FINANCE RESEARCH SEMINAR
SUPPORTED BY UNIGESTION

"How Effectively Can Debt Covenants Alleviate Financial Agency Problems?"

Prof. Andrea GAMBA
Warwick University, Warwick Business School

Abstract

We examine the effectiveness of debt covenants in alleviating financial agency problems. Distortions in both investment and financing policies with long-term debt are captured in a structural dynamic model where both policies are endogenously determined by shareholders. The combined and compounding effect of these distortions is shown to be large. We impose covenants that restrict the level of debt, or control the use of proceeds from asset sales or debt issuance, and analyze how, and how much, they mitigate financial agency costs. We investigate the direct and indirect impact of covenants on financing and investment policies, including at the point where covenants are violated, providing alternative interpretations of recent empirical evidence. We conclude that the presence and enforcement of debt covenants significantly alters dynamic financing and investment policies, not only at the point of covenant violations, and thus should be an important element of structural models.

Friday, November 14, 2014, 10:30-12:00
Room 126, Extranef building at the University of Lausanne

How Effectively Can Debt Covenants Alleviate Financial Agency Problems?

Andrea Gamba   Alexander J. Triantis[*]

February 10, 2014

Abstract

We examine the effectiveness of debt covenants in alleviating financial agency problems. Distortions in both investment and financing policies with long–term debt are captured in a structural dynamic model where both policies are endogenously determined by shareholders. The combined and compounding effect of these distortions is shown to be large. We impose covenants that restrict the level of debt, or control the use of proceeds from asset sales or debt issuance, and analyze how, and how much, they mitigate financial agency costs. We investigate the direct and indirect impact of covenants on financing and investment policies, including at the point where covenants are violated, providing alternative interpretations of recent empirical evidence. We conclude that the presence and enforcement of debt covenants significantly alters dynamic financing and investment policies, not only at the point of covenant violations, and thus should be an important element of structural models.

Introduction

It has long been argued in the corporate finance literature that the use of debt financing introduces debt–equity agency conflicts in the form of distortions to corporate investment and financing policies. The evolution from static to dynamic corporate finance models has expanded our perspectives on the nature and magnitude of these distortions, and the extent to which different debt contract designs can mitigate these conflicts. These have included a focus on seniority, maturity, and call and reset features.[1] However, the role of debt covenants in resolving financial agency problems has not yet been carefully explored in a dynamic setting, despite much recent interest in the empirical literature on the role and impact of covenants. In this paper, we introduce different debt covenants into a dynamic structural model of the firm to analyze how, and how well, they mitigate investment and financing distortions.

Given the prevalence of debt covenants, particularly in private debt contracts, they are generally viewed as value enhancing design features. The underlying concept is that they allow a state-contingent transfer of control from shareholders to bondholders which can mitigate financial agency problems. Smith and Warner (1979) provide the first detailed evidence on the types of covenants used, and tied these covenants to agency problems they were designed to mitigate. More recently, Bradley and Roberts (2004) and Billett, King, and Mauer (2007) provide additional empirical evidence on the types of covenants embedded into private and public debt contracts, respectively. Bradley and Roberts (2004) find that covenants have a direct effect on decreasing yields. Billett, King, and Mauer (2007) explore the relationship of different covenants to the leverage ratio, debt maturity structure, and the prevalence of growth opportunities.

Another strand of empirical literature on covenants focuses specifically on the states where covenants are violated. Beneish and Press (1993), Beneish and Press (1995), Dichev and Skinner (2002), and Sweeney (1994) document that covenants are written with triggers that are quite tight relative to the firm's financial condition at the time of debt issuance, and as a result the frequencies of covenant violations are quite high. They also explore firms' accounting choices as they come close to triggering covenant

---

[1]Hackbarth and Mauer (2012) explore optimal debt priority in a dynamic setting. Rajan and Winton (1995) discuss the role of short-term debt, but also the impact of long-term loans subject to covenants. Bhanot and Mello (2006) explore shareholder incentives when debt includes rating triggers that affect debt contract structure as a firm's credit risk increases.

violations. More recently, Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009) examine firms' investment and financing behavior once a financial covenant has been violated, finding a decrease in both investment rates and debt levels, and concluding that lenders use the opportunity of a technical default trigger to pressure firms to alter their policies and reduce credit risk.

We examine the investment and financing policies that shareholders would follow absent any covenant restrictions, and the resulting magnitude of financial agency costs. While several recent papers have focused on measuring agency costs using dynamic models, their models have imposed either a fixed level of debt (Childs, Mauer, and Ott (2005), Mello and Parsons (1992), Moyen (2007)), a specified maturity schedule for debt (Leland (1998), Parrino and Weisbach (1999)), or restrictions on the issuance of new debt (Titman and Tsyplakov (2007)). While a flexible financing policy could potentially help to mitigate investment distortions, we show how shareholders will instead game the financing policy, taking advantage of legacy debt holders. This financing distortion further exacerbates underinvestment problems, and the resulting compounding effect leads to significant value loss, far greater in magnitude than documented in the existing literature.

Using this baseline, we measure how effective debt covenants restrictions can be in moving shareholders' investment and financing policies closer towards firm value maximizing policies, and the extent to which agency costs can thus be mitigated. Within the range of covenants used in practice, we focus our attention on three representative restrictions that highlight some key and prevailing characteristics of real life covenants. The first is an asset sweep covenant that requires shareholders to use proceeds from asset sales to pay down debt, and thus discourages asset sales designed merely to fund shareholder payouts.[2] The second is a debt sweep covenant that specifies that proceeds from new debt issuance be used to pay down existing debt, and thus targets opportunistic leverage increases that expropriate current bondholders' wealth.[3] The third is a financial accounting covenant which is violated if the firm's Debt/EBITDA ratio exceeds a spec-

---

[2]Sweep provisions are highlighted by Bradley and Roberts (2004). Debt encumbered with an asset sweep covenant is similar to secured debt. Morellec (2001) explores the joint effects of asset liquidity and pledging assets as collateral on the debt capacity of a firm.

[3]It is also common to find covenants that restrict dividend payments. We do not model cash retention in our model, so the drivers of excessive dividend payments in our context would be either liquidated capital or new debt issuance, and the two sweep covenants address these potential agency problems directly.

ified threshold. Shareholders can remedy this technical default by making investment and/or financing decisions that will limit the firm's Debt/EBITDA ratio under similar profitability scenarios.

We find that these covenants are effective in different ways, and to varying degrees, in reducing investment and financing distortions and mitigating the value loss associated with financial agency problems. Due to the compounding effects of investment and financing distortions, the covenants that reduce the likelihood of new debt issues or increase the incentive to reduce debt are also quite effective in mitigating investment distortions despite not targeting investment directly. Similarly, the covenant that addresses asset liquidations also indirectly mitigates financing distortions. Furthermore, we show that covenants alter investment and financing policies across many states, and not simply at the points where the covenants are binding or violated. These policy modifications result in significant value creation, particularly when the propensity to increase leverage is controlled, and in low states of profitability due to either macroeconomic or firm-specific factors.

Our state–contingent framework is particularly well–suited to examine the impact of a debt covenant violation on investment and financing policies, and to provide insights on the recent empirical literature on this topic. A key advantage of our model is that we can conduct parallel simulations where debt is, or is not, subject to a financial covenant. We can thus examine in a controlled manner the investment and financing policies at the specific states where the covenant would get violated if it existed, and see whether the existence of the covenant indeed makes a difference.

From this controlled experiment, we find that while the investment rate drops following a covenant violation, once other factors related to these low states are controlled for in a multivariate regression, the violation itself actually results in higher investment. We put these results in the context of conflicting empirical conclusions in Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009). In contrast, the empirically documented drop in debt levels at the time of a financial covenant violation appears to be an unambiguous consequence of the violation and its remedy. Our simulations also produce a significant drop in net payout once the covenant is violated.

Our model is closest in structure to Brennan and Schwartz (1984) and Titman and Tsyplakov (2007).[4] In their path-breaking article, Brennan and Schwartz (1984) laid the foundations for models that incorporate both dynamic investment and dynamic financing. Their model also includes a financial accounting covenant. However, the main purpose of their paper is to show how a contingent-claims framework can be applied to value a firm allowing for dynamic corporate policies. Their results regarding the effect of the financial covenant are thus very limited.

More recently, Titman and Tsyplakov (2007) present a fully dynamic model of endogenous investment and leverage decisions that incorporates the effects of taxes, financial distress costs, and adjustment costs for both financing and investment. While our model shares many similar structural characteristics with theirs, they do not explicitly incorporate debt covenants. They do impose a debt restructuring condition similar to that in Fischer, Heinkel, and Zechner (1989) which requires all debt to be repurchased and new debt to be issued when the firm changes its debt level, even when it reduces debt. Our debt sweep covenant shares some commonality with this condition, but is less restrictive, and its effects are explicitly explored.

In a simpler dynamic setting that leads to closed form solutions, Leland (1994) examines the effect of a positive net-worth covenant on the propensity of shareholders to increase the riskiness of assets, and on the firm's capital structure. Consistent with Leland (1994) and the related dynamic corporate finance literature, we do not seek to derive optimal security designs, but rather to examine the quantitative effect of common debt features on dynamic investment and financing strategies and the resulting firm value. A different strain of the corporate finance literature studies the optimal allocation of control, and derives the design of covenants endogenously (within the class of debt contracts). For example, Garleanu and Zwiebel (2009) examine entrepreneur-lender conflicts under asymmetric information, while Mao (2011) models managerial-shareholder conflicts and derives debt covenants that address the moral hazard problem.[5]

---

[4]Other papers such as Childs, Mauer, and Ott (2005)) examine dynamic investment in the presence of debt, but allow only for a single growth option that can be exercised once rather than allowing for switching between various capital levels, and they do not incorporate dynamic debt policies.

[5]Earlier related papers on debt covenants include Berlin and Mester (1992), Rajan and Winton (1995), and Sridhar and Magee (1997). In an interesting departure to the conventional literature on debt covenants, Murfin (2012) examines the issue of covenant strictness from the supply side rather than from the firm's perspective. He finds evidence that defaults in a bank's loan portfolio tends to

The next section presents a simple example in which we provide the economic intuition of some of the results of the quantitative model. Section 2 details the quantitative model. Section 3 provides results regarding the measurement of agency costs, the impact of covenants on investment and financing policies in general and at covenant violation points, and the overall effectiveness of covenants in reducing agency costs. Section 4 concludes the paper.

1 A Simple Example

To illustrate our main results, we present a simple example featuring some key assumptions of the general model. Assume there are three dates, $t = 0, 1, 2$, and the state variable is the firm's productivity, θ, whose dynamics are described in the binomial lattice in Figure 1, with a constant conditional probability of $\frac{1}{2}$ for upward jumps. The firm's cash flow (EBITDA) is θk, where k is the capital stock, with constant return to scale. For simplicity, we assume no taxes, no depreciation, a zero discount rate, and that debt is *pari passu*. Debt matures at $t = 2$ and is paid off with the current cash flow only (i.e., the company does not save its cash flows from prior periods and there is no salvage value of capital).

At $t = 0$ the firm has capital $k = 1$ and debt $b = 2.5$, as per a prior decision, and the shareholders can decide to either keep the same debt ($b' = 2.5$) or increase it to $b' = 4$ with immediate effect. In the latter case, they receive the market value of the new debt net of a fixed adjustment cost of 0.1.

At $t = 1$, contingent on the outcome for θ, the firm can adjust the capital stock, assuming it can be either doubled ($k' = 2$) or halved ($k' = 0.5$).[6] If the equity holders decide to increase the production capacity, they pay 2.6 to buy a unit of new capital, and to sell a previously purchased unit of capital they get only 0.4 for it (and hence 0.2 for a half–unit of capital sold). The new capital, k', affects the cash flow at $t = 2$ only.

---

lead to increased strictness of covenants on subsequent portfolio loans, likely due to learning about the lender's screening ability.

[6]We exclude $k' = 0$ on the assumption that this extreme asset stripping is prohibited.

1.1 Investment distortion: asset stripping

We first analyze the investment policy assuming the firm cannot change debt ($b = b' = 2.5$). It is easy to show (and thus excluded for brevity) that the policy that maximizes the total firm value is to double capital ($k' = 2$) if $\theta_1 = 3$ and to keep it at $k' = 1$ if $\theta_1 = 1$. The resulting value of the firm at $t = 0$ (explained below) is

$$
\begin{aligned}
V = {} & 2 \cdot 1 \\
& + \frac{1}{2}\left(3 \cdot 1 - 2.6 + \frac{1}{2}(4 \cdot 2) + \frac{1}{2}(2 \cdot 2) \right) \\
& + \frac{1}{2}\left(1 \cdot 1 + \frac{1}{2}(2 \cdot 1) + \frac{1}{2}(0 \cdot 1) \right) \\
= {} & 4 + \frac{1}{2}\left(-2.6 + \frac{1}{2}(4 \cdot 2) + \frac{1}{2}(2 \cdot 2) \right) + \frac{1}{2}\left(\frac{1}{2}(2 \cdot 1) + \frac{1}{2}(0 \cdot 1) \right) = 6.2,
\end{aligned}
$$

where the first line is cash flow at $t = 0$ ($\theta_0 = 2$ and $k = 1$), the second line is the value in the upper branch at $t = 1$, when the capital stock is optimally increased to $k' = 2$, and the third line is the value in the lower branch at $t = 1$, when the capital is optimally kept at $k' = 1$. In the fourth line, we have collected 4, the value of the cash flows at $t = 0$ and $t = 1$, which does not depend on firm's decisions. So, these \$4 will be later on included without explanation.

Differently from the firm value maximizing policy, the optimal policy from the equity's perspective is to double capital if $\theta_1 = 3$ and to halve it (*asset stripping*) if $\theta_1 = 1$, with an equity value of

$$
\begin{aligned}
E = {} & 4 \\
& + \frac{1}{2}\left(-2.6 + \frac{1}{2}\max\{4 \cdot 2 - 2.5, 0\} + \frac{1}{2}\max\{2 \cdot 2 - 2.5, 0\} \right) \\
& + \frac{1}{2}\left(0.2 + \frac{1}{2}\max\{2 \cdot 0.5 - 2.5, 0\} + \frac{1}{2}\max\{0 \cdot 0.5 - 2.5, 0\} \right) \\
= {} & 4.55,
\end{aligned}
$$

where the third line is the value to equity in the lower branch at $t = 1$, when the capital is optimally halved to $k' = 0.5$. Notably, the final payoff to equity in $t = 3$ is positive

only if $k' = 2$ in $t = 1$. All other choices for k' are dominated by this one and thus not presented.

