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The Timing and Frequency of Corporate Disclosure

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The Timing and Frequency of Corporate Disclosures*

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Abstract

This paper studies disclosure dynamics and its implications for stock returns. Because disclosure is costly, the firm may withhold information for some time even when information is favorable. In equilibrium, the firm adopts a regular time-pattern of disclosure. Breaking this regularity, by failing to issue a disclosure when expected, leads to a sharp drop in the stock price and to a period of relatively low asymmetry of information.

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1 Introduction

Managers often disclose private information in a voluntary fashion. These discretionary disclosures constitute a significant source of information to capital markets whose relative importance has been growing over time.[1] Yet, not all the managers' private information is disclosed in a timely fashion. Since the seminal contributions of Grossman (1981); Milgrom (1981) the literature has recognized that, on the one hand, managers may strategically withhold private information that is likely to have negative price consequences and, on the other hand, that markets understand the strategic behavior of managers thus penalizing their silence (see e.g., Jovanovic (1982); Verrecchia (1983); Dye (1985)).

This disclosure game between managers and markets is in essence dynamic. In reality, a manager who wishes to maximize his firm's stock price must select not just whether to disclose information but also when to do it and how often. After all, the manager's private information sooner or later will become public information even if he withholds it forever. Unfortunately, very little is known about the dynamics of disclosure and it is fair to say that, in this area, measurement is ahead of theory. While a vast number of empirical papers study disclosure dynamics and its implications for the time-series of stock returns, (see e.g., Kothari, Shu and Wysocki (2009)) disclosure theories are for the most part static.

This paper follows the lead of Acharya, DeMarzo and Kremer (2011) in an attempt to fill this gap. As Acharya, DeMarzo and Kremer (2011) we suppose that the firm's manager maximizes the present value of the firm's future stock prices, perhaps because his compensation at each point in time is proportional to the market value of the firm. The evolution of asset values is described by a continuous time Markov chain that fluctuates between two possible states: low asset value and high asset value. The distribution of asset values is known but the manager privately observes the evolution of actual values, namely he only can tell whether the asset has experienced a temporary impairment or, on the contrary, has recovered from one. Yet, the manager can disclose his private information to the market at any point in time and as many time as he so wishes. Unraveling is however not possible in equilibrium because –as in Jovanovic (1982); Verrecchia (1983)– disclosing information (specially, good news) is costly.

The baseline model generates the following dynamics. At the beginning of the game, or for that matter after any disclosure, there is a *blackout period* where no disclosure is made. During that period, stock prices experience a downward drift (driven by the possibility of

[1]Earnings guidance explains a large portion of the variation in stock returns (Ball and Shivakumar 2008; Beyer, Cohen, Lys, and Walther 2009). More than 15 percent of the variation in quarterly stock returns occurs around guidance announcements, compared to less than three percent for earnings announcements and about six percent for analyst forecasts (Beyer, Cohen, Lys and Walther 2009).

an undisclosed impairment) up to a point where the undervaluation of the asset could be so severe that a disclosure becomes profitable. At this point, the manager discloses his private information if favorable, in which case the price jumps upwards and the game restarts. If, on the contrary, the manager decides to withhold his information, the market infers the asset must have low value and the stock price experiences a drastic drop. From that point onwards the stock price stays flat (at the lowest level) until the asset recovers its value and the manager discloses good news.

The length of the blackout period is affected both by the cost of disclosure and, more importantly, by the time-series properties of asset values, specially the cash flows' mean reversion. A higher mean reversion means that information becomes more transitory. Consequently, the stock price drifts faster toward its long term value. That, in turn, gives the manager an incentive to accelerate his disclosure so as to mitigate the undervaluation the asset experiences when the asset is in the hight state. This effect would seem to strengthen the incentives of the manager to disclose good news. However, a higher mean reversion, also means the price effect of disclosing good news will be shorter-lived which weakens the incentives for such disclosures. The interaction between these two effects results in the length of the blackout period being non-monotonic in the cash flows' mean reversion.

We then consider how the presence of a public news process, correlated with asset values but observed at random times, affects the manager's disclosure incentives. Specifically, we model the public news as a Poisson process whose arrival intensity depends on the value of assets. If arrivals are more likely in the bad state, a news arrival conveys bad news. The arrival thus triggers a price drop whose magnitude depends on the information quality of the news. Conversely, the absence of arrivals mitigates the price's downward drift –relative the case without public news– because the absence of arrivals is perceived by the market as good news. At first, this suggests that the presence of the public news process should moderate the propensity of the manager to disclose his information. But, public news also have an opposing effect: the observation of a news arrival by the market induces a drastic price drop which naturally stimulates the manager to disclose good news as soon as the asset recovers its value. We show however that the former effect dominates, so that the higher the frequency of news arrivals the lower is the frequency of managerial disclosures. In this setting, public news substitute managerial disclosures.

In the previous model, the manager may only disclose good news; bad news are eventually observed but only from the public news process. In the real world, however, bad news disclosures are prevalent (see e.g., Kothari, Shu and Wysocki (2009)) perhaps because the realization that the manager withheld adverse information has important legal implications (see e.g., Skinner (1997)). To capture this feature of the disclosure environment, we consider the possibility that a news arrival may give rise to litigation costs when the manager fails to disclose bad news and the news reveal the asset was overpriced. The presence of litigation risk, one might think, should stimulate the manager's bad news disclosures as a means to preempt the litigation costs: the manager should sometimes reveal bad news, specially when the stock price is relatively low. Yet, this idea presents a conundrum: if the manager revealed bad news with probability one, at any given point in time, then the absence of such disclosures would be perceived as perfect evidence the asset value is high, thus inducing a sharp increase in the stock price. This jump in the stock price would destroy the manager's incentives to disclose bad news in the first place. To overcome this conundrum, the equilibrium must entail disclosure randomization. When prices are sufficiently low, the manager randomizes between disclosing and not disclosing the bad news. At that point, the price remains constant over time, up until the bad news are either disclosed by the manager or revealed by the public news.

The manager's decision to disclose bad news has the flavor of the real options problem analyzed by Dixit (1989), where a firm has the option, at any point in time, to shut-down (i.e., disclose bad news) or restart a project (i.e., disclose good news), based on the project's observed profitability.[2] In our setting, when the stock price is low and the value of the asset is also low, disclosing bad news becomes profitable for the same reason shutting down a project that is making losses is optimal in Dixit's model. Also, as in Dixit's model, the decision to disclose bad news today is inherently liked with the value of the option to disclose good news in the future: if the cost of disclosing good news is higher, then the benefit from disclosing bad news today goes down, which naturally delays such disclosures. This speaks to a certain complementarity between bad and good news disclosures in the presence of legal liability.

