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# FINANCE RESEARCH SEMINAR SUPPORTED BY UNIGESTION

# "Cash and Dynamic Agency"


**Prof. Barney HARTMAN-GLASER**

*UCLA, Anderson School of Management*



**Abstract**

We present an agency model of cash dynamics within a firm. An investor contracts with a manager to operate a firm but faces two key frictions in doing so. First, the manager can divert cash for her own consumption by underreporting cash flow to the investor. Second, the investor has limited liability and can only transfer cash into the firm at a cost (possibly infinite) to cover cash flow shortfalls. The second friction restricts the promise-keeping ability of the investor. Cash in the firm takes on a special role as a commitment device that can be contracted upon, and thus the investor optimally allows cash to accumulate within the firm. However, the first friction implies that the firm cannot maintain too high a cash stock, lest the manager divert. In some cases, the optimal contract can be implemented via performance sensitive debt. In all cases, the payouts to the investors are given by a rate per unit of time, while payouts to the manager are lumpy. Conditional on making payouts to the investors, the firm pays the investor at a higher rate the lower the cash balance. Initially cash constrained firms may choose to begin operations by ceding more surplus to the manager to decrease the probability of liquidation.


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# Cash and Dynamic Agency

Barney Hartman-Glaser and Konstantin Milbradt*

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We present an agency model of cash dynamics within a firm. An investor contracts with a manager to operate a firm but faces two key frictions in doing so. First, the manager can divert cash for her own consumption by underreporting cash flow to the investor. Second, the investor has limited liability and can only transfer cash into the firm at a cost (possibly infinite) to cover cash flow shortfalls. The second friction restricts the promise-keeping ability of the investor. Cash in the firm takes on a special role as a commitment device that can be contracted upon, and thus the investor optimally allows cash to accumulate within the firm. However, the first friction implies that the firm cannot maintain too high a cash stock, lest the manager divert. In some cases, the optimal contract can be implemented via performance sensitive debt. In all cases, the payouts to the investors are given by a rate per unit of time, while payouts to the manager are lumpy. Conditional on making payouts to the investors, the firm pays the investor at a *higher* rate the *lower* the cash balance. Initially cash constrained firms may choose to begin operations by ceding more surplus to the manager to decrease the probability of liquidation.

*Hartman-Glaser: UCLA Anderson School of Management, bhglaser@anderson.ucla.edu. Milbradt: Northwestern Kellogg School of Management, milbradt@northwestern.edu

# 1 Introduction

Cash holdings and payout policy have been subject to heightened scrutiny in the process of the recent focus on executive compensation and the ammassement of large cash-holdings in several S&P 500 firms. In particular, should cash-payouts to equity holders be separate from bonus payments to executives?

In this paper, we introduce an integrated framework based on optimal contracts that delivers a separation of payouts to the principals from bonus payments to the manager. Further, the payouts to the principals are predominantly persistent and slow moving, while executive bonuses remain an impulse based instrument. Specifically, we take a continuous time principal-agent approach, but add an important and realistic restriction on the strategy space of the principal—the principal's promise keeping is restricted to cash inside the firm and possible incentive compatible refinancing. For example, with the cost of refinancing approaching infinity, any paths that lead cash holdings to be depleted to zero while there is still positive promised value to the manager will now be inaddmissable. Further, we extend the stealing technology that the agent is classically endowed with to the stock of cash within the firm as well. These simple restrictions deliver a clear separation of equity and bonus payments that are readily interpreted as dividends and bonuses. Cash-holdings of the firm are now become a state-variable in addition to the promised value to the manager, making the problem truly 2-dimensional without any homogeneity reduction. The firm accumulates cash to strengthen its promise keeping, while paying it out to overcome the agency problem.

In more detail, we introduce the following restriction on promise-keeping into the traditional continuous-time principal agent framework: the principal cannot be forced to either insert additional cash into the firm or raise additional cash on outside market unless he chooses to do so. Only cash within the firm and believable promises of raising cash can be used to fulfill the promise-keeping constraint of the manager. Thus, cash in the firm is a commitment device that can be re-interpreted as collateral for a promise. In the extreme case where raising additional funds is impossible, only collaterized promises can be used to fulfill the promise-keeping constraint. Any un-collateraized promises are empty promises.

Further, we extend the stealing technology that the agent is endowed with in regards to the cash-flows produced by the assets in place to the cash in the firm. If the agent can steal a proportion

$\lambda$ of each dollar the firm generates, he can also steal a proportion $\lambda$ of each dollar of cash the firm holds. This effectively puts a limit on how much cash the firm can hold relative to the promised value of the agent — the promised value to the agent can never fall below $\lambda$ times the cash within the firm. To keep cash-balances from becoming too large relatively to promised value, the firm needs to pay cash out to the principal (but not necessarily the agent).