To conclude, when $b = b' = 2.5$, equity holders deviate from the first best investment policy and take advantage of existing unprotected debt holders by cashing out part of the assets of the firm.

1.2 Financing distortion: claim dilution

We now analyze the debt policy, by forcing $k = k' = 1$ (i.e., the firm cannot change the capital level). As per our assumptions, we will only look at whether b' should continue to be equal to 2.5, or increase to 4. Since the debt does not mature until $t = 2$, the cash flows at $t = 0$ and $t = 1$ are identical to the ones seen before, with a combined value of $4.

As for the firm value maximizing policy, if $b' = 2.5$, the firm value is equal to $4 plus the probability weighted average cash flow at $t = 3$ given $k = 1$:

$$V = 4 + \frac{1}{4}(4) + \frac{1}{2}(2) + \frac{1}{4}(0) = 6.$$

If $b' = 4$, the total firm value is $5.9 = 6 - 0.1$, where 0.1 is the cost of issuing debt at $t = 0$. So, the first best decision is to keep $b = b' = 2.5$.

Considering now the equity value optimization case, for $b' = 2.5$, the value of equity cash flow is

$$E = 4 + \frac{1}{4}\max\{4 - 2.5, 0\} + \frac{1}{2}\max\{2 - 2.5, 0\} + \frac{1}{4}\max\{0 - 2.5, 0\} = 4.375.$$

If $b' = 4$ is selected instead, the market value of new debt issued with a $1.5 face value is

$$D = \frac{1}{4}(1.5) + \frac{1}{2}(0.75) + \frac{1}{4}(0) = 0.75,$$

where 1.5 is the payoff to debt when the firm is solvent at $t = 2$, in the upper state; 0.75 is the part of $2 of cash flow paid to new debt holders in proportion of their fraction $(37.5\% = (4 - 2.5)/4)$ of total debt given the pari passu assumption, in the intermediate

8

state when the firm is in default; and 0 is the payoff in the lower default state, in which there is no cash flow. Therefore, the value of equity in this case is

$$E = 4 - 0.1 + 0.75 + \frac{1}{4}\max\{4 - 4, 0\} + \frac{1}{2}\max\{2 - 4, 0\} + \frac{1}{4}\max\{0 - 4, 0\} = 4.65,$$

in which, at $t = 2$, there is the operating cash flow minus what shareholders pay both old and new bondholders. The value is higher than what the shareholders get by keeping b constant, since the equity holders are extracting value from the existing debt holders.[7] So, the optimal solution is $b' = 4$, and the equity holders distort debt policy by increasing debt to extract value from existing bondholders.

1.3 Investment distortion exacerbated by dynamic financing: underinvestment

Removing the constraint of the previous case, if the debt is increased at $t = 0$, this will affect the investment policy at $t = 1$, and the new bondholders anticipate the equity holders' selection of k' when buying the new debt (i.e., the effect is incorporated in the debt price).

From the previous analysis, the firm value maximizing solution is to leave b unchanged, since there is no advantage to higher debt (and there is a cost to issuing debt). Under $b' = 2.5$, the first–best policy is to double the capital ($k' = 2$) if $\theta_1 = 3$, while leaving $k' = 1$ if $\theta_1 = 1$.

However, shareholders will want to increase the debt to $b' = 4$ to expropriate wealth from the existing bondholders. If they do so, the shareholders will select $k' = 1$ (under-investment) rather than $k' = 2$ if $\theta_1 = 3$. The resulting solution is better for shareholders than keeping $b' = 2.5$, in which case they would double the capital if $\theta_1 = 3$. Sharehold-

---

[7]To see this, the original debt holders receive 1.625 ($= \frac{1}{4}(2.5) + \frac{1}{2}(2) + \frac{1}{4}(0)$) if $b' = 2.5$, but only get 1.25 ($= \frac{1}{4}(2.5) + \frac{1}{2}(2 \cdot 2.5/4) + \frac{1}{4}(0)$) if $b' = 4$. The equity holders get this difference, i.e. .375, minus the cost of issuing debt, 0.1, or a net value of .275 as additional value.

ers will sell assets ($k' = 0.5$) if $\theta_1 = 1$ for either level of debt. Therefore, the equity value is

$$
\begin{aligned}
E = 4 - 0.1 + 0.6563 + \frac{1}{2} & \left(\frac{1}{2} \max\{4 \cdot 1 - 4, 0\} + \frac{1}{2} \max\{2 \cdot 1 - 4, 0\} \right) \\
& + \frac{1}{2} \left(0.2 + \frac{1}{2} \max\{2 \cdot 0.5 - 4, 0\} + \frac{1}{2} \max\{0 \cdot 0.5 - 4, 0\} \right) = 4.6563,
\end{aligned}
$$

where the market value of the \$1.5 of additional debt is

$$
D = 0.6563 = \frac{1}{2} \left(\frac{1}{2}(1.5) + \frac{1}{2}(0.75) \right) + \frac{1}{2} \left(\frac{1}{2}(0.375) + \frac{1}{2}(0) \right).
$$

In the equation above, 0.375 is the prorated cash flow that is paid to the new bondholders (37.5% of $2 \cdot 0.5$) if the firm defaults.

1.4 Covenants mitigate investment and financing distortions

In this paper we consider three classes of covenants: asset sweeps, debt sweeps and a maximum Debt/EBITDA covenant. In what follows, we will show how these types of covenants can mitigate, both directly and indirectly, the shareholder–bondholder conflicts illustrated above in our simple example.

As for asset sweeps, in the case where b and k can both change, an asset sweep covenant will require that if $k' = 0.5$ is chosen at $t = 1$, the .2 gained from the asset sale will be used to pay down the debt. The equity holders therefore do not receive this amount, and the value to equity holders is 4.4563 rather than 4.6563, and this solution is dominated by $b' = 4$ and $k' = 1$ in both scenarios for θ_1, which delivers an equity value of $E = 4.65$, as shown before. While in this case the covenant eliminates the incentives to strip assets, it does not solve the underinvestment problem completely since the equity holders will still choose $k' = 1$ rather than $k' = 2$ when $\theta_1 = 3$. It also does not solve the claim dilution problem since the equity holders will still choose $b' = 4$ and then will select $k' = 1$.

A debt sweep covenant requires any new debt at $t = 1$ to be used to pay down the existing debt. So, the equity holders cannot extract value from the existing debt holders. Since this means that $b' = 2.5$ will be chosen at $t = 0$ instead of $b' = 4$, then

10

$k' = 2$ will be chosen rather than $k' = 1$ if $\theta_1 = 3$, thus mitigating not only the claim dilution problem, but also underinvestment. However, this covenant is ineffective against the asset stripping issue, because with $b' = 2.5$, the shareholders will optimally choose $k' = 0.5$ if $\theta_1 = 1$. The proof of this statement is in Appendix A.

Finally, we discuss the Debt/EBITDA covenant, whereby $b'/(\theta k)$ must be lower than 4 (where θk is the EBITDA at that same time in which b' is chosen). This covenant prevents the claim dilution and the underinvestment problem (but not the cashing out problem of $k' = 0.5$), and importantly this works even prior to the covenant being triggered.

In the current setup, for either choice of b', the covenant is not violated at $t = 0$,[8] or at $t = 1$, when $\theta_1 = 3$. However, the covenant condition is triggered if $b' = 4$, when $\theta_1 = 1$, as Debt/EBITDA $= 4/1$. If the penalty from this violation is severe enough, the equity holders will not choose $b' = 4$ at $t = 0$, getting a value of equity of 4.55.

Here, for simplicity, we discuss two possible penalties where the creditor exerts its control rights upon a covenant violation in order to protect itself. This type of creditor influence on investment and financing policy is documented by Chava and Roberts (2008), Nini, Smith, and Sufi (2009), Nini, Smith, and Sufi (2012), and Roberts and Sufi (2009). In the first, the shareholders are forced to buy back debt at par if the Debt/EBITDA covenant is violated. Specifically, the firm has \$1 in cash flow when $\theta_1 = 1$, and they will be buying back debt for \$1 par value, while its current market value (conditional on $\theta_1 = 1$) is

$$.25 = \frac{1}{2}\left(\frac{1}{4} \cdot 2\right) + \frac{1}{2}(0).$$

where the first term corresponds to $\theta_2 = 2$, with 1/4 meaning one dollar out of the four dollars of debt, and the second term is the payoff at $\theta_2 = 0$. This loss of .75 (even when multiplied by the probability of .5 of being at $\theta_1 = 1$) is much more than the gain they are getting from the claim dilution and disinvestment (4.6563 when $b' = 4$ and $k' = 1$ is chosen in the upper branch versus 4.55 when $b' = 2.5$ and $k' = 1$ is chosen).[9]

---

[8]One could pick Debt/EBITDA < 2, which would restrict $b' = 4$ being chosen at $t = 0$, solving both the claim dilution and underinvestment problem. However, our point is that a similar effect can be obtained also through a looser covenant.

[9]It can be shown that by reducing the amount of debt that the shareholders are forced to repurchased, the loss from the covenant violation can be lower than the gain from increasing the debt. However, our

In the second type of penalty, shareholders are forced to increase investment, whereby if $\theta_1 = 1$ and the covenant is violated, equity holders will be forced to select $k' = 2$ rather than $k' = 1$. The debt holders want to have more cash flow to pay off their debt, at least in the up node ($\theta_2 = 2$). Equityholders will have to pay \$2.6 to buy the extra unit of capital, and will get absolutely no benefit from this (the equity cash flow at $\theta_2 = 2$ is $2 \cdot 2 - 4 = 0$). This effective loss (multiplied by the $\frac{1}{2}$ probability of being at $\theta_1 = 1$) is \$1.3, which again is much greater than the benefit to equity holders of getting 4.6563 in place of 4.55.

To summarize, this simple example illustrates two important points. First, well know investment distortions due to debt–equity agency conflicts, like asset stripping and underinvestment, can be more severe in the presence of dynamic financing with unprotected debt. Second, simple covenants designed to target one specific agency distortion are indirectly effective in reducing other agency distortions, across states and also before the points in time in which these covenants are violated. The remainder of the paper will generalize these results, providing a quantitative analysis of the effects of debt–equity agency conflicts and the ability of covenants to mitigate them.

2 The Model

2.1 Economic and Financial Setting

We model investment and financing decisions in an infinite-horizon discrete-time dynamic and stochastic framework. The control variables are the book value of assets in place, k, and the face value of outstanding debt, b.

We denote macroeconomic risk by x, and assume that it is an autoregressive process[10]

$$x' = \rho_x x + \sigma_x \varepsilon_x', \quad \varepsilon_x' \sim \mathcal{N}(0,1), \quad |\rho_x| < 1 \tag{1}$$

point is not to discuss the optimality of covenants, but to show that if a sufficient penalty exists, the effects of the covenants can be also intertemporal.

[10]We describe the evolution of a system at the steady–state. Hence, only the current period, t, and the next one, $t + 1$ are relevant. We use a prime to denote the value of a variable in period $t + 1$, the same variable without a prime denoting its value in period t.

where ρ_x is the autocorrelation, σ_x is the conditional standard deviation, and the terms ε_x are serially i.i.d..

We model a finite set of heterogeneous firms in this economy, each driven by an independent company–specific risk, denoted with z_j, and assume it also follows an autoregressive process

$$z'_j = \rho_z z_j + \sigma_z \varepsilon'_j, \quad \varepsilon'_j \sim \mathcal{N}(0,1), \quad |\rho_z| < 1 \tag{2}$$

where the parameters have similar meanings as above, and ε_j are serially i.i.d.. The two contemporaneous random terms ε_x and ε_j are independent, and the random terms ε_j are cross–sectionally independent across the firms. In the rest of this section we will describe the behavior of an individual firm, so we will drop the index j, for convenience.

The EBITDA (operating cash flow before taxes) is

$$\pi(x, z, k) = e^{x+z} k^{\alpha} - \psi, \tag{3}$$

where $k > 0$ is the book value of assets, $\psi > 0$ is a fixed cost that includes all expenses, and $\alpha < 1$ models decreasing returns to scale of the asset.

We assume that capital is homogeneous and that it depreciates both economically and for accounting purposes at a constant rate $\delta > 0$. The company's debt consists of risky consol bonds of equal priority with face value $b \geq 0$ and a coupon rate equal to the risk free rate r.[11] We assume that the firm can not hold a cash balance in order to simplify the model.[12]

The Earnings Before Taxes (EBT) is equal to EBITDA minus depreciation and interest:

$$y = y(x, z, k, b) = \pi - \delta k - rb. \tag{4}$$

---

[11]Several structural corporate finance models explicitly model debt maturity using an exogenously specified debt repayment rate (e.g., Leland (1998), Dangl and Zechner (2004), and Titman and Tsyplakov (2007)). As we discuss below, debt is dynamically adjusted in our model, and thus while debt has infinite maturity on an ex-ante basis, the effective ex-post maturity is endogenously determined and state-dependent, reflecting financing and investment decisions, as well as the impact of any covenants.

[12]Gamba and Triantis (2008) and Riddick and Whited (2009) address the decision to raise external capital and simultaneously hold a cash balance.

We model corporate taxes on earnings net of personal taxes. The net corporate tax function is $g(y) = \tau y$, where τ is the marginal net tax rate on earnings.

At any date, given the current state (x, z, k, b), the firm can decide to invest or disinvest to get the capital stock for next period, k'.[13] If there is positive investment, then the capital expenditure is $k' - (1 - \delta)k$ and investment can be financed either by using current cash flows or by issuing debt or equity. On the contrary, if the firm decides to disinvest, the cash inflow is $\ell((1 - \delta)k - k')$, with $\ell \leq 1$ denoting a liquidation price.

The firm may also decide at any time to increase or reduce its debt to a new level b' for the next period. Any variation of debt from b to b' entails a direct cost, $q(b', b) = \eta|b' - b|$, where $\eta \geq 0$. Additional debt is issued at market value, and old and new debt have equal seniority. We later explore the possibility that covenants impose restrictions on new debt issuance. We assume that the debt level can be reduced by repurchasing a portion of the debt at its par value, rather than its market value. Mao and Tserlukevich (2012) show that in the absence of frictions, lenders will sell their debt back to the firm only at par value. Even with frictions, debt will be repurchased at a premium to market value, though the size of the premium depends on the nature of the frictions. Mao and Tserlukevich (2012) also detail institutional considerations, including securities and tax regulations, that make it more likely that debt is repurchased at or close to par value.