In the presence of legal liability, the public news process no longer substitutes managerial disclosures, but actually complements them. This is natural: a higher frequency of public news means that the expected litigation cost from withholding information goes up. Managers are subject to a tighter scrutiny and feel more compelled to reveal bad news. They thus accelerate the release of bad news.

1.1 Related Literature

This paper extends Jovanovic (1982) and Verrecchia (1983, 1990) to a continuous time setting. The most closely related paper is Acharya, DeMarzo and Kremer (2011). They consider a dynamic version of Dye (1985) where the manager may be privately informed about the

[2]But in our setting the profitability of the project is endogenous because it is determined by the Bayesian beliefs of the market about the asset value.

asset value. When informed, the manager may disclose his private information at one of two points in time: at the start of the game or right after a public news signal is released, at a known date. If the manager's private information is not so favorable, waiting for news has positive option value since the public signal might induce a higher price in the absence of disclosure than in the presence of it. By contrast, if the public signal turned out to be unfavorable, the manager could mitigate the negative price effect of the public signal by disclosing his own private information. Their model is able to explain clustering of disclosure in bad times: the less favorable the public signal the higher the probability of disclosure.

Dye (2010) also studies the timing of disclosure. A risk averse manager must sell his shares among a number of risk neutral investors during several trading periods following a fixed trading profile. Also, in each period the manager must acquire and disclose a signal about the asset value. Ex-ante, the manager is allowed to choose the precision profile of the signals he will be releasing, but the sum of the signals' precisions is fixed. So the manager's choice regards the timing of disclosure: namely how much precision to allocate to each period's signal. In equilibrium, the manager engages in *disclosure bunching*, namely he allocates all the precision to a single period instead of spreading the precision over time. This bang-bang solution is driven by optimal risk sharing between the manager and other traders: when the manager is too risk averse relative to other traders, very informative disclosures impose excessive risk on the manager's wealth, so the manager tends to delay them until a sufficiently high portion of his portfolio has already been off-loaded.

Beyer and Dye (2012) study a reputation model in which the manager may learn a single private signal in each of two periods. The manager can be either "forthcoming' and disclose any information he learns or "strategic.' At the end of each period, the firm's signal/cash flow for the period becomes public and the market updates beliefs about the value of the firm and the type of the agent.

Finally, Kremer, Guttman and Skrypacz (2012) consider the price consequences of the choice of disclosure timing. They study a two-period extension of Dye (1985) model, where in each period, the manager may observe any of two pieces of information (if previously unobserved) with some probability. They show that later disclosures are interpreted more favorably by the market because the probability that the manager is hiding information is perceived to be higher when partial disclosures are made earlier.

2 Baseline Model

We study a dynamic model of voluntary disclosure that extends Jovanovic (1982); Verrecchia (1983, 1990). We consider a firm that pays a terminal dividend V_{τ_M} when the firm matures at a random time τ_M that has arrival intensity γ.[3]

The value of assets V_t follows a continuous time Markov chain with state space $\{0,1\}$. The terminal dividend is thus equal to 1 if the value of assets is 1 at time τ_M, and zero otherwise. The value of the asset jumps from 0 to 1 with intensity λ_1 while it jumps back from 1 to 0 with intensity λ_0. We can think of λ_0 as the frequency with which the asset suffers an impairment. When $\lambda_1 = 0$, this impairment is permanent, otherwise the impairement is transitory.

At the outset, the asset value is known to be 1, namely $V_0 = 1$.[4] From that point onwards, the manager privately observes any shock to the asset value. However, at any point in time, the manager can disclose his private information at a cost. This disclosure cost may arise from the proprietary nature of the information (as in Jovanovic (1982); Verrecchia (1983)), the need to certify the information to make it credible –by for example hiring an auditor– or simply from the opportunity cost of the time required to prepare and disseminate the information.[5] The disclosure cost varies with the value disclosed. In particular, the cost of disclosing information is $C > 0$ when the asset value is high and 0 when the asset value is low. Hence, disclosing bad news is costless.[6]

Prices are set in a Bayesian and risk neutral manner. We normalize the market's interest rate to be zero. So if $d_t \in \{0,1\}$ denotes the disclosure decision at time t and $d = \{d_t\}_{t\geq 0}$ denotes the conjecture of the market about the manager's disclosure strategy then the firm's stock price, given the history of disclosures $\mathcal{F}_t$, is set as

$$P_t = E^d(V_{\tau_M}|\mathcal{F}_t) \tag{1}$$

where $E^d(\cdot)$ denotes the expectation operator based on the measure induced by d. Following Acharya, DeMarzo and Kremer (2011) and Benmelech, Kandel and Veronesi (2010), we assume the manager chooses a disclosure strategy σ that maximizes the present value of future prices net of disclosure expenses:

$$\mathcal{U}_t(d,\sigma) := E^d\left[\int_t^{\tau_M} e^{-\rho(s-t)}P_s ds - C\sum_{t\leq s<\tau_M} e^{-\rho(s-t)}\sigma_s\middle|\mathcal{F}_t, V_t\right], \tag{2}$$

[3]We make the assumption that the firm generates no cash flows before maturing to abstract away from the informational role of dividends and focus only on disclosures. Later, in Section 3, we consider the role of public information as a determinant of the manager's disclosures.

[4]Nothing changes if V_0 is private information at the start.

[5]A number of large investors such as Warren Buffett (1996) and analysts such as Candace Browning (2006), head of global research at Merrill Lynch, have called for managers to give up quarterly earnings guidance and hence avoid the myopic managerial behavior caused by attempts to meet market expectations.

[6]This assumption is not necessary for the results; assuming that the cost of disclosue in the low state is $C_0 > 0$ would generate exactly the same predictions.

The manager thus cares not only about the short term price implications of his disclosure decisions but also the long term implications. This concern for future stock prices is supported by the evidence. There is indeed ample evidence that managers' wealth is affected by the evolution of their firms' stock price.[7] This link between managerial wealth and future stock prices may arise from the manager's compensation (e.g., when equity grants have vesting periods) or reputation being linked to the evolution of the firm's stock price (as in career concern models). The above representation of the manager's objective function implicitly assumes that disclosure costs are borne by the manager. As it turns out, this assumption is innocuous. The results would not change if, instead, we assumed the disclosure cost is borne by the firm's shareholders, thus having a direct impact on stock prices.

Definition 1. An equilibrium is a disclosure strategy $d = \{d_t\}_{t\geq 0}$ and a price process $P = \{P_t\}_{t\geq 0}$ such that, for all $t \geq 0$,

1. The market price is $P_t = E^d(V_{\tau_M}|\mathcal{F}_t)$
2. The disclosure strategy maximizes the manager's utility given the market beliefs, that is $d \in \arg\max_\sigma \mathcal{U}_t(d, \sigma)$

Both conditions are standard. At every point in time, the price is set according to Bayes' rule, given the manager's strategy and the history of the game. Similarly, the manager's disclosure strategy maximizes the manager's expected utility at each point in time, and for all histories of the game.

