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“Intervention Policy in a Dynamic Environment: Coordination and Learning”

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Abstract

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Intervention Policy in a Dynamic Environment: Coordination and Learning*

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PRELIMINARY. COMMENTS WELCOME.

Abstract

We model a dynamic economy with strategic complementarity among investors and a government that intervenes as a large player in global games to mitigate coordination failures. We establish existence and uniqueness of equilibrium, and show interventions affect coordination both contemporaneously and dynamically. Initial intervention alters public information structure that could either facilitate or hamper subsequent coordination. Our results suggest that even absent signaling, optimal policy should emphasize early intervention. Moreover, considering dynamic coordination increases or decreases optimal early intervention, depending on relative costs across interventions. Our paper is applicable to intervention programs such as the bailouts of money market mutual funds and commercial paper market during the 2008 financial crisis.

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1 Introduction

Coordination failures are prevalent and socially costly, thus effective interventions could prove critical in ameliorating such failures. For example, financial systems, especially short-term credit markets, are vulnerable to liquidity shocks and runs by investors. The 2008 financial crisis witnessed a series of runs on both financial and non-financial institutions. In response, governments and central banks around the globe have employed an array of policy actions over time to provide liquidity. Given the novelty, the scale, and the dynamic nature of the interventions, it is natural to not only study the effectiveness of intervention in isolation, but also question how interventions dynamically relate to each other.

More broadly, how should a government formulate intervention policy in a dynamic economy with strategic complementarity? How does intervention in one market or region affect subsequent interventions in other markets or regions? This paper tackles these questions by modeling the government as a large player in sequential global games. We find a large class of observed intervention forms not only helps select welfare-improving equilibria within the period, but also dynamically affects future coordination among agents. In particular, optimal dynamic policy features an emphasis on early intervention as opposed to late intervention, but excessive intervention could adversely affect future coordination through modifying the informational environment. To our knowledge, this paper is among the first to study the dynamic coordination effect of endogenous interventions. Besides providing a foundation for the conventional wisdom of early intervention, we also add new insights regarding budget constraints, contingent policy, and information externality.

Specifically, in a two-period economy, a group of infinitesimal investors in each period choose whether to run or stay with a fund. Running guarantees a higher payoff when the fund fails, whereas staying pays more if the fund survives. The fund survives if and only if the total measure of investors who choose to stay is above fundamental threshold θ -interpreted as unhedgeable system-wide illiquidity shock or the quality of underlying investment. θ is unobservable. Following the global games framework, each investor observes a noisy signal of θ . Prior literature has established that there exists a unique equilibrium in which the fund survives as long as the true θ is below a threshold θ^* , and each investor stays if and only if his private signal is below a certain threshold.

Government can intervene in various forms, such as adjusting interest rate, injecting

liquidity, and writing guarantees. We focus on direct liquidity injection that helps a fund or market meet redemption or avoid fire sales, and discuss how the intuition and insights apply to other forms of intervention. The equilibrium θ^* increases strictly with the size of government intervention: the more liquidity injected, the more likely the fund's survival. This is the static effect of intervention on coordinating investors. Therefore, in a static economy, a benevolent government should always increase intervention till the contemporaneous marginal benefit equals to the marginal cost.

However, because the outcome in the first period is publicly observable, agents prior belief on θ in the second period is truncated. When the fund has survived in the first period, agents learn that $\theta < \theta^*$. Therefore, agents belief on θ is adjusted downwards and coordination becomes much easier. The opposite holds if the fund has failed in the first period. Since governments intervention in the first period affects θ^* , its intervention also has a dynamic coordination effect in the second period. Early intervention affects the structure of the information that future agents observe and can potentially render later interventions unnecessary. However, intervening too much early on might create the stigma that the economy is too bad. The optimal policy has to trade off these forces.

We establish results on the existence and uniqueness of equilibrium under any government intervention plan. We also study the optimal policy by a benevolent government. Under fairly general conditions, optimal policy features early intervention: the scale of intervention in the first period always exceeds that in the second period. The intuition of this result relies on the dynamic coordination effect of early intervention. If the fund has survived in the first period, the government needs less intervention to induce investors to stay in the second period, as the fund is likely to survive again. If, however, the fund has failed in the first period, it becomes more costly to intervene, and the government is less inclined to induce investors to stay in the second period, because the fund is likely to fail again. Therefore, as long as intervention costs are comparable across the periods, optimal intervention always induces an equilibrium in which the public outcomes are perfectly correlated across two periods. Intuitively, this is the best information structure early intervention can generate for the coordination in the second period. Early intervention is then important as it increases the probability of survival in both periods.

When the capacities to intervene in both periods are comparable, survival leads to sur-

vival, and consideration of dynamic coordination leads to greater intervention in the first period. However, excessive early intervention harms investors by rendering public news useless when the fund has survived, because investors infer that large size of intervention is the reason for survival. If, on the other hand, the fund still fails despite of a large intervention, investors would become really pessimistic. This adverse effect on belief harms investors welfare even when intervention is costless. This detrimental effect dominates when the capacities to intervene across the two periods differ drastically, so much so that survival in the first period does not guarantee survival in the second period (second period cost is too high relative to the first), nor does failure lead to failure (second period cost is so low that one can intervene more despite the negative update from first period's failure). Then the more the government considers the dynamic coordination effect, the more it shades intervention.

Government policy can help avoid inefficient equilibria, and our model is useful for studying and assessing policies that serve so in a dynamic economy. In particular, we highlight the role of government's action on the information structure: not only does it affects the probability of good news versus bad news, but also it affects the informativeness of news. This paper thus contributes to our understanding of the information environment during a crisis. Policy responses in a crisis are fundamentally about managing expectations, and formation of expectations is understudied in the context of financial crises. Since the onset of the crisis, Bernanke and Geithner spoke of it as a bank run and emphasized that need to combat a financial crisis with the "use of overwhelming force to quell panics" (p. 397 in Geithners Memoir), a tactic of shock and awe that often connates signaling by the government. However, government typically does not have superior information and political constraints are real at least at the onset of the crisis, making utilizing overwhelming force infeasible.¹ Therefore, information signaling alone cannot fully justify the conventional wisdom of emphasizing early intervention.

Our model considers the various constraints and limitations the government faces, and carefully examines in our baseline analysis the case in which the government does not have superior information. We identify a realistic information structure channel that more broadly validates the conventional wisdom. Even without superior information, the government should still emphasize early intervention. This channel also predicts "overwhelming" re-

¹For example, see Swagel (2015).

sponse could be detrimental. This complements the opt-discussed stigma in policy intervention which is primarily focused on information asymmetry.² Depending on the costs of intervention, early interventions could have either positive or negative externality on later interventions. The results hold regardless whether the government can commit to second period intervention ex-ante, which includes the situation that a policy has to be rolled out before observing the outcomes of a previous intervention.

The results and insights of our model apply to many situations with strategic complementarity and multiple interventions and actions by a large player. Examples include interventions in currency attacks, stock market crises, and bank runs. We use runs on money market mutual funds (MMMFs) and on the commercial paper market to illustrate: in the wake of bankruptcy of Lehman Brothers on September 15, 2008, the U.S. MMMF industry experienced massive waves of redemption. Interestingly, the first MMMF that broke the buck right after Lehman's collapse, the Reserve Primary Fund (RPF), only had about 1% of its holdings in commercial papers issued by Lehman Brothers. Although most MMMF holdings were diversified and immune to idiosyncratic liquidity risks, investors were concerned with funds' exposures to common risks, and that other investors might withdraw first. This episode thus constitutes a classic run phenomenon. Almost contemporaneous with the run on RPF, other money market funds began to see vast outflows and within a week, institutional investments plummeted by more than \$172 billion. The situation appeared so dire that during the same week the U.S. treasury announced unlimited insurance to all MMMF depositors and the Federal Reserve announced The Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facilities (AMLF) on Sept 19, 2008 to fund depository institutions and bank holding companies to finance purchases of high credit asset-backed commercial paper (ABCP) from MMMFs under certain conditions. This alleviates the pressure of MMMFs to meet this round of redemption demands without suffering fire sale costs (and provides liquidity in the ABCP and money markets), and essentially corresponds to a direct injection of liquidity to MMMFs because only MMMFs are eligible for using the facilities. Other interventions include the Money Market Investor Funding Facility (MMIFF) to provide credit to SPVs to purchase eligible money market instruments. The AMLF lent \$150 billion in its first 10 days

²When the government holds private information (potentially on the state of the economy), a larger intervention could be interpreted as a sign of weaker fundamentals.

of operation, and these measures overall quickly stopped the runs on MMMFs.³

Although government interventions immediately restored market liquidity, investors interpreted Lehman's failure as a revelation that commercial papers, issued and sponsored by financial institutions, were far riskier than previously reckoned. Shrinking demand by money market funds and the like therefore led to disruption in commercial paper market in general, causing a panic and reduction in commercial paper outstanding by \$ 330 billion within a month.⁴ Because the deepening dysfunction in the commercial paper market risked greater disruptions across the real economy, the government created the Commercial Paper Funding Facility (CPFF) that started operation on Oct 27, 2008 to reduce the difficulty of corporations in rolling over their short-term commercial papers.⁵ Investors in commercial papers including money market funds, foreign sectors, and mutual funds reacted to the intervention after having observed the outcome of AMLF. To the extent that coordinations and runs in both the MMMFs market and commercial paper markets depend on highly correlated fundamentals - the credit risk and systemic illiquidity of commercial papers, early intervention generates information about the fundamental just as our model describes. More broadly, the two periods could be viewed as two waves of runs or runs on two separate funds or markets sharing the same sponsor or economic fundamental. In that regard, our model also applies to the two runs on commercial papers, one due to the collapse of ABCP market in 2007, and the other triggered by Lehman's failure in 2008.

Literature

This paper is related to equilibrium selection and extends the global games framework in dynamic settings with government as a large player. The global game approach first introduced in Carlsson and Van Damme (1993) and Morris and Shin (1998) relaxes the assumption of common knowledge to resolve equilibrium indeterminacy. Burdzy, Frankel, and Pauzner (2001) and Frankel and Pauzner (2000) further extends this insight to dynamic coordination games. Our paper differs by explicitly model government as a large player

³See, for example, Duygan-Bump, Parkinson, Rosengren, Suarez, and Willen (2013).

⁴Primarily in financial commercial paper as opposed to ABCPs, see KACPERCZYK and SCHNABL (2010) for more details. Kacperczyk and Schnabl (2013) and Schmidt, Timmermann, and Wermers (2015) also document the run and government intervention of money market mutual funds during the 07-09 crisis.

⁵Adrian, Kimbrough, and Marchioni (2010) provides a detailed description of the program.

that endogenously selects coordination equilibrium. A few other papers such as Corsetti, Dasgupta, Morris, and Shin (2004) model large players in global games, but do not consider government policy or dynamic coordination. Other studies on dynamic global games, such as Morris and Shin (1999), Angeletos, Hellwig, and Pavan (2007) and Mathevet and Steiner (2013), model large players as passive agents, if at all. We demonstrate that large players in global games play important roles in coordinating agents through both static and dynamic channels.

This study also complements existing work on government interventions during financial crises. Strategic complementarity in financial markets is well-recognized in prior literature, notably Diamond and Dybvig (1983), Chen, Goldstein, and Jiang (2010), Hertzberg, Liberti, and Paravisini (2011), and He and Xiong (2012), and Goldstein and Pauzner (2005). While discussions on too-big-to-fail centers on large interconnected financial institutions, this paper studies dynamic intervention in markets with dispersed players and strategic complementarity. Closely related is Acharya and Thakor (2014) which considers how liquidation decisions by informed creditors of one bank signal systematic shocks to other creditors and create contagion. Angeletos, Hellwig, and Pavan (2006) considers signaling in a global game and policy traps. Two other related papers are Bebchuk and Goldstein (2011), which examines the effectiveness of various forms (rather than the extent) of exogenous government policies in avoiding self-fulfilling credit market freezes, and Sakovics and Steiner (2012), which analyzes who matters in coordination failures and how to set intervention targets. None of these studies concerns the dynamic coordination effects of policies or the robust comparative statics of intervention under general cost functions. Neither do they focus on information structure channel.