For notational convenience, where appropriate, we will denote the states $\theta = (x, z)$, as specific combinations of the macroeconomic and firm-specific states. At any date, given specific capital (k) and debt (b) levels, the state θ and the resulting cash flow will be observed, and new levels of capital (k') and debt (b') are chosen. While the state is observable to shareholders (and, as will be discussed below, to debtholders to price their claims), we assume that the debt contract is incomplete in that a state-contingent contract that imposes specific investment and financing decisions in all states is infeasible. When we later introduce covenants, we will assume that specific events are contractible.

Investment and financing decisions can be made with no restriction as long as the firm is not in financial distress or in a state of default (or as described later, restricted by, or in violation of, a covenant). We assume that financial distress takes place when

---

[13]We assume that investment instantaneously increases the productive capacity of the firm. In contrast, Tsyplakov (2008) examines the effect of the time-to-build characteristic of capital in certain industries, and shows that this can affect dynamic capital structure choices.

the after-tax operating cash flow is lower than the coupon payment, $rb > \pi - g(y)$. In this case, the firm sells the minimum amount of capital, at a discount s, to make the promised payment to debt holders. In contrast, as we shall soon define, default occurs at states which shareholders select to maximize the value of their equity given the option created by limited liability.

Based on the assumptions above, we can determine the residual cash flow to shareholders in state (θ, k, b) when the firm is solvent, given a decision (k', b'). For notational convenience, we define the function $\chi(\xi, \ell)$ as $\chi(\xi, \ell) = \xi$ if the capital stock change $\xi = k' - k \geq 0$, and $\chi(\xi, \ell) = \xi\ell$ if $\xi < 0$, where $\ell \leq 1$ is the liquidation price of capital. $D(\theta, k', b')$ is the market price of newly issued debt, assuming the firm is solvent, based on the new book value of assets, k', and the new book value of debt, b'. The residual cash flow to shareholders, where $\mathcal{I}(\cdot)$ is the indicator function for the event $b' < b$, is

$$
\begin{aligned}
\widetilde{e}(\theta, k, b, k', b') = {} & \max\left\{\pi - g(y) - rb, 0\right\} \\
& + \mathcal{I}(b')\left(b' - b\right) + \left(1 - \mathcal{I}(b')\right)\frac{D(\theta, k', b')}{b'}\left(b' - b\right) - q(b', b) \\
& - \chi\left(k' - k(1 - \delta) + \max\left\{\frac{rb + g(y) - \pi}{s}, 0\right\}, \ell\right). \quad (5)
\end{aligned}
$$

The first line of (5) is the after-tax operating profit; the second line is the cash flow from altering the debt level (first term is for a decrease in debt, and the second term is for an increase); and the last line is the cash flow due to investment or disinvestment, where the latter may be subject to a liquidation and/or distress sale discount.

If the optimal residual cash flow for shareholders is negative, then funds are raised by issuing new equity, and a proportional flotation cost λ $(0 < \lambda < 1)$ is incurred. Thus, when the firm is solvent, the net cash flow to equity holders is

$$
e(\theta, k, b, k', b') = \widetilde{e} - \lambda \max\left\{-\widetilde{e}, 0\right\} \quad (6)
$$

2.2 Optimization

We now set up the valuation of the firm (and its corporate securities) under different optimization conditions. In Section 2.2.1, we model the cases where the investment and

financing decisions are made to maximize either equity value or total firm value.[14] In Section 2.2.2, we introduce debt covenant restrictions into the second–best optimization problem.

2.2.1   Equity and Firm Value Maximization

Following Berk, Green, and Naik (1999), Zhang (2005), and others, we assume that the stochastic discount factor depends on macroeconomic conditions captured by x. We use the convenient functional form introduced by Jones and Tuzel (2012), where the one–period discount factor given the transition to state x' from the current state x is

$$M(\theta, \theta') = \beta e^{-\gamma(x)\varepsilon'_x - \frac{1}{2}(\gamma(x))^2 \sigma_x^2}, \tag{7}$$

where the state–dependent coefficient of risk–aversion is defined as $\gamma(x) = \exp(\gamma_0 + \gamma_1 x)$, with $0 < \beta < 1$, $\gamma_0 > 0$ and $\gamma_1 < 0$.[15]

We first examine equity value maximization. $E(\theta, k, b)$, the value of equity at state (θ, k, b), is the fixed point of the Bellman operator

$$E(\theta, k, b) = \max \left\{ \max_{(k', b')} \left\{ e(\theta, k, b, k', b') + \mathbb{E}_\theta \left[M(\theta, \theta') E(\theta', k', b') \right] \right\}, 0 \right\}, \tag{8}$$

where the expectation is conditional on the current state θ.

The optimal policy at (θ, k, b) is

$$(k^*, b^*) = \arg \max_{(k', b')} \left\{ e(\theta, k, b, k', b') + \mathbb{E}_\theta \left[M(\theta, \theta') E(\theta', k', b') \right] \right\},$$

as long as the resulting equity value from this policy is not negative. Otherwise, the shareholders default on servicing debt and surrender the firm to debt holders. When

---

[14]Managerial agency problems would add an interesting layer of complexity to the problem we explore. Covenants could conceivably either mitigate or exacerbate managerial agency problems, depending on the nature of those problems and the other mechanisms, such as compensation design, that are in place to resolve the conflict between managers and shareholders. We leave these interesting issues to further research given the complexities they would introduce into our framework.

[15]With this functional form of the stochastic discount factor, the return of a risk–free one–period zero coupon bond, $1/\mathbb{E}_t[M_{t+1}] = 1/\beta$, is independent of the state. This is an important feature when modelling a firm that can issue long term debt, whose par value is state dependent.

default occurs, a bankruptcy cost is incurred, which is a proportion ζ ($0 \leq \zeta < 1$) of the debt holder payoff $b(1 + r)$.

We define $\Delta = \Delta(\theta, k, b)$ to be the indicator function of the default event in the current state. When default occurs, the former bondholders become the new shareholders, taking over an unlevered firm with depreciated assets, $k(1 - \delta)$, so that the policy is $(k(1 - \delta), 0)$. Hence, the optimal policy for shareholders at (θ, k, b) is

$$\varphi(\theta, k, b) = (k^*, b^*) \cdot (1 - \Delta) + (k(1 - \delta), 0) \cdot \Delta. \tag{9}$$

To determine the current (ex-coupon) market value of debt if the firm is solvent, $D(\theta, k, b)$, we first define $d(\theta', k, b, \varphi)$ as the value to debt holders at (θ', k, b) and the firm's decision, $(k', b') = \varphi(\theta', k, b)$, is made:

$$d(\theta', k, b, \varphi) = \Delta \cdot (1 - \zeta)b(1 + r) +$$
$$(1 - \Delta) \cdot \left(rb(1 - \tau_b) + \mathcal{I}(b') \left(D(\theta', k', b') + b - b' \right) + (1 - \mathcal{I}(b')) \frac{D(\theta', k', b')}{b'} b \right). \tag{10}$$

In the event of default, $\Delta(\theta', k, b) = 1$, the debt equals its par value (including coupon) minus a bankruptcy cost which is a percentage, ζ, of debt value. If there is no default, $\Delta(\theta', k, b) = 0$, the debt value is equal to the coupon, plus: if the debt level is reduced ($\mathcal{I}(\cdot)$ is the indicator function for the event $b' < b$), the new debt value plus the par value for the reduced debt ($b - b'$); if the debt level is unaltered (or increased), the market (or prorated market) value of debt. Hence,

$$D(\theta, k, b) = \mathbb{E}_\theta \left[M(\theta, \theta') d(\theta', k, b, \varphi) \right]. \tag{11}$$

We now turn to the firm value maximization problem, where investment and financing policies are selected in each state to maximize total firm value rather than equity value.

The value of the firm, denoted by V, is derived from the following Bellman equation when the firm is solvent:

$$V(\theta, k, b) = \max_{(k', b')} \left\{ e(\theta, k, b, k', b') + \mathbb{E}_\theta \left[M(\theta, \theta') E(\theta', k', b') \right] \right.$$

$$\left. + rb + \mathcal{I}(b') \left(D(\theta', k', b') + b - b' \right) + (1 - \mathcal{I}(b')) \frac{D(\theta', k', b')}{b'} b \right\}. \quad (12)$$

The value of the equity given the maximand (k^*, b^*) from (12) is

$$E(\theta, k, b) = e(\theta, k, b, k^*, b^*) + \mathbb{E}_\theta \left[M(\theta, \theta') E(\theta', k^*, b^*) \right]. \quad (13)$$

If E is strictly positive, then (k^*, b^*) is the optimal solution. Otherwise, the firm is in default: the optimal decision is $(k(1 - \delta), 0)$, and the value of equity is set to zero (and the default indicator, Δ, is set equal to one).

To summarize, from the solution of equation (8) we have the optimal policy, φ^S, and the optimal endogenous default policy Δ^S, for the shareholder value maximization case. From the solution of equation (12), we have derived the optimal policy for the firm value maximization case, φ^F, and the optimal default policy, Δ^F. The specific optimal values (k^*, b^*) of the two policies can be different, given the different goals of the two programs. Accordingly, the value of debt is always found by applying equation (11) using either the optimal policy φ^F (with default Δ^S) or φ^S (with Δ^F).

2.2.2 Equity Value Maximization Subject to Debt Covenants

We now analyze the impact of covenants on investment and financing policies and the resulting firm value. While debt contracts are incomplete, we assume that particular events are verifiable and thus contractible. Covenants that are tied to these events allow creditors to gain control even in the absence of a missed debt payment, following the paradigm of Aghion and Bolton (1992). Motivated by the empirical evidence (e.g., Bradley and Roberts (2004), Billett, King, and Mauer (2007), Chava and Roberts (2008), Roberts and Sufi (2009)), we select three different type of representative restrictions, analyzing each on its own: i) an asset sweep covenant; ii) a debt sweep covenant; and

iii) a financial accounting covenant. For each case, we solve a constrained shareholder value optimization problem.[16]

When an asset sweep covenant is imposed, shareholder value is maximized subject to the restriction that if shareholders voluntarily disinvest, at least a portion $0 < \nu \leq 1$ of the sales proceeds must be used to pay down existing debt. If the voluntary portion of any asset sales, which is the total reduction in capital after netting out depreciation and any asset sales required to cover interest payments and operating losses, is negative, i.e.,

$$k' - k(1 - \delta) + \max \left\{ \frac{rb + g(y) - \pi}{s}, 0 \right\} < 0$$

then the following condition is imposed on the selection of any new debt level $b' > 0$:

$$b' - b \leq \nu \ell \left(k' - k(1 - \delta) + \max \left\{ \frac{rb + g(y) - \pi}{s}, 0 \right\} \right).$$

If all debt is repaid, i.e., $b' = 0$, the remaining proceeds from the asset sales can be distributed to shareholders.

The set of feasible decisions is denoted $\mathcal{A}^a(\theta, k, b)$, and the optimal policy if the firm is solvent is

$$(k^*, b^*) = \arg \max_{(k', b') \in \mathcal{A}^a(\theta, k, b)} \left\{ e(\theta, k, b, k', b') + \mathbb{E}_\theta \left[M(\theta, \theta') E(\theta', k', b') \right] \right\}.$$

We denote the optimal policy as $\varphi^{S,a}$ and the related default indicator as $\Delta^{S,a}$, which in general will be different from φ^S and Δ^S.

Bradley and Roberts (2004) document that asset sweeps are very common in private debt contracts, particularly in the latter years of their sample (e.g., 93.8% of the loans in their 2001 sample had asset sweeps). By forcing proceeds from assets to be used to pay down debt, shareholders will be prevented from liquidating assets in order to receive large dividends, which may be particularly enticing to shareholders as the firm approaches default. However, the asset sweep covenant may also lead shareholders to refrain from divesting assets in certain cases where such sales would have increased total firm value. While in practice negotiations between shareholders and bondholders might

---

[16]While other types can be selected from the classes of covenants found in reality, in our model they would be isomorphic to the ones considered here. This is for instance the case of a covenant restricting dividend payments, as previously discussed.

enable this additional value to be captured, we do not introduce this complexity into our model. Thus, the asset sweep modeled here may understate the potential increase in value that could be achieved in practice through imposing such a covenant, but allowing for renegotiation.

Debt sweeps are similar to asset sweeps, but with a portion of the proceeds from new debt issuance, rather than asset sales, going towards paying back existing debt. For simplicity, and for consistency with similar restrictions imposed in the capital structure literature (e.g., Fischer, Heinkel, and Zechner (1989)), we will assume that the full proceeds from new debt issuance must be applied to retiring existing debt. As a result, in order to increase the debt level, the firm must buy back all debt (at par) and reissue new debt.[17] The direct cost associated with this transaction will thus be $q(b', b) = \eta(b' + b)$. As with the asset sweep case, there will be a restricted set of feasible decisions denoted by $\mathcal{A}^d(\theta, k, b)$ which constrains the second–best optimization, and thus results in a distinct optimal policy $\varphi^{S,d}$ and default, $\Delta^{S,d}$.

Bradley and Roberts (2004) and Billett, King, and Mauer (2007) report that covenants based on financial statement metrics are extremely common in private and public debt contracts, respectively. Some covenants are based solely on balance sheet values (e.g., a minimum net worth) or income statement items (e.g., minimum interest coverage ratios). We impose a covenant that combines both balance sheet and income statement metrics, namely a maximum Debt/EBITDA ratio. Chava and Roberts (2008) document that the maximum Debt/EBITDA covenant is the most prevalent covenant in their sample of private loans, even more common than minimum net worth and interest coverage covenants. We have also found it to be a good proxy for alternative covenants that impose state-contingent restrictions on financial policies.

