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"Information Spillovers in Asset Markets with Correlated Value"

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We study information spillovers in a dynamic setting with privately informed traders and correlated asset values. A trade of one asset (or lack thereof) can provide information about the value of other assets. The information content of this trading behavior is endogenously determined in equilibrium. We show that this endogeneity leads to multiple equilibria when the correlation between asset values is sufficiently high. The equilibria are ranked in terms of both trade volume and efficiency. We study the implications for policies that target market transparency as well as the market's ability to aggregate information. Total welfare is higher in any equilibrium of a fully transparent market than in a fully opaque one. However, both welfare and trading activity can decrease in the degree of market transparency. If traders have asymmetric access to transaction data, transparency levels the playing field, reduces the rents of more informed traders, but may also reduce total welfare. Moreover, even in a fully transparent market, information is not necessarily aggregated as the number of informed traders becomes arbitrarily large.

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Information Spillovers in Asset Markets
with Correlated Values

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Abstract

We study information spillovers in a dynamic setting with privately informed traders and correlated asset values. A trade of one asset (or lack thereof) can provide information about the value of other assets. The information content of this trading behavior is endogenously determined in equilibrium. We show that this endogeneity leads to multiple equilibria when the correlation between asset values is sufficiently high. The equilibria are ranked in terms of both trade volume and efficiency. We study the implications for policies that target market transparency as well as the market's ability to aggregate information. Total welfare is higher in any equilibrium of a fully transparent market than in a fully opaque one. However, both welfare and trading activity can decrease in the degree of market transparency. If traders have asymmetric access to transaction data, transparency levels the playing field, reduces the rents of more informed traders, but may also reduce total welfare. Moreover, even in a fully transparent market, information is not necessarily aggregated as the number of informed traders becomes arbitrarily large.

1　Introduction

In many markets, asset values are positively correlated and sellers often have (correlated) private information about the value of their asset. For example, sellers of homes on the same street have information about the desirability of the location and neighborhood trends. Similarly, venture capital firms that own a stake in similar start-ups have information about the challenges faced to make these companies succeed. Or, consider two banks who own different tranches of an asset-backed security, but have similar information about the underlying collateral. Importantly, in all these environments, trade of one asset can be informative about the value of other related assets. Thus, transaction transparency can be important for both informational efficiency and the efficiency with which assets are reallocated. Indeed, the empirical literature has documented that the degree of market transparency matters, and there is an ongoing policy debate about whether to require transactional transparency for a variety of asset classes in financial markets.[1]

Our goal in this paper is to develop a theoretical framework which aims to understand the role of information spillovers and transparency in such markets. The basic model involves two sellers (i and j), each with an indivisible asset that has a value which is either low or high. Asset values are positively correlated and each seller is privately informed about the value of her asset, but does not know the value of the other seller's asset. There is common knowledge of gains from trade, but buyers face a lemons problem (Akerlof, 1970). Trading takes place via a competitive decentralized market over the course of two periods. In the first period, potential buyers can approach a seller and make offers. If a seller rejects all offers in the first period, then she can entertain more offers from new buyers in the second period. In this setting, inefficiencies can arise from delays in trade or a failure to trade altogether.

In addition to asset correlation, the key novel ingredient of the model is that if seller i (j) trades in the first period, then with probability $\xi \in [0, 1]$, the trade is observed by potential buyers of seller j's (i's) asset, prior to them making offers in the second period. We refer to ξ as the *degree of market transparency*, where $\xi = 0$ corresponds to a fully opaque market and $\xi = 1$ corresponds to a fully transparent market.

Provided that there is at least some degree of transparency, a trade of one asset can provide information to buyers about the value of the other asset. Importantly, the information content of observed trading behavior is endogenous and interacts with the degree of market transparency. For example, suppose that, in the first period, seller j trades with a high probability if she owns a low-value asset and does not trade if she owns a high-value asset. Then, because the asset values are correlated, observing whether seller j trades has information content about

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the value of seller i's asset and the degree of market transparency plays a role in determining i's trading strategy and therefore its information content. On the other hand, if seller j plans to sell the asset in the first period regardless of its value, then observing a trade by this seller is completely uninformative about the quality of seller i's asset and the degree of market transparency is irrelevant.

An important first result (Proposition 2) is that because the quality of information is endogenously determined by trading behavior, there must be positive probability of trade in each period. This result is in contrast to Daley and Green (2012, 2015), who show that when news quality is exogenous, the unique equilibrium involves periods of no trade in which both sides of the market wait for more news to be revealed. The intuition for our result is that if there was no trade, then there would be no news and hence nothing to wait for.

In equilibrium, low-value assets are more likely to trade in the first period,[2] and therefore observing a transaction of one asset in the first period is "bad news" about the other asset. This introduces an interdependence in the sellers' strategies that can be decomposed into two separate effects, which we refer to as a bad news effect and a good news effect. The bad news effect is that, as seller j trades more aggressively, it becomes more likely from seller i's perspective that bad news will be revealed, which induces seller i to trade more aggressively. The good news effect is that, conditional on not observing a trade by seller j, the market beliefs about seller i are more favorable, which leads to higher prices and induces seller i to trade less aggressively. Because these two effects push in opposite directions, the optimal trading behavior of seller i is non-monotonic in seller j's behavior. It is this non-monotonicity that lies behind our main result (Theorem 1), which shows that when asset values are sufficiently correlated, a high degree of transparency leads to multiple equilibria.

To provide more intuition as to why this multiplicity obtains, consider the case in which the assets are perfectly correlated and the market is fully transparent. Suppose that the low-type seller j trades with probability one in the first period and the high-type seller j trades with probability zero. If seller i delays trade in the first period, then her type will be perfectly revealed by whether seller j trades. Conditional on observing a trade by seller j in the first period, buyers will correctly infer that seller i has a low value asset and offer a low price in the second period. Therefore, a low-type seller i has no incentive to delay trade and strictly prefers to trade in the first period. Hence, there exists an equilibrium in which both low-value assets trade with probability one in the first period.

Next, suppose that the low-type seller j trades with some intermediate probability in the first period. From seller i's perspective, there is still positive probability that her asset will be

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[2]This feature is common in dynamic models with adverse selection and often referred to as the "skimming" property.

revealed if buyers observe a trade by seller j (the bad news effect), but there is also some chance that seller j does not trade, in which case buyers correctly infer that seller i is more likely to have a good asset making them willing to offer a high price in the second period (the good news effect). The potential for getting a high price in the second period makes seller i indifferent between trading in the first period, and hence she is willing to trade with some intermediate probability. Thus, there also exists an equilibrium in which both low-value assets trade with an intermediate probability in the first period.

In fact, we show there can exist three equilibria of the model, all of which are symmetric. The equilibria are ranked both in terms of the volume of trade that takes place and the total welfare. The higher is the volume of trade in the first period, the more efficiently assets are reallocated and the higher is the total welfare. Three equilibria exist provided that asset values are sufficiently correlated and the market is sufficiently transparent. When either of these conditions breaks down, the (expected) information content revealed by seller j's trade is insufficient to induce the moderate or high-volume equilibrium and only the low-volume equilibrium exists. Therefore, both the correlation of asset values and the degree of market transparency can impact total welfare.

We analyze the welfare implications in more detail by conducting comparative statics on both the degree of transparency and correlation of asset values. Provided the adverse selection problem is severe enough to prohibit the first-best outcome (i.e., the *lemons condition* holds), some degree of transparency ($\xi > 0$) always weakly increases welfare relative to a fully opaque market. However, starting from $\xi \in (0,1)$, increasing transparency is not guaranteed to increase welfare. For instance, welfare can be decreasing in ξ in the moderate-volume equilibrium and is independent of ξ in the low-volume equilibrium. Furthermore, if the lemons condition does not hold, then the unique equilibrium in a fully opaque market is efficient, while there exist inefficient equilibria in a transparent market.

We extend the model to a setting with an arbitrary number of assets, $N > 1$, where the correlation across assets is driven by an unknown underlying aggregate state (low or high).[3] Conditional on the aggregate state, asset values are independently drawn, but they are more likely to be of high value in the high state. This extension serves not only as a robustness check, but allows us to investigate the model's implications for whether transparency facilitates information aggregation about the underlying aggregate state. We first show that indeed our results are robust to an arbitrary N; there exist multiple equilibria that are ranked in terms of trading volume and welfare. Second, we ask whether traders are able to learn the underlying

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[3]This extension bears several interpretations. The number of assets can be interpreted literally as the number of relevant correlated assets in the marketplace. Alternatively, N can be interpreted as the degree of *market integration*: the number of different assets regarding which traders can have information (e.g., the number of assets that trade on a given platform).

aggregate state by observing trading behavior as $N \to \infty$. Interestingly, we show there are natural conditions under which information aggregation is impossible. While the number of sources of information grows with N, the likelihood of trade and thus the informativeness of each source declines fast enough so as to limit the informativeness of the overall market, despite full transparency. When these conditions do not hold, there exists an equilibrium in which information is successfully aggregated; however there can be other equilibria, in which information aggregation fails. These results suggest a limitation of (even fully) transparent markets. They also point to the importance of studying how information is generated in markets, as the extent of market informativeness can depend on the expectations of market participants.

1.1 Policy Implications

Our findings help contribute to the debate on mandatory transaction transparency, which has received significant attention from policy makers in recent years. In July 2002, the corporate bond market underwent a significant change when FINRA (then NASD) mandated that prices and volume of completed transactions be publicly disclosed. Since then, TRACE has been expanded to include other asset classes including Agency-Backed Securities and some Asset-Backed Securities. There are also ongoing efforts by regulators to increase transparency in the markets for numerous derivatives (Title VII of Dodd-Frank) and European corporate bonds (Learner, 2011).

Opponents have objected to mandatory transparency arguing that it is unnecessary and potentially harmful. For example, if price transparency reduces dealer margins, dealers will be less willing to commit capital to hold certain securities thereby reducing liquidity.[4] There is mixed empirical evidence as to whether increased transparency can reduce liquidity. Asquith et al. (2013) find that increased transparency led to a significant decline in trading activity for high-yield bonds. This is in contrast to a controlled study by Goldstein et al. (2007), who find no conclusive evidence that increased transparency causes a reduction in trading activity. Our theoretical framework helps to reconcile these findings. For example, we show that increasing transparency can increase or decrease trading activity depending on the initial degree of transparency, asset correlation, and which equilibrium is played.

Regulators such as FINRA are strong proponents of mandated transparency. They argue that it "creates a level playing field for all investors" (NASD, 2005). To investigate this claim, we extend our model to a setting where some traders have access to transaction data (e.g., broker-dealers) and some do not (e.g., retail or institutional investors). We confirm that a policy of mandatory transparency indeed reduces the trading profits of broker-dealers, which may help

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[4]In a letter to the SEC, the Bond Market Association argued that adverse effects of mandatory transparency are likely to be exacerbated for lower-rated and less frequently traded bonds.

explain their resistance to the proposed changes. Mandated transparency also mitigates the trading losses of investors who are naive about the fact that they face competition from traders with access to better information (e.g., retail investors). However, if investors are sophisticated (e.g., institutional), then mandatory disclosure has no effect on their welfare and can lead to an overall reduction in efficiency (relative to an opaque market) if traders coordinate on the low-trade equilibrium. Thus, "leveling the playing field" can come at a cost and the desirability of such a policy depends on the composition of market participants.

1.2   Related Literature

Our work is related to Daley and Green (2012, 2015), who study a setting in which information is exogenously revealed to uninformed buyers. They show that when information (or news) quality is exogenous, the unique equilibrium involves periods during which liquidity completely dries up and no trade occurs. In contrast, we show that when information is endogenously revealed by the trading behavior of other market participants, there can exist multiple equilibria all of which require trade to occur with strictly positive probability in each period.

The role of transparency in offers has been studied by Nöldeke and van Damme (1990), Swinkels (1999), Hörner and Vieille (2009) and Fuchs et al. (2015). The two most important differences with respect to these papers are that: first, we consider the transparency of transactions while they all consider the transparency of offers. Second, we explore the strategic considerations of multiple sellers whose assets have correlated values, which are inherently absent in previous work. These two considerations are crucial for understanding trading behavior since they induce complementarities in the sellers' strategies and can lead to multiple equilibria.

In contemporaneous work, Duffie et al. (2014) analyze the role of published benchmarks (e.g., LIBOR), which reveal dealers' cost of supplying the good. In our setting, past trades in related assets play a similar role to benchmarks. As in Duffie et al. (2014), we find that more transparent markets can yield higher welfare by reducing information asymmetries among markets participants. However, we also show that the effect of transparency on welfare can be non-monotonic.

The usefulness of aggregate indicators for information transmission depends critically on the extent to which they aggregate information that is dispersed in the economy. Thus, our paper is also related to a literature, initiated by Hayek (1945), that studies information aggregation in 'large' markets. Early papers in this literature include Grossman (1976), Wilson (1977), and Milgrom (1979). More recent contributions have been made by Pesendorfer and Swinkels (1997), Kremer (2002), Lauermann and Wolinsky (2013, 2015), Bodoh-Creed (2013), Axelson and Makarov (2014). Ostrovsky (2012) derives conditions under which information aggregation

obtains in a dynamic setting with a single security, an uninformed market maker, and heterogeneously informed traders. We contribute to this literature by showing that in a dynamic model with heterogeneous but correlated assets, information aggregation can (and under some conditions must) fail because the information revealed is endogenous to trade. Furthermore, whether information aggregates can depend on the equilibrium on which agents coordinate.

There is also a large literature within accounting and finance studying the effect of public disclosure of firm specific information. Healy and Palepu (2001) and Verrecchia (2001) provide surveys of both the theoretical and empirical work in this area. One key difference is that this literature takes the information to be disclosed as given and studies the effects of whether, when, how much and how frequently it is made public.

Kaya and Kim (2015) look at a model with a single seller in which sequential buyers receive a private signal about the value of the asset. Instead, our focus is on the two-way interaction between trade and the information generated by it. The idea of a two-way feedback between trading activity and market informativeness is also present in Cespa and Vives (2015). They study a noisy rational expectations model and find that multiple equilibria can arise when noise-trader shocks are sufficiently persistent and informed buyers care only about their short-term returns. While our approaches are substantially different, their model also delivers equilibria that have high trading volume and market informativeness as well as equilibria in which trading volume and informativeness are low.

Finally, Drugov (2010, 2015) considers the related problem of information externalities among two bargaining pairs. There are two important differences with respect to this work. First, he considers a different market structure where there is only one buyer per seller rather than competing buyers. Second, in his model the value of the seller is independent of the value of the buyer while in ours the values are correlated giving rise to a lemons problem.

The rest of the paper is organized as follows. In Section 2, we lay out the basic framework and conduct preliminary analysis. In Section 3, we characterize the equilibria of the model. In Section 4, we analyze the welfare implications. Section 5 considers the model with an arbitrary number of assets. Section 6 analyzes the redistributive effects of transparency when some buyers have access to transaction data and others do not. Section 7 concludes. All proofs are in the Appendix.

2   The Model

In this section, we present the basic ingredients of the model, which feature two indivisible assets with correlated values that can be traded in a decentralized market.

There are two sellers, indexed by $i \in \{A, B\}$. Each seller owns one indivisible asset and is

privately informed of her asset's type, denoted by $\theta_i \in \{L, H\}$. Seller i has a value c_θ for a type θ asset, where $c_L < c_H$. Each seller has multiple potential trading partners, which we refer to as buyers. The value of a type-θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$, which can be motivated by, for example, liquidity constraints or hedging demands.

There are two trading periods: $t \in \{1, 2\}$. In each period, two or more buyers make simultaneous price offers to each seller. A buyer whose offer is rejected gets a payoff of zero and exits the game.[5] The payoff to a buyer who purchases an asset of type θ at price p is given by

$$v_\theta - p.$$

Sellers discount future payoffs by a discount factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ, who agrees to trade at a price p in period t is

$$\left(1 - \delta^{t-1}\right) c_\theta + \delta^{t-1} p.$$

If the seller does not trade at either date, his payoff is c_θ. All players are risk neutral.