Finally, this study adds to the emerging studies on information design and Bayesian persuasion. Recent papers study situations where one agent designs the informational environment include Rayo and Segal (2010), Ely, Frankel, and Kamenica (2015), and Gentzkow and Kamenica (2011), Goldstein and Leitner (2015), and Bouvard, Chaigneau, and Motta (2015). Only the last two concern financial markets and they focus on stress tests rather than crisis intervention. A closely related paper on Bayesian persuasion in coordination games is Goldstein and Huang (2016), which in a similar way endogenizes the truncation of beliefs introduced in Angeletos, Hellwig, and Pavan (2007). The authors there focus on

one coordination game where the government pre-commits to a regime change policy that conveys the information the government first accesses to the investors. This paper underscores an information structure channel in coordinating agents' behaviors in a broader class of interventions in which the government does not have superior information.

The rest of the paper is organized as follows: Section 2 lays out the basic framework and establishes a static benchmark. Section 3 characterizes the equilibrium in dynamic settings. Section 4 solves for the optimal intervention. Section 5 extends the model and discusses other properties of the equilibrium. Section 6 concludes.

2 Model

In this section, we introduce a two-period, repeated version of global games, with the government as a large player that can intervene. In the baseline model, we follow the literature and specialize to a representative intervention form: government directly infusing capital to funds subject to runs in each period. Section 5.2 and the appendix discuss how main intuition and results generalize to other popular intervention forms. We start by analyzing a static model as our benchmark in Section 2.1 and move to the dynamic setup in Section 2.2.

The dynamic links are twofold: (a) the fundamentals are identical across two periods; (b) agents in period 2 observe the outcome in the first period. We follow Goldstein and Pauzner (2005) and Bouvard, Chaigneau, and Motta (2015) by assuming that signals follow uniform distribution.

2.1 Static benchmark

Model setup

A fund has a continuum of investors indexed by i and normalized to unit measure. Each has one unit capital invested in the fund, and simultaneously choose between two actions: stay ($a_i = 1$) or withdraw ($a_i = 0$). For the remaining analysis, we interpret withdrawals as “runs” on the fund, and staying can be interpreted as rolling over short-term debts. The net payoff from running on the fund and investing the proceeds in an alternative vehicle

(such as treasury bill) is always equal to r , whereas the payoff to each investor from staying is R if the fund survives the run ($s = S$), and is 0 if the fund fails ($s = F$). Let $R > r$. Therefore, an investor finds it optimal to stay if and only if she expects the probability of survival exceeds the cost of illiquidity defined as $c \equiv \frac{r}{R}$. Table 1 (left panel) shows the net payoff of each action under different states and actions. In the right panel of Table 1, we normalize the payoff matrix by subtracting r and scaling by $\frac{1}{R}$. For notational convenience, we use the normalized net payoffs for the remainder of the paper.

Table 1: Net Payoffs and Normalized Net Payoffs

	Stay	Run		Stay	Run
Survive	R	r	Survive	$1 - c$	0
Fail	0	r	Fail	$-c$	0

Agents' decisions are complements: the fund is more likely to survive as more agents choose to stay. Specifically, the fund survives if and only if

$$A + m \geq \theta \tag{1}$$

where A represents total measure of agents who choose to stay, $m \in [0, 1]$ is the size of the government's liquidity injection to the fund. We assume it is less than one, the normalized total amount of capital in the market. $\theta \in \mathbb{R}$ summarizes the underlying fundamental. One interpretation of θ is the common component of noisy investors who must liquidate their positions immediately due to liquidity shock. It can also be understood as the liquidity commitment of the financial sponsor to the fund. For the fund to survive, liquidity $m + l - \theta$ must dominate the redemption $1 - A$, where l is existing liquidity that is set to 1 WLOG.

The government cares about social welfare comprised of investors total payoff less the intervention cost $k(m)$ which is weakly increasing and quasiconvex. In Section 5, we examine other forms of intervention such as adjusting the interest rate r or providing a guarantee on investment by partially covering c if the fund fails, and modifies intervention costs accordingly.

Apparently, coordination is needed when both θ and m are commonly known by all agents. Indeed, if $\theta - m \in (0, 1)$, two equilibria coexist. In one equilibrium, all investors stay and in the other one, all investor run. Global games resolve this issue of multiple equilibria

through introducing incomplete information. We apply the same technique to assume that agents each observes a noisy private signal of θ . In particular, agent i observes,

$$x_i = \theta + \varepsilon_i \tag{2}$$

where the noise $\varepsilon_i \sim Unif[-\delta, \delta]$ is i.i.d. across investors. For simplicity, we assume that the prior distribution of θ is uniform on $[-B, B]$ where $B \gg \max\{\delta, m\}$.⁶ We also assume the government does not know the realization of the fundamental θ (and does not have private signal about it, which we relax later). Essentially we are assuming that institutional investors are typically more informed about the fundamental state of the market. For example, in Diamond and Kashyap (2015), financial institutions know better than the government about the fundamental illiquidity. This assumption also follows from earlier studies such as Goldstein and Pauzner (2005) and Sakovics and Steiner (2012), and studies assuming tax and subsidy distortions due to the government's inferior knowledge to utilize or allocate resources. Notice this does not rule out that the government owns private information not about the fundamental, and we discuss such a case later.

Partial Equilibrium Given Intervention

We restrict the equilibrium set to symmetric Bayesian Nash equilibria (BNE) in monotone strategies: all agents' strategies are symmetric and monotonic *w.r.t.* x and m . Specifically, agent i 's strategy $a_i(x_i, m)$ is non-increasing in x_i and non-decreasing in m .

Since $B \gg \max\{\delta, m\}$, it is *w.l.o.g.* to further restrict the equilibrium set to threshold equilibria denoted by (θ^*, x^*) . The fund survives if and only if $\theta \leq \theta^*$ and each investor stays if and only if his signal $x \leq x^*$. Lemma 1 below summarizes the equilibrium outcome in the static game.

Lemma 1. *In the static game, there exists a unique symmetric BNE in monotone strategies*

⁶In fact, we could use uninformative prior by taking $B \rightarrow \infty$, but technically many expressions would not be well-defined.

(θ^*, x^*) , where

$$\begin{cases} \theta^* &= 1 + m - c \\ x^* &= 1 + m - c + \delta(1 - 2c). \end{cases} \quad (3)$$

Each investor's strategy follows $a_i = 1 \{x_i \leq x^*\}$. The fund's outcome $s = 1 \{\theta \leq \theta^*\}$.

According to Lemma 1, the fund survives if and only if $\theta \leq \theta^*$. Each agent stays if and only if his private signal $x_i \leq x^*$. Note that θ^* increases in m and so is x^* . In words, the fund is more likely to survive and investors are more inclined to stay if the size of government intervention increases. This is the static effect of government intervention on coordination. In the next section, we will show that government intervention may also have dynamic coordination effects.

The effect of noise precision is ambiguous. While θ^* is independent of δ , x^* may or may not increase in δ , depending on whether $c < \frac{1}{2}$. The intuition of this result is interesting. The marginal investor's belief on θ satisfies $\Pr(\theta \leq \theta^* | x = x^*) = c$. When $c < \frac{1}{2}$, cost of staying is low so that staying is the *status quo*. The marginal investor is more concerned with those agents who decide to run. For a fixed θ^* , more agents will run as δ increase and thus the marginal investor behaves more conservatively by choosing a higher x^* . In contrast, when $c > \frac{1}{2}$, cost of staying is high and run is the *status quo*. The marginal investor is more concerned with those agents who decide to stay. For a fixed θ^* , more agents will stay as δ increase and thus the marginal investor behaves more aggressively by choosing a lower x^* .

It is noteworthy that these results are robust to the assumption that noises follow uniform distribution. If we alternatively assume that noises follow normal distribution (or any distribution whose *c.d.f.* satisfies $F(0) = \frac{1}{2}$), all above properties continue to hold (the expression of θ^* is even unchanged!).

Welfare and Optimal Intervention

Let V_i be investor i 's net payoff and $W = E \left[\int_0^1 V_i di \right]$. Then investors' welfare is

$$W = \frac{1}{2B} \left[\underbrace{\int_{-B}^{\theta^*} (1-c) d\theta}_{\text{fundamental}} - \underbrace{\int_{x^*-\delta}^{\theta^*} (1-c) \left(1 - \frac{x^* - (\theta - \delta)}{2\delta} \right) d\theta}_{\text{overrun}} - \underbrace{\int_{\theta^*}^{x^*+\delta} c \frac{x^* - (\theta - \delta)}{2\delta} d\theta}_{\text{underrun}} \right]$$

Let us interpret the above payoff function. $\frac{1}{2B}$ is the probability density of the uniform distribution. The terms inside the square bracket split into three terms. The first term, *fundamental*, equals to the net payoff if all agents stay when the fund survives. The second term, *overrun*, represents the net payoff loss due to the fact that some agents choose to run when the fund survives. The last term, *underrun*, is the net loss from agents who choose to stay when the fund fails.

Simple calculation suggests that total welfare as the sum of payoffs to the government and investors is

$$W - k(m) = \frac{(1-c)[1+B-c(1+\delta)+m]}{2B} - k(m)$$

The marginal benefit of m on W is a constant, $\frac{(1-c)}{2B}$. This result comes from the fact that an increase in m also raises θ^* linearly, making the fund more likely to survive. $\frac{(1-c)}{2B}$ is the net payoff from stay $1-c$, scaled by the probability density $\frac{1}{2B}$. Therefore, intervention improves coordination. Taking into consideration the intervention cost. There exists an optimal intervention:

$$m^* = \sup \left\{ m \in [0, 1] : \lim_{\epsilon \rightarrow 0} \frac{k(m+\epsilon) - k(m)}{\epsilon} \leq \frac{1-c}{2B} \right\}$$

For example, if $k(m) = \frac{1}{2}zm^2$, then $m^* = \min \left\{ \frac{1-c}{2zB}, 1 \right\}$.

2.2 Dynamic Economy with Contemporaneous Complementarity

We make three modifications to the foregoing static game: first, the economy now lasts for two periods, $t \in \{1, 2\}$, that represents relatively short episodes of panics and crises.

In each period, there is a continuum of agents of measure 1. We assume non-overlapping agents in the two periods in order to focus the learning in the model on whether or not a run occurred during the first period. Because the government plays a role in determining the level of this information and thus presents a trade-off. To bring the traditional Bayesian updating on new private information sheds no new light here. Each agent's receiving only one private signal also makes characterizing strategies easier.⁷ Second, we allow agents in period 2 to observe the outcome in the first period, i.e., whether the fund has survived the run; third, the intervention cost becomes $K(m_1, m_2)$ that is weakly increasing and quasiconvex in both arguments, and satisfies $K(0, \cdot) = K(\cdot, 0) = 0$ and $\{m_1, m_2\} \in \mathcal{I}$, where $\mathcal{I} \subset \mathcal{R}^2$ indicates a convex set of feasible interventions. Notice the cost function nests the static benchmark in that we can set $K(m, 0) = k(m)$.

The two periods are linked: (a) the fundamentals $\{\theta_t\}_{t=1,2}$ are identical across two periods;⁸ (b) agents in period 2 also observe the public outcome that whether investment has succeeded in the first period, indicated by $s_1 = S$ or $s_1 = F$; (c) there could potentially be interaction between the costs of intervention across the two periods. For the rest of the analysis, we will omit the subscript of θ .

