Bank Capital and Aggregate Credit

Nataliya Klimenko* Sebastian Pfeil † Jean-Charles Rochet ‡

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PRELIMINARY AND INCOMPLETE

Abstract

This paper seeks to explain the role of bank capital in fluctuations of lending and output. We build a continuous time equilibrium model of an economy in which commercial banks finance their loans by deposits and equity, while facing issuance costs when they raise new equity. The dynamics of the loan rate and the volume of lending in the economy are driven by aggregate bank capitalization. The model has a unique Markov competitive equilibrium that can be solved in closed form. We show that the competitive equilibrium is constrained inefficient: banks lend too much in upturns and too little in downturns. However, imposing a standard capital regulation does not help.

Keywords: macro-model with a banking sector, bank capital, pecuniary externalities

JEL: E21, E32, F44, G21, G28

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*University of Zurich. E-mail: nataliya.klimenko@bf.uzh.ch.

†Goethe University Frankfurt. E-mail: pfeil@finance.uni-frankfurt.de.

‡University of Zurich, SFI and TSE-IDEI. E-mail: jean-charles.rochet@bf.uzh.ch.

1 Introduction

There is an ongoing debate among scholars and practitioners about the "right" level of bank capital. While proponents of higher capital ratios emphasize the stabilizing effect of bank capital,[1] others argue that high leverage is a direct consequence of banks' intrinsic role as creators of information insensitive, liquid debt (e.g. deposits).[2] The current paper brings together both aspects in a dynamic general equilibrium model where bank capital plays the role of a loss absorbing buffer that facilitates the creation of liquid, risk-free claims while providing (risky) loans to real businesses.[3]

We consider an economy where firms borrow from banks that are financed by deposits and equity. Banks continuously adjust lending to firms whose default probability varies depending on aggregate shocks.[4] They also decide when to distribute dividends and when to issue new equity. Equity issuance is subject to deadweight costs, which represents the main financial friction in our economy.[5] The aggregate supply of bank loans is confronted with the firms' demand for credit, which is decreasing in the nominal loan rate. Finally, as depositors are infinitely risk-averse, all losses that may be generated by the banks' loan portfolio must be borne by risk-neutral bank shareholders.

In a set-up without financial frictions (i.e., no issuance costs for bank equity) the equilibrium volume of lending and the nominal loan rate is constant. Furthermore, dividend payment and equity issuance policies are trivial in this case: Banks immediately distribute all profits as dividends and issue new shares to offset losses and honor obligations to depositors. This implies that, in a frictionless world, there is no need to build up a capital buffer and all loans are entirely financed by deposits.

In the model with financing frictions, banks' dividend and equity issuance strategy becomes more interesting. In the unique competitive equilibrium, the value of aggregate bank equity, which turns out to be a sufficient statistics for all relevant macro and financial

[1]Most prominently, Admati et al. (2010), Admati and Hellwig (2013).

[2]See DeAngelo and Stulz (2014) for an argument along these lines. More generally, the perception of financial intermediaries as creators of liquidity ("inside money") has a long tradition within the financial intermeditation literature (see, e.g. Diamond and Dybvig (1983), Diamond and Rajan (2001), Gorton (2010), Gorton and Pennachi (1990), Holmstrom and Tirole (1998, 2011)).

[3]As we abstract from incentive effects of bank capital ("skin in the game"), our model captures best the features of more traditional commercial banks whose business model makes them less prone to risk-shifting than investment banks. The incentive effects of bank capital in a setting allowing for risk-shifting are analyzed for instance in Martinez-Miera and Suarez (2012), DeNicolo et al. (2014), and van den Heuvel (2008).

[4]Aggregate shocks are i.i.d. and, given that banks hold diversified loan portfolios, they represent the only relevant source of risk in the economy.

[5]We follow the literature (see e.g. Décamps et al. (2011) or Bolton et al. (2011)) by assuming that issuing new equity entails a deadweight cost proportional to the size of the issuance. Empirical studies report sizable costs of seasoned equity offerings (see e.g. Ross et al. (2008), Hennessy and Whited (2007)).

variables, follows a Markov diffusion process reflected at two boundaries. Banks issue new shares at the lower boundary where book equity is depleted and its marginal value equals marginal issuance costs. When the book value of equity reaches its upper boundary, any further earnings are paid out to shareholders as dividends. At this upper reflecting boundary, the marginal value of equity, or *market-to-book value*,[6] equals the shareholders' marginal value of consumption. Between the two boundaries, the level of equity changes only due to retained earnings or absorbed losses. That is, banks retain earnings in order to increase the loss-absorbing equity buffer and thereby reduce the frequency of costly recapitalizations.

The fluctuations of aggregate bank equity drives the cost of credit. The risk-adjusted spread for bank loans turns out to be strictly positive (except at the dividend payout boundary). To get an intuition for this result, note that the changes in equity of an individual bank are mirrored by changes in aggregate equity, which in turn affects the market-to-book value of bank equity. Since loss absorbing equity is most valuable when it is scarce and is least valuable when it is abundant, a negative (positive) shock to bank equity increases (decreases) the market-to-book value of equity. Therefore, the original shock gets amplified and banks will only lend to firms if the loan rate incorporates an appropriate premium. As the market-to-book value, this premium is strictly decreasing in aggregate bank equity.

Based on our analytical solutions, we are able to study the long-run behavior of the economy, which can be described by an ergodic density function. Our analysis shows that the long-run behavior of the economy is mainly driven by the (endogenous) volatility of aggregate bank capital. In particular, the economy spends the most time in states with low endogenous volatility, and, for high recapitalization costs and a low elasticity of demand for bank loans, severe credit crunches can arise. The occurrence of credit crunches is caused by a simple mechanism. Assume that a series of adverse shocks has depleted aggregate bank capital. Since banks require a larger loan rate when aggregate capital is low, this drives down firms' credit demand. As a consequence, banks' exposure to macro shocks is reduced and thus also the endogenous volatility of aggregate equity, which finally leads to persistently low levels of equity (high bank leverage), high loan rates and low volumes of lending.

A welfare analysis of the competitive equilibrium shows that lending decisions made by banks are socially inefficient. Specifically, credit is excessive in upturns and insufficient in downturns.[7] The source of this two-sided inefficiency is rooted in the difference

[6]Since the model is homothetic in the level of book equity, the marginal value of equity can be interpreted as the market-to-book value.

[7]Similar results are found by He and Kondor (2014) in a model of corporate liquidity management and investment and by Gersbach and Rochet (2012) in a two-period model of banking.

between social and private cost of lending. Furthermore, the fact that we find total welfare to be increasing in aggregate bank capital suggests that welfare could be increased by implementing a minimum capital ratio (or leverage constraint). However, although a simple minimum capital ratio does, indeed, raise aggregate bank capitalization, it also drives up loan rates, thereby, depressing demand for credit and total output in the economy.[8] This result shows that a simple capital ratio might be not the right tool to address the identified two-sided inefficiency, as it aggravates the underinvestment problem when aggregate capital is low.

Related Literature. From the technical perspective, our paper is most closely related to the macroeconomic models with financial frictions that study the formation of asset prices in a dynamic endowment economy (see, e.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2013)). As in the above-mentioned papers, our model lends itself to studying the full equilibrium dynamics of the economy, in contrast to traditional macroeconomic models that only allow for analyzing the equilibrium around a steady state. At the same time, the problem of individual banks with respect to dividend distribution and recapitalization in our model shares some similarities with the liquidity management models in Bolton et al. (2011, 2013) and Décamps et al. (2011).

The dynamic effects in our model are driven by an endogenous leverage constraint based on the loss absorbing capacity of book equity in the presence of external financing frictions. Via this transmission channel, temporary shocks can have persistent effects on loan rates, which in turn amplifies the initial shock on book equity. This amplification mechanism is similar in spirit to the collateral constraint in Kiyotaki and Moore (1997) or the limitation of pledgeable income as in Holmstrom and Tirole (1997).

Since the reflection property of aggregate equity in our model generates quasi-cyclical patterns of lending, our paper is also related to the literature on credits cycles that has brought forward a number of alternative explanations for their occurrence. Fisher (1933) identified the famous debt deflation mechanism, that has been further formalized by Bernanke et al. (1996) and Kiyotaki and Moore (1997). It attributes the origin of credit cycles to the fluctuations of the prices of the assets that are used as collateral by borrowers. Several studies also place emphasis on the role of financial intermediaries, by pointing out the fact that credit expansion is often accompanied by a loosening of lending standards and "systemic" risk-taking, whereas materialization of risk accumulated on the balance sheets of financial intermediaries leads to the contraction of credit (see e.g. Aitken et al. (2013), Dell'Ariccia and Marquez (2006), Jimenez and Saurina (2006)).

Finally, our paper relates to the literature on pecuniary externalities. Recent con-

[8] In that sense, the general equilibrium effects that are at work in our dynamic model of bank capital give rise to similar implications as the ones brought forward by the critics of capital regulation (e.g. Institute for International Finance 2010 or Pandit (2010)), albeit for quite different reasons.

tributions by Lorenzoni (2008), Bianchi (2011), Jeanne and Korinek (2011) show that collateral price fluctuations can be the source of welfare decreasing pecuniary externalities, which could justify countercyclical public policies. Such pecuniary externalities can also be generated by agency problems (see e.g. Gersbach and Rochet (2014)) or market incompleteness (e.g. He and Kondor (2014)). In our model, the source of pecuniary externalities is rooted in the endogenous leverage constraint for banks: higher (lower) credit supply depreciates (pushes up) loan rates, banks' profitability and ultimately banks' capacity to lend in future periods.

The rest of the paper is structured as follows. Section 2 presents the discrete-time version of the model and discusses two useful benchmarks. In Section 3 we solve for the competitive equilibrium in the continuous-time set up and analyse its implications on financial stability and welfare. In Section 4 we study the impact of minimum capital requirements on bank policies. Section 5 concludes. All proofs and computational details are gathered in the Appendix.