A key feature of our model is that asset values are positively (but imperfectly) correlated. To model this correlation, let the unconditional distribution of θ_i be given by $\mathbb{P}(\theta_i = L) = 1 - \pi \in (0, 1)$.[6] The distribution of θ_i conditional on θ_j is then $\mathbb{P}(\theta_i = L | \theta_j = L) = \lambda \in (1 - \pi, 1)$.[7]

Importantly, asset correlation introduces the possibility that trade in one market contains relevant information about the asset in another market. We capture information spillovers across markets as follows. Buyers who can make offers to seller i are distinct from those who can make offers to seller j and henceforth we refer to market i and market j to clarify this distinction. This allows us to capture a feature of decentralized markets; a trader may not observe trades that take place on other platforms. To capture the degree of transparency across markets, we assume that there is a probability $\xi \in [0, 1]$ that a transaction in market i at $t = 1$ is observed by buyers in market j prior to them making offers in the second period. We refer to the parameter $\xi \in [0, 1]$ as the degree of market transparency, where $\xi = 1$ stands for fully transparent markets and $\xi = 0$ for fully opaque ones. For simplicity, we assume that offers are made privately; a rejected offer is not observed by other buyers within or across markets.[8] Our primary interest is to explore how the correlation of asset values (λ) and transparency

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[5]The assumption that buyers can make only a single offer simplifies the analysis by eliminating the possibility of experimentation.

[6]We have adopted this convention so that higher π (or beliefs) corresponds to more favorable beliefs.

[7]With perfect correlation (i.e., $\lambda = 1$), the set of equilibria is sensitive to the specification of off-equilibrium path beliefs. Nevertheless, in Proposition 9 (see the Appendix), we show that the set of equilibria with perfect correlation is the limit of the set of equilibria as $\lambda \to 1$.

[8]Fuchs et al. (2015) show that this specification is without loss in a setting with a single asset.

across markets (ξ) affect equilibrium trade dynamics. To do so, we focus on primitives which satisfy the following assumptions.

Assumption 1. $\pi v_H + (1-\pi)v_L < c_H$.

Assumption 2. $v_L < (1-\delta)c_L + \delta c_H$.

The first assumption, which we refer to as the "lemons" condition, asserts that the adverse selection problem is severe enough to rule out the first-best efficient equilibrium in which both sellers trade in the first period with probability one (w.p.1) regardless of their type.[9] The second assumption rules out the fully separating equilibrium in which the low type trades in the first period w.p.1 and the high type trades in the second period w.p.1. Together, these two conditions rule out trivial equilibria in which information spillovers are irrelevant for equilibrium behavior.[10]

2.1 Strategies, information sets, and "news"

A strategy of a buyer is a mapping from his information set to a probability distribution over offers. In the first period (i.e., at $t = 1$), a buyer's information set is empty. In the second period, buyers in market i know that seller i did not trade in the first period.[11] They also observe a noisy signal about whether seller j traded in the first period. This signal or "news" is commonly observed across all buyers in market i and is denoted by $z_i \in \{b, g\}$ (the reason for this notation will become apparent in Section 2.3.2).[12] If $z_i = b$, then a trade occurred in market j and was revealed to participants in market i. If $z_i = g$ then either no trade took place in market j, or (if $\xi < 1$) trade occurred in market j, but it was not observed by players in market i due to lack of transparency.

The strategy of each seller is a mapping from her information sets to a probability of acceptance. Seller i's information includes her type, the set of previous and current offers as well as the information set of buyers in market i.

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[9]If the lemons condition does not hold, then the first-best equilibrium exists regardless of the degree of transparency. Without transparency, this is the unique equilibrium outcome. With sufficient transparency, there can exist another equilibrium in which trade is not fully efficient. Thus, transparency can distort an otherwise efficient market by introducing the possibility of learning and provide incentives for the high type to wait for a better price. This finding is similar to results in Daley and Green (2012) in which the introduction of exogenous news can reduce overall efficiency.

[10]Strictly speaking, to rule out fully separating equilibria we only need $v_L < (1-\delta)c_L + \delta v_H$; this stronger condition simplifies exposition without affecting our main results.

[11]If seller i trades at $t = 1$ then buyers in market i do not make offers at $t = 2$.

[12]We could easily extend the model to allow for the transaction price to be part of buyers' information set. In equilibrium, no additional information is revealed by the transaction price in the first period.

2.2 Equilibrium Concept

We use Perfect Bayesian Equilibria (PBE) as our equilibrium concept. This has three implications. First, each seller's acceptance rule must maximize her expected payoff at every information set taking buyers' strategies and the other seller's acceptance rule as given (*Seller Optimality*). Second, any offer in the support of the buyer's strategy must maximize his expected payoff given his beliefs, other buyers' strategy and the seller's strategy (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated according to Bayes rule whenever possible (*Belief Consistency*).

2.3 Preliminary Analysis

It is convenient to establish some basic properties all equilibria must satisfy. Because there are multiple buyers, their individual offers are not uniquely pinned down by PBE. We refer to the *bid* in market i at time t as the maximal offer made across all buyers in market i at time t.

Let $V(\tilde{\pi}) \equiv \tilde{\pi} v_H + (1 - \tilde{\pi}) v_L$ denote buyer's expected value for an asset given an arbitrary belief $\tilde{\pi}$. Let $\bar{\pi} \in (\pi, 1)$ be such that $V(\bar{\pi}) = c_H$, and let π_i denote the probability that buyers in market i assign to $\theta_i = H$ just prior to making offers in the second period. In equilibrium, π_i is determined by belief consistency and the realization of news (see Section 2.3.2). Taking these beliefs as given, equilibrium play at $t = 2$ corresponds to that of the familiar static market for lemons.

Lemma 1 *Suppose that trade does not occur in market i at $t = 1$. Then, in market i at $t = 2$:*

(i) If $\pi_i < \bar{\pi}$, then the bid is v_L and only the low type seller accepts.

(ii) If $\pi_i > \bar{\pi}$, then the bid is $V(\pi_i)$ and both types accept.

(iii) If $\pi_i = \bar{\pi}$, then the bid is $c_H = V(\pi_i)$ with some probability $\phi_i \in [0, 1]$ and v_L otherwise.

A high-type seller will only accept a bid greater than c_H. Therefore, when the expected value of the asset is below c_H (i.e., $\pi_i < \bar{\pi}$), there is no way for a buyer to attract a high-type seller without making a loss. Thus buyers will trade only with the low types and competition pushes the bid price to v_L, implying (i). When the expected value is above c_H (i.e., $\pi_i > \bar{\pi}$), competition between buyers forces the equilibrium offer to be the expected value, implying (ii). In (iii), the expected value of the asset is exactly c_H and hence buyers are indifferent between offering c_H and trading with both types or offering v_L and only trading with the low type.

2.3.1 Continuation Values

It follows from Lemma 1 that for a given belief and buyer mixing probability in market i, (π_i, ϕ_i), the payoff to a high-type seller i in the second period is

$$F_H(\pi_i, \phi_i) \equiv \max\{c_H, V(\pi_i)\}$$

and a low-type's payoff in the second period is given by

$$F_L(\pi_i, \phi_i) \equiv \begin{cases} v_L & \text{if } \pi_i < \bar{\pi} \\ \phi_i c_H + (1 - \phi_i) v_L & \text{if } \pi_i = \bar{\pi} \\ V(\pi_i) & \text{if } \pi_i > \bar{\pi}. \end{cases}$$

Notice that if $\pi_i = \bar{\pi}$, the low-type seller's payoff is not uniquely pinned down as it depends on the likelihood that c_H is offered. Hence F_L can take values in the interval $[v_L, c_H]$ at $\pi_i = \bar{\pi}$. On the other hand, F_H is independent of the probability that c_H is offered.
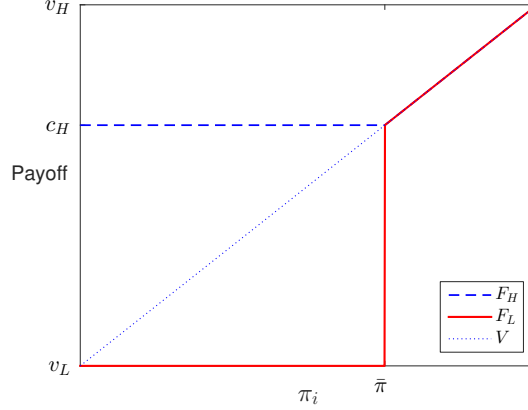
Figure 1: The set of second period payoffs of the seller as they depend on θ_i and π_i.

Given her information set in the first period, the seller's payoff in the second period is stochastic because buyers' beliefs will depend on news arriving from market j and because buyers may be mixing over offers. Fixing a candidate equilibrium, the expected continuation value from rejecting the bid in the first period of a type θ seller in market i is

$$Q_\theta^i = (1 - \delta) \cdot c_\theta + \delta \cdot \mathbb{E}_\theta \{F_\theta(\pi_i, \phi_i)\}. \tag{1}$$

Lemma 2 *In any PBE, the expected continuation value for the high type is strictly greater than that for the low type: $Q_H^i > Q_L^i$.*

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This is due to the facts that (i) $F_H \geq F_L$, (ii) flow payoff to a high type from delay is higher, and (iii) a high type rationally believes it is less likely that bad news will arrive and thus she expects a (weakly) better distribution of price offers in the second period. Note that (iii) is true regardless of the first-period trading strategies used by the seller in the other market. That is, any "good" news from market j is more likely to arrive in market i if $\theta_i = H$ than if $\theta_i = L$ and conversely.

Consider now the seller's decision in the first period. The strict ranking of continuation values implies that if a high type is willing to accept the bid in the first period, then a low type will strictly prefer to accept. Given Assumption 1, buyers' prior beliefs are sufficiently pessimistic to rule out an offer weakly above c_H; if the bid was c_H or above, even if all the high types traded, the winning buyer would not break even. Thus, buyers will make offers below c_H and trade only with the low types. Competitive forces again drive the bid to v_L.

Lemma 3 *The equilibrium outcome at $t = 1$ in market i satisfies the following.*

(i) *Buyers' bid is v_L.*

(ii) *High type seller rejects the bid, while the low type accepts with probability $\sigma_i \in [0,1)$.*

To see why σ_i must be strictly less than 1, suppose to the contrary that $\sigma_i = 1$. Then conditional on rejecting the offer, belief consistency requires that buyers believe the seller is a high type w.p.1 (regardless of any information revealed from market j) and thus the bid must be v_H in the second period (by Lemma 1). But, then a low-type seller would get a higher payoff by not trading in the first period (see Assumption 2), violating seller optimality. Thus, it must be that $\sigma_i \in [0,1)$.

2.3.2 Updating

As highlighted above, buyers' beliefs in the second period determine equilibrium play. There are two ways in which the prior is updated between the first and second periods. First, conditional on rejecting the offer in the first period, buyers' interim belief is given by

$$\pi_{\sigma_i} \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1) = \frac{\pi}{\pi + (1 - \pi)(1 - \sigma_i)} \tag{2}$$

Second, it is the key feature of our model that, before making their offers in the second period, buyers may learn that there was trade in the other market. Since asset values are positively correlated and only low types trade in the first period, news that there was trade in the other market (i.e., $z_i = b$) will lead to negative revision in beliefs and $z_i = g$ will lead to positive updating (hence, this is why we refer to these as "bad" and "good'" news).

Exactly how this news is incorporated into the posterior will depend on the degree of market transparency, ξ, and the trading strategy of the seller in the other market, σ_j. It is useful to define first the probability of news z arriving to market i conditional on the type θ_i of seller in market i, which we denote by $\rho_\theta^i(z)$. Specifically, the probability of observing the event $z_i = b$, given the seller in market i is of type θ is:

$$\rho_\theta^i(b) \equiv \mathbb{P}(z_i = b | \theta_i = \theta) = \xi \cdot \sigma_j \cdot \mathbb{P}(\theta_j = L | \theta_i = \theta). \tag{3}$$

Using equations (2) and (3), we can express the posterior probability of seller i being high type after news z arrives from market j as

$$\pi_i(z; \sigma_i, \sigma_j) \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1, z_i = z) = \frac{\pi_{\sigma_i} \cdot \rho_H^i(z)}{\pi_{\sigma_i} \cdot \rho_H^i(z) + (1 - \pi_{\sigma_i}) \cdot \rho_L^i(z)}. \tag{4}$$

To conserve on notation, we often suppress arguments of π_i. Notice that $\pi_i(z)$ has the expected property that $\pi_i(b) \leq \pi_{\sigma_i} \leq \pi_i(g)$.[13] A few additional properties are worth noting. First, $\pi_i(b)$ is increasing in σ_i and is independent of σ_j. The latter follows because only a low-type seller j trades in the first period and therefore upon observing $z_i = b$, buyers in market i know that $\theta_j = L$ regardless of how aggressively seller j trades. On the other hand, $\pi_i(g)$ is increasing in both σ_i and σ_j, since a more aggressive trading strategy for seller j implies a lower likelihood of $z_i = g$. Finally, $\pi_i(g)$ is more sensitive to changes in σ_i than σ_j since seller i's own trading strategy is always (weakly) more informative about her type than is seller j's.[14]

3   Equilibrium

From Lemmas 1 and 3 as well as the updating summarized by equations (2)-(4), an equilibrium can be characterized by the first-period trading intensity of the low type in each market and the buyer mixing probabilities conditional on $\pi_i(z) = \bar{\pi}$. Let $\gamma = \{\sigma_A, \sigma_B, \phi_A, \phi_B\}$ denote an arbitrary candidate equilibrium. In this section, we derive the set of γ that constitute equilibria and therefore the set of all PBE.

Let us briefly outline how we will proceed. We start by taking the behavior in market j as given and analyze the "partial equilibrium" in market i. We show that, for each (σ_j, ϕ_j), there is a unique (σ_i, ϕ_i) that is consistent with an equilibrium in market i, which may involve $\sigma_i = 0$ (Proposition 1). An equilibrium is then simply a fixed point, i.e., (σ_i, ϕ_i) is consistent with an

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[13]When $\sigma_j = 0$ or $\xi = 0$, we have $\rho_{i,L}(b) = \rho_{i,H}(b) = 0$. To ensure that the posterior is well-defined in this case, we adopt the convention $\pi_i(b) = \frac{\pi_{\sigma_i} \cdot \mathbb{P}(\theta_j = L | \theta_i = H)}{\pi_{\sigma_i} \cdot \mathbb{P}(\theta_j = L | \theta_i = H) + (1 - \pi_{\sigma_i}) \cdot \mathbb{P}(\theta_j = L | \theta_i = L)}$.

[14]The effects of σ_i and σ_j on $\pi_i(g)$ coincide when both there is perfect correlation ($\lambda = 1$) and the market is fully transparent ($\xi = 1$).

equilibrium in market i given (σ_j, ϕ_j) and vice versa. We argue that any fixed point must be symmetric and involves strictly positive probability of trade in the first period: $\sigma_i = \sigma_j > 0$ (Proposition 2). We then characterize the set of (symmetric) fixed points and show that multiple equilibria arise when information spillovers due to correlation and transparency are sufficiently strong (Theorem 1).

3.1 Partial Equilibrium

As mentioned above, we start by taking the behavior in market j as given and define a partial equilibrium as follows.