More specifically, we assume that there are two relevant thresholds, f^* and f^{**}, associated with our financial covenant. For positive EBITDA ($\pi(\theta, k) > 0$), if $b/\pi(\theta, k) > f^*$, i.e., if the productivity realization θ is such that the Debt/EBITDA covenant is violated for the current capital (k) and debt (b) levels, technical default occurs. Shareholders must then select new capital (k') and debt (b') levels subject to the restriction that the Debt/EBITDA ratio does not exceed f^{**} if next period's productivity, $\theta = (x, z)$, is the same as the current period's. This rule proxies for the type of outcome one would

---

[17]This is why our base case excludes that the firm repurchases all the debt when changing debt.

expect from a negotiation between the shareholders and lenders where the shareholders would agree to follow policies that reduce the credit risk in exchange for some leniency from the lenders.[18] If EBITDA is zero or negative ($\pi(\theta, k) \leq 0$), technical default is also triggered, and the only remedy is to pay off all the debt (i.e., set $b' = 0$).[19]

The rationale for using two thresholds, and for setting $f^{**} \geq f^*$ as we will, is twofold. First, the empirical literature on covenants indicates that covenants are set relatively tightly, i.e., a low f^* value in our setting, serving as tripwires when credit risk proxies increase beyond a certain level. As a result, the covenant is often repeatedly violated without the firm necessarily coming close to breaching insolvency or default barriers (Chen and Wei (1993) and Nini, Smith, and Sufi (2012)). The tripwire provides lenders with a warning signal that results in increased scrutiny of the firm's policies and prevents shareholders from taking actions that would further destroy lenders' wealth, such as increasing debt or selling off capital, which would result in a more egregious violation of the covenant (as captured by the higher f^{**} value). If productivity shocks are negative enough such that $b/\pi(\theta, k) > f^{**}$, then shareholders must be proactive to use the free cash flow and raise new equity in order to pay off debt and/or invest so that Debt/EBITDA is reduced under the current productivity state. The second rationale for the two threshold design is that it does a much better job than a single threshold level (i.e., $f^* = f^{**}$) in providing reasonable estimates of both covenant violation and default frequencies.

As with the asset and debt sweep covenants, there will be a restricted set of feasible decisions for the financial covenant case, denoted by $\mathcal{A}^f(\theta, k, b)$, which constrains the second–best optimization, and results in a distinct optimal policy $\varphi^{S,f}$ and default, $\Delta^{S,f,}$. Note that while the asset and debt sweeps explicitly restrict investment and financing decisions in all states (except when $b' = 0$), the financial covenant does not restrict these

---

[18]We also allow for the possibility that the firm can simply repurchase all of its existing debt, and then issue new debt (subject to the same financial covenant), if this alternative maximizes equity value.

[19]In practice, the lender may demand accelerated repayment of the loan upon technical default by the firm. As documented by Chen and Wei (1993) and Nini, Smith, and Sufi (2009), creditors will often waive this right, though in some cases they do demand immediate payment. While we assume in our model that shareholders decide how to respond to technical default, Chen and Wei (1993) models the lender's optimization problem, showing that if the firm is closer to bankruptcy, has higher leverage, and has unsecured debt, then the lender is less likely to waive its right to accelerate the loan. A more complete solution which is beyond the scope of our paper would involve modeling the renegotiation between lenders and shareholders.

decisions unless the firm first violates the covenant, triggering a reduction in the feasible set of policies the firm may follow.

In all three covenant cases described above, given $\varphi^{S,a}$, $\varphi^{S,d}$, and $\varphi^{S,f}$ (and the related default policies), the value of debt is found by applying equation (11) using the respective optimal policy and default indicator.

Finally, we also analyze the special case where the firm is restricted from assuming any debt ($b = 0$ always). We denote the optimal policy for this zero-debt case as φ^{Z}.

Table 1 provides a reference guide for the different optimization cases described above. Each is solved using the numerical solution approach detailed in Appendix B. In addition to obtaining the stationary policy functions (φ) that specify the investment, financing and default decisions for each (θ, k, b) state, we can also use the resulting state transition functions in order to generate a panel of simulated data. The simulation procedure we use is detailed in Appendix C. These results are presented and analyzed below.

3 Results

In this section, we explore the effects of the three covenants on investment and financing policies, and on the value of the firm. The results are based on the constrained and unconstrained optimizations and simulations described above, and detailed in the Appendices. We first present and justify the parameters used to generate our results. We then focus on how covenants mitigate agency problems by examining investment and financing policies across our different cases. We next present the resulting value effects of the covenants. Finally, we analyze the investment and financing policies around covenant violation states, tying our results to related empirical evidence.

3.1 Parameter Values

Despite the relative richness of our model, we recognize the limitations in trying to match our structural model to empirical data. The purpose of our model is to focus on the effectiveness of debt covenants and the mechanisms by which agency problems are

reduced, rather than to provide a more accurate descriptive model of corporate leverage and investment. However, it is still useful to calibrate our model to empirical data to ensure that it is reasonably representative of a typical firm.

The base case parameters for our analysis are shown in Table 2. Some are set directly to match commonly used empirical estimates, while others are based on calibrating the model, such that simulated moments of key firm level metrics approximate empirical moments. A comparison of simulated and empirical moments will be provided after briefly describing each parameter value.

The values of $\sigma_x = 1.36\%$, $\rho_x = 0.9924$, $\gamma_0 = 3.22$, and $\gamma_1 = -15.30$ are from Jones and Tuzel (2012). The parameters for the process of the firm-specific risk (z) are set to $\sigma_z = 15.80\%$ and $\rho_z = 0.6857$ based on model calibration. They directly impact metrics such as EBITDA/Asset and leverage, and indirectly affect default rates and credit spreads. The production return-to-scale parameter $\alpha = .50$ is set to the average of values used by Hennessy and Whited (2005) and Zhang (2005). The fixed production cost of $\psi = 1.03$ is set to calibrate the Q–ratio, and to a lesser extent credit spreads (no consistent fixed cost value is used in the literature, largely due to differences in model features). We use an annual depreciation rate of $\delta = .11$, similar to the 1% monthly rate in Livdan, Sapriza, and Zhang (2009) and Schmid (2008), and the 3% quarterly rate in Kuehn and Schmid (2011). The depreciation rate is a first order determinant of the investment rate. The marginal corporate tax rate (ignoring personal taxes) (τ) is set to 10%. This estimate is somewhat lower than the 13.2% rate estimated by Graham (2000), reflecting that this rate is also used for negative income, which would be subject to limited carryforwards and carrybacks in practice.

Partial irreversibility of capital investment is difficult to measure, but we assume for calibration purposes that assets are sold at a fraction $\ell = .75$ of their depreciated book value. However, when the firm is in distress, it only receives the fire-sale price of $s = .60$ of book value. The additional loss is close to the 14% fire-sale discount estimated in Pulvino (1998).[20] The cost associated with a bankruptcy restructuring (ζ) is assumed to be 60% of the debt obligation, as frequently used in credit risk models (e.g., see Guo, Jarrow, and Lin (2008)).

---

[20]It is also at the mid-point of the 5-25% distress cost range assumed by Strebulaev (2007).

The equity issuance cost ($\lambda = 6\%$) is consistent with direct cost estimates in Altinkilic and Hansen (2000) as well as indirect estimates drawn from structural models (Hennessy and Whited (2005) and Hennessy and Whited (2007)). The debt issuance cost of $\eta = 1\%$ is commonly used in this literature. We use a risk-free discounting of $\beta = 1/1.05$, which implies that $r = 5\%$, the coupon rate of long term bonds.

Finally, we specify the thresholds of the Debt/EBITDA covenant to be $f^* = 2.6$ and $f^{**} = 3.6$. Nini, Smith, and Sufi (2009) report an average value of 4 for the Debt/EBITDA maximum specified in such covenants in their sample, which is close to our assumption for f^{**}. Furthermore, as shown below, the violation trigger at Debt/EBITDA $= 2.6$ allows us to calibrate our violation frequency in line with recent evidence.

Simulated values of common firm level metrics from our model are compared to corresponding empirical values in Table 3. As described in more detail in Appendix C, simulation results are based on 50 economies (macroeconomic risk paths) with 200 firms (firm specific paths) each, over a 100 year horizon, for a total of one million data points. The first five columns of Table 3 show sample means based on the five different optimization cases described earlier.[21] All cases are based on the same simulation paths of θ for consistency. Since firms manage financial agency costs to varying degrees, it would be wrong to argue that any of the five cases on its own should correspond directly to the empirical data. Rather, we seek a calibration where sample means across the cases are close to, or bracket, empirical estimates. For some metrics, the sample means are highly sensitive to the way in which financial agency problems have (or have not) been mitigated.

The firm's profitability as a fraction of capital, as measured by EBITDA/Assets, is very consistent across the five cases, driven by the underlying characteristics of the firm's productivity and the stochastic state variables. The simulated mean of 22% matches the midpoint between the 15% reported in Hennessy and Whited (2005) and the 29%

---

[21]For all the metrics except the default and violation frequencies, the means are calculated by finding the cross-sectional mean for a particular economy at a particular point in time, then taking the time-series average of these cross-sectional means in the economy, and then averaging these means across the 50 economies. To calculate the default rate, we first measure the number of defaulted firms as a percentage of the number of non-defaulted firms in the previous period, for each point in time in a particular economy. We then take the time-series mean of this rate, and then average across the 50 economies. The covenant violation frequency is calculated in the same way.

in Gomes (2001). The investment rate $((k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\})/k)$, has greater variation across the cases, ranging from 12% for firm value maximization and 21% for the shareholder value optimization, and this variation will be explored in greater detail in subsequent results. The debt sweep and financial covenant cases have mean investment rates that match closely to the 14.5% empirical rate reported in Gomes (2001) and the 15% rate in Zhang (2005). The Q–ratio (equal to firm value divided by capital, V/k) is between 1.95 and 2.12 across the five cases, consistent with values around 2.1 reported in Whited and Wu (2006).

With regards to financing measures, the average quasi-market leverage ratios vary widely across the five cases. As we shall analyze more carefully below, in the firm value optimization case, the firm takes full advantage of the flexibility to adjust the debt level by maintaining high leverage levels in good or average profitability states and then paring down debt in bad states to limit distress and bankruptcy. In contrast, the high potential agency costs in the shareholder value optimization case lead shareholders to select much lower levels of leverage, even though they don't fully internalize the higher cost of debt when seeking additional leverage in the presence of existing debt. With the asset and debt sweep covenants, agency problems are partially resolved and leverage lies somewhere between the firm and equity value optimization cases. While agency problems are also mitigated with the financial covenant, the debt level is limited by the covenant to a large extent, leading to a much lower leverage. The leverage ratio of 18% in this case is somewhat near the empirical estimate of 23% in Korteweg (2010) based on net debt, which is appropriate in our setting since there is no cash balance.

While empirical default rates and credit yield spreads vary depending on bond type and time period analyzed (e.g., see Kuehn and Schmid (2011)), we use benchmarks of a 1% yearly default rate and a 100 basis point yield spread, roughly in line with BBB rated publicly traded corporate bonds. Not surprisingly, the firm value maximization case has a much lower default rate and credit spread. In contrast, debt is much riskier in the unconstrained equity value maximization case, reflecting the effects of agency problems. The covenants mitigate these problems, resulting in a reduction in credit risk, consistent with the empirical findings of Chava, Kumar, and Warga (2010), Goyal (2005) and Reisel (2010) for public debt issues, and Bradley and Roberts (2004) for private debt. This is particularly true in the debt sweep and financial covenant cases which restrict debt policy more directly.

Equity distribution is measured as a percentage of asset, and is positive if there is a payout to shareholders, and negative when equity is issued. The sample means vary widely across our five cases, and are generally above the empirical mean of 4% reported in Hennessy and Whited (2005). The one exception is the case of firm value optimization where there is significantly less equity in the capital structure, and thus distributions to shareholders as a percentage of capital will be lower. Since there is no cash balance in our model, any cash flow earned from operations that is not used to service or pay back debt, or to increase capital, must be paid out to shareholders. It is thus not surprising that the payout yield in our model deviates from the empirical norm. We will later gain more insight into the net equity distribution by examining equity distributions and equity issuances separately.

Finally, Table 3 shows the frequency of covenant violation for the case where the Debt/EBITDA financial covenant is imposed.[22] Nini, Smith, and Sufi (2009) report that the annual incidence of financial covenant violations hovered between 10-20% over the duration of their sample. In the last five years of their sample (2004-2008), the incidence is around 12-13%, in line with the 13.91% annual frequency of violation in our calibrated model. Chava and Roberts (2008) report similar frequencies of 14% for their net worth sample and 15% for their current ratio sample.[23]

3.2   The Impact of Covenants on Corporate Policies

We begin by analyzing the investment policy across the five different optimization cases that include debt financing. Table 4 shows the optimal investment decision, $(k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\})$, at various points in the state space, examining different values of z for a high x (good economy) and low x (bad economy). In each case, we show investment when the current capital k is equal to 9.4 (close to the median asset value for most optimization cases), and at two levels of debt: $b = 7.6$ (the median debt level under the debt sweep case), and $b = 2.8$ (the median debt level under

---

[22]In the case of the other two covenants, there is no violation. The sweep covenants constrain the joint investment and financing opportunity set in each state (by requiring that a portion of the proceeds of asset sales and new debt capital be allocated towards paying back debt), while the Debt/EBITDA covenant imposes constraints only if the covenant is first violated (i.e., Debt/EBITDA $> f^*$)

[23]We have analyzed alternative combinations of f^* and f^{**} values. These affect both firm value and covenant violation frequency in non-monotonic ways. We have selected the specification that corresponds most closely to empirical violation rates, rather than the one that optimizes value.

the shareholder value maximization case). One can first see that in all five optimization cases, the investment rate increases with higher state realizations, as expected. However, investment policies differ significantly across the five cases. Specifically, shareholders underinvest relative to the firm value maximizing case, as would be expected. Furthermore, when not constrained by covenants, they sell off capital in the lowest profitability states, despite the liquidity discount incurred in doing so, and distribute the resulting cash flow, rather than attempting to replace some of the depreciating capital that would benefit debt holders to a greater extent. These effects are more pronounced in a bad economy and when leverage is higher, highlighting that a recession magnifies the negative effects of debt overhang.

Turning to the cases where covenants are imposed, we first note that for low firm-specific states, all three covenants eliminate the propensity of shareholders to sell off capital. While this is to be expected from the asset sweep covenant, which is designed precisely to limit such sales, the debt sweep and financial covenant are also effective in this regard by limiting excessive leverage in these states and thus the tendency for investment distortions to be magnified. All three covenants also reduce the tendency for shareholders to underinvest in higher firm-specific states. The debt sweep covenant is more effective in doing so when the debt level is low, while the asset sweep covenant mitigates underinvestment more effectively at higher debt levels. These differences reflect the concurrent debt changes made at high states in these two different cases, as will be seen shortly.

The resulting effects of the incremental investment decisions at all states can be seen by examining the distribution of capital (k) which emerges from our simulations, represented by the key distribution metrics shown in Table 5. The moments of the distribution are shown for the six optimization cases, including the unlevered firm case. The debt sweep and financial covenant are effective in increasing capital levels relative to the pure shareholder value case, bringing them closer to those in the first-best case. The resulting capital levels are close to those in the unlevered case, where there are no agency costs, but yet shareholders can not benefit from the lower costs of issuing debt relative to equity, nor from the incremental interest tax shield benefit associated with investment. In contrast, the asset sweep does not result in overall higher levels of capital. Shareholders are less inclined to invest in this case knowing that they may be limited from selling off some of their capital in the future.