The government chooses the interventions to maximize investors' welfare lest the intervention cost $K(m_1, m_2)$. In each period, agents simultaneously choose between stay with the fund ($a_t = 1$) or run ($a_t = 0$). The period-by-period normalized payoff structure is identical to the static game: stay ($a_t = 0$) always guarantees 0 payoff whereas run ($a_t = 1$) pays off $1 - c$ in survival and $-c$ in failure. Agents' decisions within the same period are complements: investment in period t succeeds if and only if

$$A_t + m_t \geq \theta, \tag{4}$$

where A_t is the total measure of investors who choose to invest, m_t denotes the size of liquidity injected by the government. Again, θ represents the fundamental. Similar to the interpretation of the static game, θ represents the market-wide illiquidity that affects both periods.

⁷For completeness, we numerically solve such a case in Section 5.

⁸We made this assumption for simplicity. More generally, we need the fundamentals to be highly correlated to have non-trivial learning, which is natural as the periods are relatively short in the setup.

The timing within each period goes as follows. First, government announces m_t . Second, each investor it receives a private signal $x_{it} = \theta_t + \varepsilon_{it}$ about the fundamental where $\varepsilon_{it} \sim Unif[-\delta, \delta]$. Lastly, investors choose whether to stay and their payoffs realize. The setup is dynamic in the sense that period 1's outcome is revealed before investors take actions in period 2. However, we do not rule out the possibility that the two periods can overlap time-wise. We thus differentiate between two scenarios in the second period, depending on whether the government chooses m_2 before or after s_1 is realized. Note that in many situations the government has to roll out policy programs before knowing the outcome of previous interventions, as getting intervention budget and implementing the programs on the go are often unrealistic. This corresponds to choosing m_2 before s_1 and we call it **committed intervention**. When the government cannot or do not commit to choosing an m_2 before observing s_1 , it is equivalent to choosing contingent plans $\{m_{2S}, m_{2F}\}$, and we call it **contingent intervention**. We show the key results in the paper hold for both committed (baseline model) and contingent interventions.

3 Dynamic Coordination Equilibrium

We first take the government intervention as given, and derive the corresponding equilibrium. We then formulate a benevolent government's optimal policy design problem. Section 4 analyzes the optimal policy for committed intervention and the equilibrium outcome in details, Section 5 extends the results to contingent interventions. Again, we restrict the equilibrium set to symmetric Bayesian Nash equilibria (BNE) in monotone strategies: all agents' strategies are symmetric and monotonic *w.r.t.* x_t and m_t . Specifically, agent it 's strategy $a_{it}(x_{it})$ is non-increasing in x_{it} and non-decreasing in m_t , $t = 1, 2$.

3.1 Equilibrium and Social Welfare in Period 1

The analysis in period 1 is identical to the static game. We relabel the unique threshold equilibrium with time subscripts $(\theta_1^*, x_1^*) = (1 + m_1 - c, 1 + m_1 - c + \delta(1 - 2c))$. The fate of the fund is $s_1 = S$ if $\theta \leq \theta_1^*$ and $s_1 = F$ otherwise. Agents adopt a threshold strategy $a_{i1} = 1 \{x_{i1} \leq x_1^*\}$.

The social welfare in period 1 is also identical to the static economy,

$$W_1 - K(m_1, 0) = \frac{(1 - c)[1 + B - c(1 + \delta) + m_1]}{2B} - K(m_1, 0).$$

3.2 Equilibrium in Period 2

In period 2, the outcome of period-1's intervention (henceforth referred to as public news) is publicly known. As a result, beliefs on θ are truncated either from above or from below.

Unless specified otherwise, we assume for the remainder of the paper $2\delta > 1$ and $\frac{1}{2\delta+1} < c < \frac{2\delta}{1+2\delta}$. These assumptions correspond to the fact that during crisis uncertainty is high and cost of illiquidity is in an intermediate range where agents do not overwhelmingly prefer staying or running. These assumptions also ensures a unique threshold equilibrium in period 2 for both $s_1 = S$ and $s_1 = F$, and for all values that m_1 and m_2 take on.

3.2.1 Survival News

If the fund in period 1 has survived ($s_1 = S$), the prior belief on θ is bounded above at θ_1^* : $\theta \sim Unif[-B, \theta_1^*]$.

If and only if $m_2 > m_1 - c$, there exists an equilibrium in which agents choose to stay regardless of their signals, that is, the threshold x_2^* that agents adopt satisfies $x_2^* \geq \theta_1^* + \delta$. We can such equilibrium *Equilibrium with Dynamic Coordination* because the government's intervention in the first period has a dominant effect on improving coordination among investors in the second period:

Lemma 2. *Subgame Equilibrium with Dynamic Coordination*

If $s_1 = S$, $(\theta_2^, x_2^*) = (\infty, \infty)$ consists an equilibrium if and only if $m_2 > m_1 - c$.*

Note that $\theta \leq \theta_1^*$ is common knowledge, any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* > \theta_1^* + \delta)$ is equivalent to $(\theta_2^*, x_2^*) = (\infty, \infty)$. Next, we turn to threshold equilibria with $\theta_2^* < \theta_1^*$ so that the fate of the MMMF in period 2 still has uncertainty. Likewise, any threshold equilibrium (θ_2^*, x_2^*) necessarily satisfies two conditions. First, when $\theta = \theta_2^*$, $A_2 + m_2 = \Pr(x_2 < x_2^* | \theta = \theta_2^*) + m_2 = \theta_2^*$. Second, the marginal agent who receives the signal x_2^* is just indifferent between stay and run, $\Pr(\theta \leq \theta_2^* | x_2 = x_2^*, \theta \in [-B, \theta_1^*]) = c$.

We analyze the equilibrium in two cases, depending on whether the marginal investor finds the public news “useful”. Ignoring the public news, the marginal investor’s posterior belief on θ is simply $\Pr(\theta|x_2 = x_2^*) \sim Unif[x_2^* - \delta, x_2^* + \delta]$. If $x_2^* + \delta < \theta_1^*$, then $\Pr(\theta \leq \theta_2^*|x_2 = x_2^*, \theta \in [-B, \theta_1^*]) = \Pr(\theta \leq \theta_2^*|x_2 = x_2^*)$ and he finds the public news useless. We call such equilibrium *Equilibrium without Dynamic Coordination* because intervention in the first period has no effect on coordination in the second period.

Lemma 3. *Subgame Equilibrium without Dynamic Coordination*

If $s_1 = S$ and $m_2 < m_1 - 2\delta(1 - c)$, there exists a equilibrium with thresholds (θ_2^*, x_2^*) where

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c) \end{cases} \quad (5)$$

Notice that when public news is useless, the dynamic game is simply a repeated version of the static game. However, if $x_2^* + \delta > \theta_1^*$, $\Pr(\theta \leq \theta_2^*|x_2 = x_2^*, \theta \in [-B, \theta_1^*]) \neq \Pr(\theta \leq \theta_2^*|x_2 = x_2^*)$ and the marginal investor finds the public news useful. We call this equilibrium *Equilibrium with Partial Dynamic Coordination*, government intervention in the first period has partially improved the coordination among investors in the second period. Therefore, we name it after .Equilibrium without dynamic coordination is an artifact of bounded noise in the private signals. For unbounded noise, there is always partial dynamic coordination.

Lemma 4. *Subgame Equilibrium with Partial Dynamic Coordination*

If $s_1 = S$ and $\min\{m_1 - c, m_1 - 2\delta(1 - c)\} < m_2 < \max\{m_1 - c, m_1 - 2\delta(1 - c)\}$, there exists an equilibrium with thresholds

$$\begin{cases} \theta_2^* = 1 + m_2 - c + \frac{c[m_2 - m_1 + 2\delta(1 - c)]}{2\delta - c(1 + 2\delta)} \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c) + \frac{c(1 + 2\delta)[m_2 - m_1 + 2\delta(1 - c)]}{2\delta - c(1 + 2\delta)}. \end{cases} \quad (6)$$

Combining Lemma 2, 3 and 4, Proposition 1 describes the equilibrium outcome given any (m_1, m_2) and $s_1 = S$.

Proposition 1. *Equilibrium in period 2 when $s_1 = S$*

1. If $m_2 < m_1 - 2\delta(1 - c)$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.
2. If $m_1 - 2\delta(1 - c) < m_2 < m_1 - c$, the unique equilibrium is the Subgame Equilibrium with Partial Dynamic Coordination.
3. If $m_1 - c < m_2$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.

3.2.2 Failed Period-1 Fund

If the fund in period 1 has failed ($s_1 = F$), the prior belief on θ is bounded below at θ_1^* : $\theta \sim Unif[\theta_1^*, B]$.

The equilibrium outcome in this case can be derived similarly, summarized by Proposition 2 below. The detailed derivation can be found in Appendix B.

Proposition 2. *Equilibrium in period 2 when $s_1 = F$*

1. If $m_2 < m_1 + 1 - c$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.
2. If $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, the unique equilibrium is the Subgame Equilibrium with Partial Dynamic Coordination.
3. If $m_1 + 2c\delta < m_2$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.

3.2.3 Investors' Welfare and Dynamic Coordination

Let $W_{2S} = E \left[\int_0^1 V_{2i} di | s_1 = S \right]$ be the total expected payoff in period 2 conditional on $s_1 = S$. Also, let $W_{2F} = E \left[\int_0^1 V_{2i} di | s_1 = F \right]$ be the total expected payoff in period 2 when $s_1 = F$. Applying results from Proposition 1 and 2, we are able to obtain W_{2S} and W_{2F} for different values of m_1 and m_2 . Corollary 1 below show the results.

Corollary 1. *Investors' Welfare in Period 2 when the MMMF*

1. If $s_1 = S$

- (a) If $m_2 < m_1 - 2\delta(1 - c)$, $W_{2S}^{nc} = \frac{(1-c)[1+B-c(1+\delta)+m_2]}{B+\theta_1^*}$.
- (b) If $m_1 - 2\delta(1 - c) < m_2 < m_1 - c$, $W_{2S}^{pc} = \frac{1-c}{\theta_1^*+B} \left[\theta_1^* + B + \frac{\delta c(c-m_1+m_2)^2 + 2\delta(c-m_1+m_2)[2\delta-c(1+2\delta)]}{[2\delta-c(1+2\delta)]^2} \right]$.
- (c) $m_2 > m_1 - c$, $W_{2S}^c = (1 - c)$.

2. If $s_1 = F$

- (a) If $m_2 < m_1 + 1 - c$, $W_{2F}^c = 0$.
- (b) If $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^*} \frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2}$.
- (c) If $m_2 > m_1 + 2c\delta$, $W_{2F}^{nc} = \frac{1-c}{B-\theta_1^*} (m_2 - m_1 - c\delta)$.

The superscripts of W_{2S} and W_{2F} refer to equilibrium types. nc , pc and c respectively stand for equilibrium without dynamic coordination, with partial coordination, and with coordination.

The left panel of Figure 1 plots W_{2S} against m_2 , including the welfare function in all three different types of equilibria. Given m_1 , W_{2S} is continuous, increasing in m_2 , and convex in the region that involves partial dynamic coordination. Unlike in the first period, the marginal effect of m_2 on W_{2S} is no longer a constant. Initially, W_{2S} increases linearly in m_2 , in which case the intervention in the first period is useless. When $m_1 - 2\delta + 2c\delta < m_2 < m_1 - c$, the marginal effect of m_2 is increasing. Here, the marginal effect of m_2 depends on m_1 , due to the dynamic coordination effect of government coordination. When $m_2 > m_1 - c$, the dynamic coordination effect is maximized and all agents' decisions are well coordinated towards an equilibrium without any run. In that case, further increasing m_2 has no effect.

Similarly, the right panel of Figure 1 plots W_{2F} against m_2 , including the welfare function in all three different types of equilibria. Given m_1 , W_{2F} is continuous, increasing in m_2 , and convex when the equilibrium involves partial dynamic coordination. The effect of m_2 on W_{2F} is not a constant either. When $m_2 < m_1 + 1 - c$, the failed intervention in period 1 makes all agents very pessimistic. A slight increase in m_2 does not change people's belief and therefore, the marginal effect of m_2 on W_{2F} is zero. When $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, the marginal effect of m_2 on W_{2F} is positive and increasing. Finally, when $m_2 > m_1 + 2c\delta$, the dynamic effect is zero and W_{2F} increases linearly in m_2 .