Definition 1 *We say that (σ_i, ϕ_i) is a **partial equilibrium** in market i given (σ_j, ϕ_j) if buyers' beliefs in market i are updated according to (4) and the strategies induced by (σ_i, ϕ_i) satisfy buyer and seller optimality in market i.*

Note that this definition does not require play in market j to satisfy equilibrium conditions given (σ_i, ϕ_i), hence the "partial" moniker. To characterize partial equilibria, it will be useful to write the continuation value of a seller i of type θ explicitly as it depends on the equilibrium strategies:

$$Q_\theta^i(\sigma_i, \sigma_j, \phi_i) \equiv (1 - \delta)c_\theta + \delta \sum_{z \in \{b,g\}} \rho_\theta^i(z) F_\theta(\pi_i(z), \phi_i). \tag{5}$$

Notice that seller i's expected continuation value is independent of ϕ_j, but it depends crucially on σ_j because seller j's trading strategy determines the distribution of news in market i and hence the distribution over π_i.

Lemma 4 *Fix an arbitrary $(\sigma_j, \phi_j) \in [0, 1]^2$. Then $(\sigma_i, \phi_i) \in [0, 1]^2$ is a partial equilibrium in market i if and only if $Q_L^i(\sigma_i, \sigma_j, \phi_i) \geq v_L$, where the inequality must hold with equality if $\sigma_i > 0$.*

To understand the necessity of the inequality, suppose that $Q_L^i < v_L$. In this case, a low-type seller i would strictly prefer to accept the bid in the first period and therefore seller optimality requires $\sigma_i = 1$, which violates Lemma 3.

Proposition 1 (Unique Partial Equilibrium) *Fix an arbitrary $(\sigma_j, \phi_j) \in [0, 1]^2$. There exists a unique partial equlibrium in market i, which may involve $\sigma_i = 0$, in which case seller i simply "waits for news" (i.e., trades with probability zero).*

The existence and uniqueness of a partial equilibrium follow from Lemma 4, and the facts that Q_L^i is strictly increasing in σ_i and $Q_L^i(1, \sigma_j, \phi_i) = (1 - \delta)c_L + \delta v_H > v_L$ by Assumption 2. That

a partial equilibrium may involve $\sigma_i = 0$ is closely related to Daley and Green (2012), in which news is generated by an exogenous process. In their continuous-time setting with exogenous news, it is in fact necessary that equilibria involve periods of no trade. In our setting, news is endogenously generated by trade in other markets, which, as we will see, eliminates the possibility of a period with no trade once we solve simultaneously for an equilibrium in both markets.

Before doing so, consider the effect of σ_j on Q_L^i. As σ_j increases, there are two forces to consider. First, higher σ_j makes good news more valuable; conditional on good news arriving in market i, buyers' posterior about the seller is more favorable and hence the expected price is higher. Note that there is no analogous effect following bad news; conditional on $z_i = b$, buyers know that $\theta_j = L$ but their belief about seller i (and hence the expected price) is independent of σ_j. The second effect is that higher σ_j makes bad news more likely from the perspective of a low-type seller i. These two forces push in opposite directions and either one can dominate. Hence, Q_L^i may increase or decrease with σ_j. For $\pi_i(\cdot) \neq \bar{\pi}$ this can be expressed as:

$$\delta^{-1}\frac{\partial Q_L^i}{\partial \sigma_j} = \underbrace{\frac{\partial \rho_L^i(b)}{\partial \sigma_j}\left(F_L(\pi_i(b),\phi_i) - F_L(\pi_i(g),\phi_i)\right)}_{\text{bad news effect } (-)} + \underbrace{(1 - \rho_L^i(b))\frac{\partial F_L(\pi_i(g),\phi_i)}{\partial \sigma_j}}_{\text{good news effect } (+)}.$$

The upshot is that as seller j trades more aggressively, the partial equilibrium in market i may involve seller i trading more aggressively (if the bad news effect dominates) or less aggressively (if the good news effect dominates).

To illustrate these two effects graphically, we plot Q_L^i as a function of σ_i for four different levels of σ_j in the left panel of Figure 2. Notice that moving from $\sigma_j = 0$ to $\sigma_j = 0.3$, seller i must trade less aggressively in order to maintain indifference (i.e., $Q_L^i = v_L$). Further, when $\sigma_j = 0.6$, Q_L^i lies above v_L everywhere and hence seller i strictly prefers to wait. Finally, for $\sigma_j = 0.9$, the bad news effect dominates and seller i trades aggressively.

Next, let us define the mapping from seller j's trading strategy into the corresponding partial equilibrium trading strategy of seller i by $S(\cdot)$, where $S(\sigma_j) = 0$ if $Q_L^i(0, \sigma_j, 0) \geq v_L$ and S satisfies $Q_L^i(S(\sigma_j), \sigma_i, \phi_i) = v_L$ for some $\phi_i \in [0, 1]$ otherwise. The right panel of Figure 2 illustrates a plot of S, which can be decomposed into three regions.[15] For small σ_j, the good news effect dominates and S is decreasing. For intermediate σ_j, the partial equilibrium in market i involves the seller waiting for news. For large σ_j, the bad news effect dominates and S is increasing in σ_j. It is worth noting that the non-monotonicity in S obtains only when both asset correlation and transparency are sufficiently large. If either of these conditions fails, then information spillovers across markets have relatively little influence over seller's behavior.

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[15]The following parameters remain fixed throughout all figures: $c_L = 0$, $c_H = 0.2$, $v_L = 0.1$, $v_H = 0.25$.
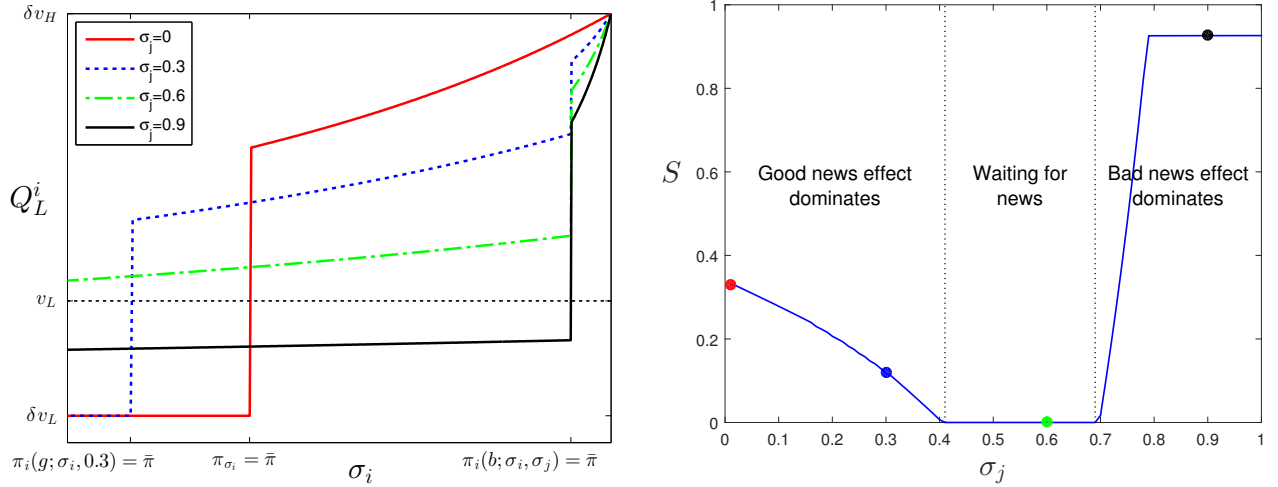
Figure 2: **Partial Equilibria and Strategic Interactions**. The left panel illustrates how the continuation value of a low-type seller i depends on both σ_i and σ_j. The right panel illustrates the set of partial equilibrium trading strategies in market i given a fixed trading strategy in market j. Parameters used: $\delta = 0.7$, $\lambda = 0.9$, $\xi = 1$.

3.2 Full Equilibria

Since markets are identical, moving from a partial equilibrium in market i to an equilibrium in both markets requires that the trading strategies also satisfy $\sigma_j = S(\sigma_i)$. The following result shows that all equilibria are symmetric and involve strictly positive probability of trade.

Proposition 2 (Symmetry and News) *In any equilibrium, $\sigma_A = \sigma_B > 0$.*

The first equality follows from noting that if $\sigma_i > \sigma_j \geq 0$ then $Q_L^i > Q_L^j$. Because Q_L^j must be weakly bigger than v_L (Lemma 4), the low-type seller i must strictly prefers to wait, which contradicts $\sigma_i > 0$ satisfying *Seller Optimality*. The strict inequality in Proposition 2 then follows immediately: if $\sigma_A = \sigma_B = 0$, then no news arrives in either market and buyers' beliefs in the second period are exactly the same as in the first period, which would imply that $Q_L^A = Q_L^B < v_L$, violating Lemma 4. Notice the contrast of this result to the partial equilibrium (i.e., the model with exogenous news). When news is endogenously generated, it cannot be an equilibrium for either market to simply wait for news.

Having established that any equilibrium must be symmetric, we now drop the subscripts and superscripts labeling the specific market and denote an equilibrium by the pair (σ, ϕ). Furthermore, having established that any equilibrium involves $\sigma \in (0, 1)$, the low type must be indifferent between accepting v_L in the first period and waiting until the second period. Hence, any pair is an equilibrium if and only if

$$Q_L(\sigma, \sigma, \phi) = v_L. \tag{6}$$

We have thus narrowed the search for equilibria to the solutions to equation (6). It is useful to note that potential equilibria can be classified into three different types depending on the posterior beliefs: $\pi_i(g) = \bar{\pi} > \pi_i(b)$, $\pi_i(g) > \bar{\pi} > \pi_i(b)$, and $\pi_i(g) > \bar{\pi} = \pi_i(b)$.[16] Since the posterior beliefs are monotonic in the amount of trade in the first period, we label the three possible types of equilibria as *low trade, medium trade* and *high trade* respectively, and denote the equilibrium trading intensity in the first period in the three equilibria by σ^q with $q \in \{low, med, high\}$, where $\sigma^{low} < \sigma^{med} < \sigma^{high}$.

Theorem 1 (Multiple Equilibria) *An equilibrium always exists and there are at most three. The equilibria fall into the following categories:*

1. *Low trade: There is at most one equilibrium in which $\pi_i(g) = \bar{\pi} > \pi_i(b)$. Given λ and ξ, there exists a $\widehat{\delta}_{\lambda,\xi}$ which can be uniformly bounded above by $\bar{\delta} < 1$ such that this equilibrium exists iff $\delta \geq \widehat{\delta}_{\lambda,\xi}$.*

2. *High trade: There is at most one equilibrium in which $\pi_i(g) > \bar{\pi} = \pi_i(b)$. Given δ, there exist $\overline{\lambda}_\delta, \overline{\xi}_\delta < 1$ such that this equilibrium exists if $\lambda > \overline{\lambda}_\delta$ and $\xi > \overline{\xi}_\delta$.*

3. *Medium trade: There are at most two equilibria in which $\pi_i(g) > \bar{\pi} > \pi_i(b)$. Exactly one such equilibrium exists if $\delta > \bar{\delta}$, $\lambda > \overline{\lambda}_\delta$, and $\xi > \overline{\xi}_\delta$.*

The three types of equilibria coexist when $\delta > \bar{\delta}$, $\lambda > \overline{\lambda}_\delta$, and $\xi > \overline{\xi}_\delta$.

The key insights of the theorem are illustrated in Figures 3 and 4. Figure 3 considers two different sets of parameter values. In the left panel, transparency and correlation are relatively low. Hence, the spillover effects across markets are modest, which leads to a unique equilibrium with low trade. In the right panel, both correlation and transparency are relatively high, leading to strong spillover effects and three equilibria. Thus, the importance of information spillovers hinges on two factors: market transparency and asset correlation. When both are sufficiently high, strategic interactions lead to multiple equilibria that are ranked in terms of trade. Otherwise, as we show in the next proposition, the equilibrium is unique.

Proposition 3 (Unique Equilibrium) *The equilibrium is generically unique if either λ is sufficiently close to $1 - \pi$ or ξ is sufficiently close to 0. Furthemore, as $\lambda \to 1 - \pi$ or $\xi \to 0$, the equilibrium trading probability converges to $\bar{\sigma}$ such that $\pi_{\bar{\sigma}} = \bar{\pi}$.*

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[16]The other two possible orderings of the posteriors $\pi_i(g) > \pi_i(b) > \bar{\pi}$ and $\bar{\pi} > \pi_i(g) > \pi_i(b)$ are ruled out by Lemma 1 and Assumption 2 respectively.
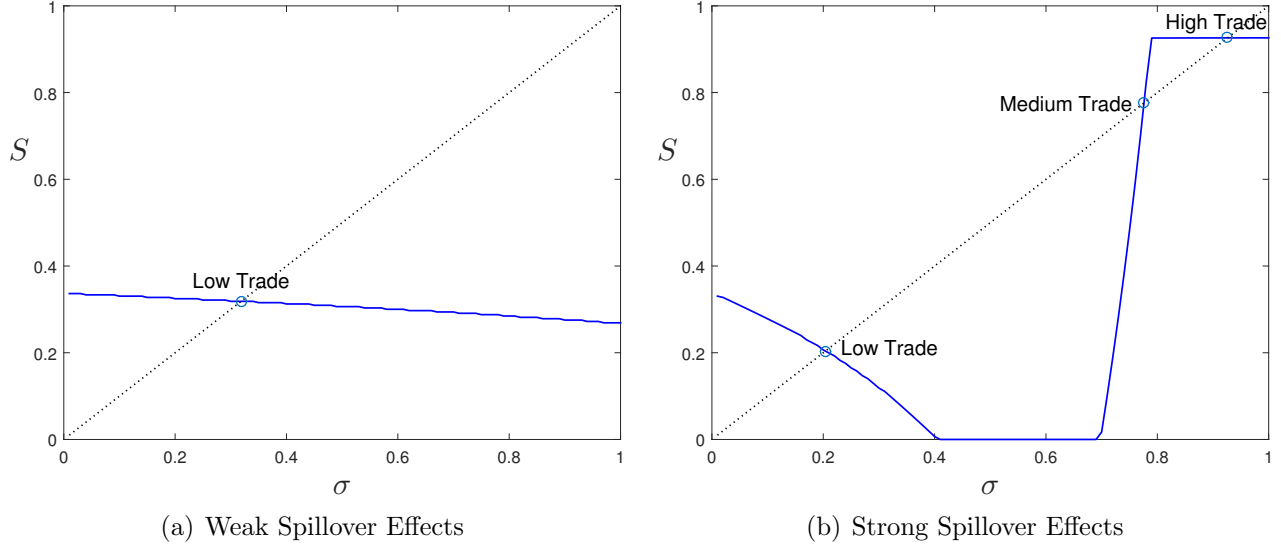
Figure 3: **Spillover Effects and Multiplicity.** This figure illustrates the set of equilibria for two different levels of transparency and correlation. In the left panel $\lambda = 0.6$ and $\xi = 0.4$, whereas in the right panel $\lambda = 0.9$ and $\xi = 1$.

Figure 4 illustrates more explicitly that it takes sufficiently high levels of both correlation and transparency for the medium and high trade equilibria to exist. It also illustrates how trade in the first period can react differently to increases in transparency or correlation depending on which equilibria we consider. For example, σ increases with both λ and ξ in the high trade equilibrium, while it decreases in both parameters in the medium and low trade equilibria. As we will see in Section 4, these findings will have important welfare implications.