27

Table 6 shows the optimal debt change, $b' - b$, at various points in the state space, examining different values of the firm–specific risk factor, z, for a high x (good economy) and low x (bad economy). As for the optimal investment policy in Table 4, we show the debt change when $k = 9.4$, and for high debt ($b = 7.6$) and low debt ($b = 2.8$). The debt policies in the firm value and equity value maximization cases stand in stark contrast to each other. In the first–best case, shareholders use debt financing to support additional investment as profitability improves. In the second–best case, shareholders add a very modest amount of debt under high profitability scenarios because they bear the agency costs of the new debt. However, when profitability is low, shareholders have a much greater incentive to issue new debt in order to transfer wealth away from existing debt holders, who are not compensated for the elevated credit risk due to the higher leverage. As with the investment distortions due to debt overhang, the financing distortions are more pronounced in the bad economy.

The three covenants are able to mitigate the financing distortion problems to different extents. The debt sweep is consistently able to remove the propensity to issue new debt in order to expropriate existing lender's wealth, by forcing shareholders to buy back existing debt before it issues new debt. The financial covenant also mitigates the new debt issuance problem, and in fact leads shareholders to reduce debt in low profitability states. We show later that debt reduction is also optimal in the first–best case under low profitability scenarios, though this happens at higher debt levels than are shown in Table 6, given that the firm chooses higher leverage when there are no agency problems. Finally, while the asset sweep covenant is not focused on addressing financing distortions, it does mitigate the debt issuance distortion as well. Since shareholders are restricted from selling off assets to directly fund payouts to themselves, debt holders are better protected and thus shareholders will have less incentive to issue new debt to expropriate wealth from existing debt holders.

The resulting effects of the optimal debt change decisions that are selected at states in the simulated paths can be seen by examining the distribution of the book value of debt (b) in the different optimization cases, as shown in Table 7. The debt sweep is most effective in eliminating financing distortions and supporting overall higher levels of debt. While the financial covenant appropriately reduces debt levels during periods of low profitability, it appears overall to manage debt within a relatively tight and conservative band that may be too restrictive.

Table 8 examines the simulation analysis from a different perspective, looking at the frequencies of positive and negative changes to capital and debt (and thus also implicitly the frequency of no change), as well as the mean of the magnitude of the changes (normalized by k) conditional on each direction. The unconditional means are shown in the last column for an overall comparison across the five optimization cases. We also analyze payout policy as an interesting byproduct of the shareholder's investment and financing decisions.

Reinforcing our results on investment distortions, we find that shareholders engage in large liquidation of assets 2% of the time, while this behavior is virtually non-existent when maximizing firm value. While the asset sweep reduces the magnitude of such liquidations when they do occur, the debt sweep and financial covenant indirectly reduce the incidence of this behavior by reducing financing distortions.

Table 8 also shows the high frequency (29%) of negative debt changes in the firm value maximization case, as the firm very actively responds to changes in profitability by adjusting its leverage. In contrast, in the equity value optimization case, shareholders never reduce the debt. The financial covenant leads to debt reductions, primarily as a result of covenant violations, as we shall explore more carefully later. In contrast, the debt sweep covenant virtually shuts down the flexibility to adjust the debt policy by imposing a costly penalty on debt increases.

Finally, note the implication of these investment and financing policies on the firm's payout and issuance policies (where the mean is again normalized by k). It is rare that the current period cash flow is precisely equal to the amount of investment and/or debt paid back, and so the frequency of a zero payout is never greater than 3% in any of the cases. Equity issuance is most prevalent in the firm value maximization and debt sweep cases. In the former, the firm seeks to avoid bankruptcy costs and will repurchase debt more frequently using equity. In the latter, the firm keeps debt relatively constant given the increased cost of adjusting debt, and thus there is greater reliance on equity as the source of outside financing. The equity maximization case has both the highest payout rate to shareholders as well as the highest average equity issuance amount. The former is the agency problem discussed earlier that shareholders funnel back cash flow into their own pockets rather than invest in capital where the return would be split with bondholders. The latter is a consequence of the agency problems. Since shareholders are unable to raise debt under favorable terms, they are forced to raise additional equity in

order to invest in higher profitability states. The covenants mitigate these problems, as argued earlier, and thus the payout and issuance policies can be seen to be less extreme than the equity value maximization case.

3.3   The Impact of Covenants on Firm Value

Table 9 shows firm values for the optimization cases as a fraction of the corresponding value for the firm-value maximizing case at various points in the state space, based on different values of the firm–specific risk factor (z), a high x (good economy) or low x (bad economy), three different debt levels ($b = 7.6$, $b = 2.8$, and $b = 0$), and in all cases with $k = 9.4$. Examining first the bottom two panels where the firm is currently unlevered ($b = 0$), the value under firm value maximization (second row) is 4-5% higher than the unlevered firm value (first row) across all the different productivity cases. This reflects the present value of the net corporate interest tax shields and lower issuance costs of debt financing relative to equity, net of bankruptcy costs and the costs associated with changing debt over time.

Unfortunately, this value enhancement from debt financing is significantly compromised by the presence of financial agency problems if there are no covenants in place. Without any mechanisms to bind or control their financing and investment policies, shareholders will do no better than select policies that maximize their own claim, incurring a higher cost of debt financing, and missing opportunities to create additional firm value. Table 9 shows that in the absence of any covenants, the value loss due to agency costs from the shareholder value maximizing case (S), varies widely depending on the current state. The percentage drop in value lies within a range of 4-17% across the various states.[24] Two key factors lead to a higher level of agency costs: a higher current level of debt overhang, and a worse state of the economy. This observation is consistent with the more significant investment and financing distortions under these conditions, as documented earlier. While Childs, Mauer, and Ott (2005), Moyen (2007), and others have found that the value loss due to financial agency problems is much smaller than what we find, this is likely due to the fact that these models hold debt constant. Thus, they do not account for the large agency costs due to distortions in financing policies,

---

[24]Note that while proportional agency costs are on the low end of this range when there is currently no leverage, the agency costs nonetheless are close to the value attainable from introducing debt financing, and thus the net value attributable to debt financing in the absence of covenants is quite small.

and they only capture part of the value loss attributable to investment distortions, since the financing distortions in turn exacerbate investment distortions.[25]

Table 9 illustrates that imposing covenants can restore some of the value loss attributable to agency problems. Given the effectiveness of the debt sweep in mitigating financing distortions, it is not surprising that this covenant generally leads to the most significant reduction in agency costs. In contrast, the asset sweep covenant is much less effective in restoring value lost due to agency problems given that shareholders may forego reductions in capital that could increase firm value but not shareholder value.[26] Furthermore, even if shareholders follow the firm value maximization investment policy and optimally disinvest in particular states, the resulting forced reduction in debt due to the asset sweep may then need to be reversed at a cost, thus reducing value. Our results in Table 9 indicate that these negative effects generally offset the benefits from reducing payout driven disinvestment, except if the economy is bad and debt levels are low.[27]

The value effect of the Debt/EBITDA covenant is shown in the last row of each panel in Table 9. Like the debt sweep covenant, the financial covenant addresses financing distortions, but restricts debt policy in a different manner. It does not directly preclude the issue of new *pari passu* debt, and thus is not as effective as the debt sweep from this perspective. However, it does result in shareholders reducing debt in low profitability states, thus eliminating considerable risk from the debt holders' perspective.

Table 10 provides statistics regarding the distribution of firm values in each of the six cases we analyze. These results provide consistent evidence of the large magnitude of agency costs and the relative ability of the covenants to reduce these costs. However, one must be careful in comparing value distributions across different optimization cases when the payout policies differ significantly (which we show to be the case in Tables 3 and 8),

---

[25]In addition, inferring the magnitude of agency costs from moments of a simulated distribution, as Moyen (2007) does, can misestimate agency costs if the payouts of firms in first–best and second–best cases differ, as we discuss below.

[26]This tradeoff between the benefits and costs of covenants is borne out in the empirical data. Billett, King, and Mauer (2007) find that companies with more growth options issue debt with fewer restrictive covenants.

[27]We have examined the effects of using lower values of ν (0.25 and 0.50). While the problems associated with the restrictions imposed by the asset sweep covenant are ameliorated, this covenant still does not provide a net benefit in increasing firm value relative to the shareholder optimization case. Since this covenant is neither state-contingent, nor subject to negotiation, it overly restricts the shareholders' decisions.

and when the path-dependent policies result in quite different steady state capital and debt levels (as shown in Tables 5 and 7). A firm with higher payouts to its shareholders will naturally have a lower remaining value in any given state, and thus value differences across cases will not accurately reflect differences in agency costs.

Simple covenants on their own appear to be reasonably effective in reducing financial agency problems, and thus helping firms reap the benefits of debt financing. However, the net benefit attained from debt financing will ultimately depend on the nature of the covenant restrictions, and the ability to appropriately enforce them. We have modeled asset and debt sweeps in a rather strict fashion, while in practice they may be more flexible to allow value-enhancing debt increases and asset decreases. Similarly, violations of financial covenants in practice may involve renegotiations that are more successful in resolving agency problems than the remedy we impose. Furthermore, multiple covenants exist in most debt contracts, leading to potentially higher gains. Recent attempts to empirically estimate the net value associated with debt financing include van Binsbergen, Graham, and Yang (2010), who find a median value increase close to 4% of the book value of assets, and Korteweg (2010), who estimates a median increase of 5.5% of firm value. As discussed earlier, we find a 4-5% difference between first–best and unlevered firm value across different profitability states, which is consistent with these empirical observations. This suggests that firms may be able in practice to successfully address potential financial agency problems by using a more nuanced design of debt covenants.

3.4 Policy Changes When the Financial Covenant is Violated

We have examined the effect of covenants on investment and financing policies at particular states, and overall along steady state equilibrium paths. These results broadly capture how covenants work in mitigating key financial agency problems. In the asset sweep and debt sweep cases, constraints are placed that directly limit the distortions in investment and financing policies followed in the equity value maximization case, though they may also limit policies that would otherwise have been followed under the firm value maximization case.

This is largely true as well for the Debt/EBITDA covenant, though the mechanism is somewhat more flexible. Rather than imposing a rule for what would happen if the firm were to sell assets or issue new debt, the Debt/EBITDA covenant does not interfere

with any investment and financing decisions unless this particular metric that proxies for the firm's credit risk exceeds a set threshold ($f^* = 2.6$). At that point, investment and financing policies are indirectly restricted by the requirement that the firm's Debt/EBITDA not exceed the prespecified level ($f^{**} = 3.6$) under the same profitability (e^{x+z}) state. This prevents shareholders from engaging in significant increases in debt or decreases in capital, and if the higher f^{**} threshold is breached, forces shareholders to reduce debt and/or increase investment.

Much of the effect of this covenant restriction occurs away from the violation point since shareholders make decisions that reduce the likelihood of triggering a violation.[28] However, it is also useful to examine how the firm reacts when the covenant is violated, particularly as this has been carefully analyzed in recent empirical work. Given the controlled nature of our simulations, we can provide new perspectives on these analyses.

Table 11 reports univariate statistics on investment, financing and payout policies at points in our simulations where the financial covenant is violated (Debt/EBITDA $> f^*$). Specifically, we provide the frequencies and means of investment, debt change, and payout, breaking these down into negative and positive changes. We similarly examine these statistics at points where there is a more serious violation, specifically where Debt/EBITDA $> f^{**}$. In these cases, shareholders have no choice but to change the capital or debt levels. We also include for comparison purposes the unconditional statistics for the financial covenant case (i.e. for all values of Debt/EBITDA), which were shown earlier (for S,f) in Table 8. Recall from Table 3 that the Debt/EBITDA covenant is violated 13.9% of the time.[29] Debt/EBITDA exceeds f^{**} 3% of the time.

There are three important things to note in Table 11. First, both the probability and the magnitude of positive investment are lower when the covenant is violated, and much lower when the covenant is severely violated, as compared to the corresponding averages across all the states. This is consistent with Chava and Roberts (2008) and Nini, Smith, and Sufi (2009), who both document that the investment rate drops significantly when a covenant is violated. Second, the likelihood of a decrease in debt is much higher at a

---

[28]This type of effect where constraints imposed at one state have an impact on actions at other states is a general property of the steady state equilibrium solution of constrained dynamic programs (e.g., as in Aiyagari (1995)).

[29]While we allow the shareholders to simply pay off their debt when the covenant is violated instead of adjusting their investment and financing policies, the shareholders never choose this option under the parameter specifications we report here.

covenant violation point than at other states, and almost certain when there is a serious violation of the covenant. This decrease in debt is consistent with the empirical findings that firms decrease leverage when covenants are violated, as documented in Nini, Smith, and Sufi (2009) and Roberts and Sufi (2009). Third, the likelihood of issuing equity and the proportional size of equity issuance are both higher when the covenant is violated than otherwise, while the mean level of a positive payout is much lower. Again these effects are magnified in the case of a serious violation. These policy adjustments are required in order to reduce debt at a time when cash flow is also low.[30]

Table 11 clearly illustrates that shareholders adjust debt and capital following a violation in ways that differ from adjustments made in non-violation states. But, it does not address whether these policy adjustments would also occur for low Debt/EBITDA states even in the absence of the covenant. This question is one that empirical studies on this issue have not, and can not readily, address. A strength of our structured simulation approach is that we are able to isolate the effects of the covenant by comparing firms that are identical other than the presence of the covenant. Conceptually, this is a diff-in-diff type of approach that allows us to analyze whether the decrease in investment and leverage is a result of the paths that result in Debt/EBITDA falling below particular thresholds, rather than simply the result of a covenant being violated in those states.

To carefully explore the marginal impacts of a covenant violation on the investment and financing decisions of the firm, we perform multivariate regressions of investment, financing, and payout in Table 12, Table 13, and Table 14, respectively, against common explanatory determinants of these policies, as well as a dummy variable for a covenant violation (Debt/EBITDA $> f^*$), or a dummy variable for a more serious covenant violation (Debt/EBITDA $> f^{**}$). The regressions are based on simulated samples for the firm value maximization (F), shareholder value maximization (S), and financial covenant (S,f) cases. Given that the simulated paths of θ states are common across these three cases, one can view F and particularly S as control samples, and S,f as the treated sample that allows improved identification of the effects of the financial covenant.

---

[30]We know of no empirical studies that seek to analyze issuance and payout policy in the presence of covenants, but these would be interesting results to confirm empirically.