As a preliminary analysis, we consider how the market belief about V_t evolves in the absence of disclosures. Using standard results (Karlin and Taylor, 1981), the probability that $V_t = 1$ evolves according to

$$dp_t = \kappa(\bar{p} - p_t)dt \tag{3}$$

where

$$\bar{p} := \frac{\lambda_1}{\lambda_0 + \lambda_1}$$

is the stationary probability that the value of the asset is 1 and $\kappa := \lambda_0 + \lambda_1$ represents the asset's mean reversion, namely the speed at which market belief reverts to the stationary point $\overline{p}$ in the absence of disclosure. Let

$$\phi_t(p) = \bar{p} + e^{-\kappa t}(p - \bar{p}).$$

[7]For example, Graham, Harvey and Rajgopal (2005) note, in their famous survey, that because of the severe market reaction to missing an earnings target, firms are willing to sacrifice economic value in order to meet a short-run earnings target. They find that managers make voluntary disclosures to reduce information risk associated with their stock but try to avoid setting a disclosure precedent that will be difficult to maintain.

be the solution to equation (3) given an initial condition $p_0 = p$. Then, the price of the firm at time t is given by

$$P_t = \int_t^\infty \phi_{s-t}(p_t)\gamma e^{-\gamma(s-t)}ds = \frac{\kappa}{\gamma+\kappa}\bar{p} + \frac{\gamma}{\gamma+\kappa}p_t. \tag{4}$$

The price P_t is affine in beliefs p_t. By virtue of this relation, in the following we use the terms "price" and "beliefs" interchangeably. Moreover, defining $r := \rho + \gamma$ and the normalized cost $c := (\gamma+\kappa)C/\gamma$, the manager's objective function can be re-written as

$$U_t(d,\sigma) := E\left[\int_t^\infty e^{-r(s-t)}p_s ds - c\sum_{s\geq t} e^{-r(s-t)}\sigma_s \middle| \mathcal{F}_t, V_t\right], \tag{5}$$

where $\mathcal{U}_t$ and U_t satisfy the following relation

$$\mathcal{U}_t = \frac{\kappa}{r(\gamma+\kappa)}\bar{p} + \frac{\gamma}{\gamma+\kappa}U_t.$$

The manager's disclosure strategy σ maximizes (5) given the asset value V_t and the market belief.

We begin by considering what would the manager's payoff be if the equilibrium entailed no disclosure. Since no disclosue expense would ever be incurred, the manager's payoff, given any initial belief p_0, would be equal to the present value of future prices. By the law of interated expectations this can be computed as

$$U^{ND}(p_0) = \int_0^\infty e^{-rt}\phi_t(p_0)dt = \frac{\bar{p}}{r} + \frac{p_0 - \bar{p}}{r+\kappa}.$$

We can now consider the actual equilibrium. We study Markov equilibria. Markov equilibria are characterized by a disclosure threshold p_* such that

$$d_t = \mathbf{1}_{\{p_{t-}\leq p_*\}}V_t.$$

That is, the manager discloses at time t if and only if both the price is lower than or equal to p_* and the value of asset is high. The idea that the propensity of disclosure is negatively correlated with the level of stock prices is natural, and has some empirical support. For example, Sletten (2009) argue that "stock price declines prompt managers to voluntarily disclose firm-value-related information (management forecasts) that was withheld prior to the decline because it was unfavorable but became favorable at a lower stock price.

Anticipating this strategy, the market expects no disclosure when the price is above p_*.

As a consequence, for any $p_t > p_*$ the price evolves according to (3). By contrast, for $p_t \leq p_*$, we have that $p_t = d_t$. That is, if the manager does not disclose his information when the price hits the threshold p_*, then the market infers the asset must be low. As a result, the price falls sharply from p_* to zero and remains there until the manager discloses good news (i.e., $V_t = 1$), which happens immediately after the asset value returns to the high state.

The dynamics of market beliefs are noteworthy: at the beginning of the game, the market belief drifts downwards until disclosing the asset value may become profitable for the manager. At that point, the price jumps upward if high value is disclosed or downwards if no disclosure is observed. Kothari, Shu and Wysocki (2009) empirically document a similar pattern. They find evidence consistent with the view that managers withhold bad news to investors and that prices tend to drift downward absent disclosure, and jump upward once the firm announces good news. Note that the failure to disclose at $p_t = p_*$ is followed by a period where (i) the price remains flat up until a disclosure is observed and (ii) the information becomes symmetric. By contrast, the period following a disclosure is characterized by the price (mean) reverting towards its stationary level $\overline{p}$, and by the manager being privately informed about the true asset value.

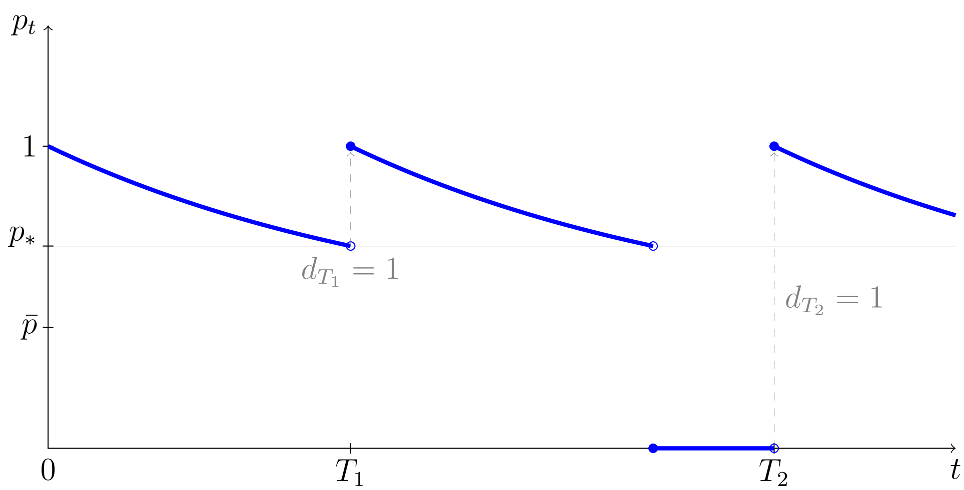


Figure 1: Example of a sample path of the share price.