2 The discrete-time model

To elucidate the main mechanisms that will be at work in the continuous time set-up, we start by formulating our model in a discrete time and then will let the length Δt of each period go to zero.

2.1 Model set-up

There is one physical good, taken as a numeraire, which can be consumed or invested. There are three types of agents: (i) depositors, who only play a passive role, (ii) investors, who own and manage the banks, and (iii) entrepreneurs, who manage the productive sector. Depositors are infinitely risk averse and discount the future at rate r. Investors and entrepreneurs discount the future at rate $\rho > r$ (i.e., they are more impatient) but they are risk neutral.[9]

2.1.1 Productive sector

The productive sector consists of a continuum of entrepreneurs controlling investment projects that are parametrized by a productivity parameter x. The productivity parameter x is privately observed[10] by each entrepreneur and is distributed according to a

[9]The difference between the discount factors ρ and r can be interpreted as a preference for liquidity: depositors accept a lower rate of return in exchange for perfect liquidity

[10]This assumption is only made to facilitate exposition, as it prevents contracts in which loan rate depends on x. Even when productivities were publicly observable, competition between banks would

continuous distribution with density function $g(x)$ defined on a bounded support $[0, \overline{R}]$.

Entrepreneurs' projects are short lived and each of them requires an investment of one unit of good. If successful, a project yields $(1+x\Delta t)$ units of good in the next period and zero otherwise. Entrepreneurs have no own funds and finance themselves via bank loans. Thus, the entire volume of investment in the economy is determined by the volume of bank credit. Entrepreneurs are protected by limited liability and default when their projects are not successful. Given a nominal loan rate $R\Delta t$ (for a loan of duration Δt), only the projects such that $x > R$ will demand financing. Thus, the total demand for bank credit in the economy will be

$$D(R) = \int_R^{\overline{R}} g(x)dx.$$

For simplicity, all projects are assumed to have the same default probability:

$$p\Delta t + \sigma_0\sqrt{\Delta t}\varepsilon_t,$$

where p is the unconditional probability of default per unit of time, ε_t represents an aggregate shock faced simultaneously by all firms and σ_0 reflects the change in the default probability caused by the aggregate shock. For simplicity ε_t is supposed to take only two values $+1$ (recession) and -1 (boom) with equal probabilities. Then, the net expected return per loan for a bank after an aggregate shock ε_t is

$$(R - r - p)\Delta t - \sigma_0\sqrt{\Delta t}\varepsilon_t, \tag{1}$$

where the first term reflects the expected earnings per unit of time and the second term captures the exposure to aggregate shocks.

At the equilibrium of the credit market, the net aggregate output per period in the economy is

$$[F(D(R)) - pD(R)]\Delta t - \sigma_0 D(R)\sqrt{\Delta t}\varepsilon_t, \tag{2}$$

where

$$F(D(R)) = \int_R^{\overline{R}} xg(x)dx$$

is the aggregate production function.

Note that $F'(D(R)) = R$ so that the total expected surplus, $F(D(R)) - pD(R) - rD(R)$, is maximized for $R_{fb} = r + p$. Thus, in the first best allocation of credit, the cost of funding for firms has two components: the riskless rate and the unconditional

lead to identical loan rates for all borrowers.

probability of default. Consequently, banks make zero expected profit, and the total volume of credit in the economy is given by $D(R_{fb})$.

2.1.2 Banking sector

Banks behave competitively and finance loans to businesses by a combination of deposits and equity.[11] Since we focus on credit, we do not introduce explicit liquidity provision activities associated with bank deposits. These deposits are modeled in a parsimonious fashion: depositors are infinitely risk averse and have a constant discount factor r. This implies two things: first, deposits must be absolutely riskless (all the risks will thus be borne by bank shareholders); second, depositors are indifferent as to the level of deposits and timing of withdrawal, provided that they receive an interest rate r. In sum, banks can collect any amount of deposits (i.e., deposits represent an infinitely inelastic source of funding), provided that they offer depositors the interest rate r and fully guarantee deposit value. As we will show in the sequel, the depositors' preference for safety is an important driver of the cost of credit in the economy.

The main financial friction in our model is that banks face a proportional issuance cost γ when they want to issue new equity.[12] Because of this deadweight issuance cost, banks will be reluctant to issue new equity too often and will mostly rely on retained earnings as a way to accumulate capital. For simplicity, we will neglect other external frictions such as adjustment costs for loans or fixed costs of issuing equity.[13] This implies that our economy exhibit a homotheticity property: all banks' decisions (lending, dividends, recapitalization) will be proportional to their equity levels. In other words, all banks will make the same decisions at the same moment, up to a scaling factor equal to their equity level. This entails an important simplification: only the aggregate size of the banking sector, reflected by aggregate bank capitalization, will matter for our analysis, whereas the number of banks and their individual sizes will not play any role.

2.2 One-period benchmark

To introduce the main tools of our analysis and to provide the basic intuitions behind the effects that we will encounter in a full-fledged continuous-time version of the model, we start with a simple static set-up in which banks only live one period. At the beginning of the period, bank shareholders have an exogenous amount of equity e_t. Given this equity

[11] Note that, in our setting, holding liquid reserves on top of risky loans is not optimal, because banks can continuously adjust the volume of deposits.

[12] On top of direct costs of equity issuance, γ may also captures inefficiencies caused by asymmetric information, which are not modeled here.

[13] We also disregard any frictions caused by governance problems inside the banks or government explicit/implicit guarantees.

level, they choose the volume of deposits d_t and lend $k_t = e_t + d_t$ to the productive sector. At the end of the period, the aggregate shock is realized, and the bank collects profit

$$k[(R - r - p)\Delta t - \sigma_0 \sqrt{\Delta t}\varepsilon_t].$$

For Δt small enough, this profit is negative (i.e., the bank incurs losses) when $\varepsilon_t = 1$. Since depositors are infinitely risk-averse, they will deposit their money into the bank only if they are certain to get it back at the end of the period. This implies that any potential losses must be absorbed by the bank's equity, i.e.,

$$e \geq k[\sigma_0 \sqrt{\Delta t} - (R - r - p)\Delta t]. \tag{3}$$

In other words, the depositors' need for safety imposes a leverage constraint on each bank, which must also be satisfied at the aggregate level:

$$E \geq K[\sigma_0 \sqrt{\Delta t} - (R - r - p)\Delta t]. \tag{4}$$

The competitive equilibrium of this static set-up is characterized by a loan rate $R(E)$ and a lending volume $K(E)$ that are compatible with the expected profit maximization by each individual bank:

$$\max_k \quad e + k(R - r - p)\Delta t \quad \text{s.t.} \quad (3)$$

and the loan market clearing condition

$$K(E) = D(R(E)).$$

It is easy to see that, depending on the level of initial capitalization, two cases are possible.

Case 1: well-capitalized banking sector. When aggregate bank capitalization is sufficiently high, the leverage constraint does not bind and thus the equilibrium loan rate is given by $R^*(E) = r + p \equiv R_{fb}$, which corresponds to the First-Best allocation of credit. Specifically, this case is feasible for such E that satisfies

$$E \geq E^* \equiv D(R_{fb})\sigma_0 \sqrt{\Delta t}.$$

Case 2: undercapitalized banking sector. When aggregate bank capitalization is low, the leverage constraint binds and the equilibrium loan rate is defined implicitly by the value $R^*(E)$ such that

$$D(R^*)[\sigma_0\sqrt{\Delta t} - (R^* - r - p)\Delta t] = E. \tag{5}$$

Since the left-hand side of (5) is decreasing in R, the loan rate $R^*(E)$ resulting from (5) is decreasing in E (for $E < E^*$), which implies that $R^*(E) > R^*(E^*) = r + p$. Thus, this parsimonious model illustrates an important idea: it is the limited loss absorbing capacity combined with the need to guarantee riskless investment to depositors that drives the loan rate away from its First-Best level.

Note that the shareholder value of any individual bank is proportional to its book value e and depends on aggregate capitalization. This implies that the market-to-book ratio is the same for all banks:

$$\frac{v(e,E)}{e} \equiv u(E) = 1 + \frac{(R(E) - r - p)\Delta t}{\sigma_0\sqrt{\Delta t} - (R(E) - r - p)\Delta t}.$$

It is easy to see that, when the banking sector is well capitalized (i.e., $E \geq E^*$), the market-to-book ratio equals one. In the alternative case ($E < E^*$), it is strictly higher than one and a decreasing function of aggregate bank capitalization:[14]

$$u(E) = \frac{\sigma_0\sqrt{\Delta t}}{\sigma_0\sqrt{\Delta t} - (R(E) - r - p)\Delta t} > 1, \quad E < E^*.$$

As will become apparent below, these properties of the market-to-book value also hold in the continuous time set-up.

Finalizing the discussion of this one-period benchmark, it is worthwhile to note that, in this static set-up, the competitive equilibrium is constrained efficient. To see this property, consider a maximization problem of a social planner. A social planner would choose a volume of lending $K(E)$ that maximizes social welfare under the aggregate leverage constraint:

$$W(E) = \max_K \quad E + [F(K) - (r+p)K]\Delta t \quad \text{s.t.} \tag{6}$$

$$E \geq K[\sigma_0\sqrt{\Delta t} - (F'(K) - r - p)\Delta t]. \tag{7}$$

Note that the social welfare function $W(E)$ is increasing and concave. Moreover, $W'(E) \geq 1$, with strict inequality when $E < E^*$. Given these observations, it is easy to see that the solution of the social planner's problem coincides with the competitive equilibrium allocation, which is therefore constrained efficient. However, we will show

[14] Recall that $R'(E) < 0$ when $E < E^*$.

below that it is no longer true in a setting with multiple periods.