Clearly, m_1 affects both W_{2S} and W_{2F} . Since W_{2S} and W_{2F} are piecewise in m_1 and thus not everywhere differentiable, we define left-hand derivative of W_{2S} and W_{2F} *w.r.t.* m_1 as

the dynamic coordination effect. Figure 2 illustrates the following result:

Proposition 3. *The Conditional Information Effect*

Through dynamic coordination, m_1 negatively affects the conditional investors' welfares W_{2S} and W_{2F} .

The proposition follows from the fact that W_{2S} and W_{2F} decrease with m_1 for fixed m_2 . However, the overall effect of m_1 on W_2 is non-monotone. Indeed, the probability of W_{2S} increases linearly with m_1 . Figure 3 shows this non-monotonic property.

Figure 3 shows the dynamic coordination effect of m_1 . It plots $E[W_2] = \Pr(s_1 = S)W_{2S} + (1 - \Pr(s_1 = S))W_{2F}$ against m_1 , taking m_2 as given. Obviously, the coordination effect attains its highest level at $m_1 = m_2 + c$, and starts to decline afterwards. Intuitively, a further increase in m_1 jams the coordination effect conditional on $s_1 = 1$, since investors would infer that large size of intervention is the reason that the fund has survived. If, on the other hand, $s_1 = 0$ follows an increase in m_1 , then it causes trouble as investors become really pessimistic, when they found out that the large intervention wasn't even effective.

3.2.4 Equilibrium comparison

It is interesting to compare thresholds across different types of equilibria. When $s_1 = S$ and $m_2 \in (m_1 - 2\delta(1 - c), m_1 - c)$, both x_2^* and θ_2^* in the Equilibrium with Partial Dynamic Coordination exceed their counterparts in the Equilibrium without Dynamic Coordination. Indeed this shows the dynamic coordination effect of government intervention. When the marginal agent finds the public news useful and realizes that certain values suggested by his signal are too high, he behaves more aggressively by choosing a higher threshold. As a result, θ_2^* is also higher and the MMMF is more likely to survive.

Matters are the opposite when $s_1 = F$, in which case both x_2^* and θ_2^* are lower in the Equilibrium with Partial Dynamic Coordination. The same intuition carries through. When the marginal agent finds the public news useful and realizes that certain values suggested by his signal are too low, he behaves more conservatively by choosing a lower threshold. As a result, θ_2^* is also lower and the MMMF is less likely to survive.

3.3 Welfare and Intervention Policy

Next, we examine the government's intervention decisions. While key economic mechanism and results hold more generally, for simplicity in exposition, we assume K is twice-differentiable in the feasible range of intervention \mathcal{I} . This specification includes cases of budget constraint and separable quadratic intervention costs. The government maximizes welfare by solving

$$\max_{m_1, m_2} E \left[\int_0^1 V_{1i} di + \int_0^1 V_{2i} di \right] - K(m_1, m_2). \quad (7)$$

Given \mathcal{I} is compact, an optimal policy exists in general, which the next section analyzes.

4 Optimal Policy and Implications

Given the costs and constraints of intervention, how should the government allocate the resources across the two periods, and how the information structure channel affects the scale of intervention? To illustrate the main tradeoffs in explicit closed-forms, we consider first the case of a simple case of budget constraint without costs when intervention is within budget.

4.1 An Example: Government with Budget Constraint

The government has a total budget M that can be costless used across the two periods. In other words, $K(m_1, m_2) = \frac{\mathbb{I}_{\{m_1+m_2>M\}}}{1-\mathbb{I}_{\{m_1+m_2>M\}}}$. Government is benevolent in the sense that it maximizes the social welfare. We do care about welfare as the welfare improvement is directly distributed to the investors. Government has a fixed budget for intervention, which can be used costlessly across two periods. The government's problem is,

$$\max_{m_1, m_2} E \left[\int_0^1 V_{1i} di + \int_0^1 V_{2i} di \right] \quad (8)$$

$$s.t. m_1 + m_2 = M. \quad (9)$$

Let $W = W_1 + W_2$ be the total welfare of all agents across both periods. The above results show that while W_1 increases linearly with m_1 , W_2 is non-monotonic in m_1 and increases

with m_2 in a non-linear matter. This is the information channel that arises from the dynamic coordination effect of government intervention.

Meanwhile, since the government also faces a hard budget constraint $m_1 + m_2 = M$, an increase in m_1 necessarily crowds out m_2 , the remaining liquidity available. This is the budget channel. When the government optimally allocate resources in two periods, it needs to consider both the information channel and the budget channel.

Figure 4 plots a typical social welfare W as m_1 varies. The pattern delivered by the figure holds for general parameters. (a) W is always flat for either small or large m_1 . (b) W always attains its maximum at $m_1 = \frac{M+c}{2}$. Therefore, whenever M is large, the government should invest $m_1^* = \frac{M+c}{2}$. Lemma 6 in the Appendix summarizes the aggregate social welfare and the net benefit of early intervention. When we impose both $m_1 \in [0, M]$ and $m_2 \in [0, M]$, certain cases in Lemma 6 no longer exist.

Therefore, the optimal intervention plan also depends on M , the total resources available to the government. When M is small ($M < \frac{M+c}{2}$), it is optimal to set $m_1 = M$. In contrast, when M gets larger, increasing m_1 may actually decrease the total payoff and the optimal $m_1 = \frac{M+c}{2}$.

Proposition 4 below characterizes the optimal intervention under different M s.

Proposition 4. *Optimal Intervention*

The optimal intervention is $\min(\frac{c+M}{2}, M)$. Optimal intervention always features early intervention: $m_1^ > m_2^*$.*

At the optimal intervention level, when the fund in period 1 has survived, the fund in period 2 will survive as well. However, when the fund in period 1 has failed, the one in period 2 always fails, too.

The intuition for the result that $m_1^* > m_2^*$ holds very generally. To see this, assume that government equally splits the budget and invest $\frac{M}{2}$ in each period. Then if the fund in period 1 has survived, the one in period 2 will succeed with probability one. On the other hand, if the period-1 fund has failed, the period-2 fund will also fail with probability one. Knowing this, government always has incentives to increase m_1 , which increases the chances that period-1 fund survives. Therefore, the optimal intervention plan satisfies $m_1^* > m_2^*$.

Finally, we note that the ration $\frac{m_2^*}{m_1^*}$ is weakly increasing in M and weakly decreasing in c , thus the tilt towards early intervention is most significant when the government has small

budget or the illiquidity cost is high.

4.2 Emphasis on Early Intervention

We have illustrated the information channel in the previous discussion. However, imposing the budget constraint takes away the flexibility of m_2 after m_1 is chosen. By specifying $K(m_1, m_2)$ differently, we can look beyond the case of budget constraint, and show that emphasizing early intervention is a very robust phenomenon. Let K_i denote the partial derivative w.r.t. m_i .

We consider in this section the case where the government separately chooses m_1 and m_2 all before s_1 is realized. This corresponds to situations where governments have to setup funding facilities or provide subsidies even before the outcomes of earlier interventions are known yet. For this, let's consider the general intervention cost function.

Proposition 5. *Committed Intervention*

If $K(m_1, m_2)$ is symmetric in m_1 and m_2 , committed interventions weakly emphasizes early intervention, in other words, $m_1^ \geq m_2^*$; if $K(m_1, m_2)$ is increasing in $|m_1 - m_2|$ (consistency criterion) when holding $m_1 + m_2$ to be an arbitrary constant, early intervention is strictly emphasized $m_1^* > m_2^*$.*

It is worth pointing out that this proposition is not about comparing the absolute sizes of the interventions. Given that we have normalized the total capital in the economy to one in both periods, we are really talking about a notion of intervention relative to the market size. Therefore, the conclusion could apply more broadly than it first appears, especially when the coordination games are scale-invariant, i.e., the normalized intervention, cost, and participation scale proportionally with the market size. Indeed, the eligible ABCPs for AMLF constitutes less than half of the commercial paper markets, thus the scale of AMLF (\$150 billion in the first 10 days) relative to market size is higher than CPFF (\$144 billion in the first week, peak usage \$350 billion Jan 2009) that targets almost the entire commercial paper markets. AMLF and its success also seem to have helped later interventions. For example, CPFF was also effective and even generated \$5 billion in net income for the government.

4.3 Information Structure and Intervention Externality

In the partial equilibrium given a policy, and in the case of budget constraint, we have seen that $\mathbb{E}[W_2]$ is nonmonotone in m_1 (Figure 3). In particular, increasing m_1 hurts W_{2S} and W_{2F} (Figure 2). Although increasing m_1 increases the probability of $s_1 = S$, beyond a certain level, the overall impact on W_2 is non-increasing.

The intuitive interpretation is that intervention in the first period has the benefit of increasing the probability of good news ($s_1 = S$), but it decreases information quality when $s_1 = S$, because one would not learn much about θ as one attributes the survival more to the intervention; conditional on $s_1 = F$, a larger m_1 also enhances the information quality for bad news, as one updates extremely negatively on θ upon failure. This may very well hurt the coordination in the second period.

This phenomenon is not an artifact of our specification of the cost, as we show below that in general, if the intervention costs are comparable across two periods, considering the information structure externality leads to increasing the intervention; however, if the intervention costs disproportionate. To isolate the externality of the first intervention on the second through the information structure channel from the externality through the budget channel, let us specialize to shut down the budget channel by setting $K_{12}(m_1, m_2) = 0$. Basically, we want to focus on how considering the information externality and coordination effect on the second period affects the optimal choice of m_1 .

In general, the government chooses $\{m_1, m_{2S}, m_{2F}\}$ to maximize the expected welfare. For a given m_1 , define the objective as

$$\begin{aligned}
 Y(m_1; \chi) = & W_1 - K(m_1, 0) + \chi \left[\frac{B + m_1 + 1 - c}{2B} \max_{m_{2S}} [W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))] \right. \\
 & \left. + \frac{B - m_1 - 1 + c}{2B} \max_{m_{2F}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \right] \quad (10)
 \end{aligned}$$

Here $\chi \in [0, 1]$ measures how much the government considers the dynamic externality of the first intervention on the second. In particular, $\chi = 0$ corresponds to the static benchmark, and $\chi = 1$ corresponds to the case where dynamic coordination is completely taken into consideration. In the case of committed intervention, we have the additional constraint $m_{2S} = m_{2F}$. Often, $\chi < 1$ because of short-termism of the government, or in the context of global economy and the dynamic coordination between interventions in two countries

with highly correlated fundamentals, one country's government typically do not consider the externality it imposes on the other country.

According to Theorem 2.1 in Athey, Milgrom, and Roberts (1998), $m_1^* \equiv \operatorname{argmax}_{m_1} Y(m_1, \chi)$ is non-increasing in χ iff Y has decreasing differences in χ and m_1 , and is non-decreasing in χ iff Y has increasing differences in χ and m_1 .