The market's conjecture d must be consistent with the manager's optimal strategy σ. With some abuse of notation, let $U_v(p)$ be the manager's payoff given that the market belief is $p_t = p$ and the asset value is $V_t = v \in \{0, 1\}$. The manager's payoff in equilibrium can be represented by the Hamilton-Jacobi-Bellman (HJB) equation:

$$rU_v(p) = p + \frac{1}{dt} E\left[dU_v\right].$$

when $p > p_*$. On the other hand,

$$\begin{aligned}
\frac{1}{dt}E\left[dU_1\right] &= \lambda_0[U_0\left(p\right) - U_1\left(p\right)] + U_1'\left(p\right)\frac{dp}{dt} \\
\frac{1}{dt}E\left[dU_0\right] &= \lambda_1[U_1\left(p\right) - U_0\left(p\right)] + U_0'\left(p\right)\frac{dp}{dt}
\end{aligned}$$

The interpretation of the value function is standard; we can think of the manager's job as an asset, whose cost of capital in a competitive market $rU_v\left(p\right)$ must equal the rate of return on the asset, as given by its instantaneous flow p, and its expected capital gains $E\left[dU_v\right]/dt$. The latter may come in two forms: the deterministic evolution of investors' beliefs, as described by (3), and the possibility the asset experiences an impairment.

We obtain the following HJB equations:

$$rU_1(p) = p + \kappa(\bar{p} - p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{6}$$

$$rU_0(p) = p + \kappa(\bar{p} - p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] \tag{7}$$

with boundary conditions

$$U_1(p_*) = U_1(1) - c \tag{8}$$

$$U_0(p_*) = \frac{\lambda_1}{r + \lambda_1}[U_1(1) - c]. \tag{9}$$

Moreover the following parametric restrictions are required for an equilibrium where the probability of disclosure is positive:

$$\begin{aligned}
U_1(1) - c &\geq 0 \\
U_1(p) \geq U_1(1) - c \quad &\text{for} \quad p > p_* \\
U_1(p) \leq U_1(1) - c \quad &\text{for} \quad p \leq p_*.
\end{aligned}$$

In essence, the manager must solve an optimal stopping problem where the stopping time must be consistent with the market's rational expectations.

The following proposition provides the solution in closed form.

Proposition 1. *Suppose that $p_* \in (\bar{p}, 1)$ satisfies*

$$U_1\left(1\right) - c \geq 0 \tag{10}$$

and

$$U_1'\left(p_*\right) \geq 0 \tag{11}$$

Then, there exists an equilibrium with threshold p_*. The manager's payoff is given by

$$U_0(p) = U_1(p) - \frac{r}{r+\lambda_1}\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right)^{1+\frac{r}{\kappa}}\Big(U_1(1) - c\Big) \tag{12}$$

$$U_1(p) = \int_0^{T(p)} e^{-rt}\phi_t(p)dt + \delta(p)\Big(U_1(1) - c\Big), \tag{13}$$

where

$$U_1(1) = U^{ND}(1) - \frac{\delta(1)}{1-\delta(1)}c$$

and

$$\delta(p) := \left(\frac{p_* - \bar{p}}{p - \bar{p}}\right)^{\frac{r}{\kappa}}\left[\frac{r\bar{p} + \kappa\bar{p}}{r + \kappa\bar{p}} + \frac{r(1-\bar{p})}{r+\kappa\bar{p}}\frac{p_* - \bar{p}}{p - \bar{p}}\right],$$

$$T(p) = -\frac{1}{\kappa}\log\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right).$$

It is instructive to consider the manager's payoff at the start of the game, namely when the market beliefs are $p = 1$. Let's define

$$\mathcal{C}(c) := \frac{\delta(1)}{1-\delta(1)}c.$$

Hence, the manager's payoff at the outset is given by

$$U_1(1) = U^{ND}(1) - \mathcal{C}(c).$$

The first component, $U^{ND}(1)$, is the payoff the manager would obtain had he been able to commit to never disclose.[8] The second component $\mathcal{C}(c)$ is the present value of the disclosure expense the manager expects to bear over his lifetime, given his lack of commitment.[9] The manager's payoff is thus bounded from above by the non disclosure payoff $U^{ND}(1)$. This is natural: in our setting information has no (social) value, hence the disclosure expense is a deadweight loss, which the manager bears ex-post only because he cannot avoid disclosing asset values when market beliefs are severely depressed. But ex-ante, the average trajectory of future prices is not affected by the manager's disclosure policy: although in equilibrium the event of disclosure drives the price up, the failure to disclose drives it down.

Observe that there are multiple equilibria, given the discrete support of V_t. In particular,

[8] Weak commitments are sometimes observed in the real world. On December 13, 2002, the Coca Cola Company announced that it would stop providing quarterly earnings-per-share guidance to stock analysts, stating that the company hopes the move would focus investor attention on long-run performance.

[9] As a mirror image, one can think of this term as the profits of a certifier who, at the outset, commits to sell his certification services for a fee c

when the cost of disclosure is not so high, there exists a continuum of thresholds p_* satisfying conditions (10) and (11). The following proposition characterizes the set of equilibrium thresholds. We refer to an equilibrium in which disclosure happens with probability zero (at any point in time and for any history) as a non disclosure equilibrium.

With some abuse of notation, we let $U_v(p|p_*)$ be the manager's expected payoff in an equilibrium with disclosure threshold p_*, when the state is v and the market belief is p.

Proposition 2. *Let*

$$\overline{c} := \frac{\lambda_1 + r}{r\,(r+\kappa)}.$$

If $c < (1-\bar{p})\overline{c}$, then any equilibrium has a positive probability of disclosure. In particular:

1. *If $c < (1-\bar{p})\overline{c}$, there are disclosure thresholds $p_*^- < p_*^+$ satisfying the boundary conditions*

$$U_1\left(1|p_*^+\right) - c = 0 \tag{14}$$

$$U_1'\left(p_*^-|p_*^-\right) = 0, \tag{15}$$

   such that, for any $p_* \in [p_*^-, p_*^+]$, there is an equilibrium with disclosure threshold p_*.

2. *If $(1-\bar{p})\overline{c} \le c < \overline{c}$, then for any $p_* \in [\bar{p}, p_*^+]$, where p_*^+ satisfies* (14)*, there is an equilibrium with disclosure threshold p_*.*

3. *If $c \ge \overline{c}$, the only equilibrium entails no disclosure.*

Hence, the *most transparent* equilibrium, in terms of the probability of disclosure, arises when condition (10) is binding. By contrast, the *most opaque* equilibrium arises when condition (11) is binding. Confronted with this multiplicity of equilibria, it is natural to focus on the Pareto dominant one.

Definition 2. The equilibrium threshold $p_*^\dagger$ is Pareto dominant if and only if $U_v(p|p_*^\dagger) \ge U_v(p|p_*)$ for all $p \in [0,1], p_* \in [p_*^-, p_*^+]$ and $v \in \{0,1\}$.

This selection criterion is natural but somewhat arbitrary because in practice there is no guarantee the manager and the market will coordinate in any particular equilibrium. On the other hand, one can think of the Pareto dominant equilibrium as the natural outcome when, at the outset, the manager informally announces his firm's disclosure policy to the market. Though the manager cannot fully commit to disclose information regularly he can issue a cheap talk message along the lines of "we will try to provide guidance on a quarterly basis".[10]

Indeed, this type of announcement is common in practice: firms often announce what their disclosure policy will be. Of course these announcements are non-binding, but they still help set market's expectations about the firm's disclosure policy.