2.3 Two-period benchmark

Suppose now that banks live two periods ($t = 0, 1$), and assume for simplicity that there is no shock in period 0 (i.e., $\varepsilon_0 = 0$). In that case, the banks are not subject to a leverage constraint. The only loan rate R_0 that is compatible with the profit maximization is $R_0 = r + p$ and the volume of credit is then given by $K_0 = D(R_0)$. Thus, in the initial period, banks make zero profit and aggregate equity remains constant ($E_0 = E_1 \equiv E$). The equilibrium loan rate at date $t = 1$ is then set to $R^*(E)$, as in the one-period benchmark.

However, in contrast to the one-period benchmark where a social planner's choice of the volume of lending coincides with the competitive outcome, in the current setting a social planner would choose a higher volume of lending in period 0. Indeed, intertemporal welfare W_0 now includes a term corresponding to the firms' profit in period 0:[15]

$$W_0(E_0) = \max_K \quad [F(K) - KF'(K)]\Delta t + W(\underbrace{E_0 + K[F'(K) - r - p]\Delta t}_{E_1}) \tag{8}$$

The profit of the banks in period 0 is retained and appears in the argument E_1 of the welfare function for period 1. The first-order condition of this problem is

$$0 = -KF''(K) + W'(E_1)[F'(K) - r - p + KF''(K)],$$

which enables us to express the corresponding loan rate as follows:

$$R_{SB} \equiv F'(K_{sb}) = r + p + \Big(- \frac{K_{SB}F''(K_{sb})}{F'(K_{sb})} \Big) \Big[1 - \frac{1}{W'(E_1)} \Big].$$

The above expression suggests that, when $W'(E_1) > 1$ (undercapitalized banking sector) and $F''(K) < 0$ (elastic loan rate), $R_{sb} > R_{fb} \equiv r + p$. The reason is that the reduction of lending below $K^* = D(R_{fb})$ allows the banks to increase their capitalization in period 1. When there are frictions on financial markets, and banks cannot recapitalize costlessly, a reduction in lending in period 0 increases banks' profit and thus relaxes their leverage constraint in period 1.

Two important remarks are in order at this stage. First, the leverage constraint in our model comes from the depositors' need for a safe investment. In such a context, bank equity plays the role of a loss absorbing buffer, very much in the spirit of the

[15] For simplicity, we do not allow for discounting and set ρ to zero for this subsection only.

"loss absorbing capacity" concept that is put forward by regulators. This justification of leverage constraints differs from the other ones that are put forward by academics. One of them relates to the limitation on pledgeable income by bank insiders (Holmstrom and Tirole (1997)) and backs the concept of "inside" equity that is supposed to play the role of "the skin in the game". The other academic justification for leverage constraints stems on the limited resalability of collateral (Kiyotaki and Moore (1997)), thereby, placing emphasis on the asset side of borrowers' balance sheets, whereas we focus here on the banks' liabilities.

Second, it is worthwhile to note that, even though the leverage constraint only binds in the period 1, it also impacts the banks' choices in the period 0, i.e., there is an intertemporal effect. If we also allow for a shock in period 0 (we ruled it off only for a simpler exposition), then a leverage constraint also appears in period 0, but it is not always binding, in contrast to period 1. The reason is that the market-to-book value of banks $u(E)$ fluctuates with E, and bankers are aware that they are exposed to the same aggregate shocks. Therefore, each bank's loss is magnified by an increase in $u(E)$ and, vice-versa, each bank's gain is reduced because of the decrease in $u(E)$. As we will see in the continuous-time setting, there exists a particular value of the loan spread $R(E) - p$ that exactly compensates for the fluctuation of the market-to-book value, and allows for an interior choice of lending volume by banks.

3 The continuous-time model

In this section we turn to the full-fledged continuous-time model. For the rest of the paper we assume that $r = 0$ and solve the model for the case $r > 0$ in Appendix D.

Taking the continuous time limit of net bank income per loan, we obtain:

$$(R - p)dt - \sigma_0 dZ_t, \tag{9}$$

where $\left\{Z_t, t \geq 0\right\}$ is a standard Brownian motion.

Given the homotheticity property of our economy, it is legitimate to anticipate the existence of a Markovian competitive equilibrium, where all aggregate variables depend on a single state variable following a Markov process. Aggregate bank equity, E_t, turns out to be a natural candidate for the state variable in our framework. Indeed, as we could clearly see in the discrete-time benchmarks, it is the very level of bank capitalization that drives the cost of credit. The dynamics of aggregate equity E_t satisfies

$$dE_t = K(E_t)[(R(E_t) - p)dt - \sigma_0 dZ_t] - d\Delta_t + dI_t, \tag{10}$$

where $K(E_t)$ is the aggregate lending, $d\Delta_t \geq 0$ reflects aggregate dividend payments and $dI_t \geq 0$ captures aggregate equity injections.

Definition 1 *A Markov competitive equilibrium consists of an aggregate bank capital process E_t, a loan rate function $R(E)$ and a credit volume $K(E)$ that are compatible with the individual banks' maximization and market clearing condition $K(E) = D[R(E)]$.*

In the following subsections we solve for the competitive equilibrium and study its implications on financial stability and social welfare.

3.1 The competitive equilibrium

To characterize the competitive equilibrium, we have to determine the optimal recapitalization and financing decisions of banks at the individual and aggregate levels, as well as the mapping $R(E)$ between the aggregate level of bank equity E_t and the loan rate R_t.

Consider first the optimal decision problem of an individual bank that takes the loan rate $R_t = R(E_t)$ as given and makes its decisions based on the level of its own equity e_t and aggregate equity E_t. Bank shareholders choose lending $k_t \geq 0$, dividend $d\delta_t \geq 0$ and recapitalization $di_t \geq 0$ policies so as to maximize the market value of equity:[16]

$$v(e, E) = \max_{k_t, d\delta_t, di_t} \quad \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(d\delta_t - (1+\gamma)di_t\right)|e_t = e, E_t = E\right], \tag{11}$$

where aggregate equity E_t evolves according to (10) and

$$de_t = k_t[(R(E_t) - p)dt \quad - \sigma_0 dZ_t] - d\delta_t + di_t. \tag{12}$$

A fundamental property of the individual decision problem of a bank is that the feasible set, in terms of trajectories of $(k_t, d\delta_t, di_t)$, and the objective function are homogenous of degree one in the individual equity level e_t. Therefore, the value function itself must satisfy:

$$v(e, E) = eu(E),$$

where $u(E)$ can be thought of as the market-to-book value of equity for banks, which reflects the market assessment of banks' profitability.

Using the above property and applying standard dynamic programming methods (see Appendix A), it can be shown that it is exactly the market-to-book value that drives all

[16]Throughout the paper, we use lower case letters for individual variables and upper case letters for aggregate variables.

bank policies in our framework. The optimal dividend and recapitalization policies turn out to be of the so-called "barrier type".[17] In particular, dividends are distributed only when $E_t = E_{max}$, where E_{max} is such that $u(E_{max}) = 1$. In other words, distribution of dividends only takes place when the marginal value of equity capital equals the shareholders' marginal value of consumption. Recapitalizations occur only when $E_t = E_{min}$, where E_{min} satisfies $u(E_{min}) = 1 + \gamma$, i.e., when the marginal value of equity equals the total cost of equity issuance. As long as aggregate bank equity E_t remains in between E_{min} and E_{max}, fluctuations of the individual bank's equity are only caused by retained earnings or absorbed losses. Given that the market-to-book value is the same for all banks, bank recapitalizations and dividend payments in our economy are perfectly synchronized in time.

Maximization with respect to the level of lending k_t shows that the optimal lending policy of the bank is indeterminate, i.e., bank shareholders are indifferent with respect to the volume of lending. Instead, the latter is entirely determined by the firms' demand for credit.[18] We show in Appendix A that the maximization problem (11) has a non-degenerate solution, if and only if the market-to-book ratio simultaneously satisfies two equations:

$$\frac{u'(E)}{u(E)} = -\frac{R(E) - p}{K(E)\sigma_0^2}, \tag{13}$$

and

$$\rho = K(E)[R(E) - p]\frac{u'(E)}{u(E)} + \frac{\sigma_0^2 K^2(E)}{2}\frac{u''(E)}{u(E)}, \tag{14}$$

where $K(E) = D(R(E))$.

Combining these two equations, we obtain that $R(E)$ satisfies a first-order differential equation:

$$R'(E) = -\frac{1}{\sigma_0^2}\frac{2\rho\sigma_0^2 + (R(E) - p)^2}{\Big(D[R(E)] - [R(E) - p]D'[R(E)]\Big)}. \tag{15}$$

Given that $D'(R) < 0$, it is easy to see that $R'(E) < 0$, so that in the states with higher aggregate capital, banks charge lower loan rates, which leads to higher volume of credit and output in the economy. In contrast, when aggregate bank capital gets scarce

[17] The barrier-type recapitalization and payout policies are well explored by the corporate liquidity management literature (see e.g. Jeanblanc and Shiryaev (1996), Milne and Robertson (1996), Décamps et al. (2011), Bolton et al. (2012, 2013) among others) that places emphasis on the loss-absorbing role of corporate liquid reserves in the presence of financial frictions. In our model, the role of book equity is very similar to the role of liquidity buffers in those models. However, we differentiate from this literature by allowing for the feedback loop between the individual decisions and the dynamics of individual book equity via the general equilibrium mechanism that determines the loan rate and thus the expected earnings of a bank.