Proposition 6. *Information Externality*

Absent the budget channel, the government's committed intervention is weakly increasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^}{\partial \chi} \geq 0$, when*

$$\text{EITHER } K_1(c, \cdot) > \frac{1-c}{B} \text{ and } K_2(\cdot, 1-c) \geq \frac{1-c}{B} \frac{c\delta}{2c\delta+c-1}$$

$$\text{OR } K_1(c, \cdot) < \frac{1-c}{B} \frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)} \text{ and } K_2(\cdot, 1-c) \leq \frac{1-c}{2B}.$$

The government's committed intervention is weakly decreasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^}{\partial \chi} \leq 0$, when*

$$\text{EITHER } K_1(c, \cdot) > \frac{1-c}{B} \text{ and } K_2(\cdot, c(1+2\delta)) < \frac{1-c}{2B} \frac{\delta}{1+2\delta}$$

$$\text{OR } K_1(c, \cdot) < \frac{1-c}{B} \frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)} \text{ and } K_2(\cdot, 0) > \frac{1-c}{2B} \frac{2\delta}{2\delta-c(1+2\delta)}.$$

When the intervention capacities in the two periods are either both small or large, the endogenous intervention outcomes are highly correlated. Basically when the first period cost is sufficiently high, the intervention amount is no greater than c , we are in the region of survival leads to survival and no intervention in the second period is optimal. Alternatively, when the intervention cost for the second period is sufficiently high, by optimally deciding between not intervening or intervening $m_1 - c$, we are back in the region of survival leads to survival. Here increasing the first period's survival probability increases the survival for second period one for one because the good news quality is the best we get. It is therefore more important to increase the probability of survival by increasing m_1 . So here we see that the information structure channel emphasizes m_1 not relative to m_2 , but relative to the case where the intervention externality is absent.

The above proposition has important implications when policy makers are myopic or policy makers are uncoordinated. For example an EU membership country intervening in domestic market and does not fully internalize its impact on interventions in other countries, the relative capability to intervene and the dynamic externality of early interventions are crucial factors to consider. If the government is to be replaced at the next election and only concerns itself with current period policy, it would fail to formulate the optimal policy that

maximizes the welfare.

Interestingly, failure to consider dynamic coordination could also result in excessive intervention through the information structure it creates. For example, this happens when the cost for first intervention is sufficiently small such that the intervention is large scale, yet the second intervention is sufficiently costly, that survival does not always lead to survival. At the same time, a high m_1 reduces the quality of good news, reducing the marginal benefit of m_2 . This effect dominates the increase in survival probability in the first period. This results only highlights that the intervention has a negative dynamic externality, Theorem 7 still holds. When the intervention capacities in the two periods are rather disproportionate, either we have large scale intervention in the first period, but second period intervention is so costly that survival does not lead to survival, or we have really small scale intervention in the first period, and second period intervention is sufficiently cheap that even after failure, we may want to intervene a lot to have a chance for survival in the second period. In either cases, outcomes are less correlated, and the information quality effect dominates, shading m_1 makes it easier to intervene in the second period no matter fund survives or fails in the first period.

In the case of AMLF and CPFF, because the capacity to intervene using CPFF is comparable to that in AMLF, the later intervention was able to fully capture the benefit from investors' learning of earlier intervention. According to the above proposition, this provides additional justification for the overwhelming scale of AMLF.

5 Discussions and Extensions

5.1 Alternative Government Specification: Contingent Intervention

Now let us consider the case where m_2 is determined after s_1 is realized. The government chooses $\{m_1, m_{2s}, m_{2f}\}$ to maximize the expected welfare.

Proposition 7. *Contingent Intervention*

If $K_1(c, 0) < \frac{2\delta+c-2}{2\delta-1}$ or $K_2(0, 1-c) > \frac{(1-c)^2}{B-1}$, then contingent interventions strictly emphasizes early intervention, in other words, $m_1^ > m_{2s_1}^*$. If $K_1(\frac{B-1+c}{2B+2}, 0) < \frac{2\delta+c-2}{2\delta-1}$, then*

contingent interventions in expectation emphasizes early intervention.

This result says that if the cost for the early intervention (first period) is small enough, or for the late intervention (second period) is big enough, then we want to emphasize early intervention. While this is intuitive, it is not trivial because this still includes situations where cost in the early intervention is higher than the cost of late intervention.

Proposition 8. *Information Externality (Contingent Case)*

The government's contingent intervention is weakly increasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^}{\partial \chi} \geq 0$, when EITHER $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta-1+c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ and $K_1(c, \cdot) \geq \frac{1-c}{B}$ OR $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta-1+c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ and $1-c - K(\cdot, 1-c) + K(\cdot, 0) - (2+B-c)K_2(\cdot, 1-c) > 0$.*

The government's contingent intervention is weakly decreasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^}{\partial \chi} \leq 0$, when $K_2(\cdot, 0) > \frac{1-c}{B+c-2} \frac{2\delta}{2\delta-c(1+2\delta)}$ and $K_1(c, \cdot) < \frac{1-c}{B} \frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)}$.*

5.2 Various forms of Intervention in Financial Markets

There are a host of situations in which the government can dynamically coordinate economic agents into more efficient equilibria. Our main contribution, a good understanding of dynamic coordination, is often particularly relevant. In this section we explore a few representative examples to which our insights apply, providing more institutional details and empirical relevance, and discuss their policy implications.

As a motivating example of our model, intervention in financial markets during crises deserves greater discussion. In our base model we have used a reduced form of liquidity provision, but in reality government interventions come in various forms that often have differential impacts (see, for example, Bebcuk and Goldstein (2011) and Diamond and Rajan (2011)). While we cannot claim that our stylized model capture the subtleties of various forms of intervention, we argue below that it represents reasonably well interventions of liquidity provision in nature.

Direct lending and investment in borrower funds. This is essentially the approach in our baseline model. During the financial crisis of 2008-2009, US government

directly entered into the market for commercial paper and purchased that of some non-financial firms. There are two subtle differences. First, such programs could introduce some inefficiency if the agents are better at screening projects than the government (as discussed in Bebhuk and Goldstein (2011)). Moreover, the government also gets paid if the project is successful, which makes later intervention less costly. A general cost function to a large extent incorporates these considerations.⁹

Direct Capital Infusion to Investors. As many financial institutions are lenders to illiquid funds and projects that are subject to runs, governments around the globe have injected capital to banks and other financial institutions so that they can keep the extension of credits. The U.S. Troubled Asset Relief Program (TARP) provided about US\$250 billion to banks, and the UK about US\$90 billion to several major banks. Tax breaks and related measures represent capital infusion to retail investors directly. Suppose government inject a fraction α investors' capital in the economy (normalized to one in our model), this changes the capital of each investor from one unit to $1 + \alpha$ without altering the optimization problem investors face. The one period survival threshold becomes $\theta^* = (1 - c)(1 + \alpha) = 1 - c + (1 - c)\alpha$. We can relabel $m = (1 - c)\alpha$ and the model solutions are equivalent. The intervention again increases the probability of success and the expected payoff of investing relative to not investing.

Government guarantees. During the financial crisis of 2008-2009, governments such as those of the US and the UK used guarantees mainly to limit the potential losses of the lenders (on existing loans, we can view that as rolling over the debt, similar to originating new loans). Specifically, suppose that the government guarantees a proportion λ of a lender or investors losses, then the lender who stays (rolls over) receive the return R when project succeeds, and $-(1 - \lambda)c$ if it fails. Since our investors are risk neutral, this is equivalent to an intervention that increases the probability of success. Solving for the survival threshold, we get $\theta^* = \frac{1-c}{1-c\lambda} = 1 - c + \frac{c(1-c)\lambda}{1-c\lambda}$. Again, we can relabel $m = \frac{c(1-c)\lambda}{1-c\lambda}$.

Interest Rate Reduction. During the financial crisis of 2008, Fed Reserve Board cut the fed funds rate from 4.25% in Jan 2008 to 1% in Oct 2008. Many other other countries took similar measures in the face of a global contraction in lending. This is equivalent to reducing r —the payoff for not investing. Under risk-neutrality, it is equivalent to increasing

⁹An earlier version of this paper also explicitly models the government as an investor and the main results go through.

the success probability through changing c , which is exactly what m does in our model.

5.3 Moral Hazard

One big concern with government bailouts is that they may increase moral hazard. Indeed, fund managers may take unverifiable actions which are against investors' interests. For example, they may divert the capital injected by government to their private accounts, or gamble by investing in more risky assets. Indeed, fund manager's moral hazard behavior provides a micro-foundation to the intervention cost $K(m_1, m_2)$ that we introduced in the last section.

To be more specific, assume the fund manager is able to divert a constant fraction $\eta \in (0, 1)$ for any amount of liquidity μ injected by the government. Among the diverted capital, the fund manager privately can consume $\lambda(\eta\mu) < \eta\mu$, and the rest $\eta\mu - \lambda(\eta\mu)$ is inefficiently lost (iceberg costs). We assume standard utility function over consumption with increasing and concave $\lambda(\cdot)$ and $\lambda(0) = 0$. Under this setup, the optimal intervention problem is isomorphic to the problem in the last section, where intervention incurs a cost $k(m)$.

To see this, note that the government is aware of the diverting technology. Therefore, to effectively inject m to the MMMF, the government need to spend μ such that $(1 - \eta)\mu = m$. Combine the social welfare with the utility from the fund manager's private diversion, the efficient loss from intervention equals to $\eta\mu - \lambda(\eta\mu)$. Equivalently, injecting m into the MMMF costs the government $k(m) = \frac{\eta m}{1-\eta} - \lambda\left(\frac{\eta m}{1-\eta}\right)$. Given the above regularity conditions on $\lambda(\cdot)$, it follows that the effective cost function $k(m)$ is increasing and convex in m , and earlier results continue to hold.

5.4 Government Information Set

So far we have assumed that the government has less information than each individual investor. In this subsection, we discuss the implications when the government has equal or more precise information. To do this, we assume that the government is either a mediocre type or a superior type, with the latter type more capable of deploying intervention resources. While a mediocre government has the same intervention effect as before, a superior government has a superior effect in intervention. In particular, by intervening m_i^s , investment

succeeds if and only if

$$A_t + m_t^s + \alpha \geq \theta, \quad (11)$$

where $B \gg \alpha > 0$ represents the superior capability of the government in intervention. Note that α may also be interpreted as the government's information on the distribution θ . In other words, the government is aware that the fundamental is better than the common prior. Below we will show that the superior government always prefers to "signal" itself by intervening even less in the first period, $m_1^s < m_1^m$, which incurs a loss of efficiency. We discuss the results in the context of a budget constraint M which applies to both government. The superscripts s and m respectively represent superior and mediocre.

[workout the payoff for alpha type first] [intuitive criteria]

5.5 Multiplicity, Distribution, and Model Robustness

So far, we have restricted that $2\delta > 1$ and $\frac{1}{1+2\delta} < c < \frac{2\delta}{1+2\delta}$, which enable us to obtain unique equilibrium. Outside this parameter, multiple equilibria exist. Proposition 9 below is complementary to Proposition 1 and 2. It also provides the results when $\delta \rightarrow 0$.

Proposition 9. *Equilibria with general δ and c*

1. If $s_1 = S$ and $\frac{2\delta}{2\delta+1} < c < 1$,
 - (a) If $m_2 < m_1 - c$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.
 - (b) If $m_1 - c < m_2 < m_1 - 2\delta(1 - c)$, all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold θ_2^* decreases with m_2 ,
 - (c) If $m_1 - 2\delta(1 - c) < m_2$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.
2. If $s_1 = F$ and $0 < c < \frac{1}{2\delta+1}$
 - (a) If $m_2 < m_1 + 2c\delta$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.

(b) If $m_1 + 2c\delta < m_2 < m_1 + 1 - c$, all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold θ_2^* decreases with m_2 .

(c) If $m_2 > m_1 + 1 - c$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.

Multiple equilibria resurface because we can apply the argument of iterated deletion of dominated regions only from one end of θ space. Despite this, with slight modifications on the intervention cost functions, the main intuitions for the results from earlier sections still apply as long as we are consistent with equilibrium selection.

5.6 Normally Distributed Signals

In this subsection, we discuss the robustness of our model when the individual signals follow Normal distribution, i.e., $\varepsilon_i \sim N(0, \delta)$ and that the prior distribution of θ is uninformative¹⁰. Similar results are found at Goldstein and Huang (2016).

To keep matters comparable, we stick to the assumption that investors have no overlap between two periods. The case when investors overlap in two periods can be similarly analyzed. We will characterize the equilibrium in each period and emphasize that government intervention in period 1 still has a dynamic coordination effect in period 2.

Lemma 5 below summarizes equilibrium outcomes in two periods. Detailed analysis can be found in Appendix C.