Proposition 3. *Suppose that $c < (1-\bar{p})\bar{c}$, then the Pareto dominant equilibrium is the least transparent equilibrium, that is, $p_*^\dagger = p_*^-$. On the other hand, if $c \geq (1-\bar{p})\bar{c}$, then the Pareto dominant equilibrium has no disclosure.*

This result is intuitive. Given that disclosure is a deadweight cost, the most efficient equilibrium and the one the manager prefers ex-ante, is the equilibrium that minimizes the frequency of disclosure, since this equilibrium also minimizes the present value of the disclosure expense. In this model, disclosure reduces the firm's long term value and is driven by the manager's short term incentives to manager the stock price.[11]

Notice that the least transparent equilibrium is the preferred equilibrium for the manager for any initial belief p, and any asset value. Hence the manager's incentives to coordinate in the least transparent equilibrium will remain the same for all the histories of the game.

Reputational Equilibrium

Graham, Harvey and Rajgopal (2005) argue that managers limit their voluntary disclosures to avoid setting a disclosure precedent that will be difficult to fulfill. This type of disclosure precedent effect would arise if investors formed their expectations about the frequency of disclosures based on the firm's history. Such phenomenon cannot arise in Markov equilibria which, by definition, are independent of the game's history.

In this section we consider whether the phenomenon described by Graham, Harvey and Rajgopal (2005) can take place in non-Markov equilibrium. The main result presented here is that there is a non-Markov equilibrium with no disclosure for any positive but arbitrarily

[10] For example, Chen, Matsumoto and Rajgopal (2011) note that on December 13, 2002, the Coca Cola Company announced that it would stop providing quarterly earnings-per-share guidance to stock analysts, stating that the company hopes the move would focus investor attention on long-run performance. Shortly thereafter, several other prominent firms such as AT&T and McDonalds made similar announcements renouncing quarterly earnings guidance.

[11] In 2007, the Commission on the Regulation of U.S. Capital Markets in the 21st Century (the 21st Century Commission) — an independent, bipartisan commission established by the U.S. Chamber of Commerce — recommended that public companies stop issuing earnings guidance or, at a minimum, move away from providing quarterly earnings per share guidance as a point estimate to providing annual guidance as a range of earnings per share numbers. The 21st Century Commission believed that quarterly earnings per share guidance caused companies to focus too much on short-term performance and the pressure for companies to meet short-term estimates created "adverse incentives to forgo value-added investments in long-term projects."

small disclosure cost c. Moreover, a direct implication of Proposition 1 is that this equilibrium yields the maximum profit for the manager.

Proposition 4 (Folk Theorem)**.** *For all $c > 0$, there is a non-Markov equilibrium with no disclosure.*

Proof. If $c \geq \bar{c}$, then the only equilibrium involves no disclosure and there is nothing to prove. If $c < \bar{c}$, consider the following trigger strategy: the firm never discloses unless it has disclosed in the past, in which case it uses a disclosure strategy with threshold p_*^+. Accordingly, the market expectations are that the firm never disclose private information unless it has disclosed in the past. Then, for any $p_t \geq 0$ we have

$$U^{ND}(p_t) \geq U(1|p_*^+) - c = 0.$$

Hence, there is no incentive to deviate and disclose. Moreover, from Proposition 1, $U(1|p_*^+)$ is the equilibrium payoff in the continuation game following disclosure. ∎

Proposition 4 shows the existence of a discontinuity with respect to c in the disclosure game. If disclosure is costless, then there is unraveling and full disclosure obtains. However, for any arbitrarily small cost of disclosure, there exists an equilibrium without disclosure.

The previous result shows that the manager's concern for setting a disclosure precedent can be a powerful deterrent of disclosure, and may eliminate the incentives to disclose altogether, as if the manager could commit ex ante to a non disclosure policy.

A major limitation of the previous equilibrium is the degree of coordination required to sustain the no-disclosure equilibrium.[12] For this reason, we focus on Markov equilibria hereafter.

2.1 The Frequency of Disclosure

There is considerable variation in the frequency of voluntary disclosures across different industries. In fact, there is even variation within industries (see e.g., Chen, Subramanyam, Zhang, 2007). The purpose of this section is to understand why some firms disclose more often than others, even when their managers seem to face similar incentives.[13] We focus,

[12]This is a standard critiques in the dynamic games literature to the use of non-Markov equilibria.

[13]Cheng, Subramanyam and Zhang (2007) note that firms in the wholesale and retails industries have the highest guidance frequency with mean of 5.93 (median of 7) quarters—out of 12 quarters—having at least one earnings forecast. In addition, firms in the industries of chemicals, consumer non-durables and business equipment provide relatively frequent quarterly earnings guidance as well. On the other hand, firms in the energy sector have the lowest guidance frequency with mean of 1.98 (median of 0) quarters, followed by telecommunications and healthcare industries. The within-industry standard deviations range from 3.00 to 4.12, suggesting that there is also within-industry variation in guidance frequency.

particularly, on the role of mean reversion as a determinant of the frequency of disclosure. We argue that mean reversion plays a key role insofar as it captures the extent to which information has a permanent effect on asset values.

Given the Markov structure of the problem, the sequence of disclosure times is a renewal process. Hence, it suffices to focus on the expected time of the first disclosure to derive the frequency of disclosure. If we let $T = \inf\{t > 0 | d_t = 1\}$ be the timing of the first disclosure, we are interested in computing $\overline{T}_v(p) := E(T|p_0 = p, v)$. Given $\overline{T}_v(\cdot)$, the frequency of disclosures is simply given by $1/\overline{T}_1(1)$.