[18] This situation is analogous to the case of an economy with constant returns to scale, in which the equilibrium price of any output is only determined by technology (constant marginal cost), whereas the volume of activity is determined by the demand side.

after a long series of negative aggregate shocks, banks raise the loan rate, which triggers the reduction in credit and output.[19]

It is important to emphasize that, for any level of bank capitalization $E \in [E_{min}, E_{max})$, the risk-adjusted credit spread $R(E) - p$ remains *strictly positive*. Indeed, expressing the loan rate $R(E)$ from equation (13) immediately shows that, for any $E > E_{max}$, bank shareholders require a strictly positive premium for accepting to lend:

$$R(E) = p + \underbrace{\sigma_0^2 K(E)\Big[-\frac{u'(E)}{u(E)}\Big]}_{\text{“lending premium”}}. \tag{16}$$

To understand the “raison d'être” for this lending premium, consider the impact of the marginal unit of lending on shareholder value $eu(E)$. A marginal increase in the volume of lending increases the bank's exposure to aggregate shocks. However, note that the aggregate shock not only affects the individual bank's equity e_t but also aggregate equity E_t and thus the market-to-book ratio $u(E)$ that is decreasing in E.[20] Thus, if there is a negative aggregate shock $dZ_t > 0$ that depletes the individual bank's equity, the effect of this loss on shareholder value gets amplified via the market-to-book ratio. Symmetrically, a positive aggregate shock ($dZ_t < 0$), while increasing book equity, translates into a reduction of $u(E_t)$, which reduces the impact of positive profits on shareholder value. This mechanism gives rise to a kind of *induced risk aversion* with respect to variation in aggregate capital, which explains why risk-neutral bankers require a positive spread for accepting to lend.

The following proposition summarizes our findings.

Proposition 1 *There exists a unique Markov equilibrium, in which the loan rate function $R(E)$ satisfies the ordinary differential equation*

$$R'(E) = -\frac{1}{\sigma_0^2}\frac{2\rho\sigma_0^2 + (R(E)-p)^2}{\Big(D[R(E)] - [R(E)-p]D'[R(E)]\Big)}, \tag{17}$$

with the boundary condition $R(0) = R_{max}$. Aggregate bank capital evolves according to:

$$dE_t = K(E_t)[(R(E_t) - p)dt - \sigma_0 dZ_t], \qquad E_t \in (0, E_{max}). \tag{18}$$

Banks recapitalize when $E_t = 0$ and distribute dividends when $E_t = E_{max}$.

[19] Another remark to be made in light of the negative relation between the loan rate and aggregate equity is that recapitalizations occur when the bank makes a strictly positive profit in expectation, whereas dividends are distributed when the bank makes a zero expected profit.

[20] Intuitively, having an additional unit of equity reduces the probability of facing costly recapitalizations in the short-run, so that the marginal value of equity, $u(E)$, is decreasing with bank capitalization.

The numbers E_{max} and R_{max} are uniquely determined by two conditions:

$$R(E_{max}) = p \quad \textit{and} \quad \int_0^{E_{max}} \frac{R(E)-p}{\sigma_0^2 D(R(E))} dE = \log(1+\gamma).$$

The typical patterns of the loan rate $R(E)$ and the market-to-book value $u(E)$ that emerge in the competitive equilibrium are illustrated in Figure 1.

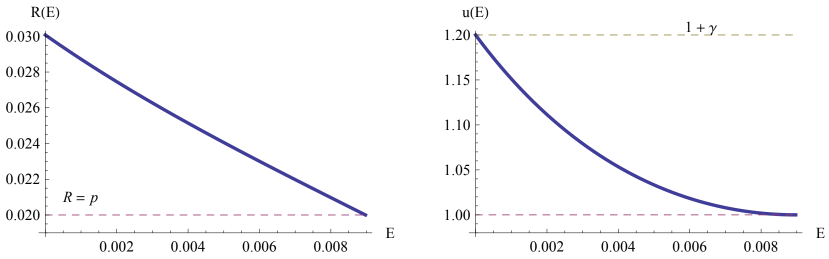

Figure 1: Loan rate and market-to-book ratio in the competitive equilibrium

Note that the loan rate function $R(E)$ cannot generally be obtained in closed form. However, it turns out that the dynamics of the loan rate $R_t = R(E_t)$ is explicit.

Proposition 2 *The loan rate $R_t = R(E_t)$ follows the process*

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t, \qquad p \leq R_t \leq R_{max}, \tag{19}$$

with reflections at both ends of the support. The volatility function is given by

$$\sigma(R) = \frac{2\rho\sigma_0^2 + (R-p)^2}{\sigma_0\Big(1-(R-p)\frac{D'(R)}{D(R)}\Big)}. \tag{20}$$

The drift function is

$$\mu(R) = \sigma(R)(R-p)H(R), \tag{21}$$

where

$$H(R) = \frac{(R-p)}{\sigma_0[D(R)-(R-p)D'(R)]}D'(R) + \frac{[(R-p)^2+2\rho\sigma_0^2]}{2\sigma_0[D(R)-(R-p)D'^2}D''(R). \tag{22}$$

Moreover, R_{max} satisfies

$$\int_p^{R_{max}} \frac{(R-p)}{\sigma_0\sigma(R)}dR = \log(1+\gamma) \quad \textit{and} \quad \int_p^{R_{max}} \frac{\sigma_0 D(R)}{\sigma(R)}dR = E_{max}. \tag{23}$$

To sum up, the dynamics of E_t and $R_t = R(E_t)$ in the competitive equilibrium depends on the credit demand function $D(R)$ and four parameters: exposure to aggregate shocks (or fundamental volatility) σ_0, the unconditional probability of default p, discount factor ρ and financial frictions γ. In equilibrium, the loan rate R_t fluctuates in between its first-best level p and R_{max}. Reflection of the loan rate at the both boundaries of its support generates a pseudo-cyclical behavior of the economy in our model. Moreover, it can be easily seen from the first expression in (23) that the magnitude of these "credit cycles", captured by the wedge $R_{max} - p$, is increasing with the magnitude of financial frictions, γ.[21] Thus, our model predicts that loan rates, lending and, thereby, output will be more volatile in the economies with stronger financial frictions. At the same time, the second expression in (23) shows that the target level of bank capitalization, E_{max}, is also increasing with the magnitude of financial frictions. By contrast, in the absence of financial frictions, i.e., when $\gamma = 0$, one would have $R_{max} = p$ and $E \equiv 0$.

3.2 Financial (in)stability

To study the properties of the competitive equilibrium and the behavior of the economy in the long run, we will focus at the dynamics of the loan rate, which is explicitely determined in (19).[22] We start by applying the impulse response methodology to study the stability of the deterministic steady state. Then, we discuss the ergodic properties of the system, showing that the system behavior in a stochastic environment can be in sharp contrast to the behavior predicted by the analysis conducted in a deterministic setting.

3.2.1 Impulse response analysis

The usual methodology to analyze the long-term behavior of macro-variables in a DSGE model is to linearize around the deterministic steady-state and perturb the system by a single unanticipated shock. The equivalent here would be to look at the case where $dZ_t \equiv 0$ for $t > 0$. The dynamics of the system then becomes deterministic and can be described by the ordinary differential equation (linearization is not needed here):

$$dR_t = \mu(R_t)dt,$$

where the initial shock determines $R_0 > p$.

It is easy to see from expression (21) that $\mu(p) = 0$. Hence, the frictionless loan rate ($R_t \equiv p$) is an equilibrium of the deterministic system that is further referred to as the

[21] Interestingly, financial frictions only affect R_{max}, without having any impact on $\mu(R)$, $\sigma(R)$ and $u(E)$.

[22] Working with R_t instead of E_t enables us to provide an analytical characterization of the system's behavior, because the drift and volatility of R_t are closed-form expressions. By contrast, in a general case, the drift and volatility of the process E_t cannot be expressed in a closed form, since $R(E)$ has an explicit expression only for the particular specifications of the credit demand function.

deterministic steady-state (DSS). It is *locally* stable when $\mu'(p) < 0$ and is *globally* stable when $\mu(R) < 0$ for all R. After some computations, it can be shown that

$$\mu'^2 \sigma_0^2 \frac{D''(p)}{D(p)}.$$

Hence, the DSS is locally stable when $D''(p) < 0$. Moreover, it also follows from (22), that condition $D''(R) < 0$ ensures global stability.

Illustrative example. To illustrate the properties of the equilibrium, consider the following specification of the loan demand function:

$$D(R) = (\overline{R} - R)^{\beta}, \tag{24}$$

where $\beta > 0$ and $p < \overline{R}$.

Under the above specifications, the volatility of the loan rate is

$$\sigma(R) = \frac{[2\rho\sigma_0^2 + (R-p)^2]\,(\overline{R} - R)}{\sigma_0[\overline{R} + (\beta - 1)R - \beta p]}. \tag{25}$$

The drift of the loan rate is given by

$$\mu(R) = \sigma(R)\frac{\beta(R-p)Q(R)}{2\sigma_0[\overline{R} + (\beta - 1)R - \beta p]^2}, \tag{26}$$

where $Q(R)$ is a quadratic polynomial:

$$Q(R) = (1-\beta)((R-p)^2 - 2\rho\sigma_0^2) - 2(R-p)(\overline{R} - p). \tag{27}$$

Given the above specification, it can be easily shown that, when $\beta < 1$ (which is equivalent to $D''(R) < 0$), $\mu'(p) < 0$ and $\mu(R) < 0$ in the entire interval $[p, \overline{R}]$. Thus the DSS is locally and globally stable. By contrast, when $\beta > 1$ (which is equivalent to $D''(R) > 0$), the DSS is locally unstable, i.e., $\mu'(p) > 0$, and there exists a unique $R^* \in (p, \overline{R})$ such that $\mu(R)$ is positive in the region $(0, R^*)$ and negative in the region $(R^*, \overline{R})$ (see Figure 2).