Lemma 5. *Equilibrium when signals follow Normal distribution*

1. Given m_1 , there exists unique equilibrium thresholds in period 1:

$$\begin{aligned}\theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta\Phi^{-1}(c).\end{aligned}$$

2. Given (m_1, m_2) and $s_1 = S$,

¹⁰The assumption of uninformative prior is analogous to the previous assumption that $\theta \sim Unif[-B, B]$, since the for normally distributed noise is $(-\infty, \infty)$

- (a) When $m_2 > m_1 - c$, $(\theta_2^* = \theta_1^*, x_2^* = \infty)$ consists a threshold equilibrium strategies.
- (b) Equilibrium strategies (θ_2^*, x_2^*) which satisfy $\theta_2^* < \theta_1^*$ and $x_2^* < \infty$ may or may not exist. Moreover, there can exist multiple thresholds.

Table 2 presents the local comparative statics when there exists a unique equilibrium strategy. Clearly, when m_1 increases from 0.7 to 0.9, both θ_2^* and x_2^* decrease. Other relevant parameters are $c = 0.5, \delta = 0.5, m_2 = 0.1$

Table 2: θ_2^* as a function of m_1 ($s_1 = S$)

m_1	0.7	0.75	0.8	0.85	0.9
θ_2^*	0.7602	0.6981	0.6693	0.6511	0.6384
x_2^*	0.9667	0.8222	0.7565	0.7152	0.6866

6 Conclusion

How should a benevolent government intervene into a dynamic economy with strategic complementarity? Through the lens of a stylized model, we analyze real life phenomena as sequential global games in which government is a large player that intervenes to mitigate coordination failures, we establish general results on the existence and uniqueness of equilibrium, and show that government intervention can affect coordination both contemporaneously and dynamically. Our result suggests that optimal intervention always features stronger early intervention, validating conventional wisdom in realistic settings where government does not have superior information. However, excessive intervention in the early period may harm investors as it adversely alters public information structure. Therefore depending on the capacities to intervene, early intervention could have either positive or negative externality on subsequent coordination. Our paper thus has policy relevance to various intervention programs, such as the bailout of money market mutual funds during the financial crisis, and subsidies to the alternative energy sector.

The learning mechanism and information structure externality also applies to broader contexts, such as interventions in currency attacks, credit market freezes, cross-sector industrialization, and green energy development, where multiple rounds of coordination efforts

significantly affect agents' learning and actions. Our discussion therefore opens several avenues for future research. For example, we have discussed how government needs to consider the signaling aspect with private knowledge about intervention cost or economic fundamentals. This effect is worth exploring further especially in dynamic settings, because a sequence of interventions would affect the beliefs of the agents on the underlying state of the economy and their subsequent behaviors. Moreover, this paper has only considered forms of interventions we typically observe in real life, understanding the optimal contingent intervention not only is of theoretical interests, but also adds new insights to policy-making.

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Appendix

A Derivations and Proofs

A.1 Proof of Lemma 1

We prove the thresholds here. We still need to show that monotone Bayesian Nash equilibria are equivalent to threshold equilibria in this context.

Suppose there is a threshold $x^* \in \mathbb{R}$ such that each agent invests if and only if $x \leq x^*$. The measure of agents who invest is thus,

$$A(\theta) = \Pr(x \leq x^* | \theta) = \begin{cases} 0 & \text{if } x^* < \theta - \delta \\ \frac{x^* - (\theta - \delta)}{2\delta} & \text{if } \theta - \delta \leq x^* \leq \theta + \delta \\ 1 & \text{if } x^* > \theta + \delta. \end{cases} \quad (12)$$

It follows that the investment succeeds if and only if $\theta \leq \theta^*$ where θ^* solves

$$A(\theta^*) + m = \theta^*. \quad (13)$$

By standard Bayesian updating, the posterior distribution about θ conditional on the private signal is also uniform distribution with bandwidth 2δ . Therefore, the posterior probability of investment success is

$$\Pr(R = 1 | x) = \Pr(\theta \leq \theta^* | x) = \begin{cases} 0 & \text{if } x > \theta^* + \delta \\ \frac{\theta^* - (x - \delta)}{2\delta} & \text{if } \theta^* - \delta \leq x \leq \theta^* + \delta \\ 1 & \text{if } x < \theta^* - \delta. \end{cases} \quad (14)$$

For the marginal investor who is indifferent between investing or not, his signal x^* satisfies

$$\Pr(R = 1 | x^*) = c \quad (15)$$

Jointly solve equations (13) and (15), we obtain the two thresholds

$$\begin{cases} \theta^* = 1 + m_1 - c \\ x^* = 1 - c + \delta - 2c\delta + m_1. \end{cases} \quad (16)$$

A.2 Proof of Lemma 2

Proof. "if" \Leftarrow

If $m_2 > m_1 - c$, and if all agents know that other agents will adopt a threshold strategy $x_2^* = \infty$, then

$$A_2 + m_2 = 1 + m_2 > 1 + m_1 - c = \theta_1^* > \theta. \quad (17)$$

Therefore, the investment succeeds with probability 1. Therefore, it is individually rational for each agent to set $x_2^* = \infty$.

"only if" \Rightarrow

We prove by contradiction. Suppose that an equilibrium in which all agents adopt a threshold $x_2^* = 1 + m_1 - c + \delta$ when $(m_1 - c) - m_2 = \Delta > 0$. Therefore, any agent with a signal $x_2 < \theta_1^* + \delta$ will invest. In other words,

$$\Pr(\theta < 1 + m_2 | x_2, \theta < \theta_1^*) \geq c$$

holds for any x_2 .

Consider an agent who observes $\hat{x}_2 = m_1 + 1 - c + \delta - \frac{\Delta}{2}$. Such an agent exists when $\theta \in (m_1 + 1 - c + \delta - \frac{\Delta}{2}, m_1 + 1 - c + \delta)$. Apparently,

$$\Pr(\theta < 1 + m_2 | x_2 = \hat{x}_2, \theta < \theta_1^*) \geq c = 0 < c$$

which violates the assumption that all agents invest irrespective of their signals. □

A.3 Proof of Proposition 4

Plugging in the government's budget constraint, we are able to obtain the aggregate social welfare as a function of m_1 . As a by-product, we are also able to calculate the net benefit of early intervention. Lemma 6 summarizes the results.

Lemma 6. *Aggregate Social Welfare W and Net benefit of early intervention $\left. \frac{\partial W}{\partial m_1} \right|_{m_1+m_2=M}$*

1. If $m_1 > \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{nc} + \Pr(s_1 = F) W_{2F}^c \\ &= \frac{1-c}{2B} [2 + 2B - 2c(1+\delta) + M] \\ \frac{\partial W}{\partial m_1} &= 0. \end{aligned}$$

This case only exists for $M > 2\delta(1-c)$.

2. If $\frac{M+c}{2} < m_1 < \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{pc} + \Pr(s_1 = F) W_{2F}^c \\ &= \frac{1-c}{2B} [2 + 2B - c(2+\delta) + 2m_1 \\ &\quad + \frac{\delta c(c+M-2m_1)^2 - 2\delta[c-2(1-c)\delta](c+M-2m_1)}{[c-2(1-c)\delta]^2}] \\ \frac{\partial W}{\partial m_1} &= \frac{(1-c)[2c(1+2\delta)[c-2(1-c)\delta] - 4c\delta(c+M-2m_1)]}{2B[c-2(1-c)\delta]^2} < 0. \end{aligned}$$

This case only exists for $M > c$.

3. If $\frac{M-(1-c)}{2} < m_1 < \frac{M+c}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^c + \Pr(s_1 = F) W_{2F}^c \\ &= \frac{1-c}{2B} [2 + 2B - c(2 + \delta) + 2m_1] \\ \frac{\partial W}{\partial m_1} &= \frac{1-c}{B}. \end{aligned}$$

This case always exists.

4. If $\frac{M-2c\delta}{2} < m_1 < \frac{M-(1-c)}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^c + \Pr(s_1 = F) W_{2F}^{pc} \\ &= \frac{1-c}{2B} \left[2 + 2B - c(2 + \delta) + 2m_1 + \frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} \right] \\ \frac{\partial W}{\partial m_1} &= \frac{1-c}{2B} \left[2 - \frac{4c\delta(c-2m_1+M-1)}{(2c\delta+c-1)^2} \right]. \end{aligned}$$

This case only exists for $M > 1 - c$.

5. If $m_1 > \frac{M-2c\delta}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^c + \Pr(s_1 = F) W_{2F}^{nc} \\ &= \frac{1-c}{2B} [2 + 2B - 2c(1 + \delta) + M] \\ \frac{\partial W}{\partial m_1} &= 0. \end{aligned}$$

This case only exists for $M > 2c\delta$.

A.4 Proof of Proposition 5

Proof. When K is symmetric, suppose in equilibrium $m_1^* < m_2^*$, then we can interchange m_1 and m_2 because $W_1 + E[W_2]$ weakly increases after the change, and the intervention cost is unchanged. This outcome contradicts the intervention policy being optimal. When K requires consistency criterion, suppose $m_1^* < m_2^*$, then we can make a profitable deviation by changing the intervention amounts to $\{\frac{1}{2}m_1^* + \frac{1}{2}m_2^*, \frac{1}{2}m_1^* + \frac{1}{2}m_2^*\}$, which reduces the cost, and improves the $W_1 + E[W_2]$, again this is a contradiction to m_1^* and m_2^* being optimal. \square

A.5 Proof of Proposition 6

Proof. Given $\mathbb{E}[W_2]$ as a function of m_2 , if $m_1 \geq c$, we know $m_1 - c + 1 \geq 1 \geq m_2$, thus $m_2^* \leq m_1 - c$; else $m_1 < c$, then either $m_2^* = 0$ or $m_2^* > m_1 + 1 - c$.

$$\begin{aligned}
& \frac{\partial}{\partial m_1} \frac{\partial}{\partial \chi} Y(m_1; \chi) \\
= & \frac{d}{dm_1} \max_{\{m_2\}} [\mathbb{E}[W_2(m_1, m_2)] - (K(m_1, m_2) - K(m_1, 0))] \\
= & (\mathbb{I}_{\{m_1 < c\}} \mathbb{I}_{\{m_2^* > m_1 + 1 - c\}} + \mathbb{I}_{\{m_1 \geq c\}} \mathbb{I}_{\{m_2^* < m_1 - c\}}) \frac{\partial}{\partial m_1} [\mathbb{E}[W_2(m_1, m_2^*)] - (K(m_1, m_2^*) - K(m_1, 0))] \\
& + \mathbb{I}_{\{m_1 \geq c\}} \mathbb{I}_{\{m_2^* = m_1 - c\}} \frac{\partial}{\partial m_1} \left[\mathbb{E}[W_2(m_1, m_1 - c)] - \frac{\partial}{\partial m_1} (K(m_1, m_1 - c) - K(m_1, 0)) \right] \\
& + \mathbb{I}_{\{m_1 < c\}} \mathbb{I}_{\{m_2^* = 0\}} \frac{\partial}{\partial m_1} \mathbb{E}[W_2(m_1, 0)] \tag{18}
\end{aligned}$$

The four indicator products are mutually exclusive. As shown later, the first term is non-positive while the last two are non-negative. If $K_1(c, \cdot) > \frac{1-c}{B}$, we have $m_1^* < c$ because the maximum marginal benefit of m_1 on investors' welfare is $\frac{1-c}{B}$, $K_1(c, \cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Therefore $m_2^* > m_1 - c$ for sure and $W_{2S} = (1 - c)$. Given $K_2(\cdot, c(1 + 2\delta)) < \frac{1-c}{2B} \frac{\delta}{1+2\delta}$, we have $K(\cdot, c(1 + 2\delta)) < \frac{c(1-c)\delta}{2B}$, then $\frac{B-m_1-1+c}{2B} W_{2F} > K(m_1, m_1 + 2\delta c) - K(m_1, 0)$. Thus the increase in W_{2F} exceeds the intervention cost at $m_2 = m_1 + 2\delta c$, $m_2^* > m_1 + 1 - c$. Similarly, $k_2(\cdot, 1 - c) \geq \frac{1-c}{B} \frac{c\delta}{2c\delta+c-1}$ is a sufficient condition for $m_2^* = 0$ because this implies the cost exceeds the benefit at both $m_2 = 1 - c + m_1$ and $m_2 = m_1 + 2c\delta$, and the convexity of K in m_2 excludes $m_2^* > 1 - c + m_1$. Note in Figure 5 increasing m_2 does not increase welfare in $[0, m_1 + 1 - c]$. We could alternatively use $K(\cdot, 1 - c) \geq \frac{c(1-c)\delta}{2B}$ as a sufficient condition on cost, rather than on the derivative.