The coefficients shown are obtained by averaging the estimated regression coefficients obtained on 50 simulated panels (each with 200 firms and 100 years).[31] The standard errors of the estimated coefficients are calculated as the standard deviations of the 50 estimates. We have also calculated standard errors as the average of the 50 clustered standard errors (clustered at the firm level) and obtained values that, while somewhat different, had no meaningful impact on statistical significance. Both the adjusted-R^2 and t-statistics are high in our regressions, which is to be expected given that our simulations are based on a model with a limited set of decision variables and two sources of uncertainty, and use a large number of runs.

For the investment regression in Table 12, the dependent variable is the investment rate at time t, i.e., $(k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\})/k$, and the independent variables are: current profitability (EBITDA/ASSET(t)), the lagged Q-ratio (Q-ratio(t-1)), lagged leverage (Book Leverage(t-1)), and lagged Debt/EBITDA (Debt/EBITDA(t-1)). These variables are frequently chosen as highly predictive of firms' investment rates. Our focus, however, is on the dummy variables, Debt/EBITDA(t) $> f^*$, and Debt/EBITDA(t) $> f^{**}$. In the first two sets of regressions for the F and S cases, neither of the dummy variables is significant, as expected. After controlling for key variables, there is nothing special about passing through covenant violation points in the cases where no covenants apply. However, interestingly, the coefficients on the dummies in the regressions for the financial covenant (S,f) case are positive, not negative as we found in the univariate statistics. The coefficient is larger and more significant for the covenant violation case overall, as compared to the case of more serious violations.

Chava and Roberts (2008) and Nini, Smith, and Sufi (2009) both find a significant drop in investment. Since we too find this relationship in our univariate tests, this suggests that having the proper controls in a multivariate regression may be important in properly isolating the effect of a covenant violation. However, it is also possible that the empirical studies are capturing other agency problems that are not in our model, such as overinvestment in risky assets or empire building, and that lenders may be able to curb these tendencies as a result of their negotiating power when covenants are violated.

In fact, the undercurrent of the empirical literature is that managers are investing at times when they should instead be keeping cash as a buffer, which will benefit lenders

---

[31]We drop the points in the simulation where there is default, and hence there are fewer than 1 million points.

directly by ensuring there is cash available to service the debt. However, given the difficulty in practice in ensuring that this cash remains in the firm, investment may be better security for lenders. Since there is no cash balance in our model, and the alternative to investment is paying money out to shareholders, debtholders benefit from additional investment that can generate some cash flow even in low states, and serves as a store of liquidity, albeit a costly one given the liquidation discount. The increase in investment does not show up in the univariate statistics since covenant violations occur in low states, but once we control for proxies related to the state and other drivers of credit risk, as we have in Table 12, it becomes apparent that additional investment is prescribed when the covenant is violated.

We repeat a similar exercise for leverage in Table 13. The dependent variable is quasi-market leverage, computed using the book value of debt, i.e. $b/(b + E)$. The independent variables are: the change in profitability from $t-1$ to t (Δ EBITDA/Asset), the lagged investment rate (Investment/Asset), and the lagged quasi-market leverage (Lagged Leverage), since several studies (e.g., Welch (2004)) find it to have the strongest predictive power, as we do. The covenant violation dummies are included as in the investment regressions.

In the case of firm value maximization, the independent variables pick up the impact of the drop in profitability on leverage decreases, and thus the dummy variables are either not statistically or economically significant. In contrast, the violation dummy coefficients become significant in the shareholder value maximization case even though there is no financial covenant in place. Note, however, that the coefficients are positive. This is picking up a non-linearity in the effect of profitability on debt changes. For higher states, debt changes become more positive given the ability to support more debt. But, the debt change also increases when profitability becomes low since shareholders try to expropriate wealth from existing bondholders. This u-shaped effect results in positive coefficients for the dummy variables.

When a financial covenant is present, and violated, shareholders pursue a debt policy which is different than what they otherwise would have followed in the unconstrained equity value maximization case. While they are reticent to decrease debt, and will try to first address violations using increased investment to grow EBITDA, when the covenant violation is serious enough (Debt/EBITDA $> f^{**}$), they must resort to buying back debt. These results from our simulation approach thus provide further support for

the findings in Nini, Smith, and Sufi (2009) and Roberts and Sufi (2009) that covenant violations trigger leverage decreases, even after controlling for other factors that could drive leverage changes.

Finally, we repeat a similar exercise for the payout policy (e/k) in Table 14, using the same regressors as in Table 12. The last two columns show that the payout drops when the covenant is violated, which is not the case in the absence of the covenant. Whether shareholders respond to a covenant violation by increasing investment or decreasing debt, or both, they will have to decrease the cash payout, or potentially issue new equity. The latter is very likely when the covenant is seriously violated and cash flow from operations is low, and this is clearly seen in the large, and highly significant, negative coefficient for the violation dummy variable in the regression in the last column.

4   Conclusions

In this paper, we examine the effectiveness of debt covenants in alleviating financial agency problems. While distortions in investment policies have been the central focus of the financial agency cost literature, we show that distortions in financing policies can also be large, and have an indirect effect on further exacerbating investment distortions. The covenants we explore mitigate financial agency costs to varying degrees, and in different ways. In all cases, there are direct effects of limiting distortions on the policy that is directly targeted, but also indirect effects of addressing the other policy distortion. While the empirical literature has recently focused on the policy changes when covenants are violated, we find that covenants alter policies more generally, even in states distant from the covenant violation states. We also find that policy changes that occur when a covenant is violated may differ significantly from those that would occur in the same state even when the covenant does not exist. Given that debt covenants significantly alter dynamic financing and investment policies, we conclude that covenants should be an important element of structural and empirical models of not only debt and investment policy, but also dividend and issuance policies. Further research to model alternative remedies to covenant violations, including the negotiation process between shareholders and bondholders at these points, would enrich this literature.

Appendix

A. Simple example: proof of some statements

We relegate in this appendix the proof that in the context of our simple example it is optimal, under the debt sweep covenant, to keep $b' = 2.5$.

To prove it, we show below that if the shareholders use the proceeds of new debt to buy back the old debt at par, then their value becomes lower than 4.55, the optimal value for $b' = 2.5$. Under this covenant, if they issue \$1.5 of new debt with market value $D \leq 1.5$, they have to repurchase D of par value of the old debt, so that the total residual obligation is $4 - D$. The equity value from this decision is

$$E = 4 - 0.1 - D + M + \frac{1}{2}\left(\frac{1}{2}\max\{4 \cdot 1 - (4 - D), 0\} + \frac{1}{2}\max\{2 \cdot 1 - (4 - D), 0\}\right)$$
$$+\frac{1}{2}\left(0.2 + \frac{1}{2}\max\{2 \cdot 0.5 - (4 - D), 0\} + \frac{1}{2}\max\{0 \cdot 0.5 - (4 - D), 0\}\right) = 4 + \frac{D}{4} - D + M,$$

where M are the proceeds from debt issuance (which will be soon equated to D). The above equation is based on the choice of $k' = 0.5$ if $\theta_1 = 1$ and $k' = 1$ if $\theta_1 = 3$. It can be shown that this policy dominates all the other feasible choices.

This is easy to prove as far as $k' = 0.5$ is concerned, because, if $k' = 1$ in the lower branch of the tree, there would be a zero value to equity at $t = 2$ when $D \leq 3$ (and actually $D \leq 1.5$), while not getting the \$.2 from the asset sale. The considered policy is optimal also in the upper part of the tree, because the equity payoff in that scenario is $\frac{1}{2}D$ if $k' = 1$, and it would be $D - 0.6$ if $k' = 2$. It results that $\frac{1}{2}D > D - 0.6$ if $D < 1.2$. It turns out that, given the default probability of the firm, this is actually the case.

The proceeds from debt issuance are

$$M = \frac{1}{2}\left(\frac{1}{2}(1.5) + \frac{1}{2}\left(2 \cdot \frac{1.5}{4 - D}\right)\right) + \frac{1}{2}\left(\frac{1}{2}\left(1 \cdot \frac{1.5}{4 - D}\right) + \frac{1}{2}(0)\right) = 0.3750 \cdot \left(1 + \frac{3}{4 - D}\right),$$

where $1.5/(4 - D)$ is the fraction of debt that is owned by new debt holders. Because $M = D$, then the solution is $D = 0.7178$ and the corresponding value of equity is $E = 4 + D/4 = 4.1794$, which is below 4.55, so the debt sweep is such that the equity

holders will prefer $b' = 2.5$ rather than try to issue 1.5 units of new debt and effectively make $b' = 3.2822$, because they will have to pay the 0.1 debt issuance cost, and they also will be buying back .7178 of the existing debt at par, thus diminishing the value of their default option on this .7178 of debt.

B. Numerical solution

In Section 2, we introduced the valuation problem for equity and debt under different optimization scenarios. The numerical solution approach is briefly described here only for the equity maximization case, with the other cases being similar.

The solution to equation (8), with constraint (11) is found numerically by a discrete state-space method based on value function iteration done simultaneously on both equations. The discretization method proposed by Rouwenhorst (1995) is used to approximate the dynamics of the two independent components of θ with a finite state Markov chain. This method is preferable to standard Gaussian quadrature given the high persistence of the systematic risk variable. We discretize the state variable k and b by setting the upper bound for capital, $\bar{k}$, and the upper bound for debt, $\bar{b}$, in a way that they are never binding when we simulate the optimization problem. The debt is equally spaced within the interval. The capital stock is discretized so that the $\log k$ points are spaced $(1 - \delta)$ from each other. We solve the model using 21 points for θ (3 for x, and 7 for z), and 51 points for each of k and b. The iteration on the two simultaneous equations is stopped when the maximum absolute difference between the old and the new value of both securities (equity and debt) is lower than 10^{-6}.

C. Monte Carlo simulation

Given the optimal policy function φ from the solution of each valuation problem, we use Monte Carlo simulation to generate a sample of possible future paths for the state variables. We obtain the simulated dynamics of the state variable x by applying the recursive formula (1) for $t = 0, \ldots, T - 1$, where ε'_x are independent draws from a Normal distribution with a mean of zero and variance σ^2_x. There are Ω_x possible future paths for x, each referred to as an economy. For each of these economies, there are

Ω_z paths of z, obtained appying equation (2), each representing a different firm. The simulated dynamics for z follows the same procedure as for x.

For each $\omega_z = 1, \ldots, \Omega_z$, we set as initial point for the ω_z-th simulated company (θ_0, k_0, b_0). Then, the subsequent steps are a recursive application of the same principle: $(k_{t+1}, b_{t+1}) = \varphi(\theta_t, k_t, b_t)$, if the firm is solvent and $(k_{t+1}, b_{t+1}) = (k_t(1 - \delta), 0)$, in case of default, for $t = 1, \ldots, T - 1$. This is repeated for $\omega_z = 1, \ldots, \Omega_z$, and for each $\omega_x = 1, \ldots, \Omega_x$. Given the independence across the different paths of the realizations of θ, and the presence of investment and financing frictions, we obtain Ω_x simulated panels of ex–post heterogenous firms, from which we determine all the quantities of interest (e.g., value of equity, capital and debt levels) and the derived quantities (leverage, Q-ratio, credit spreads, default frequency).

In our numerical experiments, we generate simulated samples with $\Omega_x = 50$ economies, $\Omega_z = 200$ firms within each economy, and $T = 150$ years (steps) for each. To limit the dependence of our results on the initial conditions, we drop the first 50 steps, yielding a total of $50 * 200 * 100 = 1$ million data points.

References

Aghion, P., and P. Bolton, 1992, An incomplete contracts approach to financial contracting, *Review of Economic Studies* 59, 473–494.

Aiyagari, S. R., 1995, Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting, *Journal of Political Economy* 103, 1158–1175.

Altinkilic, O., and R. S. Hansen, 2000, Are there economies of scale in underwriting fees? Evidence of rising external financing costs, *Review of Financial Studies* 13, 191–218.

Beneish, M., and E. Press, 1993, Costs of technical violation of accounting-based debt covenants, *The Accounting Review* 68, 233–257.

Beneish, M., and E. Press, 1995, The resolution of technical default, *The Accounting Review* 70, 337–353.

Berk, J., R. Green, and V. Naik, 1999, Optimal Investment, Growth Options, and Security Returns, *Journal of Finance* 54, 1553–1607.

Berlin, M., and L. Mester, 1992, Debt covenants and renegotiation, *Journal of Financial Intermediation* 2, 95–133.

Bhanot, K., and A. Mello, 2006, Should corporate debt include a rating trigger?, *Journal of Financial Economics* 79, 69–98.

Billett, M., D. King, and D. Mauer, 2007, The effect of growth opportunities on the joint choice of leverage, maturity and covenants, *Journal of Finance* 62, 697–730.

Bradley, M., and M. Roberts, 2004, The structure and pricing of corporate debt contracts, Working paper, Duke and Wharton.

Brennan, M. J., and E. S. Schwartz, 1984, Optimal financial policy and firm valuation, *Journal of Finance* 39, 593–603.

Chava, S., P. Kumar, and A. Warga, 2010, Managerial agency and bond covenants, *Review of Financial Studies* 23, 1120–1148.

Chava, S., and M. Roberts, 2008, How does financing impact investment? The role of debt covenants, *Journal of Finance* 63, 2085–2121.

Chen, K., and K. Wei, 1993, Creditors' decisions to waive violations of accounting-based debt covenants, *The Accounting Review* 68, 218–232.

Childs, P. D., D. C. Mauer, and S. H. Ott, 2005, Interactions of corporate financing and investment decisions: The effects of agency conflicts, *Journal of Financial Economics* 76, 667–690.

Dangl, T., and J. Zechner, 2004, Credit risk and dynamic capital structure choice, *Journal of Financial Intermediation* 13, 183–204.

Dichev, I., and D. Skinner, 2002, Large sample evidence on the debt covenant hypothesis, *Journal of Accounting Research* 40, 1091–1123.

Fischer, E.O., R. Heinkel, and J. Zechner, 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance* 49, 19–40.

Gamba, A., and A. J. Triantis, 2008, The value of financial flexibility, *Journal of Finance* 63, 2263–2296.

Garleanu, N., and J. Zwiebel, 2009, Design and renegotiation of debt covenants, *Review of Financial Studies* 22, 749–781.

Gomes, J. F., 2001, Financing investment, *American Economic Review* 91, 1263–1285.

Goyal, V., 2005, Market discipline of bank risk: Evidence from subordinated debt contracts, *Journal of Financial Intermediation* 14, 318–350.