3.2.2 Long run behavior in the stochastic set-up

After studying the properties of the deterministic equilibrium, we consider the full dynamics of the stochastic equilibrium. It turns out that the system is ergodic and thus the long run behavior of the economy can be described by the ergodic density function. This ergodic density measures the average time spent by the economy in the

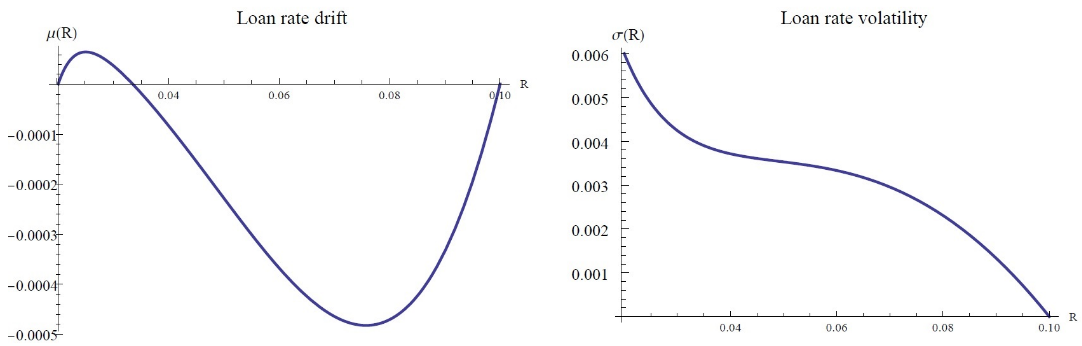

Figure 2: The loan rate drift and volatility, $\beta > 1$.

neighborhood of each possible loan rate R: the states with lower R (and, respectively, high aggregate capital E) can be interpreted as "boom" states and the states with higher R (and, respectively, low aggregate capital E) can be thought of as "bust" states. The ergodic density function can be computed by solving the Kolmogorov forward equation.

Proposition 3 *If $\sigma(R) > 0$ for $\forall R \in [p, R_{max}]$, there exists a unique ergodic distribution of R characterized by the density function*

$$f(R) = \frac{C_0}{\sigma^2(R)} exp\Big(\int_p^R \frac{2\mu(s)}{\sigma^2(s)} ds\Big), \tag{28}$$

where the constant C_0 is such that $\int_p^{R_{max}} f(R)dR = 1$.

It can easily be seen from the expression of $\sigma(R)$ provided in (20), for any loan demand specifications such that $D'(R) < 0$ and $D(R) > 0$ in the region $[p, R_{max}]$, the volatility of the loan rate remains strictly positive, which guarantees the existence of an ergodic distribution of R. By differentiating the logarithm of the ergodic density defined in (28), we obtain:

$$\frac{f'(R)}{f(R)} = \frac{2\mu(R)}{\sigma^2(R)} - \frac{2\sigma'(R)}{\sigma(R)}. \tag{29}$$

Using the general formulas for $\sigma(R)$ and $\mu(R)$, it can be shown that $\sigma(p) = 2\rho\sigma_0$, $\sigma'(p) = 2\rho\sigma_0 \frac{D'(p)}{D(p)} < 0$ and $\mu(p) = 0$. Hence, $f'(p) > 0$, which means that the state $R = p$ corresponding to the DSS is definitely *not* the one at which the economy spends most of the time in the stochastic set up. To get a deeper understanding of the determinants of the system behavior in the long run, we resort to the numerical example.

Figure 3 reports the typical patterns of the endogenous volatility $\sigma(R)$ (left panel) and the ergodic density $f(R)$ (right panel) for the loan demand specification defined in (24). It shows that the extrema of the ergodic density almost coincide with those of the volatility function, i.e., the economy spends most of the time in the states with the lowest

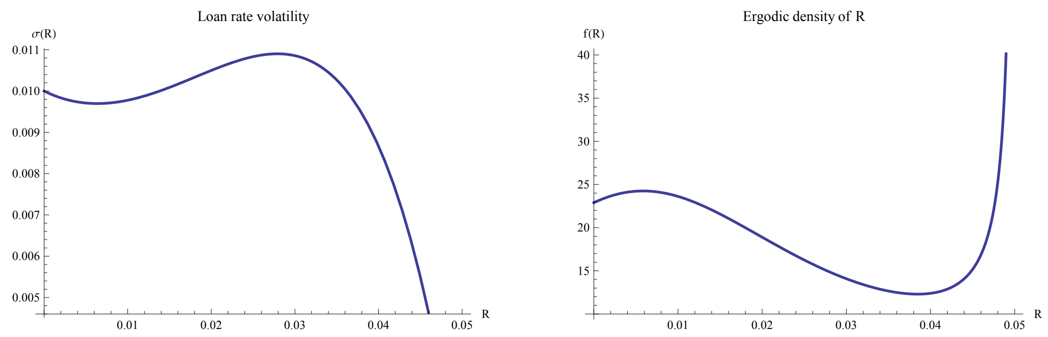

Figure 3: Endogenous volatility and ergodic density functions

loan rate volatility. Intuitively, the economy can get "trapped" in the states with low loan rate volatility because the endogenous drift is generally too small to move it away from these states. In fact, $\sigma(R)$ turns out to be much larger than $\mu(R)$ for any level of R, so that the volatility impact always dominates the drift impact.[23]

Note that functions $\sigma(.)$ and $f(.)$ must be truncated (and, in the case of the ergodic density, rescaled) on $[p, R_{max}]$, where R_{max} depends on the magnitude of issuing costs γ. For the chosen specification of the loan demand function, $D(R) = (\overline{R} - R)^{\beta}$, we always have $R_{max} < \overline{R}$.[24] However, R_{max} can be arbitrary close to $\overline{R}$, which typically happens under strong financial frictions and low elasticity of credit demand. In that case the economy will spend quite some time in the region where the loan rate is close to R_{max}. We interpret this situation as a persistent "credit crunch": it manifests itself via scarce bank equity capital, high loan rates, low volumes of lending and output.

This "credit crunch" scenario is reminiscent to the "net worth trap" documented by Brunnermeier and Sannikov (2014). In their model, the economy may be brought into recession because of the inefficient allocation of productive capital between more and less productive agents, which they call “experts” and “households” respectively. This allocation is driven by the dynamics of the equilibrium price of capital, which depends on the fraction of the total net worth in the economy that is held by experts. After experiencing a series of negative shocks on their net worth, experts have to sell capital to less productive households, so that the average productivity in the economy declines. Under a reduced scale of operation, experts may struggle for a long time to rebuild net worth, so that the economy may be stuck in a low output region. In our model, the output in the economy is driven by the volume of credit that entrepreneurs can get from banks, whereas the cost of credit depends on the level of aggregate bank capitalization. When

[23]Formally, this can be observed from the expression (26): in fact, $\mu(R) \equiv \sigma(R)H(R)$, where $H(R)$ is typically very small.

[24]To prove this property, it is sufficient to show that the integral $\int_p^{\overline{R}} \frac{R-p}{\sigma_0 \sigma(R)} dR$ diverges.

the banking sector suffers from a series of adverse aggregate shocks, its loss absorbing capacity deteriorates. As a result, the amplification mechanism via the market-to-book value becomes more pronounced and bankers thus require a larger lending premium. The productive sector reacts by reducing its demand for credit and the banks have to shrink their scale of operations, which makes it even more difficult to rebuild equity capital.

Overall, the analysis conducted in this subsection suggests that the long run behavior of the economy in the stochastic environment is determined by the volatility of the loan rate, rather than by its drift. Thus, relying on the results of the impulse response analysis in order to infer the long-run behavior of the economy in the stochastic environment might be misleading.

3.3 Welfare analysis

Having solved for the competitive equilibrium, it is natural to question its efficiency. We show below that the competitive allocation of credit emerged in our continuous-time framework does not maximize social welfare, provided that the social planner is subject to the same frictions as the private investors (i.e., she cannot directly transfer wealth through taxes and subsidies between the productive and the banking sectors).

In our simple set-up where deposit taking does not generate any surplus, social welfare can be computed as the sum of the market value of the firms (i.e., the expected discounted profit of the productive sector) and the market value of the banks' equity:[25]

$$W(E) = \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\pi_F[K(E)]\right] + \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(d\Delta_t - (1+\gamma)dI_t\right)\right] \equiv V_F(E) + V(E), \tag{30}$$

where $\pi_F[K(E)] = F(K) - KF'(K)$ is the expected profit of firms per unit of time.

Note that the social welfare function can be seen as the value of a "claim" that is contingent on the underlying "asset" - aggregate bank equity E. Thus, we can apply standard pricing methods to compute the social welfare function. Recall that, in the region $(0, E_{max})$, banks neither distribute dividends nor recapitalize, so that the available cash flow consists uniquely of the firms' profit. Therefore, for $E \in (0, E_{max})$, the social welfare function at the competitive equilibrium, $W(E)$, must satisfy the following differential equation:

$$\rho W(E) = \pi_F[K(E)] + K(E)[R(K(E)) - p]W'(E) + \frac{\sigma_0^2}{2}K^2(E)W''(E). \tag{31}$$

Note that dividend distribution and bank recapitalizations only affect the market

[25] We assume that the social planner has the same discount factor as equity investors.

value of banks, without producing any immediate impact on the firms' profit. This observation yields two boundary conditions, $W'(0) = 1+\gamma$ and $W'(E_{max}) = 1$. Thus, the welfare function corresponding to the competitive allocation of credit can be computed numerically.[26] Figure 4 depicts a typical pattern of social welfare in the simple case where the credit demand function is linear (see Appendix B for the computation details): maximum welfare is attained at a maximum level of aggregate bank capitalization and the maximum variation of welfare depends on the magnitude of financial frictions.