If $K_1(c, \cdot) < \frac{1-c}{B} \frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)} \leq \frac{\partial Y}{\partial m_1}$, we have $m_1^* \geq c$. On the one hand, if in addition, $\min \left\{ \frac{1-c}{2B}, \frac{1-c}{2B} \frac{2\delta(1-c)}{2\delta-c(1+2\delta)} \right\} \geq \frac{1-c}{2B} \geq K_2(\cdot, 1 - c)$, we have $\mathbb{E}[W_2] - (K(m_1, m_2) - K(m_1, 0))$ increasing in the entire region of $[0, m_1 - c]$, therefore $m_2^* = m_1 - c$. On the other hand, $K_2(\cdot, 0) > \frac{1-c}{2B} \frac{2\delta}{2\delta-c(1+2\delta)}$ implies $\frac{1-c}{2B} \frac{2\delta}{2\delta-c(1+2\delta)} < K_2(m_1, m_1 - c)$, which means $m_2^* < m_1 - c$.

When $m_1^* < c$ and $m_2^* > m_1^* + 1 - c$, OR $m_1^* \geq c$ and $m_2^* < m_1^* + 1 - c$, then we should increase intervention as externality is taken into consideration because the last two terms in equation 18 are both non-negative by direct calculation; otherwise, we should shade down because the first term is non-positive. To see this, we note m_2^* is interior, thus we can apply envelop theorem to compute the partial derivative in m_1 of $\mathbb{E}[W_2(m_1, m_2)] - (K(m_1, m_2) - K(m_1, 0))$. Because $K_{12} = 0$, we know the partial derivative must be negative in the corresponding regions from figure 3. \square

A.6 Proof of Proposition 7

Proof. If $s_1 = S$, the left panel in Figure 1 shows $m_{2S}^* \leq m_1^* - c$, otherwise increasing m_2 incurs additional cost without increasing $\mathbb{E}[W_2]$. If $s_1 = F$, $K_1(c, 0) < \frac{2\delta+c-2}{2\delta-1}$ or $K_2(0, 1-c) > \frac{(1-c)^2}{B-1}$ would ensure either $m_1^* > c$ or $m_2^* < 1 - c$, which in turn implies $m_{2F}^* = 0$ (according to the right panel in Figure 1). $K_1(\frac{B-1+c}{2B+2}, 0) < \frac{2\delta+c-2}{2\delta-1}$ would ensure $\mathbb{E}[m_2^*] < m_1^*$. To see this, we know if $m_1^* \geq c$, $m_{2F}^* = 0$; if $m_1^* < c$, $m_{2F}^* \leq 1$, so when we compute the expected m_2 , we get it is less than m_1^* . \square

A.7 Proof of Proposition 8

Proof. Because the maximum marginal benefit of m_1 on the investors' total welfare is $\frac{1-c}{B}$, $K_1(c, \cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Figure 1 implies when $m_{2S} > m_1 - c$, welfare is weakly decreasing in m_2 , thus $m_{2S}^* = 0$.

$$\begin{aligned}
& \frac{\partial}{\partial m_1} \frac{\partial}{\partial \chi} Y(m_1; \chi) \\
&= \frac{d}{dm_1} \left[\frac{B + m_1 + 1 - c}{2B} \max_{\{m_{2S}\}} [W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))] \right. \\
&\quad \left. + \frac{B - m_1 - 1 + c}{2B} \max_{\{m_{2F}\}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \right] \\
&= \frac{1-c}{2B} + \frac{\partial}{\partial m_1} \left[\frac{B - m_1 - 1 + c}{2B} \max_{\{m_{2F}\}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \mathbb{I}_{\{m_{2F} > m_1 + 1 - c\}} \right] \\
&= \frac{1-c}{2B} + \frac{1}{2B} [K(m_1, m_{2F}) - K(m_1, 0)] \mathbb{I}_{\{m_{2F}^* > m_1 + 1 - c\}} - \frac{1-c}{2B} \mathbb{I}_{\{m_{2F}^* > m_1 + 2c\delta\}} \\
&\quad - \frac{1-c}{2B} \frac{2c\delta(-1+c-m_1+m_{2F}^*)}{(-1+c+2c\delta)^2} \mathbb{I}_{\{m_{2F}^* \in (m_1+1-c, m_1+2c\delta)\}} \\
&\geq 0
\end{aligned} \tag{19}$$

the second equality holds by Envelope Theorem and by the fact that if $m_{2F} \leq m_1 + 1 - c$, then taking $m_{2F} = 0$ dominates as seen in Figure 1. When $K(\cdot, 1-c) - K(\cdot, 0) > \frac{c\delta(1-c)}{B-(1-c)}$, $W_{2F}(m_2 = m_1 + 2\delta c) < K(m_1, m_1 + 1 - c)$, thus $m_{2F}^* = 0$; when $K(\cdot, 1-c) - K(\cdot, 0) > \frac{(1-c)^2}{2c\delta-1+c}$, the last two terms on the RHS of the third equality is dominated by the first two term as $m_{2F}^* = 0$. In either case, we have the whole expression being non-negative.

From $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2+B-c)K_2(\cdot, 1-c) > 0$, we have $K_2(\cdot, 1-c) < \frac{1-c}{B+2-c} \frac{2\delta}{2\delta-(1+2\delta)c}$, the marginal benefit for increasing m_{2S} in W_{2S} exceeds the cost as long as $m_{2S} < m_1 - c$, therefore $m_{2S}^* = [m_1 - c]^+$. When $m_1 \leq c$, $m_{2S}^* = 0$, the local derivative is the same as above, thus is positive. When $m_1 \geq c$, $m_{2S}^* = m_1 - c$, $m_{2F}^* = 0$, the local derivative is

$$\begin{aligned}
& \frac{\partial}{\partial m_1} \frac{\partial}{\partial \chi} Y(m_1; \chi) \\
&= \frac{1-c}{2B} - \frac{1}{2B} [K(m_1, m_{2S}) - K(m_1, 0)] - \frac{m_1 + B + 1 - c}{2B} \frac{\partial}{\partial m_1} [K(m_1, m_1 - c) - K(m_1, 0)] \\
&= \frac{1-c}{2B} - \frac{1}{2B} [K(m_1, m_{2S}) - K(m_1, 0)] - \frac{m_1 + B + 1 - c}{2B} [K_2(m_1, m_1 - c) \\
&\quad + K_1(m_1, m_1 - c) - K_1(m_1, 0)], \quad \text{note } K_1(m_1, m_1 - c) = K_1(m_1, 0) \\
&= \frac{1-c}{2B} - \frac{1}{2B} [K(m_1, m_{2S}) - K(m_1, 0)] - \frac{m_1 + B + 1 - c}{2B} [K_2(m_1, m_1 - c)] \\
&\geq \frac{1}{2B} [1 - c - K(m_1, 1-c) + K(m_1, 0) - (2+B-c)K_2(m_1, 1-c)] \geq 0
\end{aligned} \tag{20}$$

The last two inequalities come from the fact $m_1 \leq 1$, and the fact $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2+B-c)K_2(\cdot, 1-c) > 0$. Therefore we have Y has increasing differences in (m_1, χ) .

Now to prove the second half of the theorem, Note $K_2(\cdot, 0) > \frac{1-c}{B+c-2} \frac{2\delta}{2\delta-c(1+2\delta)}$, thus $K(\cdot, 1-c) > \frac{1-c}{B+c-2} \frac{1-c}{1-c(1+\frac{1}{2\delta})} > \frac{1-c}{B-2+c}$. And W_{2F} at $m_2 = m_1 + 2c\delta$ is still less than $K(m_1, m_1 + 1 - c) - K(m_1, 0)$. Consequently $m_{2F}^* = 0$. $K_2(\cdot, 0) > \frac{1-c}{B+c-2} \frac{2\delta}{2\delta-c(1+2\delta)}$ also implies $\frac{1-c}{B+m_1+1-c} \frac{2\delta}{2\delta-c(1+2\delta)} < K_2(m_1, m_1 - c)$,

which means $m_{2S}^* < m_1 - c$.

$$\begin{aligned}
& \frac{\partial}{\partial m_1} \frac{\partial}{\partial \chi} Y(m_1; \chi) \\
= & \frac{d}{dm_1} \left[\frac{B + m_1 + 1 - c}{2B} \max_{\{m_{2S}\}} [W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))] \right. \\
& \left. + \frac{B - m_1 - 1 + c}{2B} \max_{\{m_{2F}\}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \right] \\
= & \frac{\partial}{\partial m_1} \left[\frac{B - m_1 - 1 + c}{2B} \max_{\{m_{2F}\}} [W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))] \right] \\
= & \frac{\partial}{\partial m_1} W_{2S}(m_{2S}^*) + \frac{B - m_1 - 1 + c}{2B} \frac{\partial}{\partial m_1} [K(m_1, m_{2S}^*) - K(m_1, 0)] \\
& - \frac{1}{2B} (K(m_1, m_{2S}) - K(m_1, 0)) < 0
\end{aligned} \tag{21}$$

The first term is negative as $m_{2S}^* < m_1 - c$. The second term is non-positive as K is weakly increasing in second argument. Finally, the third term is zero as K has zero cross-partials.

Finally, the above argument would not work if $m_1^* \leq c$. But this can be ruled out in that the minimum $\frac{\partial Y}{\partial m_1} = \frac{1-c}{2B} \left[1 - \frac{c(1+2\delta)}{2\delta-c(1+2\delta)} \right] = \frac{1-c}{B} \frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)}$. Notice we have used the fact that $m_{2F}^* = 0$. This is bigger than the marginal cost $K_1(c, \cdot)$, thus $m_1^* > c$, and we indeed have an interior m_{2S}^* .

□

B Full Analysis of Section 3.2.2

Is there any equilibrium that agents choose to run irrespective of their signals? In other words, the threshold x_2^* that agents in period 2 adopt satisfy $x_2^* \leq \theta_1^* - \delta$. It turns out that such an equilibrium exists if and only if $m_2 < m_1 + 1 - c$. In this type of equilibrium, government intervention in the first period has a dominant effect on coordination among investors in the second period. Therefore, we name it after *Subgame Equilibrium with Dynamic Coordination*.

Lemma 7 describes this type of equilibrium. Since it is common knowledge that $\theta > \theta_1^*$, any equilibrium with $(\theta_2^* < \theta_1^*, x_2^* < \theta_1^* - \delta)$ is equivalent to $(\theta_2^*, x_2^*) = (-\infty, -\infty)$.

Lemma 7. *Subgame Equilibrium with Dynamic Coordination*

If $s_1 = F$, $(\theta_2^*, x_2^*) = (-\infty, -\infty)$ consists an equilibrium if and only if $m_2 < m_1 + 1 - c$.

Next, we turn to threshold equilibria with $\theta_2^* > \theta_1^*$ so that the fate of the fund in period 2 still has uncertainty. Similar to the analysis when $s_1 = S$, we consider two types of equilibria, depending on whether the marginal investor find the public news useful.