Graham, J. R., 2000, How big are the tax benefits of debt?, *Journal of Finance* 55, 1901–1941.

Guo, X., R. Jarrow, and H. Lin, 2008, Distressed debt prices and recovery rate estimation, *Review of Derivatives Research* 11, 171–204.

Hackbarth, D., and D. Mauer, 2012, Optimal priority structure, capital structure, and investment, *Review of Financial Studies* 25, 747–796.

Hennessy, C. A., and T. M. Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165.

Hennessy, C. A., and T. M. Whited, 2007, How costly is external financing? Evidence from a structural estimation, *Journal of Finance* 62, 1705–1745.

Jones, C., and S. Tuzel, 2012, Inventory investment and the cost of capital, *Journal of Financial Economics (forthcoming)*.

Korteweg, A., 2010, The net benefits to leverage, *Journal of Finance* 65, 2137–2170.

Kuehn, L., and L. Schmid, 2011, Investment-based corporate bond pricing, Working paper, Tepper School of Business and Fuqua School of Business.

Leland, H., 1998, Agency costs, risk management, and capital structure, *Journal of Finance* 53, 1213–1243.

Leland, H. E., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213–1252.

Livdan, Dmitry, Horacio Sapriza, and Lu Zhang, 2009, Financially Constrained Stock Returns, *Journal of Finance* 64, 1827–1862.

Mao, L., 2011, Debt covenants, capital structure, and managerial incentives, Working paper, HKUST.

Mao, L., and Y. Tserlukevich, 2012, Repurchasing debt, Working paper, HKUST and Arizona State University.

Mello, S. A., and J. E. Parsons, 1992, Measuring the agency cost of debt, *Journal of Finance* 47, 1887–1904.

Morellec, E., 2001, Asset liquidity, capital structure, and secured debt, *Journal of Financial Economics* 61, 173–206.

Moyen, N., 2007, How big is the debt overhang problem?, *Journal of Economic Dynamics and Control* 31, 433–472.

Murfin, J., 2012, The supply-side determinants of loan contract strictness, *Journal of Finance* 67, 1565–1601.

Nini, G., D. Smith, and A. Sufi, 2009, Creditor control rights and firm investment policy, *Journal of Financial Economics* 92, 400–420.

Nini, G., D. Smith, and A. Sufi, 2012, Creditor control rights, corporate governance, and firm value, *Review of Financial Studies* 25, 1713–1761.

Parrino, R., and M. Weisbach, 1999, Measuring investment distortions arising from stockholder-bondholder conflicts, *Journal of Financial Economics* 53, 3–42.

Pulvino, T. C., 1998, Do asset fire-sales exist? An empirical investigation of commercial aircraft transactions, *Journal of Finance* 53, 939–978.

Rajan, R., and A. Winton, 1995, Covenants and collateral as incentives to monitor, *Journal of Finance* 50, 1113–1146.

Reisel, N., 2010, On the value of restrictive covenants: Empirical investigation of public bond issues, Working paper, SMU Cox Business School.

Riddick, L., and T. Whited, 2009, The corporate propensity to save, *Journal of Finance* 64, 1729–1766.

Roberts, M., and A. Sufi, 2009, Control rights and capital structure: An empirical investigation, *Journal of Finance* 64, 1657–1695.

Rouwenhorst, K. Geert, 1995, Asset Pricing Implications of Equilibrium Business Cycle Models, in Thomas F. Cooley, eds.: *Frontiers of Business Cycle Research* (Princeton University Press, Princeton - NJ).

Schmid, L., 2008, A Quantitative Dynamic Agency Model of Financial Constraints, Working paper, Duke Fuqua School.

Smith, Jr., C. W., and J. Warner, 1979, On financial contracting: an analysis of bond covenants, *Journal of Financial Economics* 7, 281–308.

Sridhar, S., and R. Magee, 1997, Financial contracts, opportunism, and disclosure management, *Review of Accounting Studies* 1, 225–258.

Strebulaev, I. A., 2007, Do tests of capital structure theory mean what they say?, *Journal of Finance* 62, 1747–1787.

Sweeney, A., 1994, Debt covenant violations and managers' accounting responses, *Journal of Accounting and Economics* 17, 281–308.

Titman, S., and S. Tsyplakov, 2007, A dynamic model of optimal capital structure, *Review of Finance* 11, 401–451.

Tsyplakov, S., 2008, Investment frictions and leverage dynamics, *Journal of Financial Economics* 89, 423–443.

van Binsbergen, J. H., J. R. Graham, and J. Yang, 2010, The cost of debt, *Journal of Finance* 65, 2089–2136.

Welch, I., 2004, Capital structure and stock returns, *Journal of Political Economy* 112, 106–131.

Whited, T. M., and G. Wu, 2006, Financial constraints risk, *Review of Financial Studies* 19, 531–559.

Zhang, L., 2005, The value premium, *Journal of Finance* 55, 67–103.

$\theta = 2$

3

4

2

1

0

$t = 0$ 1 2
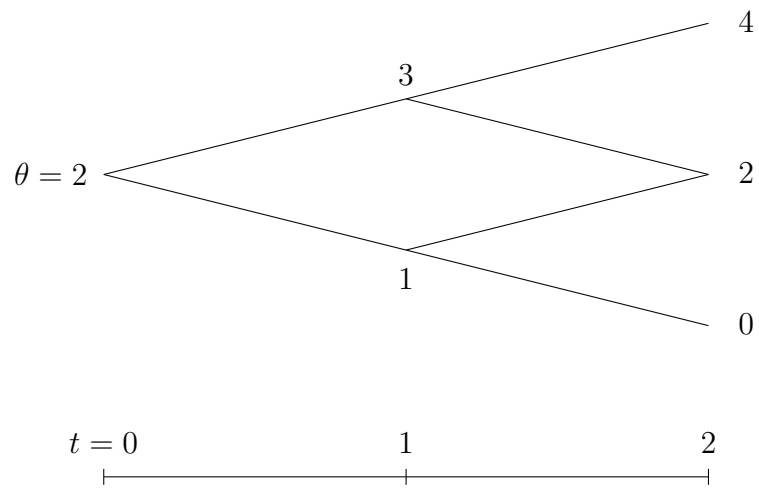
Figure 1: Simple example. Firm's productivity

Symbol	Description
F	Firm value maximization
S	Equity value maximization
Z	Unlevered firm value maximization
S,a	Second best case subject to asset sweep covenant
S,d	Second best case subject to debt sweep covenant
S,f	Second best case subject to financial (Debt/EBITDA) covenant

Table 1: **Notation for Different Optimization Cases.** This table provides the symbols used to represent each of the constrained and unconstrained optimization cases examined in the paper.

σ_x	conditional volatility of systematic risk	1.36%
ρ_x	persistence of systematic risk	0.9224
γ_0	constant price of risk parameter	3.22
γ_1	time varying price of risk parameter	-15.30
σ_z	conditional volatility of idiosyncratic risk	15.80%
ρ_z	persistence of idiosyncratic risk	0.6857
β	time discount factor	1/1.05
α	capital share	0.50
ψ	fixed production cost	1.03
δ	annual depreciation rate	11%
τ	marginal net corporate tax rate	10%
ℓ	liquidation price for disinvestment	0.75
s	fire-sale discount for asset sales	0.60
ζ	proportional bankruptcy costs	0.60
λ	flotation cost for equity	0.06
η	debt adjustment cost	0.01
f^*	trigger for Debt/EBITDA covenant violation	2.6
f^{**}	Debt/EBITDA limit for covenant resolution	3.6
ν	Asset sweep fraction used to pay back debt	100%

Table 2: **Base Case Parameter Values**. This table provides the base case parameters used in the optimizations and simulations.

	F	S	S,a	S,d	S,f	Empirical
EBITDA/Assets	0.22	0.22	0.22	0.22	0.22	0.22
Investment Rate	0.12	0.21	0.18	0.14	0.15	0.15
Q ratio	1.97	2.12	2.04	1.95	1.98	2.10
Leverage	0.73	0.19	0.33	0.43	0.18	0.23
Credit Spread (bps)	2.20	208.55	173.28	40.54	77.77	100.00
Default (%)	0.02	3.52	2.50	0.49	1.51	1.00
Equity Dist./Assets	0.03	0.11	0.08	0.07	0.14	0.04
Violation (%)	–	–	–	–	13.91	13.00

Table 3: **Simulated Means of Key Metrics.** This table provides unconditional sample means for the following variables: EBITDA/Assets (π/k); Investment Rate ($(k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\})/k$); Q–ratio ($V/k$); Leverage ($b/(b + E)$); Credit Spread (the yield to maturity on debt minus the yield of the corresponding risk-free rate debt, in basis points); Default (the annual frequency of shareholders defaulting on the debt); Equity Dist./Assets (the payout to shareholders, i.e., positive distribution, or amount of equity raised, i.e., negative distribution, as a fraction of k); and Violation (the percentage of times the firm violates the Debt/EBITDA covenant). The sources of the empirical estimates are provided in the paper. All means are reported on an annual basis. The columns represent the five optimization cases: F (firm value maximization), S (shareholder value maximization), S,a (asset sweep covenant), S,d (debt sweep), and S,f (financial covenant). The means reported are based on the simulation described in Appendix C using the base parameters shown in Table 2.

e^z	0.59	0.70	0.84	1.00	1.19	1.43	1.70
			Bad economy, $b = 7.6$				
F	0.00	0.00	0.00	0.00	2.19	4.20	6.60
S	-7.26	-7.26	0.00	0.00	0.00	2.19	3.49
S,a	0.00	0.00	0.00	0.00	0.00	2.82	8.45
S,d	0.00	0.00	0.00	0.50	1.59	2.19	3.49
S,f	0.00	0.00	0.00	0.00	1.59	2.19	3.49
			Good economy, $b = 7.6$				
F	0.00	0.00	0.00	1.59	4.20	6.60	9.45
S	-7.26	0.00	0.00	1.03	2.82	4.20	4.96
S,a	0.00	0.00	0.00	1.03	2.82	4.20	10.52
S,d	0.00	0.00	0.50	1.59	2.19	2.82	4.20
S,f	0.00	0.00	0.50	1.59	2.19	2.82	4.20
			Bad economy, $b = 2.8$				
F	0.00	0.00	0.00	0.00	2.19	4.20	6.60
S	-7.26	0.00	0.00	0.00	1.59	2.82	4.20
S,a	0.00	0.00	0.00	0.00	1.59	2.82	4.20
S,d	0.00	0.00	0.00	0.50	2.19	4.20	6.60
S,f	0.00	0.00	0.00	0.00	1.59	2.82	3.49
			Good economy, $b = 2.8$				
F	0.00	0.00	0.00	1.59	4.20	6.60	9.45
S	0.00	0.00	0.00	1.59	2.82	3.49	4.96
S,a	0.00	0.00	0.00	1.59	2.82	4.20	4.96
S,d	0.00	0.00	0.50	2.19	4.20	6.60	8.45
S,f	0.00	0.00	0.50	1.59	2.19	5.75	8.45

Table 4: **Investment Policy**. This table shows the values of optimal investment ($k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\}$) at different values of the firm-specific risk factor, at high and low levels of the macroeconomic risk factor, and at two given levels of current debt ($b = 2.8$, the median debt for S, and $b = 7.6$, the median debt for S,d). The current level of capital in all cases is $k = 9.4$, which is close to the median capital level for most of the cases. Results are presented for five cases: F (first–best), S (second–best), S,a (asset sweep covenant), S,d (debt sweep covenant), and S,f (financial covenant). The values are based on the solution of the optimal program described in Appendix B using the base parameters shown in Table 2.

	Median	Mean	s.d.	Quantiles			
				5th	25th	75th	95th
Z	9.38	9.62	2.04	6.61	8.35	11.17	13.30
F	9.94	10.10	2.19	6.61	8.35	11.17	14.10
S	8.85	9.04	2.60	5.88	7.43	10.54	13.30
S,a	8.85	9.06	2.56	5.55	7.43	10.54	13.30
S,d	9.38	9.57	2.09	7.01	8.35	10.54	13.30
S,f	9.38	9.69	2.28	7.01	8.35	11.17	13.30

Table 5: **Distribution of Capital**. This table shows the unconditional median, mean, standard deviation, and 5%, 25%, 75% and 95% quantiles for the book value of assets (capital stock) under the following cases: Z (zero debt), F (first–best), S (second–best), S,a (asset sweep covenant), S,d (debt sweep covenant), and S,f (financial covenant). The values are based on the simulation described in Appendix C using the base parameters shown in Table 2.

e^z	0.59	0.70	0.84	1.00	1.19	1.43	1.70
	Bad economy, $b = 7.6$						
F	0.00	0.00	0.00	2.40	4.80	6.40	7.60
S	12.40	12.40	0.00	0.00	0.00	0.80	0.40
S,a	11.60	0.40	0.40	0.40	0.40	0.80	12.40
S,d	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S,f	-5.60	-4.40	-3.20	-1.60	0.00	0.00	0.00
	Good economy, $b = 7.6$						
F	0.00	1.60	3.20	4.80	6.80	8.00	9.60
S	12.40	0.00	0.00	0.00	0.80	2.00	2.00
S,a	0.00	0.00	0.00	0.00	0.80	2.00	12.40
S,d	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S,f	-5.20	-4.00	-2.00	0.00	0.00	0.00	0.40
	Bad economy, $b = 2.8$						
F	2.80	4.00	4.80	7.20	9.60	11.20	12.40
S	17.20	0.40	0.40	0.40	0.00	0.40	0.80
S,a	0.00	0.00	0.00	0.00	0.00	0.40	0.80
S,d	0.00	0.00	0.00	4.80	4.80	4.80	4.80
S,f	-0.80	-0.40	0.00	0.00	0.00	0.00	0.00
	Good economy, $b = 2.8$						
F	4.80	6.40	8.00	9.60	11.60	12.80	14.40
S	0.00	0.00	0.00	0.00	0.40	0.40	0.80
S,a	0.00	0.00	0.00	0.00	0.40	1.20	1.20
S,d	0.00	0.00	4.80	4.80	4.80	4.80	4.80
S,f	-0.40	0.00	0.00	0.00	0.00	2.80	4.80

Table 6: **Debt Policy**. This table shows the values of the optimal debt change, $b' - b$, at different values of the firm-specific risk factor, at high and low levels of the macroeconomic risk factor, and at two given levels of current debt ($b = 2.8$, the median debt for S, and $b = 7.6$, the median debt for S,d). The current level of capital in all cases is $k = 9.4$, which is close to the median capital level for most of the cases. Results are presented for five cases: F (first–best), S (second–best), S,a (asset sweep covenant), S,d (debt sweep covenant), and S,f (financial covenant). The values are based on the solution of the optimal program described in Appendix B using the base parameters shown in Table 2.