4 Welfare

In this section, we study how the degree of market transparency and the level of correlation among assets affect the welfare of market participants. First note that because buyers are identical and compete via Bertrand competition, they make zero expected profits (Section 6 analyzes the model with non-identical buyers). Next, note that in any equilibrium the low-type sellers are indifferent between trading in the first period at a price of v_L or waiting and trading in the second period. Hence, their ex-ante equilibrium payoff is v_L regardless of market transparency or correlation. In order to study the welfare implications of transparency and correlation, it is therefore sufficient to consider the equilibrium payoff of a high-type seller, which we denote by Q_H^q, where $q \in \{low, med, high\}$ denotes the equilibrium. Furthermore, any welfare improvement for the high type is a Pareto improvement.
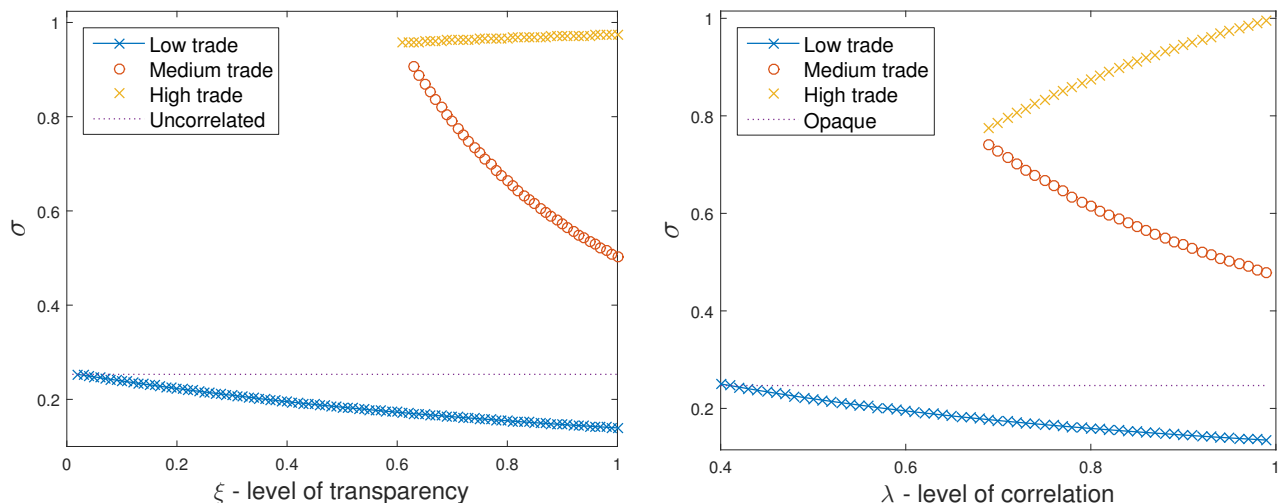
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Figure 4: **Market Transparency and Asset Correlation.** This figure illustrates the set of equilibrium σ as they depend on the level of correlation and degree of market transparency. The parameters used to compute the three equilibria are $\lambda = 0.9$ and $\delta = 0.6$ in the left panel and $\xi = 1$ and $\delta = 0.6$ in the right panel. The dotted black line in the left (right) panel is the unique equilibrium when asset values are uncorrelated (markets are perfectly opaque).

Proposition 4 (Welfare) *Welfare in transparent markets is weakly greater than in opaque markets. Whenever the three types of equilibria coexist, we have $Q_H^{low} < Q_H^{med} < Q_H^{high}$. Moreover,*

- *Q_H^{high} is increasing in both ξ and λ.*

- *Q_H^{med} is decreasing in both ξ and λ.*

- *$Q_H^{low} = c_H$ for all ξ and λ.*

In the low trade equilibrium, buyers mix between c_H and v_L after good news, and they offer v_L after bad news. In this equilibrium, following both good and bad news, the high-type seller's equilibrium payoff is c_H. In both the medium and high trade equilibria, after good news the buyers' beliefs satisfy $\pi_i(g) > \overline{\pi}$. Hence, after good news the price offered is strictly above c_H and after bad news the high type is no worse off. This immediately implies that in both of these equilibria, the high-type seller is strictly better off than in the low trade equilibrium.

We illustrate these results graphically in Figure 5. In the left panel, when transparency is low, there is a unique equilibrium, which involves low trade. In this equilibrium, the high type is indifferent whether to trade or not following good news and any increase in transparency results in a decrease in σ keeping total welfare unchanged. When transparency becomes sufficiently large, multiplicity kicks in. Welfare in the low-trade equilibrium remains independent of the
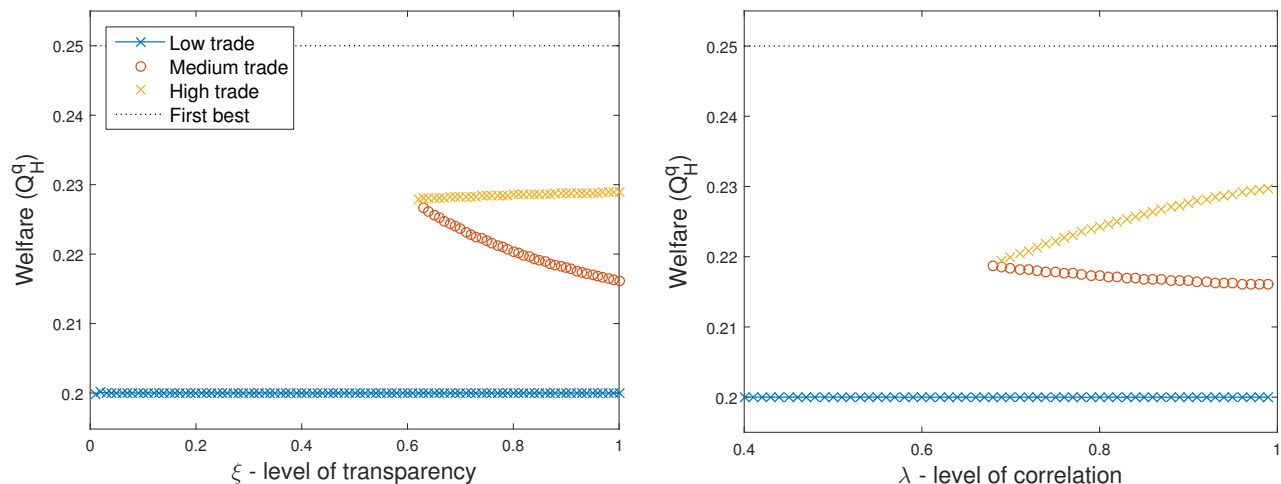
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Figure 5: **Effect of Transparency and Correlation on Welfare.** The left panel has $\lambda = 0.9$; right panel has $\xi = 1$.

degree of transparency. Welfare is strictly higher in both the medium and high trade equilibria and depends on the degree of transparency.

As we can see from Figure 5, both transparency and correlation reduce welfare along the medium trade equilibrium. This is because the good news effect dominates and the low type trades less aggressively as ξ and λ increase (see Figure 4), leading to less efficient trade at $t = 1$, more adverse selection at $t = 2$ and lower high-type welfare. On the other hand, in the high trade equilibrium, the bad news effect is dominant and the low type trades more aggressively as ξ and λ increase, leading to less adverse selection at $t = 2$ and higher welfare.

5   Many Assets and Market Informativeness

In this section, we extend our analysis to an economy with an arbitrary number of assets. This is an important exercise not only to capture a broader set of economic environments and demonstrate robustness, but because it allows us to study the implications for information aggregation. In particular, we model correlation by supposing that each asset is correlated with an unobservable aggregate state of nature, and we ask whether market participants learn this aggregate state as the number of assets in the economy grows to infinity.

We show that indeed our results pertaining to multiplicity are robust by characterizing three types of equilibria that coexist under analogous parametric restrictions. We then use the characterization to show that as $N \to \infty$, information about the underlying state may or may not be aggregated. More specifically, we derive a necessary and sufficient condition under which information is *not* aggregated along any sequence of equilibria as $N \to \infty$. Thus, although the

number of sources of information becomes arbitrarily large, the informativeness of each source can decline fast enough so as to limit the market's ability to aggregate information.

As in the case with two assets, some degree of transparency and correlation is necessary for information spillovers to lead to interesting strategic interactions. In order to simplify the exposition of this section, we focus on the case in which markets are perfectly transparent ($\xi = 1$). This specification strengthens our results pertaining to the impossibility of information aggregation: market participants do not necessarily learn the underlying state even if the market is fully transparent.

5.1 Equilibrium

The economy has $N + 1$ sellers, where $N \geq 1$, and seller i is endowed with an asset of type $\theta_i \in \{L, H\}$ that, as before, has payoffs c_{θ_i} to the seller and v_{θ_i} to the buyers and where $\mathbb{P}(\theta_i = H) = \pi \in (0, 1)$ for $i = 1, 2, ..., N + 1$. As before, the type of asset i is private information of seller i. The correlation among asset values arises due to correlation with an unobservable underlying aggregate state ω that takes values in $\{l, h\}$.

Asset types are mutually independent conditional on the realization of state ω, and their conditional distributions are given by $\mathbb{P}(\theta_i = L | \omega = l) = \lambda \in (1 - \pi, 1)$. To allow for arbitrarily high level of correlation, we set $\mathbb{P}(\omega = h) = \pi$. Notice that as a result of the correlation structure, a seller has private information about the underlying state but does not observe the state or know it with certainty. Finally, we maintain Assumptions 1 and 2 throughout and, as mentioned earlier, assume that the market is perfectly transparent ($\xi = 1$).

It is straightforward to show that generalized versions of Lemmas 1–3 and Proposition 2 hold with arbitrary number of assets. Thus, the equilibrium continues to feature low-type sellers trading with symmetric probability $\sigma_N \in (0, 1)$ at $t = 1$ and the analogue of equation (6) must hold. The key difference between this economy and the setting with only two assets is the information that arrives to each market prior to the second trading period. In any equilibrium, this information can be summarized by how many trades occurred at $t = 1$ among the N other assets. We thus denote by z_k the event that exactly k trades (at a price of v_L) occurred at $t = 1$.

Let X_i be the indicator for a trade having occurred in market i at $t = 1$. Conditional on the state, ω, we have

$$X_1, ..., X_N | \omega \sim^{iid} \text{Bernoulli}(p_\omega),$$

where, by conditional independence, $p_\omega = \sigma_N \cdot \mathbb{P}(\theta_i = L | \omega)$. Thus, we can summarize the

distribution of news from the perspective of a type-θ seller in market $N+1$ as

$$\rho_\theta(z_k) \equiv \mathbb{P}\left(\sum_{i=1}^{N} X_i = k | \theta_{N+1} = \theta\right) = \sum_{\omega \in \{l,h\}} \binom{N}{k} \cdot (p_\omega)^k \cdot (1-p_\omega)^{N-k} \cdot \mathbb{P}(\omega | \theta_{N+1} = \theta). \quad (7)$$

By symmetry, equation (7) also characterizes the distribution of news in markets $i \in \{1, ..., N\}$. Buyers' posterior belief in an arbitrary market following event z_k is in turn given by

$$\pi_i(z_k) = \frac{\pi_{\sigma_N} \cdot \rho_H(z_k)}{\pi_{\sigma_N} \cdot \rho_H(z_k) + (1 - \pi_{\sigma_N}) \cdot \rho_L(z_k)}, \quad (8)$$

where $\pi_{\sigma_N} = \frac{\pi}{\pi + (1-\pi) \cdot (1-\sigma_N)}$ is the interim belief about the seller before the arrival of news.

As in Section 3, we will subdivide the set of equilibria into three categories, which we again call the low, medium, and high trade equilibria. The low trade equilibria are now defined by the ordering of posteriors $\pi_i(z_0) = \bar{\pi} > \pi_i(z_k)$ for $k > 0$. That is, in the second period a high type trades only following the best possible news in a low trade equilibrium. At the other extreme, the high trade equilibrium is defined by the ordering of posteriors $\pi_i(z_k) > \bar{\pi} = \pi_{\sigma_N}(z_N)$ for $k < N$, i.e., in the second period a high type trades following any news realization. Finally, the remaining equilibria, all of which will feature posterior ordering $\pi_i(z_0) > \bar{\pi} > \pi_{\sigma_N}(z_N)$, we term medium trade equilibria. In contrast to the two-asset setting, there can now be many medium trade equilibria. As the number of assets grows, the number of potential equilibria grows as well. For expositional simplicity, we will not distinguish between them.

The following proposition extends our main result in Theorem 1 to the setting with an arbitrary number of assets:

Proposition 5 *Fix $N \geq 1$. An equilibrium always exists. The equilibria fall into the following categories:*

1. <u>*Low trade:*</u> *There is at most one equilibrium in which $\pi_i(z_0) = \bar{\pi} > \pi_i(z_k)$ for $k > 0$. Given λ, there exists a $\widehat{\delta}_{\lambda,N}$ which can be uniformly bounded above by $\bar{\delta}_N < 1$ such that this equilibrium exists iff $\delta \geq \widehat{\delta}_{\lambda,N}$.*

2. <u>*High trade:*</u> *There is at most one equilibrium in which $\pi_i(z_k) > \bar{\pi} = \pi_i(z_N)$ for $k < N$. Given δ, there exist $\overline{\lambda}_{\delta,N} < 1$ such that this equilibrium exists if $\lambda > \overline{\lambda}_{\delta,N}$.*

3. <u>*Medium trade:*</u> *There can be many equilibria in which $\pi_i(z_0) > \bar{\pi} > \pi_i(z_N)$. If $\delta > \bar{\delta}_N$ and $\lambda > \overline{\lambda}_{\delta,N}$, then at least one such equilibrium exists.*

The three types of equilibria coexist when $\delta > \bar{\delta}_N$ and $\lambda > \overline{\lambda}_{\delta,N}$ and are Pareto ranked with $Q_H^{low} < Q_H^{med} < Q_H^{high}$.

Having characterized the set of equilibria, we now use the model to ask questions about the informational efficiency of markets, as they become large. Though we do not model them explicitly, one can imagine a variety of reasons why information aggregation is valuable (e.g., better allocation of capital).

5.2 Does Information Aggregate?

Let us focus on the information available to an arbitrary market i. Let $\pi_i^{state}(z) \equiv \mathbb{P}(\omega = h|z)$ denote buyers' posterior belief about the state following the realization z of news arriving from other markets (i.e., how many other sellers have traded). Notice that as in the two-asset setting, buyers' posterior beliefs about the state are imperfect. However, if the news becomes sufficiently informative in the limit, then the noise in posteriors should disappear. In this case, we say that there is information aggregation about the state. To formalize this notion, consider a sequence of trading probabilities $\{\sigma_N\}_{N=1}^{\infty}$ where, for each N, σ_N is an equilibrium trading probability in the economy with $N + 1$ assets.

Definition 2 *There is information aggregation about the state along a sequence $\{\sigma_N\}_{N=1}^{\infty}$ if along this sequence $\pi_i^{state} \to^p 1_{\{\omega = h\}}$ as $N \to \infty$.*

Clearly, if the information content of news from a single market is fixed and non-trivial, i.e, σ_N is uniformly bounded above zero, then information aggregation obtains. The reason, of course, is that by the law of large numbers the fraction of trades that buyers would observe would concentrate around its population mean $\sigma_N \mathbb{P}(\theta_i = L|\omega = \widehat{\omega})$, which would be greater in the low state than in the high state since $\mathbb{P}(\theta_i = L|\omega = l) > \mathbb{P}(\theta_i = L|\omega = h)$.

However, because the information contained in trading behavior is endogenous, it is possible that the trading behavior becomes less informative as the number of assets grows (i.e., $\sigma_N \to 0$). In this case, the rate of convergence determines whether aggregation obtains. To see this possibility, it is useful to consider the 'fictitious' limiting economy where buyers learn the state of nature in the second period w.p.1. In this fictitious economy, buyers have exogenous information in the second period. As shown in Proposition 1, with exogenous news, it is in fact possible for trade to completely collapse in the first period. Indeed, under the following two conditions the low type seller in this limiting economy would strictly prefer to delay trade to the second period.

Condition 1. $1 - \frac{(1-\lambda)(1-\pi)}{\pi} > \bar{\pi}$.

Condition 2. $v_L < (1 - \delta) \cdot c_L + \delta \cdot \left(\lambda \cdot v_L + (1 - \lambda) \cdot V \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi} \right) \right)$.

Condition 1 guarantees that the price conditional on good news would be above c_H, while Condition 2 requires that the expected continuation value of the low type be strictly above

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v_L. This implies that an offer of v_L in the first period would be rejected w.p.1. The following proposition utilizes these two conditions to state the main result of this section.