With these two restrictions in place, we can now solve for the optimal contract. We first observe that the promised value and the cash-holding of the firm are perfectly correlated due to the fact that cash-holdings within the firm grow at a risk-less rate and only the cash generated by the assets under management feature risk. This means that cash payouts to the principal triggered by the stock-stealing constraint are of a $dt$ order, or in other words locally riskless They are further persistent in that if there was a payout to principals yesterday, then there is a high likelihood a similar payment will occur today. In contrast, bonus payments take the form of an impulse control. Only where the bonus boundary overlaps with the stock-stealing constraint do we have a coincidence of bonus and payments to the principal which are both of the impulse control kind. Importantly, away from the stock-stealing constraint, if bonus payments occur they can be understood as a way to manage the implementability of promise keeping.

For certain parameter restrictions, and with sufficient initial cash within the firm, the value function can achieve the DeMarzo-Sannikov value regardless of what the cost of raising outside financing are. Nevertheless, even though the total value is the same, not any implementation that is discussed in DeMarzo-Sannikov will be admissible, with the admissibility always a function of the cost of raising outside financing and the value function of the principal across the whole state-space. When there are prohibitive cost of raising additional cash, there exists a unique implementation of the DeMarzo-Sannikov solution that results in placing the firm on the stock-stealing boundary and keeping it there — there is an immediate and continuous dividend payments to the principals, and only occasional impulse based bonus payments that coincide with impulse based dividends.

With sufficiently restricted initial cash available to form the firm (but no restriction on promised value to the agent besides promise keeping), we can show that the firm is initialized away from the stock-stealing boundary and there is no immediate dividend payments or bonus payments. This solution cannot be achieved in the DeMarzo-Sannikov framework because it relies on the separation of the cash holdings within the firm from the promised value to the agent. The firm promises the agent more than he could possibly steal to decrease the probability of liquidation.

**Related Literature** This paper contributes to the growing literature on optimal dynamic contracting in continuous time pioneered by Sannikov (2008) and DeMarzo and Sannikov (2006) (hereafter DS). Like DS, we consider a risk neutral manager who may privately divert cash from firm owned by a risk neutral shareholder. They derive an optimal contract that can be implemented via equity, long term debt, and a credit line. Importantly, they assume that investors can make long term commitments to inject cash into the firm to cover cash flow short falls, i.e. via a credit line. We depart from this setting by assuming that investors cannot transfer cash into the firm after time zero. A number of other recent papers have expanded upon the continuous time principal-agent model of DS, including He (2009), He (2011), DeMarzo, Livdan, and Tchistyi (2014). Feng (2014) considers a continuous time principal agent problem with principal limited commitment. Unlike our setting, limited commitment in his paper means that the principal may liquidate the firm at any time and renege on the agents agreed upon contract.

Another branch of the theoretical literature has considered the role of cash within a firm. Décamps, Mariotti, Rochet, and Villeneuve (2011) present a model in which a firm accumulates cash to save on issuance costs. In their model, agency costs are exogenously specified as a reduction on the rate of return of cash held within the firm. In contrast, the agency costs in our model are fully endogenous. This difference leads us to find very different implications about the timing of investor payouts. In their paper, dividends are paid when the value of delaying refinancing costs equals the opportunity cost of keeping cash in the firm. In out model, payouts are made when the firm can no longer retain more cash do to agency problems.

There has also been a voluminous empirical literature on cash within the firm. Nikolov and Whited (2014) present a structural model of cash holdings integrating managerial perquisite consumption, limited managerial ownership, and size-based compensation. They find that perquisite cash diversion is an important determinant of cash holdings.

# 2 The Model

The model builds on the continuous time dynamic contracting framework of DS. As in their model, we consider an investor who contracts with a manager to operate a firm. Cash flow is unobservable and can be diverted by the manager for her private consumption. Our main point of departure is that we assume limited liability for the investor in the following sense. Negative cash flow shocks, payments to the manager, and dividends to share holders must be funded entirely through cash within the firm. The investor can transfer cash to the firm, i.e. by issuing new equity, but at some cost. As a result, the investor will optimally accumulate cash within the firm from retained earnings. At the same time, cash inside the firm is subject to diversion in the same manner as cash flow. Which in turn implies that the investors cannot accumulate too much cash within the firm, lest the manager have an incentive to abscond with it.

## 2.1 Technology, Preferences, and the Moral Hazard Problem

Time is continuous and measured by $t$. A risk-neutral investor contracts a risk-neutral manager to operate a firm. The investor has the discount rate $r$ while the manager has the discount rate $\gamma > r$. The firm produces cumulative operating cash flow $X_t$ according to the following dynamics

$$dX_t = \mu dt + \sigma dZ_t \tag{1}$$

where $Z_t$ is a standard Brownian motion. We refer to $dX_t$ as simply cash-flow shocks. The firm can also maintain a positive cash balance $C_t$ that returns the risk-free rate $r$. For the moment, we focus the case when the firm is unable to raise cash after inception, i.e. there are infinitely costly financial constraints on the firm. In this case, all payouts, including negative cash flow shocks, must be paid out of the firm's cash balance. We will relax this assumption in Section **??** Thus the total cash balance of the firm has the following dynamics

$$dC_t = (rC_t + \mu)dt + \sigma dZ_t - dB_t - dD_t \tag{2}$$

where $dB_t$ denotes cash payouts to the manager and $dD_t$ denotes dividends paid to the investors. We assume limited liability for both the manager and the investor so that $dB_t \geq 0$ and $dD_t \geq 0$.

