, the empirical autocorrelation plot provides a direct way to estimate a_θ. For daily returns, Figure 5 suggests a relatively high persistence. The persistence of liquidity shocks required by the model to approximately match the decaying autocorrelation pattern for the first lags in the data seems lower for intraday returns than for daily returns (Figures 1 and 5). This evidence is inconsistent with a single liquidity trading process driving both intraday and daily returns. For instance, a combination of low-frequency and high-frequency liquidity shocks would result in a more complicated process than an AR(1). Still, the AR(1) assumption represents a clean benchmark to evaluate the main results. Furthermore, the infrequent rebalancing mechanism does not require any persistence in liquidity shocks (Section 4.1). Prior literature does not provide precise guidance about liquidity shocks volatility, which is hard to estimate. I set σ_θ to a lower value than the equivalent value estimated by Campbell et al. (1993). Liquidity shocks correlation is set to zero for simplicity.