Proposition 6 (Information Aggregation) *If conditions* (1) *and* (2) *hold, then there does not exist a sequence of equilibria along which information aggregates. Conversely, if either* (1) *or* (2) *is reversed, there exists a sequence of equilibria along which information aggregates.*

The proof uses the fact that information aggregation implies that for N large enough, the payoffs of the sellers in the economy with finitely many assets can be bounded below by the payoff in the limiting economy, thus making delay also optimal when there are a large but finite number of assets.

Conversely, when the limiting economy has an equilibrium with positive trade, we can always construct a sequence of equilibria that converge to an equilibrium with that trading probability, and information is clearly aggregated along such a sequence. However, a violation of Condition 1 or 2 is not sufficient to ensure aggregation along every sequence of equilibria due to multiplicity. In the Appendix (see Proposition 10), we illustrate this finding with an example where multiple equilibria coexist, in some information aggregates, while in others information aggregation fails. These results suggest yet another reason why it is important to understand how information is generated in markets, as the extent of market informativeness can depend on the expectations of market participants.

6   Transparency with Asymmetric Buyers

Pro-transparency policies are sometimes motivated as a way to "level the playing field" between traders with heterogenous access to information. In this section we evaluate the effects of such policies. In order to do so, we extend the model to allow for buyers with differential access to transaction data. This is meant to capture the fact that some traders (e.g., broker-dealers) are active in many markets, which may give them an informational advantage over investors who participate in fewer markets.

Let us return to the model with two sellers and suppose that there are $M > 2$ buyers for each seller in each period, and that one of the buyers in each market is able to observe what occurs in the other market regardless of whether there is transparency.[17] We refer to this buyer as a "dealer." The remaining buyers, whom we refer to as "investors" only learn about trades if there is transparency. In this section, we assume that in each period the seller holds a second

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[17]Our results extend to the case in which the dealer present with probability $\epsilon \in (0, 1)$.

price auction with a secret reserve price.[18] For simplicity, we focus on a setting with only two assets and compare the case of fully opaque ($\xi = 0$) vs fully transparent ($\xi = 1$) markets.

Corollary 1 (Fully Transparent) *When the market is fully transparent, investors and dealers have identical information and the set of equilibrium is the same as in the fully transparent case of Theorem 1. Therefore, with full transparency, both investors and dealers make zero (expected) trading profits.*

When the market is fully opaque, investors' degree of sophistication is an important determinant of whether they benefit from transparency. Below, we consider two different specifications regarding the degree of sophistication of investors. They can be either "naive" or "sophisticated" regarding the fact that they are facing competition from a trader with access to better information. One can think of naive investors as utility maximizing agents with an incorrect prior belief about the probability that a dealer is present (i.e., the true probability is one but naive investors believe it is zero). Sophisticated investors are fully rational agents. In either case, several familiar features arise in equilibrium. The bid in the first period is v_L. The high-type seller sets a secret reserve price above c_H and the low-type seller mimics this reserve price with some probability $1 - \sigma_i$. Thus, the offer is rejected by the high-type seller w.p.1 and accepted with probability σ_i by the low-type seller. We characterize what happens in the second period separately for each case below.

Naive Investors. If the market is fully opaque then, in the second period, naive investors bid according to Lemma 1, where their posteriors are conditioned only on σ_i. A dealer bids the expected value if its posterior is above $\bar{\pi}$ and v_L otherwise. The naive investors thus fall prey to a winner's curse. They will win the auction only when the dealer receives bad news from the other market (i.e., that the other market has traded). Conditional on a trade in the other market, investors' bid underestimates the probability of the asset being low value and hence, on average, they experience trading losses. On the other hand, when the other market does not trade, then naive investors are always outbid by the dealer who is thus able to capture information rents. Also, since the second highest bid always originates from investors, the seller faces exactly the same situation as if he were facing only investors. This implies that any rents made by the dealer are exactly offset by the losses of the investors. Thus, in addition

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[18]Adopting this trading mechanism (rather than Bertrand competition) is primarily to simplify the equilibrium analysis and intuition. When buyers are symmetric (or the market is fully transparent), both trading mechanisms lead to exactly the same equilibrium outcomes. In an opaque market with asymmetric buyers and Bertrand competition, the equilibrium construction is more complex because the optimal bidding strategy depends on the distribution of other bids, which can require that buyers mix over a continuum of offers. Nevertheless, the key insights in Propositions 7 and 8 are robust to Bertrand competition (formal results available upon request).

to multiplicity and the potential welfare gains for the seller, transparency has a *redistributive* effect from the dealer to the naive investors.

Proposition 7 (Naive) *If investors are naive and markets are fully opaque, there exists a unique equilibrium which generates the same total surplus as the low-trade equilibrium in Theorem 1. However, dealers make positive trading profits while naive investors experience trading losses.*

Sophisticated Investors. When investors understand that they are competing against a dealer, they are aware of the winner's curse. Therefore, when the market is opaque, sophisticated investors bid in the second period as if a trade occurred in the other market. Note that a bid of v_L w.p.1 in the second period cannot be part of an equilibrium. Therefore, it must be that $\pi_i(b) = \bar{\pi}$. This requires a higher equilibrium σ than if buyers are symmetric. This increased probability of trade in the first period enhances efficiency relative to the symmetric buyer case. Yet, all the additional surplus goes to the dealer and not the seller who still faces the same distribution of offers in the second period. On the other hand, when markets become fully transparent, the dealer now faces competition from the sophisticated investors and thus loses his rents. Unlike with naive buyers, the reduction in the dealer's rents is not purely redistributive.

Proposition 8 (Sophisticated) *When investors are sophisticated and markets are fully opaque, there exists a unique equilibrium that Pareto dominates the low-trade equilibrium in Theorem 1. The additional surplus is captured entirely by dealers.*

Importantly, in both cases, dealers prefer the opacity. This may help explain why insiders in financial markets lobby strongly against making transaction data freely available to all market participants. It also shows that although enhancing transparency might help protect naive investors it runs the risk of actually reducing welfare if applied in an environment where investors are sophisticated. Thus, in the complex derivatives markets where most participants are highly sophisticated, transparency has the potential to reduce overall welfare. It is worth noting here the contrast with respect to our benchmark model, in which transparency cannot yield lower welfare than opacity (see Proposition 4). The difference in the results stems from the fact that when traders are asymmetric and markets are opaque, there is effectively less competition in the second period due to the informational advantage of the dealer. This makes the seller more pessimistic about the future and thus increases her willingness to trade early. Thereby, opacity has the potential to mitigate the adverse selection problem and increase welfare provided that investors are sophisticated.

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7   Conclusions

Since the seminal work of Hayek (1945), the role of markets in aggregating information has been well understood. In this paper, we have highlighted feedback effects between the information content in markets and the incentive to trade in a dynamic setting. Our model delivers several novel theoretical insights, which have implications for both policy and empirical work.

Our main theoretical insights are as follows. First, contrary to an economy with exogenously revealed information, when the information revealed is endogenously determined by trading behavior, there cannot be periods of no trade. Second, the endogenous nature of information introduces interdependence in the trading behavior across markets. In particular, when correlation and transparency are sufficiently high, this interdependence can be sufficiently strong so as to lead to multiplicity of equilibria that differ in their trading volume, prices, information content, and welfare. Perhaps surprisingly, we also find that markets can fail to aggregate information as the number of participants becomes large, even if information is made available to market participants (i.e. even if it is fully transparent).

The fact that multiplicity of equilibria can arise once markets are made transparent has important implications for how we interpret and conduct empirical work. Our results suggest that a change in transparency can lead to either little change in market behavior (if the market remains in the low trade equilibrium) or the market behavior can change dramatically (if the market has switched to the high trade equilibrium). This can help explain why Bessembinder et al. (2006) and Edwards et al. (2007) find that market participants gain from the introduction of TRACE, while Goldstein et al. (2007) see no effect within a subclass of securities, and Asquith et al. (2013) find significantly different results for bonds that were part of the different phases of the TRACE program.

The existence of multiple, welfare-ranked equilibria is also important from a policy perspective. In order to actually achieve the welfare gains associated with increased transparency, it is important to steer market participants to coordinate on the high-trade equilibrium. When market participants have differential access to transaction data, transparency indeed "levels the playing field" if investors are naive; it reduces both dealer's trading profits and investors' trading losses. However, transparency does not benefit sophisticated investors and can potentially reduce total surplus. Therefore, it is important to take into account the type of market participants (e.g., retail or institutional) when considering policies aimed at transparency.

References

Akerlof, G. (1970). The Market for Lemons: Quality Uncertainty and the Market Mechanism. *Quarterly Journal of Economics 84* (3), 488–500.

Asquith, P., T. Covert, and P. Pathak (2013). The Effects of Mandatory Transparency in Financial Market Design: Evidence from the Corporate Bond Market. *National Bureau of Economic Research Working Paper Series No. 19417.*

Axelson, U. and I. Makarov (2014). Informational Black Holes in Auctions. *Working Paper.*

Bessembinder, H., W. Maxwell, and K. Venkataraman (2006). Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds. *Journal of Financial Economics 82* (2), 251–288.

Bodoh-Creed, A. (2013). Efficiency and Information Aggregation in Large Uniform-Price Auctions. *Journal of Economic Theory 146* (6), 2436–2466.

Cespa, G. and X. Vives (2015). The Beauty Contest and Short-Term Trading. *Journal of Finance 70* (5), 2099–2154.

Daley, B. and B. Green (2012). Waiting for News in the Market for Lemons. *Econometrica 80* (4), 1433–1504.

Daley, B. and B. Green (2015). An Information-Based Theory of Time-Varying Liquidity. *Journal of Finance (forthcoming).*

Drugov, M. (2010). Contracts with Informational Externalities. *Working Paper.*

Drugov, M. (2015). Bargaining with Informational and Real Externalities. *Working Paper.*

Duffie, D., P. Dworczak, and H. Zhu (2014). Benchmarks in Search Markets. *Working Paper.*

Edwards, A. K., L. E. Harris, and M. S. Piwowar (2007). Corporate Bond Market Transaction Costs and Transparency. *Journal of Finance 62* (3), 1421–1451.

Fuchs, W., A. Öery, and A. Skrzypacz (2015). Transparency and Distressed Sales under Asymmetric Information. *Cowles Foundation Working Paper.*

Goldstein, M. A., E. S. Hotchkiss, and E. R. Sirri (2007). Transparency and Liquidity: A Controlled Experiment on Corporate Bonds. *Review of Financial Studies 20* (2), 235–273.

Grossman, S. (1976). On the efficiency of competitive stock markets where trades have diverse information. *The Journal of Finance 31* (2), 573–585.

Hayek, F. A. (1945). The Use of Knowledge in Society. *American Economic Review 35* (4), 519–30.

Healy, P. M. and K. G. Palepu (2001). Information Asymmetry, Corporate Disclosure, and the

Capital Markets: A Review of the Empirical Disclosure Literature. *Journal of Accounting and Economics 31*, 405–440.

Hörner, J. and N. Vieille (2009). Public vs. Private Offers in the Market for Lemons. *Econometrica 77*(1), 29–69.

Kaya, A. and K. Kim (2015). Trading Dynamics with Private Buyer Signals in the Market for Lemons. *Working Paper*.

Kremer, I. (2002). Information Aggregation in Common Value Auctions. *Econometrica 70*(4), 1675–1682.

Lauermann, S. and A. Wolinsky (2013). A Common Value Auction with Bidder Solicitation. *Working Paper*.

Lauermann, S. and A. Wolinsky (2015). Search with Adverse Selection. *Econometrica (forthcoming)*.

Learner, H. (2011). An Examination of Transparency in European Bond Markets. Technical report, CFA Institute.

Milgrom, P. R. (1979). A Convergence Theorem for Competetive Bidding with Differential Information. *Econometrica 47*(3), 679–688.

NASD (2005). NASD's Fully Implemented "TRACE" Brings Unprecedented Transparency to Corporate Bond Market.

Nöldeke, G. and E. van Damme (1990, January). Signalling in a Dynamic Labour Market. *The Review of Economic Studies 57*(1), 1–23.

Ostrovsky, M. (2012). Information Aggregation in Dynamic Markets with Strategic Traders. *Econometrica 80*(6), 2595–2649.

Pesendorfer, W. and J. M. Swinkels (1997). The Loser's Curse and Information Aggregation in Common Value Auctions. *Econometrica 65(6)*, 1247–1281.

Swinkels, J. M. (1999). Education Signaling with Preemptive Offers. *Review of Economic Studies 66*(4), 949–970.

Verrecchia, R. E. (2001). Essays on Disclosure. *Journal of Accounting and Economics 32*, 97–180.

Wilson, R. (1977). A Bidding Model of Perfect Competition. *The Review of Economic Studies 44*(3), 511–518.

A   Appendix

Proof of Lemma 1.   Notice that the reservation price of the low type seller is c_L and the reservation price of the high type seller is c_H. Thus, if a price offer above c_H is accepted, the expected profits of the buyer are $\pi v_H + (1 - \pi) v_L - p$, and if an offer strictly below c_H is accepted, the expected profits are $v_L - p$ since only the low types would accept.

(i) If $\pi_i < \bar{\pi}$, then $\pi v_H + (1 - \pi) v_L < c_H$. Hence, any offer that is sufficiently high, above c_H, to attract the high type sellers would lead to strictly negative profits in expectation. Since any offer below c_H would only be accepted by the low types. Thus, any offer $p \in (v_L, c_H)$ would also lead to sure loses if accepted. Any offer below v_L would lead to strictly positive profits; thus in equilibrium buyers would deviate to slightly higher offers to capture the whole market. Thus, in equilibrium v_L will be offered in this case. The same argument rules out any mixed strategy equilibrium that has a mass point anywhere other than v_L. Finally, mixing continuously over some interval of offers cannot be an equilibrium. We show this by contradiction. If one player mixes over some interval $[\underline{b}, \bar{b}]$ with $\bar{b} = v_L$, then the other player must be offering v_L with probability 1, because otherwise he would never want to offer v_L which leads to zero profits with probability 1. If instead $\bar{b} < v_L$, the other player's best response can never have $\underline{b}$ (or anything below) as part of its support. This bid will loose with probability 1 and thus earn zero profits, while bidding $\frac{\bar{b} + v_L}{2}$ would lead to strictly positive profits.

(ii) If $\pi_i > \bar{\pi}$, then $\pi v_H + (1 - \pi) v_L > c_H$. Hence, any offer between c_H and $\pi v_H + (1 - \pi) v_L$ would lead to strictly positive profits. Buyers would want to increase their offer to capture the whole market until the offer reaches $p = \pi v_H + (1 - \pi) v_L$; increasing the offer beyond leads to sure loses. Notice that no other offer can be part of an equilibrium. Any offer between v_L and c_H would lead to negative profits if accepted, since only low types would accept such an offer. Any offer below v_L cannot be an equilibrium because profits would be strictly positive and the seller would have an incentive to offer either an extra small amount and trade with low types or jump to an offer of c_H and trade with both types. Similarly, v_L cannot be an equilibrium since a deviation to c_H would be profitable. The same arguments as in (i) rule out mixed strategy equilibria.