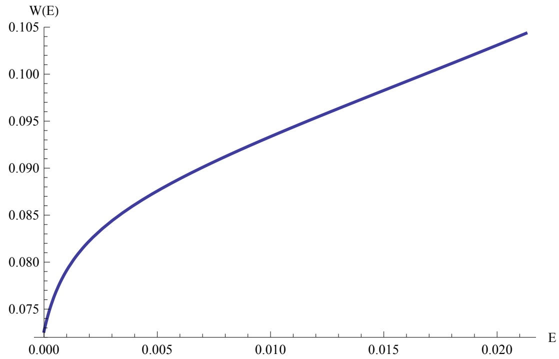

Figure 4: Welfare as a function of aggregate bank capital

Consider now the marginal impact of lending on social welfare, by taking the first derivative of the right-hand side of equation (31) with respect to $K(E)$:

$$\mathcal{L}(E) = -K(E)F''(K(E)) + [K(E)F''(K(E)) + R(K(E)) - p]W'(E) + \sigma_0^2 K(E)W''(E), \tag{32}$$

where $F''(K(E)) = R'(K(E))$.

The constrained optimal allocation of credit could be obtained by maximizing the right-hand side of (31) in $K(E)$ and solving the associated Bellman equation. However, it is more reasonable to consider that the social planner has no real power to *directly* control the volume of credit in the economy. Thus, instead of looking for the socially optimal allocation of lending, we exploit expression (32) to shed light on the source and the sign of inefficiency at the competitive equilibrium.

Note that $\mathcal{L}(E) = 0$ and thus the socially optimal allocation of credit would be achieved if the loan spread would equal the social costs of lending, i.e., when

$$R(E) - p = \sigma_0^2 K(E)\Big[-\frac{W''(E)}{W'(E)}\Big] - R'(K(E))K(E)\Big[1 - \frac{1}{W'(E)}\Big]. \tag{33}$$

Comparing the social costs of lending in (33) with the private costs of lending to

[26]Given $W(E)$, one could easily compute the expected social welfare loss as a function of the bank capital loss, i.e., $[W(E_0) - W(E_0 - \Delta E)]/W(E_0)100\%$, where $\Delta E > 0$ is the aggregate loss of bank capital and E_0 is the level of bank capitalization before the loss.

banks (see (16)) suggests that the allocation of credit in the competitive equilibrium is distorted in two ways: first, because of the difference in the private and social induced risk aversion, i.e., $-\frac{u'(E)}{u(E)}$ and $-\frac{W''(E)}{W'(E)}$; second, because an increase in lending decreases the loan spread (recall that $R'(K) < 0$) and thus reduces the marginal earnings of banks. Indeed, since $W'(E) > 1$, the second term at the right-hand side of expression (33) is positive. This is exactly the difference between the social and private cost of lending that drives the inefficiency of the competitive allocation of credit.

To illustrate this inefficiency and to have a precise picture of the relation between the levels of bank capitalization and welfare distortions, we numerically estimate the sign of the marginal cost of lending $\mathcal{L}(E)$ in the particular case where the credit demand function is linear (see Figure 5). It turns out that, in this case,[27] $\mathcal{L}(E)$ is positively signed for the lower levels of bank capitalization and becomes negative for the higher level of capitalization. This suggests that, for the lower levels of bank capital, welfare can be improved by increasing credit to the productive sector, whereas for the higher level of bank capitalization welfare can be improved by reducing credit. Put differently, competitive banks lend too much when things go well (high equity), and too little when things go badly (low equity). In our economy where credit entirely determines investment, such an allocation of credit would lead to underinvestment in downturns and overinvestment in upturns.[28]

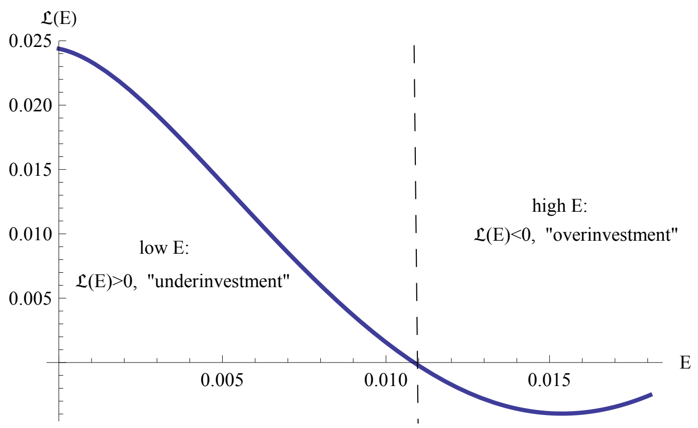

Figure 5: Market failure

[27]We do not know if this result extends to more general specifications of credit demand.

[28]A similar result is obtained in He and Kondor (2014) albeit in a different setting without an explicit banking sector

4 Capital regulation

So far our analysis has been focused on the "laissez-faire" environment in which banks face no regulation. Our objective in this section is to understand how *capital regulation* does affect the banks' lending policies and the equilibrium behavior of the economy. We assume that public authorities enforce a minimum capital requirement, under which each bank must maintain equity capital above a certain fraction of loans, i.e.,

$$e_t \geq \Lambda k_t,$$

where Λ is the minimum capital ratio.

It is worthwhile to note that banks have two options to comply with minimum capital requirements. The first option would be to immediately recapitalize as soon as the regulatory constraint starts binding. The second option consists in cutting on lending and reducing deposit taking. Anecdotal evidence suggests that bank shareholders usually prefer to use the latter option, rather than to undertake costly recapitalizations. As will become apparent below, such a strategy turns out to be privately (but not socially) optimal in the presence of financial frictions.

To solve for the regulated equilibrium, we again start by looking at the maximization problem of an individual bank. As in the unregulated set-up, bank shareholders maximize the market value of their claim by choosing the lending, recapitalization and dividend policies, yet, facing a restriction on the volume of lending:

$$v_\Lambda(e, E) \equiv eu_\Lambda(E) = \max_{k_t \leq \frac{e}{\Lambda}, d\delta_t, di_t} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(d\delta_t - (1+\gamma)di_t\right)|e_t = e, E_t = E\right]. \quad (34)$$

To have the intuition on the problem solution, recall that, in the unregulated case, bank recapitalizations take place only when equity is completely depleted. Thus, it is natural to expect that the regulatory constraint will be binding for relatively low levels of equity. Indeed, in the general case, the bank may find itself in one of two regions: (i) when the level of equity is relatively high, the regulatory constraint is not binding and the volume of lending is still determined by the firms' demand for credit; (ii) in the states with low equity, the regulatory constraint binds and the volume of lending is determined by $k_t = e_t/\Lambda$. Due to the homotheticity property, at each moment of time, all banks have the same leverage ratio. Thus, it is legitimate to anticipate the existence of the critical level of bank capital E_Λ, such that the regulatory constraint binds on the aggregate and individual levels for any $E \in [E^\Lambda_{min}, E_\Lambda]$ and is slack for any $E \in (E_\Lambda, E^\Lambda_{max}]$. This critical threshold E_Λ must satisfy

$$\frac{K(E_\Lambda)}{E_\Lambda} = \frac{1}{\Lambda}.$$

For Λ high enough, E_Λ tends to E^Λ_{max} and the unconstrained region disappears entirely.

Proposition 4 *For all Λ, there exists a unique regulated equilibrium, characterized by one of two regimes:*

a) for Λ high enough, the regulatory constraint binds over $[E^\Lambda_{min}, E^\Lambda_{max}]$. The loan rate is explicitly given by

$$R(E_t) = D^{-1}[E_t/\Lambda], \tag{35}$$

where D^{-1} is the inverse function of the loan demand. The dynamics of the aggregate bank capital is given by:

$$\frac{dE_t}{E_t} = \frac{(D^{-1}[E_t/\Lambda] - p)dt - \sigma_0 dZ_t}{\Lambda}, \qquad (E^\Lambda_{min}, E^\Lambda_{max}).$$

b) for relatively low Λ, capital constraint is binding or $E \in [E^\Lambda_{min}, E_\Lambda]$ and is slack for $E \in (E_\Lambda, E^\Lambda_{max}]$. When $E \in [E^\Lambda_{min}, E_\Lambda]$, the dynamics of aggregate equity and the loan rate function are defined as in the regime a). When $E \in (E_\Lambda, E^\Lambda_{max}]$, the loan rate must satisfy differential equation (15) with the boundary condition $R(E_\Lambda) = D^{-1}[E_\Lambda/\Lambda]$ and the dynamics of the aggregate equity is described by the same equation as in the unregulated set-up.

In either regime, banks distribute dividends when $E_t = E^\Lambda_{max}$ and recapitalize when $E_t = E^\Lambda_{min}$.

We show in the Appendix C that, in the unconstrained region $(E_\Lambda, E^\Lambda_{max}]$, the market-to-book value still simultaneously satisfies (13) and (14), whereas in the constrained region $[E^\Lambda_{min}, E_\Lambda]$ it must satisfy

$$\rho = \frac{E(D^{-1}[E/\Lambda] - p)}{\Lambda}\frac{u'_\Lambda(E)}{u_\Lambda(E)} + \frac{\sigma_0^2 E^2}{2\Lambda^2}\frac{u''_\Lambda(E)}{u_\Lambda(E)}, \tag{36}$$

under condition

$$\frac{u'_\Lambda(E)}{u_\Lambda(E)} \geq -\frac{D^{-1}[E/\Lambda] - p}{E/\Lambda\sigma_0^2}, \tag{37}$$

with strict equality at $E = E^\Lambda_{min}$.

The optimal decisions are characterized by the recapitalization and payout boundaries, E^Λ_{min} and E^Λ_{max}, such that $u_\Lambda(E^\Lambda_{min}) = 1 + \gamma$ and $u_\Lambda(E^\Lambda_{max}) = 1 + \gamma$. In Appendix C we provide the detailed description of the computational procedure that enables us to numerically solve for the regulated equilibrium.