The manager observes the true cash-flow shocks $dX_t$, while the investors can only observe the manager's report $d\hat{X}_t$. The manager consumes the difference between the manager's report and the true cash-flow for her own benefit at the rate $\lambda \leq 1$.[1] Given a cash payout process $dB_t$ to the manager, she has the following utility from working at the firm

$$W_t = \left[ \int_t^{\tau} e^{-\gamma s} (dB_s + \lambda(dX_s - d\hat{X}_s)) \right], \tag{3}$$

where $\tau$ is the liquidation time of the firm conditional on the history of reported shocks and stipulated in the contract. Note there are two reasons that the investors liquidate the firm in this model. The first is that the investors choose to terminate the employment of the manager. This is the standard reason for liquidation in the dynamic contracting literature. The second is that the firm runs out of cash, in which case it can not cover negative cash-flow shocks. We call a contract $(\tau, B_t)$ *incentive compatible* if the managers always finds it optimal to report the true cash flow. Given the report of the manager $\hat{X}_t$, the investor receives the value

$$V_0 = \left[ \int_0^{\tau} e^{-rs} dD_t \right]. \tag{4}$$

In order to simplify the contracting problem, we restrict attention to incentive compatible contracts by virtue of the following revelation principle result.

**Lemma 1.** *For any contracts with implements the reporting strategy $\hat{X}_t$, there is an alternative contract which implements $X_t$ and gives the investors at least as much value.*

## 2.2 Incentive Compatibility, Promise Keeping, and Optimal Contracting

We now turn to characterizing the set of incentive-compatible contracts. First we note that by the martingale representation theorem and arguments that are now standard in the continuous time dynamic contracting literature, there exists a process $\beta_t$ adapted to the filtration generated by $X_t$ such that

$$dW_t = \gamma W_t + \beta_t(d\hat{X}_t - \mu dt) - dB_t \tag{5}$$

[1] We do note consider private saving for the manager, thus over-reporting is not feasible. There is no difficulty in relaxing this assumption, however it does not lead to additional results.

where $W_t$ is given by equation (3). If the manager chooses to under-report cash flow, she get the immediate gain of $\lambda(dX_t - d\hat{X}_t)$, while losing $\beta_t(dX_t - d\hat{X}_t)$ in continuation utility. Thus, the manager will find it optimal to never divert cash flow if

$$\beta_t \geq \lambda. \tag{6}$$

While setting $\beta_t$ to satisfy Equation (6) provides the manager with incentives not to divert cash out of operating cash flow, it is not necessarily feasible for the following reason. Since, the manager has limited liability, her continuation utility is bounded below by zero, so that the greatest decrease in continuation utility the investor can deliver to the manager is $W_t$. At the same time, the manager may simple report a negative cash flow shock that completely wipes out the cash balance of the firm. Doing so would lead to a decrease in continuation utility of $\beta_t C_t$. Thus, in order to guarantee that the contract is feasible, we require

$$W_t \leq \beta_t C_t. \tag{7}$$

While Equation (6) and (7) characterize incentive compatible contracts, the firm must have sufficient resources to deliver the continuation utility $W_t$ to the agent at any time. This is in contrast to the standard dynamic contracting literature in which the principal (in our case the investor) can commit to paying the agent (the manager) wages regardless of the cash flow of the project. In the current setting, all negative cash flows must be deducted from the cash balance of the firm. If there is zero cash left in the firm, then it must liquidate and cannot make any further payment to the manager. Thus $C_t = 0$ implies $W_t = 0$. Now consider what takes place when the firm has a relatively low cash balance, i.e. $C_t$ only slight greater than zero and much less than $W_t$. In this case, there is a positive probability that a sequence of negative cash-flow shocks will lead the cash balance of the firm fully depleted but not lead to zero continuation utility for the manager as prescribed by the dynamics given in Equation (5). Such a sample path must have zero probability for $W_t$ to indeed be the true continuation utility of the manager. Put simply, it must be impossible for the firm to run out of cash before completely paying of the agent. This intuition leads to the following lemma.

**Lemma 2.** *The firm must maintain sufficient cash reserves to cover promised payments to the manager:*

$$W_t \leq C_t. \tag{8}$$

Finally, we call a contract and dividend policy *optimal* if it maximizes the investors value at time zero while respecting the incentive compatibility, feasibility, and liquidity constraints.

## 3 A special case: $\lambda = 1$

In principle, the dynamic contracting problem we have specified so far requires to state variables: the firm's cash balance and the manager's continuation utility. Although we will discuss cases in which both state variables are necessary in Section 4, we begin our analysis of the optimal contracting problem with the special case when the manager can perfectly divert cash flow, i.e., $\lambda = 1$. This assumption will turn out to imply that the firm's cash balance and the manager's continuation utility deterministically co-move so that the optimal contracting problem can be characterized with only one state variable.