	Median	Mean	s.d.	Quantiles			
				5th	25th	75th	95th
F	14.40	14.41	1.42	12.40	13.60	15.60	16.80
S	2.80	3.46	3.03	0.40	1.20	4.80	9.60
S,a	3.60	4.75	3.97	1.20	2.40	6.00	11.20
S,d	7.60	8.22	1.22	7.60	7.60	10.00	10.00
S,f	2.80	3.30	1.52	1.60	2.00	4.40	6.00

Table 7: **Distribution of the Book Value of Debt**. This table shows the unconditional median, mean, standard deviation, and 5%, 25%, 75% and 95% quantiles for the book value of debt (b) under the following cases: Z (zero debt), F (first–best), S (second–best), S,a (asset sweep covenant), S,d (debt sweep covenant), and S,f (financial covenant). The values are based on the simulation described in Appendix C using the base parameters shown in Table 2.

		Negative		Positive		Overall
		mean	freq.	mean	freq.	mean
	Investment	-0.55	0.00	0.16	0.72	0.12
F	Debt change	-0.07	0.29	0.13	0.20	0.01
	Payout	-0.03	0.29	0.05	0.71	0.03
	Investment	-0.73	0.02	0.35	0.62	0.21
S	Debt change	–	0.00	1.18	0.27	0.33
	Payout	-1.00	0.10	0.24	0.87	0.11
	Investment	-0.33	0.02	0.31	0.59	0.18
S,a	Debt change	-0.26	0.02	0.81	0.24	0.19
	Payout	-0.53	0.11	0.16	0.87	0.08
	Investment	-0.71	0.00	0.18	0.76	0.14
S,d	Debt change	–	0.00	9.67	0.01	0.05
	Payout	-0.03	0.18	0.09	0.82	0.07
	Investment	-0.74	0.01	0.22	0.71	0.15
S,f	Debt change	-0.12	0.06	6.22	0.03	0.20
	Payout	-0.09	0.11	0.16	0.88	0.14

Table 8: **Investment, Financing and Payout Policies**. This table characterizes the dynamics of the firm's investment and financing policies for the five cases we examine: F (first–best), S (second–best), S,a (asset sweep covenant), S,d (debt sweep), and S,f (financial covenant). The policies are determined based on the optimizations described in the paper, and the simulation detailed in Appendix C using the base parameters shown in Table 2. For each optimization scenario, we show the frequency of investment ($(k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\})/k$), debt change ($(b' - b)/k$) and payout (also normalized by assets) being either strictly negative or strictly positive, and the conditional mean for each of these two cases. The mean is the time–series average of the cross–sectional mean. We also provide the unconditional (Overall) mean for the investment, debt change and payout for each case.

e^z	0.59	0.70	0.84	1.00	1.19	1.43	1.70
Bad economy, $b = 7.6$							
S	0.92	0.88	0.90	0.90	0.91	0.92	0.93
S,a	0.85	0.86	0.88	0.89	0.89	0.91	0.91
S,d	0.98	0.98	0.99	0.99	0.99	0.99	0.98
S,f	0.91	0.91	0.94	0.95	0.96	0.96	0.96
Good economy, $b = 7.6$							
S	0.83	0.90	0.91	0.92	0.92	0.92	0.93
S,a	0.89	0.90	0.91	0.91	0.92	0.92	0.92
S,d	0.98	0.98	0.98	0.98	0.98	0.98	0.98
S,f	0.93	0.94	0.95	0.96	0.96	0.97	0.96
Bad economy, $b = 2.8$							
S	0.92	0.93	0.94	0.94	0.95	0.95	0.95
S,a	0.94	0.94	0.95	0.95	0.96	0.96	0.95
S,d	0.98	0.98	0.98	0.98	0.98	0.98	0.99
S,f	0.93	0.93	0.94	0.95	0.95	0.95	0.95
Good economy, $b = 2.8$							
S	0.94	0.95	0.95	0.95	0.95	0.95	0.95
S,a	0.95	0.95	0.95	0.95	0.96	0.96	0.96
S,d	0.98	0.98	0.98	0.98	0.98	0.98	0.98
S,f	0.95	0.95	0.96	0.96	0.96	0.97	0.97
Bad economy, $b = 0$							
Z	0.95	0.96	0.96	0.96	0.96	0.96	0.96
S	0.96	0.96	0.96	0.96	0.96	0.96	0.96
S,a	0.96	0.96	0.96	0.96	0.96	0.96	0.96
S,d	0.98	0.98	0.99	0.99	0.99	0.99	0.99
S,f	0.96	0.96	0.96	0.96	0.96	0.96	0.97
Good economy, $b = 0$							
Z	0.95	0.96	0.96	0.96	0.96	0.96	0.96
S	0.96	0.96	0.96	0.96	0.96	0.96	0.96
S,a	0.96	0.96	0.96	0.96	0.96	0.96	0.96
S,d	0.98	0.98	0.98	0.98	0.99	0.99	0.99
S,f	0.96	0.96	0.96	0.96	0.96	0.97	0.97

Table 9: **Firm Value**. This table shows firm values as a fraction of the corresponding F value under the different optimization cases for different z values, at high and low levels of x, and at three levels of current debt ($b = 7.6$, $b = 2.8$, and $b = 0$), when $k = 9.4$.

	Median	Mean	s.d.	Quantiles			
				5th	25th	75th	95th
Z	18.90	19.31	3.48	14.32	16.72	21.49	26.00
F	20.57	20.80	3.72	15.34	18.08	23.19	27.79
S	18.21	18.41	3.94	12.08	15.89	20.84	25.31
S,a	17.85	17.96	4.15	11.10	15.25	20.54	25.19
S,d	19.18	19.44	3.64	14.08	16.94	21.66	25.93
S,f	19.02	19.37	3.70	13.47	16.79	21.66	26.24

Table 10: **Firm Value Distributions**. This table shows the unconditional median, mean, standard deviation, and 5%, 25%, 75% and 95% quantiles for firm value under the following cases: Z (zero debt), F (firm value maximization), S (shareholder value maximization), S,a (asset sweep covenant), S,d (debt sweep covenant), and S,f (financial covenant). The values are based on the simulation described in Appendix C using the base parameters shown in Table 2.

		Debt/EBITDA $> f^*$		Debt/EBITDA $> f^{**}$		Overall	
		negative	positive	negative	positive	negative	positive
	freq.	0.00	0.51	0.00	0.15	0.01	0.71
Investment	mean	-0.67	0.10	-0.66	0.08	-0.74	0.22
	freq.	0.37	0.00	0.96	0.00	0.06	0.03
Debt change	mean	-0.12	0.16	-0.17	4.12	-0.12	6.22
	freq.	0.29	0.71	0.81	0.19	0.11	0.88
Payout	mean	-0.07	0.07	-0.11	0.02	-0.09	0.16

Table 11: **Firm Policies at Covenant Violation Points.** This table reports the investment, financing and payout behavior of firms with debt subject to a financial covenant, for points where the Debt/EBITDA covenant is violated (i.e., when $b/\pi(\theta, k) > f^*$), as well as those where Debt/EBITDA is also higher than f^{**} and consequently at least one of k or b must be adjusted. We report the frequencies (freq.) and the mean changes of capital stock $((k' - k(1-\delta) + \max\{(rb + g(y) - \pi)/s, 0\})/k)$, debt $((b'-b)/k)$, and payout (normalized by assets). The "negative" and "positive" columns report the strictly negative and positive changes. "Mean" is the time–series average of the cross–sectional mean. In the last two columns, we provide the overall means for the investment, debt change and payout for firms whose debt is subject to the financial covenant, as a benchmark to compare the policies at the covenant violation points. All results are based on the simulation detailed in Appendix C using the base case parameters shown in Table 2.

Table 12: Investment Regression.

The table reports regressions of the investment rate for three of the optimization cases: firm value maximization (F), unconstrained equity value maximization (S), and constrained equity value maximization, where debt is subject to the financial covenant (S,f). The dependent variable is the investment rate at time t, i.e., $(k' - k(1 - \delta) + \max\{(rb + g(y) - \pi)/s, 0\})/k$. The independent variables are: profitability (EBITDA/Asset), the Q-ratio, leverage (Book Leverage), and one of two dummy variables: Debt/EBITDA $> f^*$ which equals one if *either* Debt/EBITDA $> f^*$ *or* Debt/EBITDA < 0; or Debt/EBITDA $> f^{**}$ which equals one if *either* Debt/EBITDA $> f^{**}$ *or* Debt/EBITDA < 0. t-statistics are shown in parentheses. All results are based on the simulation detailed in Appendix C using the base case parameters shown in Table 2. The coefficients are obtained by averaging the estimated regression coefficient obtained on 50 simulated panels (each with 200 firms and 100 years). The standard errors of the estimated coefficients are calculated as the standard deviations of the 50 estimates.

	F		S		S,f	
Constant	-0.994	-1.000	-1.052	-1.049	-1.077	-1.061
	(-47.33)	(-49.63)	(-31.48)	(-40.29)	(-18.33)	(-17.39)
EBITDA/Asset	1.820	1.823	1.673	1.589	1.327	1.253
	(69.74)	(65.38)	(35.83)	(38.87)	(13.49)	(15.19)
Q-ratio	0.033	0.033	0.427	0.431	0.465	0.460
	(1.38)	(1.39)	(26.62)	(29.23)	(14.31)	(13.58)
Book Leverage	0.444	0.443	-0.140	-0.091	0.008	0.052
	(24.30)	(24.64)	(-8.36)	(-5.28)	(0.63)	(5.47)
Debt/EBITDA $> f^*$	-0.001		0.001		0.039	
	(-0.14)		(0.03)		(3.80)	
Debt/EBITDA $> f^{**}$		0.004		-0.074		0.013
		(1.14)		(-1.30)		(1.67)
Observations	979750	979750	915462	915462	952375	952375
adjusted-R^2	0.828	0.828	0.580	0.589	0.564	0.557

Table 13: Leverage Regression.

The table reports regressions of leverage for three of the optimization cases: firm value maximization (F), unconstrained equity value maximization (S), and constrained equity value maximization, where debt is subject to the financial covenant (S,f). The dependent variable is the Quasi-Market leverage, $b/(b+E)$. The independent variables used include: the prior period's $(t-1)$ quasi-market leverage (Lagged Leverage), the change in profitability from $t-1$ to t (Δ EBITDA/ Asset(t)), the lagged investment rate (Investment/Asset), and one of two dummy variables: Debt/EBITDA $> f^*$ which equals one if *either* Debt/EBITDA $> f^*$ *or* Debt/EBITDA < 0; or Debt/EBITDA $> f^{**}$ which equals one if *either* Debt/EBITDA $> f^{**}$ *or* Debt/EBITDA < 0. t-statistics are shown in parentheses. All results are based on the simulation detailed in Appendix C using the base case parameters shown in Table 2. The coefficients are obtained by averaging the estimated regression coefficient obtained on 50 simulated panels (each with 200 firms and 100 years). The standard errors of the estimated coefficients are calculated as the standard deviations of the 50 estimates.

	F		S		S,f	
Constant	0.050	0.051	0.009	0.013	0.016	0.013
	(15.00)	(11.93)	(2.42)	(3.79)	(4.04)	(3.06)
Lagged Leverage	0.953	0.949	0.966	0.954	0.931	0.945
	(153.35)	(105.69)	(30.06)	(39.55)	(55.95)	(44.17)
Δ EBITDA/Asset	-0.792	-0.788	-0.092	-0.087	-0.117	-0.129
	(-34.47)	(-31.22)	(-15.07)	(-14.11)	(-5.69)	(-6.73)
Investment/Asset	-0.170	-0.169	-0.002	-0.003	-0.014	-0.012
	(-22.37)	(-22.74)	(-4.20)	(-6.12)	(-0.83)	(-0.61)
Debt/EBITDA $> f^*$	0.054		0.046		-0.004	
	(0.15)		(2.32)		(-1.10)	
Debt/EBITDA $> f^{**}$		0.006		0.070		-0.042
		(1.84)		(2.57)		(-21.81)
Observations	979750	979750	915462	915462	952375	952375
adjusted-R^2	0.931	0.931	0.889	0.895	0.715	0.723

Table 14: Payout Regression.

The table reports regressions of the payout rate for three of the optimization cases: firm value maximization (F), unconstrained equity value maximization (S), and constrained equity value maximization, where debt is subject to the financial covenant (S,f). The dependent variable is the payout rate at time t, i.e., e/k. The independent variables are: profitability (EBITDA/ASSET), the Q-ratio, leverage (Book Leverage), and one of two dummy variables: Debt/EBITDA $> f^*$ which equals one if *either* Debt/EBITDA $> f^*$ *or* Debt/EBITDA < 0; or Debt/EBITDA $> f^{**}$ which equals one if *either* Debt/EBITDA $> f^{**}$ *or* Debt/EBITDA < 0. t-statistics are shown in parentheses. All results are based on the simulation detailed in Appendix C using the base case parameters shown in Table 2. The coefficients are obtained by averaging the estimated regression coefficient obtained on 50 simulated panels (each with 200 firms and 100 years). The standard errors of the estimated coefficients are calculated as the standard deviations of the 50 estimates.

	F		S		S,f	
Constant	0.229	0.209	0.865	0.869	1.009	0.980
	(9.85)	(8.83)	(17.56)	(23.29)	(24.50)	(21.10)
EBITDA/Asset	0.108	0.122	-0.836	-0.719	-0.432	-0.358
	(9.56)	(10.53)	(-10.98)	(-8.81)	(-1.94)	(-1.92)
Q-ratio	0.100	0.101	-0.314	-0.321	-0.412	-0.400
	(7.27)	(7.10)	(-9.79)	(-11.47)	(-11.78)	(-9.59)
Book Leverage	-0.289	-0.291	0.113	0.040	-0.091	-0.139
	(-23.11)	(-21.52)	(3.62)	(1.22)	(-5.09)	(-7.34)
Debt/EBITDA $> f^*$	-0.000		0.040		-0.084	
	(-0.14)		(0.58)		(-2.63)	
Debt/EBITDA $> f^{**}$		0.020		0.164		-0.170
		(9.06)		(1.46)		(-8.42)
Observations	979750	979750	915462	915462	952375	952375
adjusted-R^2	0.687	0.690	0.184	0.211	0.188	0.204