(iii) If $\pi_i = \bar{\pi}$, then $\pi v_H + (1 - \pi) v_L = c_H$. Hence, any offer above c_H if accepted would lead to sure loses. Similarly, any offer strictly in (v_L, c_H) would lead to loses, since it would only be accepted by the low types. Any offer strictly below v_L would be profitable if accepted; thus buyers would raise their offers to capture the whole market. They will continue this way until they reach v_L at this point they are also indifferent to jump their offer up to c_H and sell to both types (we assume that although indifferent, the high types are willing to sell at c_H). Given that one of the buyers mixes between v_L and c_H, the other buyer is also willing to mix

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since both offers if accepted lead to zero profits, and any other offer would lead to negative profits in expectation. ∎

Proof of Lemma 2.   Most of the proof follows directly from the arguments in the text. We will therefore only prove that the high type seller expects a weakly better distribution of price offers in the second period. This is the same as saying that the high type seller expects a weakly better distribution of buyers' posteriors π_i. More formally, we claim that the distribution of the random variable π_i given $\theta_i = H$ (weakly) first order stochastically dominates the distribution of the random variable π_i given $\theta_i = L$. Note that the distribution of posteriors π_i in the second period is a function of the trading probabilities by both types in both markets and the news realization $z = \{trade, no\ trade\}$ generated in the other market. In the case of uninformative signals, there is no updating and the result follows immediately. Thus, we assume that, whatever are the strategies being played in the other market in equilibrium, one of the realizations of z must be associated with a high type more likely being present in the other market. Given the positive correlation in types, this in turn induces an updating of buyers' posteriors in the seller's own market when such event is observed. More concretely, if the high type is expected to trade with lower (higher) probability than the low type in the other market, then $z = trade$ would be bad (good) news and lead to a higher (lower) π_i. The important fact is that since types are correlated, relative to the low type seller, the high type seller in market A assigns a higher probability to the event that the seller in market B is a high type. Consistent with this, the high type seller from market A also expects the news to more likely be good than the low type seller. Which implies that the high type expects, as a result, a better distribution of posteriors and prices. Together with points (i) and (ii) argued in the text this establishes lemma. ∎

It is worth remarking that this proof can be generalized to an arbitrary set of possible news realizations. In that case, we would simply need to assign some likelihood ratio to the signal, given the equilibrium strategies.

Proof of Lemma 3. (i) From Lemma 2, we know that if the high type is willing to accept an offer with positive probability then the low type would accept it with probability 1. Thus, given Assumption 1, any bid above c_H would lead to losses. Any bid in (v_L, c_H) also leads to losses since it is only accepted by the low type. Any bid $b < v_L$ would be profitable but it would lead to the other buyer offering $b + \varepsilon$ for some small positive ε in order to capture the market. Thus, any deterministic offer strictly below v_L can be ruled out. The only deterministic bid possible is v_L, at this point there is no profitable deviation for the other buyer than offering v_L as well. The same arguments rule out any mixed strategy equilibrium that has a mass point anywhere other than v_L. Finally, mixing continuously over some interval of offers cannot be an equilibrium. We show this by contradiction. If one of the buyers mixes over some interval $[\underline{b}, \bar{b}]$

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with $\bar{b} = v_L$ then the other buyer must be offering v_L with probability 1 because otherwise he would never want to offer v_L, which leads to zero profits with probability 1. If instead $\bar{b} < v_L$, the other buyer's best response can never have $\underline{b}$ (or anything below) as part of its support. This bid will lose with probability 1 and thus earn zero profits, while bidding $\frac{\bar{b}+v_L}{2}$ would lead to strictly positive profits.

(ii) Clearly the high type would reject v_L, since $v_L < c_H$. To see that the low type must accept with probability less than one, note that if in equilibrium the low type accepted with probability 1, then the posterior belief would assign probability 1 to the type being high in the next period. The offer in the next period (as argued in Lemma 1) would be v_H but, given Assumption 2, the low type seller would then want to deviate and trade in period 2 at v_H rather than at v_L in period 1. ∎

Proof of Lemma 4. See text for the proof of necessity. Sufficiency follows from Lemmas 1 through 3. ∎

Proof of Proposition 1. For given (σ_j, ϕ_j), we evaluate $Q_L^i(0, \sigma_j, \phi_i)$. If $Q_L^i(0, \sigma_j, \phi_i) \geq v_L$, then $\sigma_i = 0$ satisfies the partial equilibrium as established in Lemma 4. Furthermore there is no other candidate value for σ_i, because monotonicity implies that in that case for all $\sigma_i > 0$ we would have $Q_L^i(\sigma_i, \sigma_j, \phi_i) > v_L$. But then the seller would deviate. If $Q_L^i(0, \sigma_j, \phi_i) < v_L$, monotonicity and upper hemicontinuity of Q_L^i with respect to σ_i and the fact that $Q_L^i(1, \sigma_j, \phi_i) > v_L$ (by Assumption 2) imply that there exists a unique σ_i such that $Q_L^i(\sigma_i, \sigma_j, \phi_i) = v_L$. ∎

Proof of Proposition 2. We show that all equilibria must be symmetric. The proof that all equilibria involve positive trade is in the text. In search of a contradiction, assume there exists an equilibrium in which $\sigma_A > \sigma_B \geq 0$. If this is the case, then notice the following: (1) the probability of bad news arriving to market B is higher than that of bad news arriving to market A; (2) the beliefs about the seller i being of high type, conditional on bad news arriving in market j, must satisfy $\pi_A(b) > \pi_B(b)$ since the posterior

$$\pi_i(b) = \frac{1}{1 + \frac{\mathbb{P}(\theta_j = L | \theta_i = L)}{\mathbb{P}(\theta_j = L | \theta_i = H)} \cdot \frac{1 - \pi_{\sigma_i}}{\pi_{\sigma_i}}}$$

is increasing in σ_i and is independent of σ_j; and (3) the beliefs about the seller i being of high type, conditional on good news arriving, must satisfy $\pi_A(g) > \pi_B(g)$ since the posterior

$$\pi_i(g) = \frac{1}{1 + \frac{1 - \xi \cdot \sigma_j \cdot \mathbb{P}(\theta_j = L | \theta_i = L)}{1 - \xi \cdot \sigma_j \cdot \mathbb{P}(\theta_j = L | \theta_i = H)} \cdot \frac{1 - \pi_{\sigma_i}}{\pi_{\sigma_i}}}$$

is increasing faster in σ_i than in σ_j. The results (1) to (3) then imply that $Q_L^A > Q_L^B$. This in turn implies that the seller in market B would have strictly stronger incentives to trade than

the seller in market A. Thus, if seller A is trading with probability $\sigma_A \in (0,1)$, which can only happen in equilibrium if $Q_L^A = v_L$, which then implies that $Q_L^B < v_L$. But this is a contradiction to Lemma 4. ∎

Proof of Theorem 1. The existence of equilibria follows from the fact that the range of the function Q_L is the interval $[v_L, v_H]$. That equilibria fall within the stated three categories follows from the fact that we can rule out the posterior orderings $\pi_i(g) > \pi_i(b) > \bar{\pi}$ and $\bar{\pi} > \pi_i(g) > \pi_i(b)$. In the latter case, the equilibrium offer in the second period would be v_L, which would imply that $\sigma = 1$, and is thus ruled out by Lemma 3. In the former case, the expected offers in the second period are strictly greater than c_H, which is ruled out by Assumption 2. Hence, the three categories in Theorem 1 are the only remaining possible orderings of posteriors.

Low trade. That there is at most one low trade equilibrium follows from the fact that the trading probability σ is fully pinned down by the requirement that $\pi_i(g) = \bar{\pi}$. Let $\bar{\sigma}$ be such that $\pi_i(g)|_{\sigma=\bar{\sigma}} = \bar{\pi}$. Then, in the second period, low type expects to receive a payoff:

$$\mathbb{E}_L\{F_L(\pi_i, \phi_i)\}|_{\sigma=\bar{\sigma}} = \xi\bar{\sigma}\lambda \cdot v_L + (1 - \xi\bar{\sigma}\lambda) \cdot [\phi_i c_H + (1 - \phi_i)v_L]$$

where $\phi_i \in [0,1]$. After bad news, the offer is v_L and after good news it is either c_H or v_L. Thus, in the low trade equilibrium, as ϕ_i varies between 0 and 1, the range of this payoff is the interval: $[v_L, \xi\bar{\sigma}\lambda \cdot v_L + (1 - \xi\bar{\sigma}\lambda) \cdot c_H]$. Since, in equilibrium, we must have $Q_L = v_L$, we can conclude that there exists a $\delta_{\lambda,\xi} < 1$ such that the low trade equilibrium exists iff $\delta \geq \delta_{\lambda,\xi}$. The existence of the uniform bound $\bar{\delta}$ follows from the fact that, for $\bar{\sigma}$ satisfying $\pi_i(g)|_{\sigma=\bar{\sigma}} = \bar{\pi}$, we have that $\sup_{\xi,\lambda} \xi\bar{\sigma}\lambda < 1$.

High trade. That there is at most one high trade equilibrium follows from the fact that the trading probability σ is fully pinned down by the requirement that $\pi_i(b) = \bar{\pi}$. Let $\bar{\sigma}$ be such that $\pi_i(b)|_{\sigma=\bar{\sigma}} = \bar{\pi}$. Then, in the second period, low type expects to receive a payoff:

$$\mathbb{E}_L\{F_L(\pi_i, \phi_i)\}|_{\sigma=\bar{\sigma}} = \xi\bar{\sigma}\lambda \cdot [\phi_i c_H + (1 - \phi_i)v_L] + (1 - \xi\bar{\sigma}\lambda) \cdot V(\pi_i(g))$$

where $\phi_i \in [0,1]$. Because for $\bar{\sigma}$ satisfying $\pi_i(b)|_{\sigma=\bar{\sigma}} = \bar{\pi}$ we have $\lim_{\lambda,\xi\to 1} \xi \cdot \sigma \cdot \lambda = 1$, we conclude that the range of this payoff, as ϕ_i varies between 0 and 1, converges to the interval $(v_L, c_H]$ as λ, ξ go to 1. Since by Assumption 2 in any equilibrium the continuation value must be strictly below c_H, this establishes the existence of thresholds $\bar{\lambda}_\delta$ and $\bar{\xi}_\delta$ such that the high trade equilibrium exists whenever $\lambda > \bar{\lambda}_\delta$ and $\xi > \bar{\xi}_\delta$.

Medium trade. We show that there are at most two medium trade equilibria, i.e., that $Q_L^{med} = v_L$ has at most two solutions. In the medium trade equilibrium, the expected second period

payoff of the low type is:

$$\mathbb{E}_L \left\{ F_L \left(\pi_i, \phi_i \right) \right\} = \rho_L \left(b \right) \cdot v_L + \left(1 - \rho_L \left(b \right) \right) \cdot V \left(\pi_i \left(g \right) \right) = \widehat{\pi} \left(\sigma \right) \cdot v_H + \left(1 - \widehat{\pi} \left(\sigma \right) \right) \cdot v_L$$

where

$$\widehat{\pi} \left(\sigma \right) = \left(1 - \rho_L \left(b \right) \right) \pi_i \left(g \right) = \frac{1 - \rho_L \left(b \right)}{1 + \frac{1 - \rho_L(b)}{1 - \rho_H(b)} \cdot \frac{1 - \pi_\sigma}{\pi_\sigma}} = \left(\frac{1}{1 - \rho_L \left(b \right)} + \frac{1}{1 - \rho_H \left(b \right)} \cdot \frac{1 - \pi_\sigma}{\pi_\sigma} \right)^{-1}$$

Thus, it suffices to show that the equation $\widehat{\pi}' \left(\sigma \right) = 0$ has at most one solution for $\sigma \in [0, 1]$, as this will imply that Q_L^{med} intersects v_L at most twice. This is equivalent to showing that the derivative of the following expression:

$$\frac{1}{1 - \rho_L \left(b \right)} + \frac{1}{1 - \rho_H \left(b \right)} \cdot \frac{1 - \pi_\sigma}{\pi_\sigma} = \frac{1}{1 - \alpha_0 \cdot \sigma} + \frac{1 - \sigma}{1 - \alpha_1 \cdot \sigma} \cdot \frac{1 - \pi}{\pi}$$

is equal to zero for at most one value of σ where plugging for $\rho_L(b)$ and $\rho_L(g)$ we note that $1 > \alpha_0 > \alpha_1$. Differentiating and setting to zero, we get

$$\frac{1}{\left(1 - \alpha_0 \cdot \sigma \right)^2} = \frac{1 - \alpha_1}{\left(1 - \alpha_1 \cdot \sigma \right)^2} \cdot \frac{1 - \pi}{\pi}$$

which holds if and only if

$$\frac{1 - \alpha_1 \cdot \sigma}{1 - \alpha_0 \cdot \sigma} = \left(\left(1 - \alpha_1 \right) \cdot \frac{1 - \pi}{\pi} \right)^{\frac{1}{2}}$$

and, indeed, the LHS is monotonic in σ for $\sigma \in [0, 1]$. This establishes the existence of at most two medium trade equilibria.

For existence, note the following: the lowest (highest) value in the range of payoff $\mathbb{E}_L \{ F_L(\pi_i, \phi_i) \}$ in the low (high) trade equilibrium is lower (higher) than the lowest (highest) value in medium trade equilibrium. This implies two things. First, a medium trade equilibrium exists whenever the low and the high trade equilibria coexist. Second, using the argument that Q_L^{med} crosses v_L at most twice, we know that there is only one medium trade equilibrium when the low and the high trade equilibria coexist. From our previous arguments, the low and the high trade equilibria indeed coexist when $\delta > \overline{\delta}$, $\lambda > \overline{\lambda}_\delta$, and $\xi > \overline{\xi}_\delta$. ∎

Proof of Proposition 3. The convergence of the trading probability to $\bar{\sigma}$ that has $\pi_{\bar{\sigma}} = \bar{\pi}$ follows from right continuity of low type's continuation value in each of λ and ξ, at $\lambda = 1 - \pi$ and $\xi = 0$ respectively, and from Assumption 2 which guarantees in equilibrium the expected prices in the second period must be below c_H. Let us now consider the uniqueness argument.

(i) We start with uniqueness for small ξ. In the medium trade equilibrium, we have

$$\mathbb{E}_L \{F_L (\pi_i, \phi_i)\} = \xi \sigma \lambda \cdot v_L + (1 - \xi \sigma \lambda) \cdot V (\pi_i (g))$$

where σ is required to satisfy $\pi_i (b) < \overline{\pi} < \pi_i (g)$. Differentiation with respect to σ (in this open set) yields:

$$\frac{d\mathbb{E}_L \{F_L (\pi_i, \phi_i)\}}{d\sigma} = \underbrace{\xi \lambda \cdot (v_L - V (\pi_i (g)))}_{(-)} + (1 - \xi \sigma \lambda) \cdot \underbrace{\frac{dV (\pi_i (g))}{d\sigma}}_{(+)}$$

The first term can be made arbitrarily small by making ξ small, while the second term is bounded below by a positive number. Hence, we conclude that the payoff $\mathbb{E}_L \{F_L (\pi_i, \phi_i)\}$ is monotonically increasing in σ in the medium trade equilibrium when ξ is sufficiently small. But then we immediately have that the payoff $\mathbb{E}_L \{F_L (\pi_i, \phi_i)\}$ is lower in the low trade equilibrium than in the medium trade equilibrium, which in turn is greater than the payoff in the high trade equilibrium. Hence, we have uniqueness for small ξ.

(ii) We now show uniqueness for λ close to $1 - \pi$. Let $\overline{\sigma}$ be such that $\pi_{\overline{\sigma}} = \overline{\pi}$, and consider $\delta_0 \in (0, 1)$ defined by

$$v_L = (1 - \delta_0) c_L + \delta_0 [\xi \overline{\sigma} \pi \cdot v_L + (1 - \xi \overline{\sigma} \pi) \cdot c_H]$$

We show that a sufficient condition for the equilibrium to become unique for λ close to $1 - \pi$ is that $\delta \neq \delta_0$ (thus, we have our qualifier that the equilibrium is generically unique). As λ approaches $1 - \pi$, the posteriors in the second period converge as well: $\pi_i (b), \pi_i (g) \to \overline{\pi}$ where we used our earlier result $\pi_\sigma \to \overline{\pi}$. In particular, we have that

$$\lim_{\lambda \to 1 - \pi} Q_L = (1 - \delta) c_L + \delta [\xi \overline{\sigma} \pi \cdot v_L + (1 - \xi \overline{\sigma} \pi) \cdot c_H]$$

But if $\delta \neq \delta_0$, then for λ close enough to $1 - \pi$, a medium trade equilibrium cannot exist, because otherwise we cannot have $Q_L = v_L$ as required in any equilibrium.