First we derive the implications of the incentive feasibility and liquidity constraints for the optimal contract and dividend policy. On the one hand, constraint (7) implies that $C_t \leq W_t$, otherwise it is impossible to prevent the manager from diverting the entire cash balance of the firm. On the other hand, constraint (8) implies that $C_t \geq W_t$, otherwise the firm may not have sufficient liquidity to deliver the manager her promised continuation utility. Thus, the incentive feasibility and liquidity constraints together imply that $W_t = C_t$ for all time $t > 0$. If firm starts life with an excess of cash, or $W_0 < C_0$, then it must pay and immediate lump sum dividend $dD_0 = C_0 - W_0$. Recall that we have assumed that $W_0 \leq C_0$, so that the firm always has sufficient liquidity at time zero to commence operations.

Now we characterize the dynamics of the manager's continuation utility and the firms cash balance after time zero. We have seen that constraints of the contracting problem require that $W_t = C_t$, i.e. these two processes must have identical dynamics. To equate the dynamics of these two processes, the firm must either disgorge cash or make cash payouts to the agent. When the return on the firm's cash plus operating cash flow exceeds the required expected return of the agent, that is when $rC_t + \mu \geq \gamma W_t$, the firm must payout the excess to the investor in order to maintain the feasibility constraint. To do so, the dividend process must follow

$$dD_t = \mu + rC_t - \gamma W_t \tag{9}$$

$$= \mu - (\gamma - r)W_t. \tag{10}$$

These payouts are only feasible when $W_t \leq \mu/(\gamma - r)$, but for the moment suppose that this is always the case. In addition to the dividend process, we must also equate the volatility terms of the cash and continuation utility dynamics to get $\beta_t = 1$.

We next determine the manager's bonus process. First note that if the firm makes a cash payment to the manager, the cash balance of the firm decreases by precisely the same amount. Thus if it is optimal for the firm to make such a payment, it must be the case that

$$V(C, W) \leq V(C - dB, W - dB) \tag{11}$$

where $V$ denotes the value function of the investor. Dividing both sides by $dB$ and letting $dB \to 0$ we see that cash payments to the agent occur whenever $V_W \leq -V_C$. We conjecture that there is given threshold $\overline{W}$ such that the manager receives a payment whenever $W_t \geq \overline{W}$ given by $dB_t = (W_t - \overline{W})^+$. To pin down $\overline{W}$ we use standard dynamic programing techniques.

In principle, both the cash balance of the firm as well as the manager's continuation utility are state variable for the contracting problem. Recall however that $W_t = C_t$ for all $t > 0$ for the current special case of $\lambda = 1$, so it is enough to consider $W_t$ as the only state variable. Let $g(W) = V(W, W)$, an application of Ito's formula for the region in which $dB = 0$ yields the following ordinary differential equation

$$rg(W) = \mu - (\gamma - r)W + \gamma W g'(W) + \frac{1}{2}\sigma^2 g''(W). \tag{12}$$

With the boundary condition at liquidation

$$g(0) = 0, \tag{13}$$

and the boundary conditions at $\overline{W}$

$$g'(\overline{W}) = 0 \tag{14}$$

$$g''(\overline{W}) = 0. \tag{15}$$

Finally note that if $\overline{W}$ satisfies Equations (12)-(15), then

$$\overline{W} = \frac{\mu - rg(\overline{W})}{\gamma - r} \leq \frac{\mu}{\gamma - r}. \tag{16}$$

since $g(W) \geq 0$. Since $C_t = W_t$ and $W_t \leq \overline{W}$, Equation (16) implies that the dividend process given by Equation (10) is always positive.

**Proposition 1.** *If the manager can costlessly divert cash, i.e. $\lambda = 1$, then the optimal contract and dividend policies are given by*

1. $dD_0 = C_0 - W_0$
2. $dD_t = (\mu - (\gamma - r)W_t)dt$
3. $dB_t = (W_t - \overline{W})^+$

*where $\overline{W}$ solves Equations (12)-(15).*

### 3.1 Relation to DeMarzo/Sannikov and Security Design

The contract given in Proposition 1 above leads to and a identical value function (net of cash) and payment boundary as in DS, however the implementation is quite different. To see why the two contracting problems lead to the same value for the investors, recall that the DS contracting problem is essentially identical to the own given about except that in their setting, the firm need maintain a cash balance. Thus, the only source of inefficiency at the optimal contract in either problem is due to early liquidation. In contrast, cash held in the firm earns the interest at the investors discount rate and is therefore not a source of inefficiency. Regardless of the this equality, the security design implementation of the optimal contract in DS does not work in our setting. It requires that the firm have access to a credit line, a form of contractually agreed upon violation of limited liability for the investor.