Note that T is a random variable with support $[T(1; p_*), \infty)$ where $T(1; p_*) > 0$. Here, $T(1; p_*)$ is then the minimum time that must elapse until the first disclosure is observed. We find $T(p; p_*)$ by solving $\phi_T(p) = p_*$, which yields

$$T(p; p_*) = -\frac{1}{\kappa} \log\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right). \tag{16}$$

Before time $T(1; p_*)$, disclosure has probability zero. On the other hand, if the manager does not disclose the value of the asset at time $T(1; p_*)$, then the expected time spell before the next disclosure is released has an exponential distribution with mean $1/\lambda_1$, that is, $\overline{T}_0(p_*) = 1/\lambda_1$. Noting that $\overline{T}_1(p_*) = 0$, we can compute $\overline{T}_v(\cdot)$ directly. The frequency of disclosures is given by $1/\overline{T}_1(1)$ where

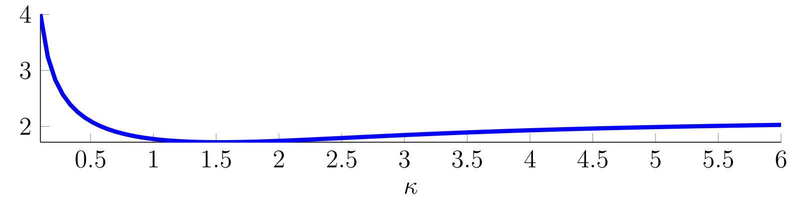
$$\overline{T}_1(1) = T(1; p_*) + \frac{1 - p_*}{\lambda_1}. \tag{17}$$

For a given threshold p_*, the frequency of disclosure increases in cash flows' mean reversion κ. This is natural: a higher mean reversion exacerbates the downward drift in market beliefs which in turn shortens the time until the market beliefs hits p_*. Of course, this is only part of the story because the disclosure threshold also depends on κ. The following proposition studies how κ affects the disclosure threshold $p_*^\dagger$.

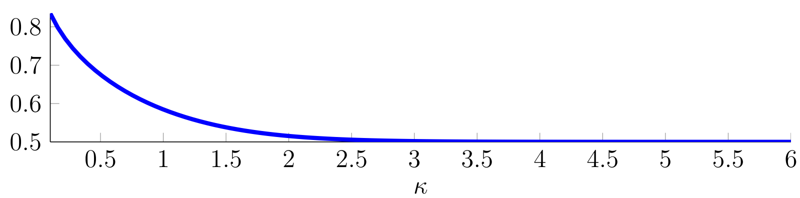
Proposition 5. *The equilibrium threshold $p_*^\dagger$ decreases in the cash flows' mean reversion κ.*

The price benefit of disclosure is weaker when mean reversion is stronger, since then the effect of a disclosure on the future prices is shorter lived which creates an incentive for the manager to reduce the frequency of disclosure. The effect of a higher κ on the frequency of disclosure could therefore be ambiguous: on the one hand, the manager has an incentive to stop the price drift by disclosing good news earlier. On the other hand, the effect of disclosures on the stock price is less permanent, which reduces the benefit of disclosing good news. This makes the overall effect ambiguous: indeed, Figure 2 shows that the duration of the blackout period, in which no disclosures are expected, $T(p; p_*)$, is non monotonic in κ.

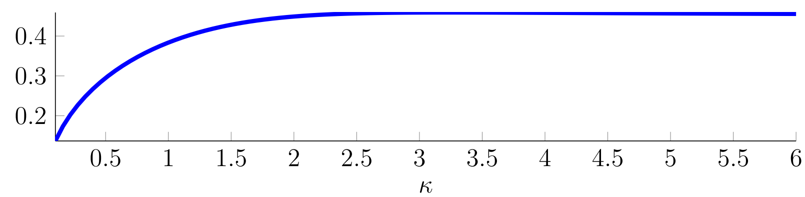
Yet, the frequency of disclosure increases monotonically in κ.



(a) Duration of "quiet period," $T(1, p_*^\dagger)$



(b) Pareto dominating equilibrium disclosure threshold, $p_*^\dagger$



(c) Expected frequency of disclosures, $1/\overline{T}_1(1)$

Figure 2: Effect of cash flow persistence, κ, on disclosures in the baseline model. A higher value of κ is associated with less persistent cash flows. Parameters: $r = 0.1$, $\bar{p} = 0.5$ and $c = 0.5$.

It is interesting to analyze the long run behavior of the beliefs p_t. This will allow us to quantify the amount of asymmetric information in the market. The process p_t has limiting distribution F. Let $p_0 = 1$ and define $\tau := \inf\{t > 0 | p_t = 1\}$, then Theorem 1.2 in Asmussen (2003, p. 170) implies that F is given by

$$F(p) = \frac{E\left[\int_0^\tau \mathbf{1}_{\{p_t \le p\}} dt\right]}{E[\tau]}. \tag{18}$$

The next proposition provides an explicit expression for F.

Proposition 6. *For any disclosure threshold $p_* > \bar{p}$, the limiting distribution of p_t exists and is given by*

$$F(p) = \begin{cases} \frac{1-p_*}{1-p_*+\bar{p}(\log(1-\bar{p})-\log(p_*-\bar{p}))} & \text{if } p < p_* \\ \frac{1-p_*+\bar{p}(\log(p-\bar{p})-\log(p_*-\bar{p}))}{1-p_*+\bar{p}(\log(1-\bar{p})-\log(p_*-\bar{p}))} & \text{if } p \ge p_*. \end{cases} \tag{19}$$

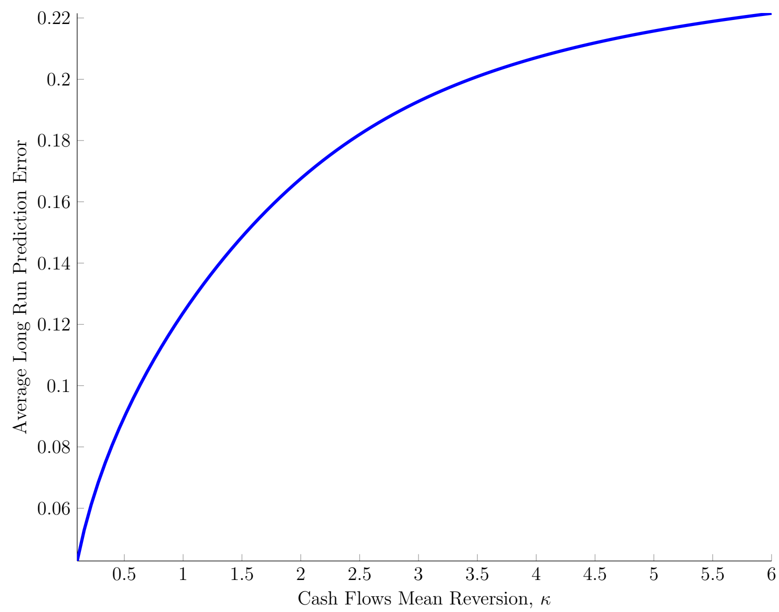


Figure 3: Effect of cash flow persistence, κ, on the long-run average mean square prediction error $\int_{p_*}^1 p(1-p)dF(p)$. A higher value of κ is associated with less persistent cash flows. Parameters: $r = 0.1$, $\bar{p} = 0.5$ and $c = 0.5$

Now, we can quantify the amount of asymmetric information in the market. One possible measure is the market expected prediction error, which is given by $E_t[(V_t - p_t)^2] = p_t(1-p_t)$. In the long run, we have[14]

$$\lim_{t\to\infty} E[p_t(1-p_t)] = \frac{E\left[\int_0^\tau p_t(1-p_t)dt\right]}{E[\tau]} = \int_{p_*}^1 p(1-p)dF(p).$$

Note that the mean error made in the long-run also corresponds to the average error made by the market over time.