We have ruled out the medium trade equilibrium. Now, we need to argue that the high and the low trade equilibria cannot coexist. For this, it suffices to show that the range of low type's expected payoff $\mathbb{E}_L \{F_L (\pi_i, \phi_i)\}$ in the two equilibria does not overlap. It can be shown that in the limit, as $\lambda \to 1 - \pi$, one of the following two equalities must be satisfied

$$v_L = (1 - \delta) c_L + \delta \left[\xi \overline{\sigma} \pi \cdot v_L + (1 - \xi \overline{\sigma} \pi) \cdot \left(\phi^{low} c_H + \left(1 - \phi^{low} \right) v_L \right) \right]$$

or

$$v_L = (1 - \delta) c_L + \delta \left[\xi \overline{\sigma} \pi \cdot \left(\phi^{high} c_H + \left(1 - \phi^{high} \right) v_L \right) + (1 - \xi \overline{\sigma} \pi) \cdot c_H \right]$$

for some $\phi^{low}, \phi^{high} \in [0, 1]$, where the RHS of the first equation is the limit of Q_L in the low trade equilibrium and the RHS of the second equation is the limit in the high trade equilibrium. But then, if $\delta > \delta_0$, then we have that

$$v_L < (1 - \delta) c_L + \delta \left[\xi \overline{\sigma} \pi \cdot v_L + (1 - \xi \overline{\sigma} \pi) \cdot c_H \right]$$

and, thus, the low trade equilibrium cannot exist. An analogous argument shows that if $\delta < \delta_0$, then a high trade equilibrium cannot exist. This establishes the uniqueness result. ∎

Proof of Proposition 4. The welfare of the high type in the low trade equilibrium is c_H, which is also the case with opaque markets. Also, note that in the medium and high trade equilibria, we have that the posterior following good news satisfies $\pi_i(g) > \overline{\pi}$, which implies that in those equilibria the high type's welfare is strictly above c_H. Thus, welfare in transparent markets is weakly greater than welfare in opaque markets, and strictly so if the high or medium trade equilibria are played.

Since we know that in high and medium trade equilibria, the welfare of the high type is higher than in the low trade equilibrium, we are left to rank the welfare in the medium and high trade equilibria. High type seller i's welfare is clearly increasing in own trading probability σ_i, so we just need to show that it is non-decreasing in σ_j. Since higher equilibrium σ means that both are higher, the result will follow. It is convenient to define $\rho_{obj}(b) = \xi \sigma_j (1 - \pi)$ as the objecctive probability of bad news arrival; higher values of σ_j corresponds to higher values of $\rho_{obj}(b)$. Notice that the high type seller i's equilibrium expected payoff in the second period is given by

$$\mathbb{E}_H \left\{ F_H (\pi_i, \phi_i) \right\} = \rho_H(b) \cdot c_H + (1 - \rho_H(b)) \cdot V(\pi_i(g))$$

with the requirement that

$$\pi_i(g) \leq \overline{\pi} \leq \pi_i(b)$$

First, note that:

$$\rho_H(b) \cdot V(\pi_i(b)) + (1 - \rho_H(b)) \cdot V(\pi_i(g)) = \underbrace{\rho_{obj}(b) \cdot V(\pi_i(b)) + (1 - \rho_{obj}(b)) \cdot V(\pi_i(g))}_{\text{unconditional expected value}}$$

$$+ \underbrace{\left(\rho_H(b) - \rho_{obj}(b) \right)}_{(-)} \cdot \underbrace{\left(V(\pi_i(b)) - V(\pi_i(g)) \right)}_{(-)}$$

is increasing in σ_j. To see this, note that by iterated expectations the unconditional expected

value is independent of the probability $\rho_{obj}(b)$; the second term, which is the difference in subjective and objective payoffs, is positive and is increasing in σ_j: (i) $V(\pi_i(g))$ is increasing in σ_j, while $V(\pi_i(b))$ is independent of σ_j, and (ii) $\rho_H(b) - \rho_{obj}(b) = \frac{\mathbb{P}(\theta_j=L|\theta_i=H)-\mathbb{P}(\theta_j=L)}{\mathbb{P}(\theta_j=L)} \cdot \rho_{obj}(b)$ is negative and decreasing in σ_j, since $\rho_{obj}(b)$ is increasing in σ_j.

Consider the high trade equilibrium, where recall that we have $c_H = V(\pi_i(b))$, which implies:

$$\mathbb{E}_H\{F_H(\pi_i, \phi_i)\} = \rho_H(b) \cdot V(\pi_i(b)) + (1 - \rho_H(b)) \cdot V(\pi_i(g))$$

First, because posterior $\pi_i(b)$ is independent of σ_j, the previous argument shows that the RHS declines as σ_j declines. In particular, it declines as we reduce σ_j to the value it takes in either the low or the medium trade equilibrium. Second, high type's welfare unambiguously decreases as σ_i decreases; in particular, it decreases as we take σ_i to the value it takes in the low or the medium trade equilibrium. This establishes the welfare ranking.

We now prove the comparative statics results. Since the low trade equilibrium has $\pi_i(g) = \overline{\pi}$, we have that $Q_H^{low} = c_H$ independently of ξ and λ, provided that the low trade equilibrium exists. We show that Q_H^{high} is increasing in λ and ξ. First, because an increase in ξ has the same effect as an increase in the trading probability of the other market, we already know that Q_H^{high} is increasing in ξ. Second, note that $\pi_i(b) = \overline{\pi}$ implies that the equilibrium trading intensity σ^{high} is increasing in λ. Thus, if we first increase σ to its new equilibrium value (holding λ fixed at its initial value), by our earlier argument Q_H^{high} must be higher. But note that now we have $\pi_i(b) > \overline{\pi}$; hence, this is not an equilibrium. Now, increasing λ until $\pi_i(b) = \overline{\pi}$ (as required by high trade equilibrium) also implies that Q_H^{high} is increasing since for all values of λ between the initial value and the value that sets $\pi_i(b) = \overline{\pi}$, we have:

$$\mathbb{E}_H\{F_H(\pi_i, \phi_i)\} = \rho_H(b) V(\pi_i(b)) + (1 - \rho_H(b)) V(\pi_i(g))$$
$$= \underbrace{\rho_{obj}(b) V(\pi_i(b)) + (1 - \rho_{obj}(b)) V(\pi_i(g))}_{\text{unconditional expected payoff}}$$
$$+ \underbrace{(\rho_H(b) - \rho_{obj}(b))}_{(-)} \underbrace{(V(\pi_i(b)) - V(\pi_i(g)))}_{(-)}$$

where again by iterated expectations the first term is independent of λ and the second term is increasing in λ: (i) $V(\pi_i(g))$ is increasing in λ, while $V(\pi_i(b))$ is decreasing in λ, and (ii) $\rho_H(b) - \rho_{obj}(b) = \frac{\mathbb{P}(\theta_j=L|\theta_i=H)-\mathbb{P}(\theta_j=L)}{\mathbb{P}(\theta_j=L)} \cdot \rho_{obj}(b)$ is negative and decreasing in λ.

We now show that Q_H^{med} is decreasing in ξ and λ. Consider the continuation values of the

low and the high types in the medium trade equilibrium:

$$Q_L^{med} = (1 - \delta) c_L + \delta [\rho_L (b) \cdot v_L + (1 - \rho_L (b)) \cdot V (\pi (g))]$$
$$= (1 - \delta) c_L + \delta [v_L + (1 - \rho_L (b)) \cdot (V (\pi (g)) - v_L)]$$

and

$$Q_H^{med} = (1 - \delta) c_H + \delta [\rho_H (b) \cdot c_H + (1 - \rho_H (b)) \cdot V (\pi (g))]$$
$$= c_H + \delta (1 - \rho_H (b)) \cdot (V (\pi (g)) - c_H)$$

where recall that the subjective distribution of news is given by: $\rho_L (b) = \xi \sigma \lambda$ and $\rho_H (b) = \rho_L (b) \cdot \frac{(1-\lambda)(1-\pi)}{\lambda \pi} < \rho_L (b)$. Two facts will be useful to keep in mind. First, recall that there is a unique medium trade equilibrium when the low and the high trade equilibria coexist (see proof of Theorem 1); but, importantly, it must be that Q_L^{med} is decreasing in σ for values close to the medium trade equilibrium. Second, it is straight-forward to show that Q_H^{med} is increasing in σ.

Thus, consider the following thought experiment. Suppose that, as ξ increases, σ decreases so that $\rho_H(b)$ (and thus also $\rho_L(b)$) remains fixed. This implies that $\pi (g)$ decreases, because it is given by

$$\pi (g) = \frac{\pi_\sigma \frac{1-\rho_H(b)}{1-\rho_L(b)}}{\pi_\sigma \frac{1-\rho_H(b)}{1-\rho_L(b)} + 1 - \pi_\sigma}$$

and π_σ is increasing in σ. Thus, we conclude that if $\rho_H(b)$ were to remain fixed, then Q_L^{med} and Q_H^{med} would be decreasing in ξ. Since Q_L^{med} is decreasing in σ, to keep $v_L = Q_L^{med}$ as required in equilibrium, σ must decrease by more than what is needed to keep $\rho_H(b)$ fixed. But this implies a further decline in Q_H^{med}. Hence, we conclude that Q_H^{med} is decreasing in ξ. The proof of the effect of correlation on Q_H^{med} is analogous. ∎

Proof of Proposition 5. This proof is analogous to that of Theorem 1.

Low trade. That there is at most one low trade equilibrium follows from the fact that the trading intensity σ in this category is fully pinned down by the requirement that $\pi_i(z_0) = \bar{\pi}$. Now, let $\bar{\sigma}$ be such that $\pi_i(z_0)|_{\sigma=\bar{\sigma}} = \bar{\pi}$, and note that then

$$\mathbb{E}_L\{F_L(\pi_i, \phi_i)\} = (1 - \rho_{i,L}(z_0)) \cdot v_L + \rho_{i,L}(z_0) \cdot [\phi_i c_H + (1 - \phi_i)v_L]$$

where $\phi_i \in [0, 1]$ and $\rho_{i,L}(z_0)$ is the belief that the low type in market i assigns to the event that no other market trades. Thus, the range of the expected payoff $\mathbb{E}_L\{F_L(\pi_i, \cdot)\}$ is the interval

$$[v_L, (1 - \rho_{i,L}(z_0)) \cdot v_L + \rho_{i,L}(z_0) \cdot c_H]$$

We thus immediately conclude that there exists a $\delta_{\lambda,N} < 1$ such that the low trade equilibrium exists iff $\delta \geq \delta_{\lambda,N}$. The uniform bound $\overline{\delta}_N$ follows from the fact that $\sup_\lambda \rho_{i,L}(z_0)|_{\sigma=\overline{\sigma}} < 1$.

<u>High trade.</u> That there is at most one high trade equilibrium follows because σ is pinned down by the requirement that $\pi_i(z_N) = \overline{\pi}$. Now, let $\overline{\sigma}$ be such that $\pi_i(z_N)|_{\sigma=\overline{\sigma}} = \overline{\pi}$, and note that then

$$\mathbb{E}_L\{F_L(\pi_i, \phi_i)\} = \rho_{i,L}(z_N) \cdot [\phi_i c_H + (1 - \phi_i)v_L] + \sum_{k=0}^{N-1} \rho_{i,L}(z_k) \cdot V(\pi_i(g))$$

where $\phi_i \in [0,1]$ and $\rho_{i,L}(z_k)$ is the belief that the low type seller i assigns to k trades by other markets being observed. Because $\lim_{\lambda \to 1} \rho_{i,L}(z_N) = 1$, the range of the expected payoff $\mathbb{E}_L\{F_L(\pi_i, \cdot)\}$, as ϕ_i varies between 0 and 1, converges to the interval $(v_L, c_H]$ as λ goes to 1. Since by Assumption 2 in any equilibrium the continuation value must be strictly below c_H, this establishes the existence of the threshold $\overline{\lambda}_{\delta,N}$ such that the high trade equilibrium exists whenever $\lambda > \overline{\lambda}_{\delta,N}$.

<u>Medium trade.</u> There can be many medium trade equilibria. In particular, since the range of function $\mathbb{E}_L\{F_L(\pi_i, \phi_i)\}$ lies between the ranges in the low and high trade equilibria, we know that at least one such equilibrium exists whenever the low and the high trade equilibria coexist, i.e., if $\delta > \overline{\delta}_N$ and $\lambda > \overline{\lambda}_{\delta,N}$.

Finally, the proof of welfare ranking across the three types of equilibria is analogous to that of Proposition 4. ∎

Proof of Proposition 6. It is useful to consider a 'fictitious' limiting economy in which buyers know the state of nature in the second period. In this setting, buyers' posteriors about the seller, conditional on state $\omega = \widehat{\omega}$ and trading probability σ, are given by

$$\pi_{\sigma,\widehat{\omega}} = \frac{\mathbb{P}(\omega = \widehat{\omega}|\theta_i = H) \cdot \pi_\sigma}{\mathbb{P}(\omega = \widehat{\omega}|\theta_i = H) \cdot \pi_\sigma + \mathbb{P}(\omega = \widehat{\omega}|\theta_i = L) \cdot (1 - \pi_\sigma)}$$

where as before π_σ denotes the interim belief and $\mathbb{P}(\omega = l|\theta_i = L) > \mathbb{P}(\omega = l|\theta_i = H)$. In equilibrium, the trading probability σ needs to satisfy

$$v_L \leq (1 - \delta) c_L + \delta \mathbb{E}_L \{F_L(\pi_{\sigma,\omega})\}$$

where

$$\mathbb{E}_L \{F_L(\pi_{\sigma,\omega})\} = \sum_{\widehat{\omega}=l,h} \mathbb{P}(\omega = \widehat{\omega}|\theta_i = L) \cdot [\phi(\widehat{\omega}) \cdot V(\pi_{\sigma,\widehat{\omega}}) + (1 - \phi(\widehat{\omega})) \cdot v_L]$$

with equality if $\sigma > 0$, and where

$$\phi(\widehat{\omega}) \begin{cases} = 1 & \text{if } \pi_{\sigma,\widehat{\omega}} > \overline{\pi} \\ \in [0,1] & \text{if } \pi_{\sigma,\widehat{\omega}} = \overline{\pi} \\ = 0 & \text{if } \pi_{\sigma,\widehat{\omega}} < \overline{\pi} \end{cases}$$

Note that Assumption 2 rules out the possibility of $\sigma = 1$, but we can still have that $\sigma = 0$.

Conditions 1 and 2 in Proposition 2 guarantee that the unique equilibrium in the limiting economy has $\sigma = 0$, since in that case we have:

$$v_L < (1-\delta) c_L + \delta \mathbb{E}_L \{F_L(\pi_{\sigma,\omega})\}|_{\sigma=0}$$

Now, let us go back to the actual economy where buyers do not know the state, but they observe news from finitely many N other markets. The equilibrium trading probability σ_N, when there are $N+1$ markets, is given by

$$v_L = (1-\delta) c_L + \delta \mathbb{E}_L \{F_L(\pi_{\sigma_N,z})\}$$
$$= \sum_{\widehat{\omega} \in \{l,h\}} \mathbb{P}(\omega = \widehat{\omega}|\theta_i = L) \cdot \sum_{z \in \Omega^N} \mathbb{P}(z_i = z|\omega = \widehat{\omega}) \cdot [\phi(z) \cdot V(\pi_{\sigma_N,z}) + (1-\phi(z)) \cdot v_L]$$

where

$$\phi(z) \begin{cases} = 1 & \text{if } \pi_{\sigma,z} > \overline{\pi} \\ \in [0,1] & \text{if } \pi_{\sigma,z} = \overline{\pi} \\ = 0 & \text{if } \pi_{\sigma,z} < \overline{\pi} \end{cases}$$

for all N. We will now show that in fact if information aggregates along an equilibrium sequence $\{\sigma_N\}_{N=1}^{\infty}$, then after large enough N we must have $v_L < (1-\delta) c_L + \delta \mathbb{E}_L \{F_L(\pi_{\sigma_N,z})\}$, which will yield a contradiction since no trade is inconsistent with an equilibrium for any finite N.