Any feasible implementation in our setting must both uphold limited liability for the investors through an internal cash balance while at the same time maintaining a small enough cash balance to keep the manager from stealing. These two conditions combine to give the payout rate (per unit of time) shown in Figure 1. We can see that as the cash balance (equal to the manager's continuation utility) decreases the payout rate to the investors actually increases. This may seem counter intuitive at first as one might expect that as the cash balance of the firm increases, it would be optimal to cut payouts to accumulate more cash and forestall liquidation. Note however that as long the manager is not receiving any payout, the liquidity and incentive feasibility constraints set the expected growth rate of the cash in the firm equal to the expected growth rate of the manager's continuation utility. Consequently, the total cash flow that can be retained while maintaining incentive feasibility is $\gamma C$. At the same time, the total expected cash flow of the firm is $rC + \mu$. The difference between these two quantities increases as the cash balance of the firm decreases. Intuitively, when the firm is close to liquidation, the manager has low continuation utility, and can therefore only be trusted with a small amount of additional cash. At the same time the firm is still expected to produce at least $\mu$ in cash flow. Thus agency problem forces the firm to increase payouts as the cash balance decreases.

The payout rate given in Figure 1 naturally leads to an implementation with performance sensitive debt given in the following proposition.

**Proposition 2.** *The contract in Proposition 1 can be implemented by paying out excess cash to the investor at time zero, granting the manager a* 100% *equity claim on the firm, and granting the investor a debt contract with face value* 1 *and the following features*

1. *(Seniority) No equity dividends may be paid until all coupon payments to debt holders have been paid*

2. *(Performance Sensitivity) The debt coupon rate is set to* $\mu - (\gamma - r)\min\{C_t, \overline{W}\}$

The roll of performance sensitive debt is important to discuss here. A leading explanation for this type of security as laid out by Manso, Strulovici, and Tchistyi (2010) is that it allows for the screening of firms. In our setting, there is no asymmetric information about firm type. Rather as the firm depletes its cash balance, it's capacity to increase that balance decreases because of the state of the agency problem. This occurs because the wedge between the expected return of the manager and the total expected cash flow increases as shown in Figure 1. The coupon rate of the bond exactly corrects for the difference.

# 4 Costly Cash flow diversion

We now relax the assumptions of the previous section and allow for cash flow diversion to entail some loss,i.e., $\lambda < 1$. The main difficulty in relaxing this assumption is that the manager's continuation utility will no longer be a sufficient state variable for the contract problem. We now provide a heuristic derivation of the optimal contract. Again let $V(C, W)$ denote the value function of the investor. We hypothesis that there exists a region, which will call the interior, such that it is optimal to set payouts to both the investors and the manager equal to zero. In this region and application of Ito's formula yields the following Hamilton-Jacobi-Bellman equation for $V$

$$rV = \max_{\beta \geq \lambda} \left\{ \gamma W V_W + (rC + \mu) V_C + \frac{1}{2}\sigma (V_{CC} + 2\beta V_{CW} + \beta^2 V_{WW}) \right\}. \tag{17}$$

The left hand side is the required return of the investor. The objective function on the right hand side is the investors expected capital gain. Note that in the interior, the investor recieves zero payouts so that the investors required return must be equal to her expected capital gain. For the moment, we assume that

$$V_{CW} + \beta V_{WW} \leq 0$$

for all $\beta \geq \lambda$, we will then verify that this is the case when we prove optimality of the contract. Given this assumption, it is clearly optimal to set $\beta = \lambda$. This leads to the following partial differential equation (PDE) for the investors value on the interior

$$rV = \gamma W V_W + (rC + \mu) V_C + \frac{1}{2}\sigma (V_{CC} + 2\lambda V_{CW} + \lambda^2 V_{WW}). \tag{18}$$

We will require a number of boundary conditions for this PDE. The first boundary condition follows from the liquidity constraint given in Lemma 1. When the managers promised utility exceeds the cash balance of the firm, the investor must liquidate the firm and immediately pay the manager in order to maintain promise keeping. This leads to the following boundary condition

$$V(W, W) = 0. \tag{19}$$

This boundary condition should be understood to mean that if $(W, W)$ is a point on the boundary of the interior, then it firm must be liquidated if this point is every reached.

The next boundary derives from the incentive feasibility constraint. Whenever the manager's continuation utility $W$ exceeds the amount she could get by stealing the entire cash stock, the firm must reduce its cash balance by making a payout to investors. Formally, if $\lambda C_t \geq W_t$, the firm must payout $C_t - W_t/\lambda_t$ to shareholders. On the boundary $C = W$, this condition states

$$V_C(\lambda C, C) = 1. \tag{20}$$

Intuitively, since the firm must make a payout to investors at $\lambda C = W$, the marginal value of a dollar of cash in the firm (the left hand side) must be equal to the marginal value of a dollar paid to investors (the right hand side).