It is already clear from expression (35) that, for any given level of aggregate bank capitalization, a higher capital ratio implies a higher loan rate, unless the bank does not immediately recapitalize.[29] To get a better understanding of the impact of capital regulation on the cost of credit in the economy, we perform a comparative static analysis by computing the equilibrium characteristics of bank policies for all values of $\Lambda \in (0,1]$. Parameter values are taken as follows: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\overline{R} = 0.1$, $\gamma = 0.2$, and credit demand function is linear, i.e., $D(R) = \overline{R} - R$.

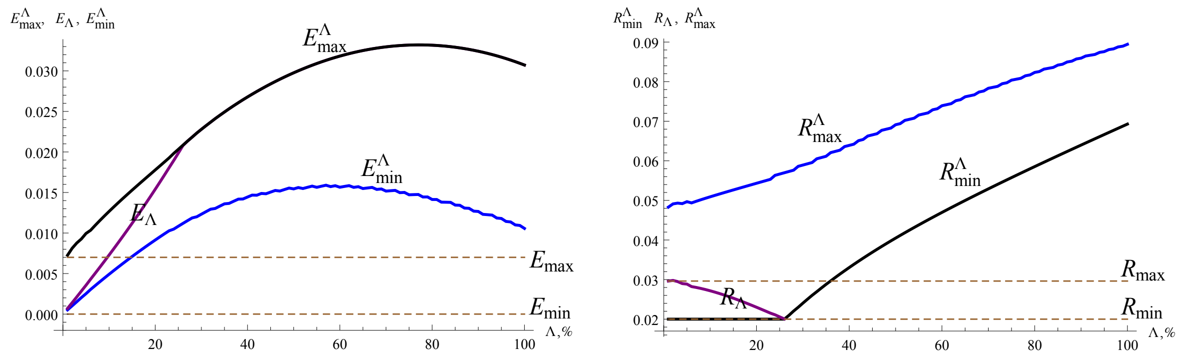

Figure 6: Impact of minimum capital ratio on bank policies

The left panel of Figure 6 reports the values of E^{Λ}_{min}, E^{Λ}_{max} and E_{Λ} (solid lines), contrasting them to the E_{min} and E_{max} computed in the unregulated setting (dashed lines). It shows that, in the regulated equilibrium, banks recapitalize at a strictly positive level of equity and generally build larger target equity buffer, as compared to the unregulated set-up. However, even though higher bank capitalization improves the loss absorbing capacity of banks, this fact does not have any positive implications for lending. In contrast, as shown in the right panel of Figure 6, tighter capital regulation drives up the cost of credit in the economy, thereby triggering a decline of lending and output. This mechanism driving this result is very simple. Recall that depositors (that have a preference for liquidity) are willing to accept a lower return than shareholders. Capital regulation induces banks to substitute cheaper deposit funding by costlier equity funding, and, as a consequence, the loan rate increases.[30]

It is worthwhile to note, however, that this adverse effect of capital regulation that

[29]Note that the credit demand function $D(R)$ is decreasing on its argument and the inverse function of a decreasing function is also decreasing.

[30]This can be seen immediately in the set-up without refinancing costs. In the fully fledged model with refinancing costs, however, we so far only claim that capital regulation shifts the support of the loan rate to the right. A more complete analysis of the welfare effects of regulation in our setting is subject to ongoing research.

manifests itself in our framework does not mean per se that capital regulation is always bad. Our results simply suggest that a *rigid capital ratio* is not a suitable tool to tackle credit market inefficiencies, so that there is room for additional research exploring alternative regulatory instruments to address this issue (e.g. countercyclical capital ratios or dividend restrictions).

5 Conclusion

This paper develops a general equilibrium model of commercial banking, in which banks satisfy households' needs for safe investment and channel financing to the productive sector. Bank capital plays the role of a loss-absorbing buffer that insulates banks from the need to undertake costly recapitalizations too often. In our model, the aggregate level of bank capitalization drives the cost and the volume of lending. Specifically, we establish a negative relation between the equilibrium loan rate and the level of aggregate bank capital. The closed-form characterization of the equilibrium dynamics of loan rates enables us to study analytically the long-run behavior of the economy. We show that this behavior is ergodic and is essentially determined by the volatility of the loan rate and the magnitude of financing frictions. The economy never spends the most of time at the boom state and, under severe financing frictions, may spend quite a lot of time in the credit crunch phase.

We also use our model to explore the impact of minimum capital regulation on bank policies and find that, while boosting the overall bank capitalization, tighter capital requirements push up the cost of credit, leading to the reduction of lending in the economy.

Our model suffers from two important limitations. First, it only considers commercial banking activities (deposit taking and lending), while neglecting market activities such as securities and derivatives trading. Second, it only considers diffusion risks that do not lead to actual bank defaults, but merely fluctuations in the size of the banking sector. A consequence of these limitations is that we cannot address the important questions of banks' excessive risk-taking and the role of capital regulation in the mitigation of this behaviour.

Appendix A. Proofs

Proof of Proposition 1. By the standard dynamic programming arguments, shareholder value $v(e, E)$ must satisfy the Bellman equation:[31]

$$\rho v = \max_{k\geq 0, d\delta\geq 0, di\geq 0} \Big\{ d\delta(1-v_e) - di(1+\gamma-v_e) + \\ + k[(R(E)-p)v_e + \sigma_0^2 K(E) v_{eE}] + \frac{k^2\sigma_0^2}{2} v_{ee} \\ + K(E)(R(E)-p)v_E + \frac{\sigma_0^2 K^2(E)}{2} v_{EE} \Big\}. \tag{A1}$$

Using the fact that $v(e, E) = eu(E)$, one can rewrite the Bellman equation (A1) as follows:

$$\rho u(E) = \max_{k\geq 0, d\delta\geq 0, di\geq 0} \Big\{ \frac{d\delta}{e}[1-u(E)] - \frac{di}{e}[1+\gamma-u(E)] + \frac{k}{e}[(R(E)-p)u(E) + \sigma_0^2 K(E)u'(E)] + \\ + K(E)(R(E)-p)u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E) \Big\} \tag{A2}$$

A solution to the maximization problem in k only exists when

$$\frac{u'(E)}{u(E)} \leq -\frac{R(E)-p}{\sigma_0^2 K(E)}, \tag{A3}$$

with strict equality when $k > 0$.

Under conjecture that $R(E) \geq p$ (which will be verified ex-post), it follows from the above expression that $u(E)$ is a decreasing function of E. Then, the optimal payout policy maximizing the right-hand side of (A2) is characterized by a critical barrier E_{max} satisfying

$$u(E_{max}) = 1, \tag{A4}$$

and the optimal recapitalization policy is characterized by a barrier E_{min} such that

$$u(E_{min}) = 1 + \gamma. \tag{A5}$$

In other words, dividends are only distributed when E_t reaches E_{max}, whereas recapitalization occurs only when E_t reaches E_{min}. Given (A3), (A4), (A5) and $k > 0$, it it easy to see that, in the region $E \in (E_{min}, E_{max})$, market-to-book value $u(E)$ satisfies:

$$\rho u(R) = K(E)(R(E)-p)u'(E) + \frac{\sigma_0^2 K^2(E)}{2} u''(E). \tag{A6}$$

Note that, at equilibrium, $K(E) = D[R(E)]$. Taking the first derivative of (A3), we

[31] For the sake of space, we omit the arguments of function $v(e, E)$.

can compute $u''(E)$. Plugging $u''(E)$ and $u'(E)$ into (A6) and rearranging terms yields:

$$R'(E) = -\frac{1}{\sigma_0^2}\frac{2\rho\sigma_0^2 + (R(E)-p)^2}{\Big(D[R(E)] - [R(E)-p]D'[R(E)]\Big)}. \tag{A7}$$

Since $D'(R(E)) < 0$, it is clear that $R'(E) < 0$ if $R(E) > p$. To verify that $R(E) > p$ for any $E \in [E_{min}, E_{max})$, it is sufficient to show that $R_{min} \equiv R(E_{max}) \geq p$.

To obtain R_{min}, let $V(E) \equiv Eu(E)$ denote the market value of the entire banking sector. At equilibrium, one must have $V'(E_{max}) = 1$ and $V'(E_{min}) = 1 + \gamma$. Given that $V'(E) = u(E) + Eu'(E)$, it must hold that $Eu'(E) = 0$. Hence, $u'(E_{max}) = 0$ and $E_{min} = 0$. Inserting $u'(E_{max}) = 0$ into the binding condition (A3) immediately shows that $R_{min} = p$, so that $R(E) > p$ for any $E \in [E_{min}, E_{max})$. The loan rate $R(E)$ is defined as a solution to the differential equation (A7) with the boundary condition $R(E_{max}) = p$.

To obtain E_{max}, we use the fact that individual banks' optimization with respect to the recapitalization policy implies $u(E_{min}) = 1 + \gamma$. Thus, integrating equation (A3) in between $E_{min} = 0$ and E_{max}, while using $u(E_{max}) = 1$, yields an equation that implicitly determines E_{max}:

$$u(0) = exp\Big(\int_0^{E_{max}} \frac{R(E)-p}{\sigma_0^2 K(E)} dE\Big) = 1 + \gamma. \tag{A8}$$

Proof of Proposition 2. Applying Itô's lemma to $R_t = R(E_t)$ yields:

$$dR_t = \underbrace{K(E_t)\left((R(E_t)-p)R'(E_t) + \frac{\sigma_0^2 K(E_t)}{2}R''(E_t)\right)}_{\mu(R(E_t))} dt \underbrace{-\sigma_0 K(E_t) R'(E_t)}_{\sigma(R(E_t))} dZ_t. \tag{A9}$$

After some computations involving the use of (A7), the drift and the volatility of $R_t = R(E_t)$ can be expressed by simple formulas stated in (20) and (21). The system of equations (23) immediately follows from the change of the variable of integration in equation (A8), i.e., $dE = E'(R)dR = \frac{dR}{R'(E)}$.