In what follows, we will use the following straight-forward implication of information aggregation. Given a sequence of trading probabilities $\{\sigma_N\}_{N=1}^{\infty}$ along which information about state ω aggregates, we also have convergence of posteriors: in state $\widehat{\omega}$, we have $\pi_{\sigma_N,z} \to^p \pi_{\sigma_N,\widehat{\omega}}$ as $N \to \infty$.

Non-Aggregation Result. We now prove our non-aggregation result. Assume that conditions (1) and (2) hold, and take a sequence $\{\sigma_N\}_{N=1}^{\infty}$ of equilibria along which information aggregates. Take $\epsilon > 0$, then using the above convergence result we know that there exists N^* such that

for $N > N^*$, we have the following set of inequalities:

$$\mathbb{E}_L \left\{ F_L \left(\pi_{\sigma_N, z} \right) \right\} =$$

$$= \sum_{\widehat{\omega} \in \{l, h\}} \mathbb{P} \left(\omega = \widehat{\omega} | \theta_i = L \right) \cdot \sum_{z \in \Omega^N} \left[\mathbb{P} \left(Z_i = z | \omega = \widehat{\omega} \right) \cdot \phi \left(z \right) \cdot V \left(\pi_{\sigma_N, z} \right) + \left(1 - \mathbb{P} \left(Z_i = z | \omega = \widehat{\omega} \right) \cdot \phi \left(z \right) \right) \cdot v_L \right]$$

$$> \sum_{\widehat{\omega} \in \{l, h\}} \mathbb{P} \left(\omega = \widehat{\omega} | \theta_i = L \right) \cdot \sum_{z \in \Omega^N} \left[\mathbb{P} \left(Z_i = z | \omega = \widehat{\omega} \right) \cdot \phi \left(z \right) \cdot V \left(\pi_{\sigma_N, \widehat{\omega}} \right) + \left(1 - \mathbb{P} \left(Z_i = z | \omega = \widehat{\omega} \right) \cdot \phi \left(z \right) \right) \cdot v_L \right] - \frac{\epsilon}{2}$$

$$\geq \lambda \cdot v_L + (1 - \lambda) \cdot V \left(1 - \frac{(1 - \lambda)(1 - \pi)}{\pi} \right) - \epsilon$$

where the first inequality follows from the fact that $\pi_{\sigma_N, z} \to^p \pi_{\sigma_N, \widehat{\omega}}$ implies that $V \left(\pi_{\sigma_N, z} \right) \to^p$ $V \left(\pi_{\sigma_N, \widehat{\omega}} \right)$, and the second inequality follows from the fact that $\pi_{\sigma_N, z} \to^p \pi_{\sigma_N, \widehat{\omega}}$ implies

$$\sum_{z \in \Omega^N} \mathbb{P} \left(Z_i = z | \omega = \widehat{\omega} \right) \cdot \phi \left(z \right) \to \begin{cases} 1 & \text{if } \pi_{\sigma_N, \widehat{\omega}} > \pi \\ 0 & \text{if } \pi_{\sigma_N, \widehat{\omega}} < \pi \end{cases}$$

combined with the fact that $\pi_{\sigma_N, h} > \overline{\pi}$ for any σ_N. Since ϵ is arbitrary, we have established that for N large enough

$$\mathbb{E}_L \left\{ F_L \left(\pi_{\sigma_N, z} \right) \right\} \geq \lambda \cdot v_L + (1 - \lambda) \cdot V \left(1 - \frac{(1 - \lambda)(1 - \pi)}{\pi} \right) - \epsilon > \frac{v_L - (1 - \delta) c_L}{\delta}$$

contradicting the requirement for equilibrium. Thus, we cannot have information aggregation under conditions 1 and 2.

Aggregation Result. We now establish our aggregation result. Suppose that condition (1) or (2) is violated. In that case, we immediately see that the equilibrium of the limiting economy must have $\sigma^* > 0$. We now find a sequence of equilibria $\{\sigma_N\}_{N=1}^{\infty}$ that is bounded below by a positive number. Along any such sequence, information clearly aggregates.

To this end, first consider a sequence $\{\widehat{\sigma}_N\}_{N=1}^{\infty}$, not necessarily an equilibrium one, such that $\widehat{\sigma}_N = \widehat{\sigma} \in (0, \sigma^*)$, i.e., this is a sequence of constant trading probabilities that are positive but below σ^*. Along such a sequence, information aggregation clearly holds. Thus, we have that if the state were $\widehat{\omega}$, then $\pi_{\widehat{\sigma}_N, z} \to^p \pi_{\widehat{\sigma}_N, \widehat{\omega}} < \pi_{\sigma^*, \widehat{\omega}}$ and there is N^* such that for $N > N^*$, we have:

$$\mathbb{E}_L \left\{ F_L \left(\pi_{\widehat{\sigma}_N, z} \right) \right\} < \mathbb{E}_L \left\{ F_L \left(\pi_{\sigma^*, \omega} \right) \right\} = \frac{v_L - (1 - \delta) \cdot c_L}{\delta}$$

Now, recall that the continuation payoff $\mathbb{E}_L \left\{ F_L \left(\pi_{\sigma, z} \right) \right\}$ is continuous in σ for all N, with a maximum value of $v_H > \mathbb{E}_L \left\{ F_L \left(\pi_{\sigma^*, \omega} \right) \right\}$. Hence, we know that for each $N > N^*$, there exists a σ_N such that both $\sigma_N \geq \widehat{\sigma} > 0$, and $\mathbb{E}_L \left\{ F_L \left(\pi_{\widehat{\sigma}_N, z} \right) \right\} = \frac{v_L - (1 - \delta) \cdot c_L}{\delta}$. This yields the desired sequence $\{\sigma_N\}_{N=1}^{\infty}$. ∎

Proof of Proposition 7 (Naive). If uninformed buyers are naive, they are unaware there is an additional informed buyer in the market and thus they bid as if they were in a symmetric environment. In the first period they bid v_L and believe the seller accepts the offer with probability $\bar{\sigma}$ such that $\pi_{\bar{\sigma}} = \bar{\pi}$. As characterized in Lemma 1, in the second period they mix between v_L and c_H where, in aggregate, the probability that c_H is chosen by at least two bidders is such that the low type seller's continuation value is v_L. The informed buyer uses the information contained in the news to determine his bid. When there is good news his expected value is strictly above c_H, in which case he bids his value. When there is bad news his expected value is below c_H, and thus he would at most bid v_L. Given that at this price he would never make any rents, we assume for simplicity he does not bid after observing bad news. Note that since there are many uninformed and just one informed, the price is always set by the uninformed and thus it coincides with the price in the symmetrically uninformed case. This directly implies that the welfare for the seller is the same as in the symmetrically uninformed case and, thus, that total welfare is also the same.

Given the bidding strategies, the informed buyer always wins the auction when he observes good news in the other market and he never wins when only he observes trade in the other market. But this implies the informed buyer is making rents since the fact he observed good news raises his posterior expected value above the price (which is either v_L or c_H). In turn, this implies that when the uninformed win the object it must have been that the informed has seen a trade in the other market. They are thus prey of a winner's curse. The true expected value of the asset is below their bid and thus they are making losses when the market is not fully transparent. Since total rents are constant and the sellers' welfare is unchanged, it immediately follows that any rents made by the informed are exactly offset by the losses of the uninformed. When markets become transparent, the rents of the informed become zero. Thus, in addition to the potential welfare gains for the seller (if the equilibrium switches to the medium trade or high trade one), we have *redistributive* effects of transparency from the informed to the naively uninformed buyers. ∎

Proof of Proposition 8 (Sophisticated). Consider first the fully opaque case, $\xi = 0$. The uninformed never get to see the news but since they are sophisticated they are aware that if they bid just based on their interim posterior, they would only win when the informed buyer observes bad news (as explained in the proof of Proposition 7). Being sophisticated, they avoid these loses by bidding as if they had actually observed bad news. This way they never overbid for the asset. This implies that they can only bid with positive probability c_H or prices weakly below v_L. The informed buyer, as in the Naive case, will bid $V(\pi(g))$ after good news and not bid (or bid v_L) after bad news. Since there is only one informed, prices will again be set by the uninformed. Next, given that the uninformed set the prices, the prices must be consistent with

the posterior belief $\pi_i(b)$ being realized w.p. 1. Thus our condition for equilibrium becomes:

$$v_L = (1 - \delta) c_L + \delta F_L(\pi_i(b), \phi_i)\}$$

Note that Assumptions 1 and 2 imply that the only possible solution for σ must result in $\pi_i(b) = \bar{\pi}$. This requires a higher σ than the one that we would have in equilibrium if buyers were symmetrically uninformed since $\pi_i(b) < \pi_{\sigma_i}$. This implies that compared to the symmetrically informed case, the high type sellers obtain the same payoff, c_H. The increased amount of trade in the first period does generate additional gains from trade. Thus the outcome is more efficient. These additional efficiency gains are all captured by the informed buyers. When we make the market fully transparent the informed buyers lose their rents. Also, if the low trade equilibrium prevails when $\xi = 1$, the outcome is actually Pareto worse than under opaqueness. This follows from noting that in the low trade equilibrium the sellers' welfare is also c_H as explained in Proposition 4. Thus, sellers are indifferent. The uninformed sophisticated buyers are also indifferent since their expected payoff is 0 in either equilibrium. ∎

Proposition 9 (Perfect Correlation) *The equilibria with perfect correlation are equivalent, in terms of welfare and trading probability σ, to the limits of equilibria with imperfect correlation as λ goes to 1.*

Proof. When correlation is perfect, we also need to specify buyers' off-equilibrium beliefs. Suppose that the equilibrium specifies that low type trades immediately at $t = 0$, but that only one of the sellers has traded. In this case, buyers can put any probability $\pi^{off} \in [0, 1]$ to the remaining seller being low type. Then the expected price that the low type seller receives upon rejection in the first period is as before given by $\mathbb{E}_L\{F_L(\pi_i, \phi_i)\}$, but if $\sigma = 1$ then $\pi_i(b) = \pi^{off}$. There are two sets of equilibria to consider depending on whether the low type plays a pure strategy of trading immediately or a mixed trading strategy. By the same reasoning as before, an equilibrium with no trade is not possible.

First, as with $\lambda < 1$, we can have the low type mix between trade at $t = 0$ and $t = 1$. In such equilibria, the low type must be indifferent whether to trade at $t = 0$ or $t = 1$. Importantly, notice that the payoffs $\mathbb{E}_\theta\{F_\theta(\pi_i, \phi_i)\}$ are left continuous at $\lambda = 1$. Hence, it follows that this equilibrium is the limit of the low and medium trade equilibria as λ goes to 1.

Second, in contrast to imperfect correlation, we can have an equilibrium in which the low type seller trades w.p.1 at $t = 1$. In that case, the low type receives a payoff v_L and the high type receives a payoff $(1 - \delta)c_H + \delta v_H$, and a sufficient condition for this equilibrium to exist is that with the most pessimistic off-equilibrium belief $\pi^{off} = 0$, we have

$$v_L \geq (1 - \delta)c_L + \delta \mathbb{E}_L\{F_L(\pi_i, \phi_i)\}|_{\sigma=1}$$

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Intuitively, if the low type expects the other low type to trade and reveal their common type, then there is no incentive to delay trade to $t = 2$. Now, despite being in pure strategies, these equilibria are the limits of the high trade equilibria with imperfect correlation. To see this, note that the latter requires that the belief following trade in the other market is:

$$\pi_i(b) = \frac{\rho_{i,H}(b) \cdot \pi_\sigma}{\rho_{i,H}(b) \cdot \pi_\sigma + \rho_{i,L}(b) \cdot (1 - \pi_\sigma)} = \frac{1}{1 + \frac{\mathbb{P}(\theta_j = L | \theta_i = L)}{\mathbb{P}(\theta_j = L | \theta_i = H)} \cdot \frac{1 - \pi_\sigma}{\pi_\sigma}} = \overline{\pi}$$

which implies that $\lim_{\lambda \to 1} \sigma = 1$. Thus, the low type's payoff is v_L and the high type's payoff converges to $(1 - \delta)c_H + \delta v_H$, which are exactly the payoffs in the equilibrium with immediate trade at $t = 1$ when correlation is perfect. ∎

Proposition 10 *Suppose that $\lambda = 1$, then for δ sufficiently high the following two equilibria coexist for any $N \geq 1$:*

- *High trade: an equilibrium in which low-type sellers trade at $t = 1$ w.p.1.*

- *Low trade: an equilibrium in which the posteriors are ordered by $\pi_i(z_k) < \pi_i(z_0) = \overline{\pi}$ for all $k > 0$.*

Moreover, information fails to aggregate along the low trade equilibria, while it aggregates along the high trade equilibria.

Proof. First, note that information aggregation holds in any equilibrium where low type seller trades immediately. This is an equilibrium since in that case we have:

$$\mathbb{E}_L \{F_L(\pi_i, \phi_i)\} = v_L < \frac{v_L - (1 - \delta) \cdot c_L}{\delta}$$

This establishes the existence of information aggregating equilibria.

We now show that there is an equilibrium sequence along which information does not aggregate. Let σ_N denote an equilibrium with $N + 1$ assets and note that $\rho_{i,L}(z_0) = (1 - \sigma_N)^N$ and $\rho_{i,H}(z_0) = 1$, which are the subjective beliefs of the low and the high type respectively about the likelihood of event z_0, i.e., that no trade occurs in any other asset, which we call the low trade equilibrium. Let us construct such an equilibrium, which requires that the trading intensity σ_N satisfy the following restriction:

$$\pi_i(z_0) = \frac{\pi_{\sigma_N} \cdot \rho_{i,H}(z_0)}{\pi_{\sigma_N} \cdot \rho_{i,H}(z_0) + (1 - \pi_{\sigma_N}) \cdot \rho_{i,L}(z_0)} = \frac{1}{1 + \frac{\rho_{i,L}(z_0)}{\rho_{i,H}(z_0)} \frac{(1 - \pi)(1 - \sigma_N)}{\pi}} = \overline{\pi}$$

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which implies that

$$(1 - \sigma_N)^N = \rho_{i,L}(z_0) = \frac{1}{1 - \sigma_N} \cdot \left(\frac{\pi}{1 - \pi} \frac{1 - \overline{\pi}}{\overline{\pi}} \right)$$

First, notice that a sequence $\{\sigma_N\}_{N=1}^{\infty}$ (not necessarily an equilibrium one) satisfying the above equality always exists. Second, note that if these trading probabilities also constitute an equilibrium, then we must have $\lim_{N \to \infty} \sigma_N = 0$ and $\lim_{N \to \infty} \rho_{i,L}(z_0) = \left(\frac{\pi}{1 - \pi} \frac{1 - \overline{\pi}}{\overline{\pi}} \right) > 0$. Hence, because $\rho_{i,L}(z_0)$ is uniformly bounded below by a positive number, if δ is large enough, the low trade equilibrium must exist independently of N. The failure of information aggregation along such a sequence of equilibria can be seen from the fact that buyers' posterior about the state has a non-degenerate distribution in the limit:

$$\pi^{state}(z_0) = \frac{\pi}{\pi + (1 - \sigma_N)^N (1 - \pi)} \to \overline{\pi} \in (0, 1)$$

and, as shown above, this event occurs with positive probability in the limit. ∎