The final set of boundary conditions pins down when optimal contract calls for cash payment to the manager. We guess that there exists a boundary to the interior, which we call the bonus boundary, given by a function $\overline{W}(C)$ such that when $W \geq W(C)$, the contract calls for an immediate payout to the manager to move the point $C, W$ back to the boundary of the interior. For such a contract to be optimal, it must be the case that

$$V(\overline{W}(C) + dB, C + dB) \leq V(\overline{W}(C), C). \tag{21}$$

Deviding both sides by $dB$ and letting $dB$ go to zero, gives the following boundary condition for $\overline{W}(C)$.

$$V_W(\overline{W}(C), C) + V_C(\overline{W}(C), C) = 0. \tag{22}$$

To guarantee the optimality $\overline{W}(C)$, we must have

$$V_{CC}(\overline{W}(C), C) + 2V_{CW}(\overline{W}(C), C) + V_{WW}(\overline{W}(C), C) = 0. \tag{23}$$

**Proposition 3.** *The optimal contract is given by*

1. $dD_0 = (C_0 - W_0/\lambda)^+$

2. $dD_t = \mathbb{1}\{W_t = \lambda C_t\}(\mu - (\gamma - r)C_t)^+ dt$

3. $dB_t = (W_t - \overline{W}(C_t))^+$

*where* $\overline{W}(\cdot)$ *solves Equations (18),(19), (22),and (23) (provided that such a solution exists).*

## 4.1 Dividend Dynamics

Let $\hat{W} = \lambda\mu/(\gamma - r)$. Note that on the dividend boundary for $W < \hat{W}$, Proposition 3, calls for an investor payout of $(\mu - (\gamma - r)W/\lambda)dt$. For $W > \hat{W}$, no dividend need be paid to the investors, as the joint dynamics of $W_t$ and $C_t$ tend toward the interior of the region. Next, let $\overline{W}^*$ be defined by the intersection point of the bonus boundary and the dividend boundary:

$$\overline{W}^* = \overline{W}\left(\frac{\overline{W}^*}{\lambda}\right)$$

If $\overline{W}^* < \hat{W}$, then if $W_t = \lambda C_t$ then $W_s = \lambda C_s$ for all $s \geq t$. In other words the dividend boundary is absorbing. This begs the question of under what circumstances will $\overline{W}^* < \hat{W}$. To address this question we examine the behavior of the value function on the dividend boundary.

Define $g(W)$ by

$$g(W) \equiv V(W, W/\lambda), \tag{24}$$

and note that

$$g'(W) = V_W(W, W/\lambda) + \frac{1}{\lambda}V_C(W, W/\lambda) \tag{25}$$

$$g''(W) = V_{WW}(W, W/\lambda) + \frac{2}{\lambda}V_{CW}(W, W/\lambda) + \frac{1}{\lambda^2}V_{CC}(W, W/\lambda) \tag{26}$$

Plugging these expressions into (18) yields an ODE on the dividend boundary for $W < \hat{W}$:

$$rg(W) = \mu - \frac{\gamma - r}{\lambda}W + \gamma W g'(W) + \frac{1}{2}\sigma^2\lambda^2 g''(W) \tag{27}$$

with boundary conditions

$$g\,(0) = 0 \tag{28}$$

$$g'(\overline{W}^*) = \frac{1-\lambda}{\lambda} \tag{29}$$

$$g''(\overline{W}^*) = 0. \tag{30}$$

To get the second boundary condition above we used the fact that at $\overline{W}$, we have $V_C = -V_W$ and $V_C = 1$ since the contract calls for dividends. The third boundary condition guarantees the optimality of $\overline{W}^*$. This leads to the following proposition

**Proposition 4.** *Let $\overline{W}^*$ solve Equations (27)-(30). If $\overline{W}^* < \hat{W}$, then the optimal contract can be by paying out excess cash to the investor at time zero, granting the manager a $\lambda$ equity claim on the firm, and granting the investor the remaining equity and a debt contract with face value* 1 *and the following features*

1. *(Payment discretion) Debt holders can choose to suspend coupon payments.*
2. *(Performance Sensitivity) The debt coupon rate is set to $\mu - (\gamma - r)\min\{C_t/\lambda, \overline{W}^*\}$*
3. *(Seniority) No equity dividends may be paid until coupon rate reaches it's minimum.*

A key difference between the contract given in Proposition 4 and DS is that the manager's continuation utility and the firms available liquidity are no longer perfectly link. From the standpoint of empirical predictions, this means that a firms financial slack is not necessary a measure of the the state the agency conflicts within the firm.

### 4.2 Numerical Solution

Although Equations (18),(19), (22),and (23) do not readily admit a closed from solution. We can solve this problem using finite difference estimation detailed in the Appendix. Figure 2 show the solution to $\overline{W}(C)$ for a given set of parameters. Some features of the boundaries important to note are as follows. First, not that $\overline{W}(C)$ is upward sloping. This means that when the firm has less cash, the manager receives bonuses earlier. The intuition for this feature of the optimal contract is that, holding $W$ constant, inefficient liquidation is more likely when $C$ is smaller. Rather than allow liquidation to become for likely , the firm reduces the magnitude of its obligation to the manager in order to bring the cash balance of the firm and its promises in line.