In the next section, we add a public information process to the baseline setting as an intermediate step toward analyzing the case of litigation costs in Section 4.

3 Public Information

In practice, managers' incentives to disclose private information, at any point in time, depend on the velocity the information will leak into the market via external sources (e.g., media coverage, analysts' recommendations, peer firms' disclosures) and the way the market interprets the absence of public information. An interesting question in this context is whether

[14]The fact that $\frac{E\left[\int_0^\tau p_t(1-p_t)dt\right]}{E[\tau]} = \int_{p_*}^1 p(1-p)dF(p)$ is a consequence of the Renewal Theorem (Asmussen, 2003, p. 170)

the presence of more public information reduces the frequency of managerial disclosures or, on the contrary, exacerbates it. Despite the enormous flow of public information that characterizes the US market some commentators argue that american CEO's are particularly inclined to providing earnings' guidance.

We model the interaction between public information and managerial disclosures as follows. The public information is represented by a Poisson process $N = \{N_t\}_{t\geq 0}$ with the following characteristics. If the value of assets is low, N has arrival rate μ, whereas if the value of assets is high, then N has arrival rate 0. Hence, observing an arrival is perfect evidence of low asset value.[15]

This information process has intuitive features. As a preliminary analysis, consider how the market belief evolves during periods where the probability of disclosure is perceived to be zero. Using Bayes' rule, the evolution of beliefs –in the absence of news arrivals– (i.e., the drift) must obey

$$dp_t = f(p_t)dt, \tag{20}$$

where

$$f(p) = \kappa(\bar{p} - p) + \mu p(1 - p). \tag{21}$$

In the absence of disclosures and news, beliefs experience a downward drift toward the stationary level $\hat{p}$ as defined by $f(\hat{p}) = 0$, where

$$\hat{p} = \frac{1}{2}\left(1 - \frac{\kappa}{\mu}\right) + \sqrt{\frac{1}{4}\left(1 - \frac{\kappa}{\mu}\right)^2 + \frac{\kappa}{\mu}\bar{p}}.$$

From these conditions, we can see that the mere presence of the news process affects not only the drift but also the stationary belief $\hat{p}$. The stationary belief $\hat{p}$ increases in the intensity of news arrivals μ. Since arrivals can only take place when the underlying state is low, the absence of arrivals is perceived by the market as good news. It is easy to verify that for $p \geq \hat{p}$ we have $f(p) < 0$, and vice versa. Of course, in the event of a news arrival, beliefs drop down abruptly to zero.

Consider how the presence of the public news process affects the manager's incentives to disclose his information. Assuming that the cost of disclosure is not too high, there exists a

[15]When public information is noisy, managerial disclosures may be triggered by a news arrival, and be used by the manager as a means to counteract the sometimes adverse price effect of noisy news. This reactive-like disclosures generate clustering of disclosure in bad times (see Acharya, DeMarzo and Kremer (2011)). For simplicity, we abstract away from this effect and instead focus on the the case where a news arrival reveal the underlying state perfectly, without noise.

disclosure threshold, above the stationary point, i.e., $p_* > \hat{p}$. The HJB equations are

$$rU_1(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{22}$$

$$rU_0(p) = p + f(p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)] \tag{23}$$

with boundary conditions

$$U_1(p) = U_1(1) - c, \; p \leq p_* \tag{24}$$

$$U_0(p) = \frac{\lambda_1}{r + \lambda_1}[U_1(1) - c], \; p \leq p_*. \tag{25}$$

Unfortunately, it is no longer possible to solve the HJB equation in closed form. The following proposition characterizes the equilibrium.

Proposition 7. *For any $p_* \in (\hat{p}, 1)$ satisfying*

$$U_1(1) - c \geq 0 \tag{26}$$

and

$$U_1'(p_*) \geq 0 \tag{27}$$

there is an equilibrium with threshold p_*.

We can also establish the converse of Proposition 2.

Proposition 8. *Suppose there are equilibrium disclosure thresholds $\hat{p} \leq p_*^- < p_*^+$ such that*

$$U_1\left(1|p_*^+\right) - c = 0 \tag{28}$$

$$U_1'\left(p_*^-|p_*^-\right) = 0, \tag{29}$$

then, p_* is an equilibrium disclosure threshold if and only if $p_* \in [p_*^-, p_*^+]$. Moreover, the least transparent equilibrium, p_*^-, is the Pareto dominant equilibrium.

The public news process adds uncertainty to the manager's payoff, specially in the low state. By revealing the asset value in the low state the news triggers a significant drop in market beliefs thus reducing the manager's payoff in the low state. Of course, ex-ante, the news process can only affect the manager's payoff if it modifies the disclosure expense by changing the disclosure frequency.

The presence of public news has the following two effects on the manager's disclosure incentives. First, both the drift $f(p)$ and the stationary belief $\hat{p}$ are altered by the presence of a public news process. A higher intensity of news arrivals μ generally leads to a slower drift and to more favorable market beliefs. This reduces the manager's disclosure incentives. Hence, even in the absence of news arrivals, the mere presence of the news process may affect the frequency of disclosures. Second, a news arrival results in a sharp price drop which of course stimulates disclosure as soon as the asset returns to the high state. The former effect dominates though: more public information, represented by a higher μ, reduces the frequency of disclosures. One can thus say that public information substitutes managerial disclosures. In the following section we examine whether this result holds in the presence of litigation costs.

4 Legal Liability and Disclosure of Bad News

The empirical literature argues that litigation costs are an important driver of firm's voluntary disclosures. For example, Lev (1992) and Skinner (1994) document that managers can reduce stockholder litigation costs by voluntarily disclosing adverse earnings news "early", namely before the mandated release date. Consistent with this view, Skinner (1994) finds that managers use voluntary disclosures to preempt large, negative earnings surprises more often than other types of earnings news. [16]

In this section, we analyze the effect of litigation on the manager's disclosure incentives. We show that the presence of litigation costs fundamentally alters the structure of the equilibrium. Technically, litigation costs introduce a signaling motive to the decision to withhold information: the firm can signal high value by not disclosing its private information.[17] We also show that the presence of litigation costs crowds-out good news disclosures. This effect may be so strong that higher litigation costs may increase the manager's payoff by reducing the firm's expected disclosure costs.

As in the previous sections, here we assume that there is a public news process N_t that arrives with intensity $\mu \mathbf{1}_{\{V_t=0\}}$. That is, arrivals only take place when the firm is in the low state, being thus bad news. The manager is subject to legal liability. If bad news arrive and the manager has not yet disclosed that the asset value is low, then the manager is penalized with positive probability. Let ℓ_t be a random variable that takes the value one in the event the manager is found liable of withholding information, and zero otherwise. The manager's

[16] Also, Skinner (1997) finds that voluntary disclosures occur more frequently in quarters that result in litigation than in quarters that do not, because managers' incentives to predisclose earnings news increase as the news becomes more adverse, presumably because this reduces the cost of resolving litigation that inevitably follows in bad news quarters.