Appendix B. Computing social welfare

Consider the simple case where the credit demand is linear, i.e.

$$D(R) = \overline{R} - R.$$

In this particular case, the loan rate $R(E)$ can be computed in a closed form:

$$R(E) = p + \sqrt{2\rho}\sigma_0 \tan\left(\frac{\sqrt{2\rho}}{\sigma_0(\overline{R}-p)}(E_{max} - E)\right), \tag{A10}$$

and thus

$$K(E) = \overline{R} - p - \sqrt{2\rho}\sigma_0 \tan\left(\frac{\sqrt{2\rho}}{\sigma_0(\overline{R}-p)}(E_{max} - E)\right). \tag{A11}$$

To recover the production function, $F(D(R))$, recall that $F'(D(R)) = R$. Using the fact that $R = \overline{R} - D$, we obtain $F'(D) = (\overline{R} - D)$ and, thereby,

$$F(D(R)) = \overline{R}D(R) - \frac{[D(R)]^2}{2}. \tag{A12}$$

At equilibrium, we have $D(R(E)) = K(E)$, so the that firm' expected profit is given by

$$\pi_F(K(E)) = F(K(E)) - K(E)F'(K(E)) = \frac{[K(E)]^2}{2}.$$

Then, social welfare follows ODE:

$$\rho W(E) = \frac{[K(E)]^2}{2} + K(E)(\overline{R} - K(E) - p)W'(E) + \frac{\sigma_0^2}{2}[K(E)]^2 W''(E), \tag{A13}$$

given that $W'(0) = 1 + \gamma$ and $W'(E_{max}) = 1$.

Differentiating the above expression with respect to E and solving the obtained equation numerically with respect to $W'(E)$ enables us uncover $W(E)$.

Appendix C. Solving for the regulated equilibrium

Consider the shareholders' maximization problem stated in (34). By the standard dynamic programming arguments and the fact that $v_\Lambda(e, E) = eu_\Lambda(E)$, the optimal bank's policies must satisfy the following Bellman equation:

$$\begin{aligned}\rho u_\Lambda(E) = \max_{d\delta\geq 0, di\geq 0} \quad & \Big\{\frac{d\delta}{e}[1 - u_\Lambda(E)] - \frac{di}{e}[1 + \gamma - u_\Lambda(E)]\Big\} + \\ + \max_{0<k\leq e/\Lambda} \quad & \Big\{\frac{k}{e}[(R(E) - p)u_\Lambda(E) + \sigma_0^2 K(E)u'_\Lambda(E)]\Big\} + \\ & + K(E)[R(E) - p]u'_\Lambda(E) + \frac{\sigma_0^2 K^2(E)}{2}u''_\Lambda(E).\end{aligned} \tag{A14}$$

The solution to (A14) exists only if

$$\frac{u'_\Lambda(E)}{u_\Lambda(E)} \geq -\frac{R(E) - p}{\sigma_0^2 K(E)}, \tag{A15}$$

whith strict equality for $0 < k < e/\Lambda$.

The optimal dividend and recapitalization policies are characterized by the barriers E^Λ_{max} and E^Λ_{min} such that $u_\Lambda(E^\Lambda_{max}) = 1$ and $u_\Lambda(E^\Lambda_{min}) = 1 + \gamma$.

Under the conjecture that there exists a certain E_Λ such that the regulatory constraint is binding for $E \in [E^\Lambda_{min}, E_\Lambda]$ and is slack for $E \in [E_\Lambda, E^\Lambda_{max}]$, let

$$\alpha_i(E) \equiv -\frac{u'_\Lambda(E)}{u_\Lambda(E)},$$

where $i = 1$ for $E \in (E_\Lambda, E^\Lambda_{max}]$ and $i = 2$ for $E \in [E^\Lambda_{min}, E_\Lambda]$.

Then, in the region $(E^{\Lambda}_{min}, E^{\Lambda}_{max})$, equation (A14) can be rewritten as follows:

$$\rho = -\Pi_i(E)\alpha_i(E) + \frac{\sigma_0^2 K_i^2(E)}{2}[\alpha_i^2(E) - \alpha_i'(E)] + \mathbb{1}_{i=2}\frac{[R_i(E) - p - \sigma_0^2 K_i(E)\alpha_i(E)]}{\Lambda}, \tag{A16}$$

where index $i = \{1, 2\}$ reflects the fact that the loan rate and the aggregate volume of lending are defined differently on each of two regions and $\Pi_i(E)$ denotes the aggregate expected banks' profit:

$$\Pi_i(E) = K_i(E)[R_i(E) - p].$$

At equilibrium, in the constrained region $i = 2$, the volume of credit is given by

$$K_2(E) = D[R_2(E)] = E/\Lambda,$$

and, hence, the loan rate is

$$R_2(E) = D^{-1}[E/\Lambda],$$

where D^{-1} is the inverse function of the demand for credit.

In the unconstrained region $i = 1$, the volume of lending is determined by the credit demand and the loan rate $R_1(E)$ satisfies equation (A7) with the boundary condition[32]

$$R_1(E_\Lambda) = R_2(E_\Lambda).$$

Moreover, it must hold that $u'_\Lambda(E_{max}) = 0$ and $u'_\Lambda(E_{min}) = 0$, which implies $\alpha_1(E_{max}) = 0$ and $\alpha_2(E_{min}) = 0$.

Numerical procedure to solve for the regulated equilibrium. This numerical algorithm solving for the regulated equilibrium can easily be implemented with the *Mathematica* software:

- pick a candidate value $\hat{E}^{\Lambda}_{min}$. Solve numerically for $\alpha_2(E)$ such that $\alpha_2(\hat{E}_{min}) = 0$;
- compute a candidate value $\hat{E}^{\Lambda}_{max}$ such that $\alpha_2(\hat{E}^{\Lambda}_{max}) = 0$;
- check the condition

$$\frac{D^{-1}[\hat{E}^{\Lambda}_{max}/\Lambda] - p}{\sigma_0^2 \hat{E}^{\Lambda}_{max}/\Lambda} \geq 0 \tag{A17}$$

  a) if (A17) holds, compute the market-to-book value $u_\Lambda(\hat{E}^{\Lambda}_{min})$ according to

$$u_\Lambda(\hat{E}^{\Lambda}_{min}) = exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{max}} \alpha_2(E)dE\Big)$$

  b) if (A17) is violated, find $\hat{E}_\Lambda$ that satisfies

$$\alpha_2(\hat{E}_\Lambda) = \frac{D^{-1}[\hat{E}_\Lambda/\Lambda] - p}{\sigma_0^2 \hat{E}_\Lambda/\Lambda}, \tag{A18}$$

[32] This is an immediate implication of condition $D[R_1(E_\Lambda)] = E_\Lambda/\Lambda$.

and compute the market-to-book value $u_\Lambda(\hat{E}^\Lambda_{min})$ according to:

$$u_\Lambda(\hat{E}^\Lambda_{min}) = C_\Lambda exp\Big(\int_{\hat{E}^\Lambda_{min}}^{\hat{E}_\Lambda} \alpha_2(E)dE\Big),$$

where

$$C_\Lambda = exp\Big(\int_{\hat{E}_\Lambda}^{\hat{E}^\Lambda_{max}} \alpha_1(E)dE\Big), \quad \text{where} \quad \alpha_1(E) = \frac{R_1(E)-p}{\sigma_0^2 K_1(E)}.$$

- if $u_\Lambda(\hat{E}^\Lambda_{min}) = 1+\gamma$, $E^\Lambda_{min} = \hat{E}^\Lambda_{min}$ and $E^\Lambda_{max} = \hat{E}^\Lambda_{max}$; otherwise, pick a different $\hat{E}^\Lambda_{min}$, repeat the procedure from the beginning.

Appendix D. Competitive equilibrium with $r > 0$

In this appendix, we solve for the competitive equilibrium in the set up where $r > 0$. In this case, the dynamics of equity value of an individual bank follows:

$$de_t = re_t dt + k_t[(R(E_t) - p - r)dt \quad - \sigma_0 dZ_t] - d\delta_t + di_t. \tag{A19}$$

The aggregate equity of the banking sector evolves according to:

$$dE_t = [K(E_t)(R(E_t) - p - r) + rE_t]dt - \sigma_0 K(E_t)dZ_t - d\Delta_t + dI_t. \tag{A20}$$

Solving the shareholders' maximization problem in the same way as we did in the proof of Proposition 1 and allowing for $k > 0$ yields us two equations:

$$\frac{u'(E)}{u(E)} = -\frac{R(E)-p-r}{\sigma_0^2 K(E)}, \tag{A21}$$

$$(\rho - r)u(E) = K(E)(R(E) - p - r)u'(E) + \frac{\sigma_0^2 K^2(E)}{2}u''(E). \tag{A22}$$

Substituting $u'(E)$ and $u''(E)$ into (A22), while allowing for $K(E) = D[R(E)]$, enables us to express $R'(E)$:

$$R'(E) = -\frac{1}{\sigma_0^2}\frac{2(\rho-r)\sigma_0^2 + (R(E)-p-r)^2 + 2(R(E)-p-r)E/D[R(E)]}{\Big(D[R(E)] - [R(E)-p-r]D'[R(E)]\Big)}. \tag{A23}$$

Applying the same arguments as in the setting with $r = 0$, we can show that $E_{min} = 0$ and $R_{min} = r+p$. The boundary R_{max} must be computed numerically by solving equation

$$\int_p^{R_{max}} E'(R)\frac{(R-p-r)}{\sigma_0^2 D(R)}dR = \log(1+\gamma), \tag{A24}$$

where $E'(R) = 1/R'(E)$.

Note that the left-hand side of the above expression is increasing in R_{max}. Hence, there exists a unique solution to (A24), which guarantees the uniqueness of the equilibrium.

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