In addition to the bonus boundary. It is also interesting to examine properties of the value function. Specifically, we ask the what is the optimal starting point for the manager's continuation utility given a fixed amount of initial cash in the firm. Figure (3) shows the value function for four different amounts of initial cash. When cash is high or low, the value function is monotonically decreasing. As such, it is optimal to start the firm with as little continuation utility granted to the manager as allowed by the participation constraint. When initial cash is neither very low or very high, the value function is upward sloping for low levels of continuation utility. It is then optimal for the investor to grant additional continuation utility above and beyond what is required by the participation constraint.

## 5 Conclusion

We have presented a model of cash dynamics within a firm plagued by an agency conflict between the manager and the owners of the firm. In addition to the agency problem, owners cannot costlessly transfer cash into the firm, creating a roll for cash within the firm. Performance sensitive debt implement can the optimal payout policy. Interestingly, the manager's continuation utility and the cash balance of the firm are not necessarily perfectly correlated. Future work will be to include

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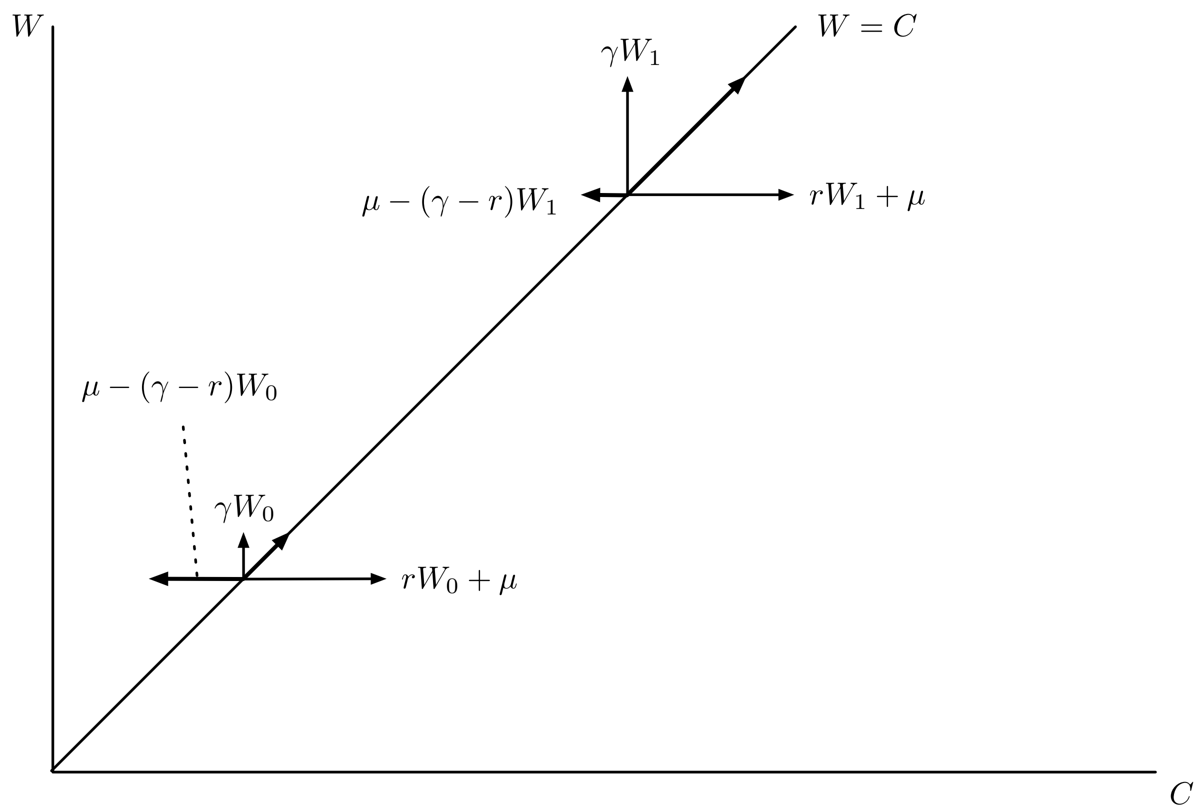


**Figure 1.** The payout rate to investors is given by the left pointing vector. When the firm has a low cash balance, it must also be the case that the manager has low continuation utility. In which case, the firm can only retain a relatively small amount of cash flow due to the incentive feasibility constraint. The remainder must be paid out to the investor in order to guarantee that incentive feasibility holds.