Lemma 8. *Subgame Equilibrium without Dynamic Coordination*

If $s_1 = F$ and $m_2 > m_1 + 2c\delta$, there exists a equilibrium with thresholds

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c). \end{cases} \quad (22)$$

Lemma 9. *Subgame Equilibrium with Partial Dynamic Coordination*

If $s_1 = F$ and $\min\{m_1 + 2c\delta, m_1 + 1 - c\} < m_2 < \max\{m_1 + 2c\delta, m_1 + 1 - c\}$, there exists an equilibrium with thresholds

$$\begin{cases} \theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}. \end{cases} \quad (23)$$

Given any (m_1, m_2) and $s_1 = F$, Proposition 2 clearly follows Lemma 7, 8 and 9.

C Full Analysis of Section 5.6

The equilibrium outcome in period 1 is characterized by two thresholds (θ_1^*, x_1^*) which satisfy

$$\begin{aligned} A_1(\theta_1^*) + m_1 &= \theta_1^* \\ \Pr(\theta < \theta_1^* | x_1 = x_1^*) &= c \end{aligned}$$

where $A_1(\theta_1^*) = \Pr(x_1 < x_1^* | \theta = \theta_1^*)$ is the measure of investors who choose to roll over. Simple calculation shows that,

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta \Phi^{-1}(c) \end{aligned}$$

The equilibrium in period 2 is again, state-independent. We discuss the outcomes when $s_1 = S$ and leave the case $s_1 = F$ to the Appendix. When the intervention in the first period has succeeded, equilibrium in the second period will be either a subgame equilibrium with full dynamic coordination (similar to Lemma 2), or one with partial dynamic coordination (similar to Lemma 4). The case without dynamic coordination vanishes as the support of the noise now spans between (∞, ∞) . The first type of equilibrium is denoted as $(\theta_2^*, x_2^*) = (\infty, \infty)$ and any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* = \infty)$ is equivalent. The necessary conditions that $(\theta_2^*, x_2^*) = (\infty, \infty)$ consists an equilibrium is

$$\begin{aligned} \Pr(1 + m_2 > \theta | \theta < \theta_1^*) &= 1 \\ \Rightarrow m_2 &> m_1 - c. \end{aligned}$$

Likewise, the necessary conditions that an equilibrium with partial dynamic coordination exists is that the solution (θ_2^*, x_2^*) to the equation system

$$\begin{aligned} A_2(\theta_2^*) + m_2 &= \theta_2^* \\ \Pr(\theta < \theta_2^* | x_2^*, \theta < \theta_1^*) &= c \end{aligned}$$

exists and satisfies $\theta_2^* < \theta_1^*$. Equivalently, we are looking for θ_2^* that solves

$$1 - (\theta_2^* - m_2) = c \Phi \left(\frac{\theta_1^* - \theta_2^* - \delta \Phi^{-1}(\theta_2^* - m_2)}{\delta} \right) \quad (24)$$

We numerically solve equation (24). Figure 6 shows that there can be no solution (top panel), one solution (middle panel) or two solutions (bottom panel), even if $m_2 > m_1 - c$.

Tables and Figures

Figure 1: W_{2S} and W_{2F} as A Function of m_2

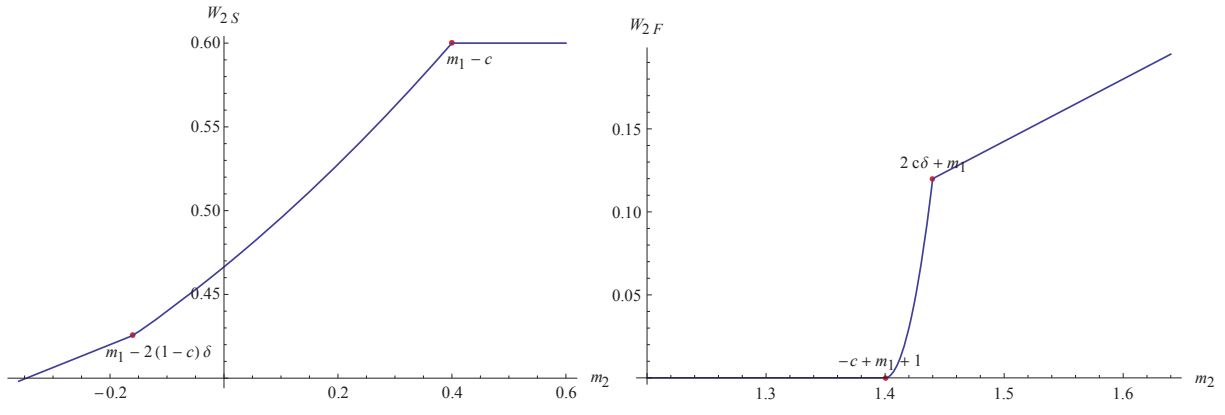


Figure 2: W_{2S} and W_{2F} as A Function of m_1

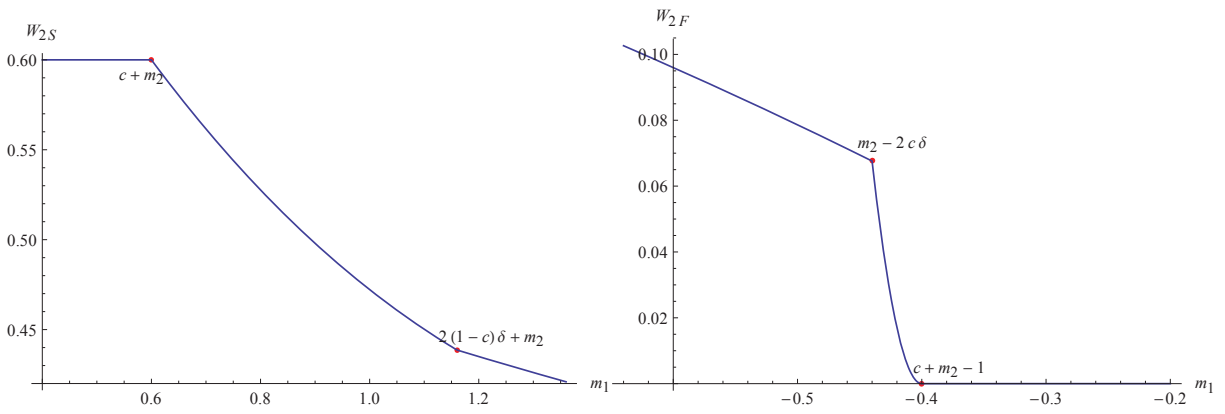


Figure 3: $E[W_2]$ as a function of m_1

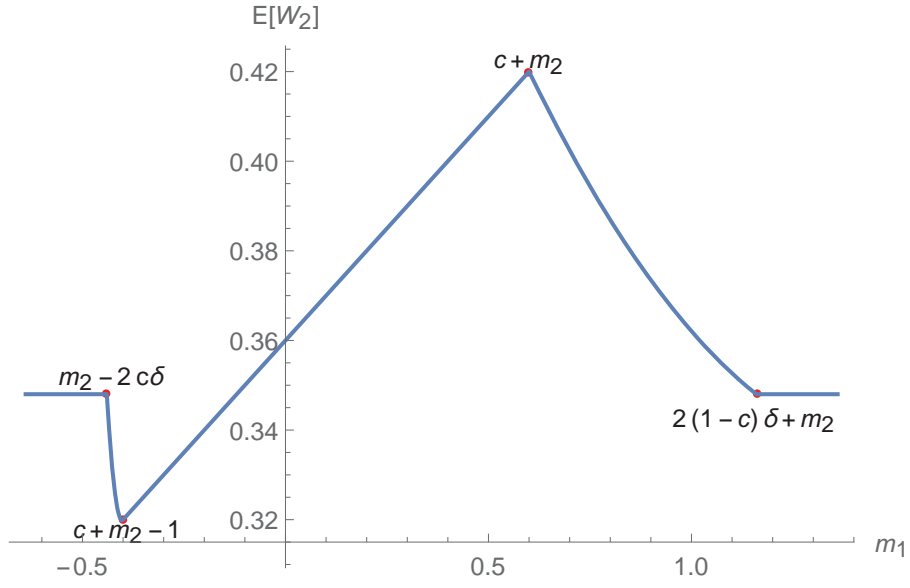


Figure 4: $W_1 + W_2$ as A Function of m_1 ($m_1 + m_2 = M > 2c\delta$)

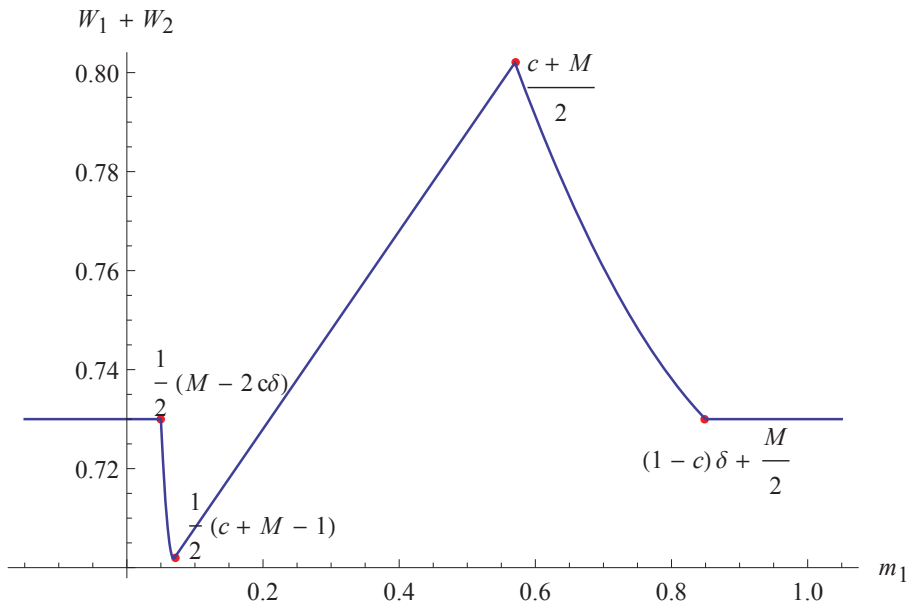


Figure 5: $E[W_2]$ as A Function of m_2

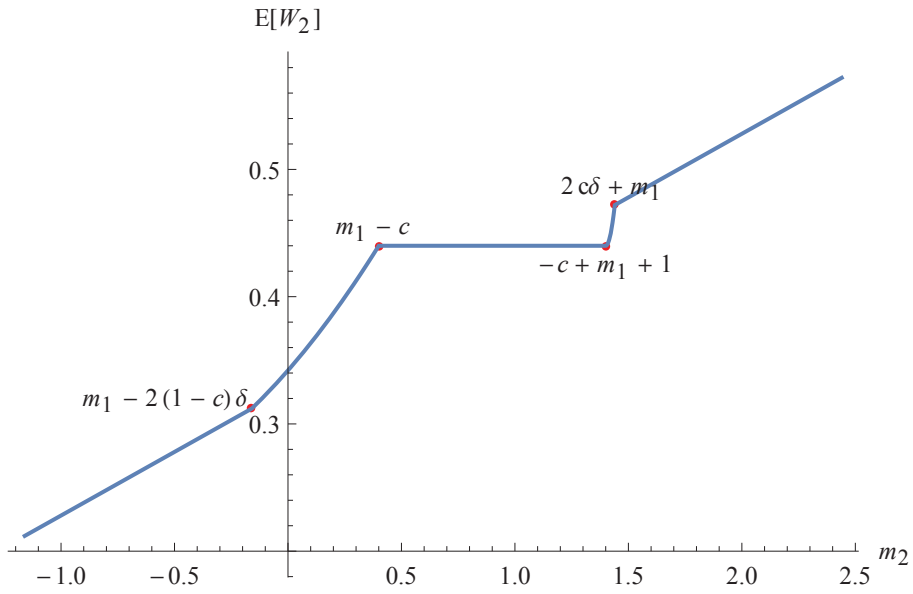


Figure 6: Normally Distributed Signals