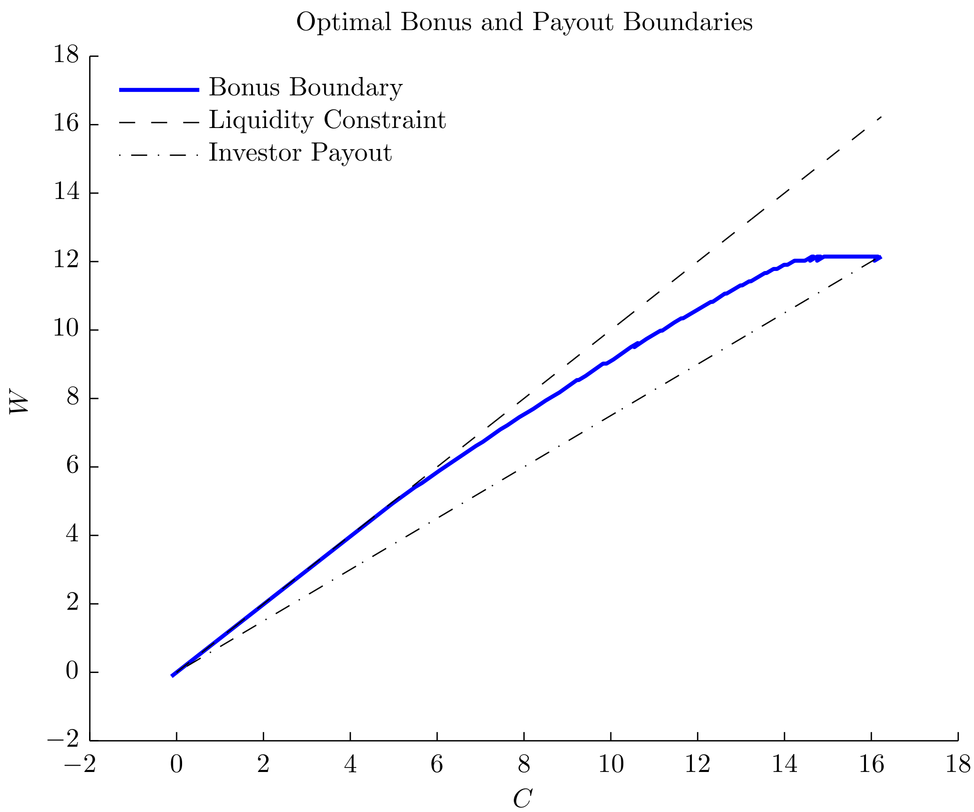


**Figure 2.** The optimal bonus boundary $\overline{W}(C)$ and payout boundary. The dashed line represents the liquidity constraint. If the cash balance of the firm $C$ falls below the manager's continuation utility $W$, the firm must liquidate. The dash-dotted line represents the incentive feasibility constraint. If the $C$ exceeds $W/\lambda$ the firm must make a payout to investors to maintain incentive compatibility. The solid curve is the bonus boundary at which the manager receives a payout. In the region bounded by the solid curve and the dash-dotted line, the firm makes no payouts.

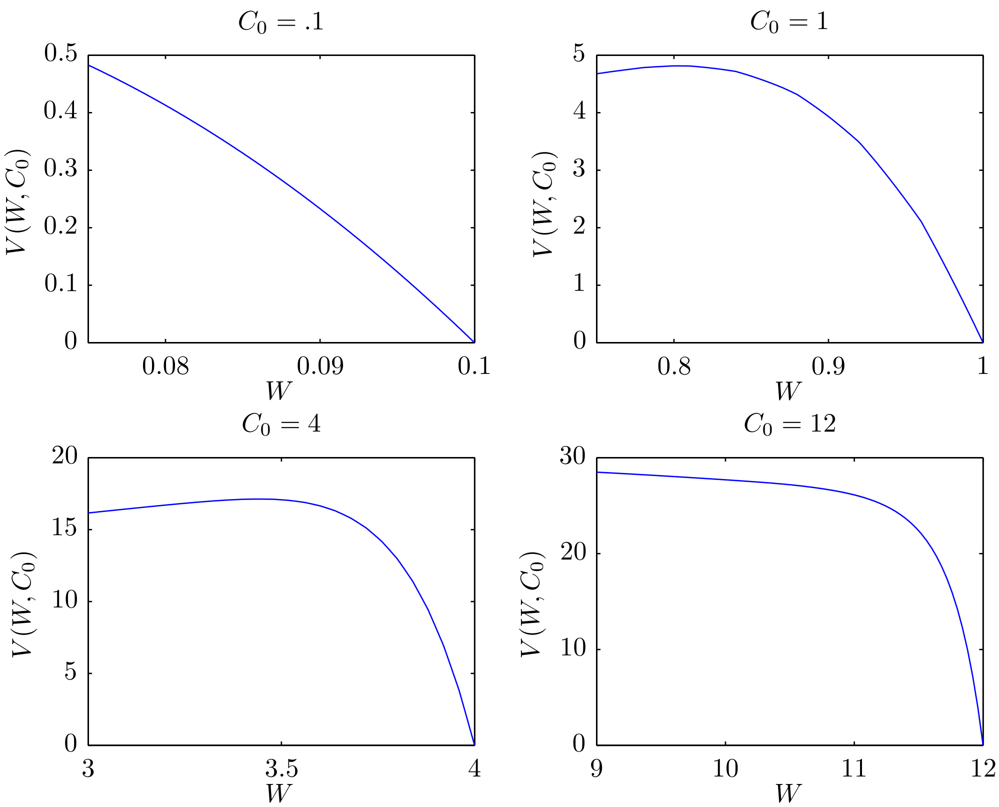


**Figure 3.** The value function for a fixed level of cash $V(C_0, W)$ for various amounts of cash. For $C_0$ low or high, the