[17] See Bar-Isaac (2003); Kremer and Skrzypacz (2007); Daley and Green (2012); Gul and Pesendorfer (2012) for examples of situation in which delaying some intervention may be a positive signal.

personal cost of legal liability is denoted c_ℓ while the probability of experiencing this cost is q if the last time the manager disclosed information he disclosed good news and zero otherwise.[18] Hence, if the manager's latest disclosure was bad news, then the manager is safe from the legal liability as he can claim he already disclosed the bad news. We denote by $\theta := c_\ell q$ the expected legal cost of not disclosing negative information, conditional on a news arrival.

Consequently, the manager's expected payoffs, given a market's conjectured disclosure strategy, d, and manager's disclosure strategy, σ, can be written as

$$U_t(d,\sigma) := E\left[\int_t^\infty e^{-r(s-t)} p_s ds - c\sum_{s\geq t} e^{-r(s-t)}\sigma_s - c_\ell \int_t^\infty e^{-r(s-t)}\ell_s dN_s \middle| \mathcal{F}_t, V_t\right] \tag{30}$$

where the first term inside the expectation is the present value of stock prices, the second term captures the present value of the expected disclosure expense, and the third component represents the present value of litigation costs.

4.1 Equilibrium Description

We restrict attention to interior equilibria where the probability of both good and bad news' disclosures is positive. Depending on the cost of disclosure c, three types of equilibria may emerge. All of them are characterized by a threshold, p_*, for the market belief such that whenever the market belief hits the threshold the manager may disclose some of his information.[19] Whether he discloses good or bad news depends, nonetheless, on the magnitude of the disclosure cost relative to the litigation cost. First, if the disclosure cost is sufficiently low, then the manager discloses his information when the price reaches $p_* \in (\mu\theta, 1)$ regardless of the value of assets (and, off equilibrium, the market assumes the manager is withholding low asset values if no disclosure is observed). In essence, this is the equilibrium characterized in the previous section, except that sometimes the firm must pay the litigation cost.

Second, at the opposite extreme, when the cost of disclosure is very high (i.e., $c > c^+ = \frac{\mu\theta}{r+\lambda_0+\lambda_1}$) the manager may disclose low asset values with positive probability, whenever the market beliefs hit the threshold p_*, but he never discloses high asset values because such disclosures are unaffordable. One can think of this case as arising when certification costs are too expensive or the highly proprietary nature of the information means that it is too

[18]Strictly speaking c_ℓ is the normalized legal cost. If C_ℓ is the cost then $c_\ell := (\gamma+\kappa)C_\ell/\gamma$.

[19]Throughout, we assume that the cost of litigation is neither too low nor too high, so that the threshold p_* is interior, namely it belongs to $(\hat{p}, 1)$.

costly for the firm to disclose good news.

For intermediate disclosure costs the manager discloses bad news with positive probability when the price hits the threshold p_* but he discloses good news only when beliefs are severely depressed (when $p_t = 0$). This is the equilibrium we focus on in the sequel. Interestingly, the presence of litigation costs results in a unique equilibrium, so no equilibrium selection criterion is necessary.

One might think that the presence of litigation costs will induce the manager to "spontaneously" disclose bad news, even when prices are relatively high. But on closer inspection, the equilibrium is not so clear: if the market expected the manager to disclose bad news at a particular point in time, then missing such announcements would be perceived as clear evidence of good news. This would lead to a sharp positive jump in the stock price. This in turn would destroy the manager's incentives to disclose the bad news in the first place: the temptation to not disclose bad news so as to benefit from the jump would offset the litigation benefits of disclosing the bad news. This suggests that the manager's disclosure strategy must entail some randomization.

We conjecture and verify that the equilibrium is given by

1. If $V_t = 1$, then $d_t = \mathbf{1}_{\{p_t < p_*\}}$.
2. If $V_t = 0$, then:
   (a) If $p_t > p_*$ we have $d_t = \emptyset$.
   (b) If $p_t = p_*$ then the manager discloses with a mean arrival rate
   $$\zeta = \kappa \frac{p_* - \bar{p}}{p_*(1 - p_*)} - \mu.$$
   (c) If $p_t < p_*$ then the manager discloses immediately with probability[20]
   $$\frac{p_t}{1 - p_t} \frac{1 - p_*}{p_*}.$$

Figure 4 shows a sample path of the stock price in this equilibrium. At the beginning, the price experiences a downward drift up until it hits the threshold p_*. This downward drift is caused purely by the increased likelihood of an undisclosed impairment. Given bad news, the manager starts randomizing between disclosing and not disclosing his private information. The price remains flat up until the manager reports bad news, at time T_1. Naturally, this

[20]This is an out-off-equilibrium event as with perfect bad news beliefs never enter the interval $(0, p_*)$ on the equilibrium path.

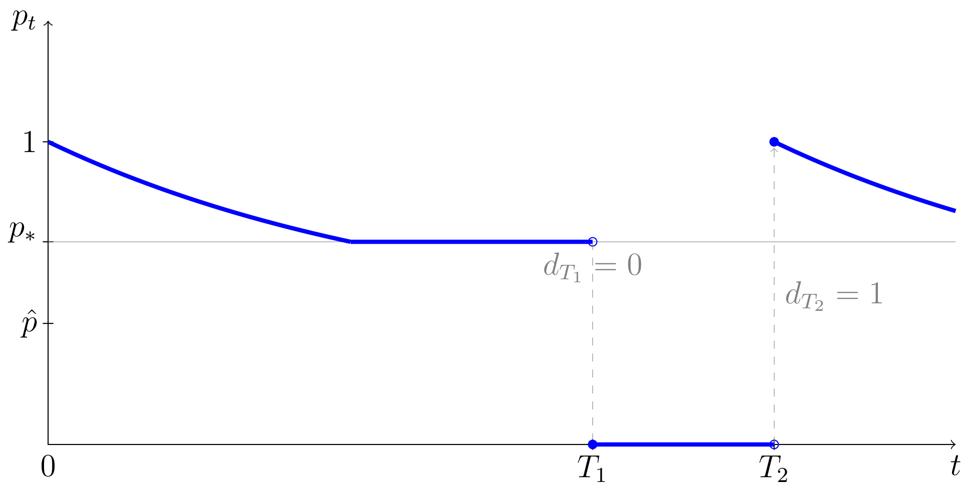


Figure 4: Example of a sample path of the share price with litigation cost.

disclosure causes the price to drop to zero and stay there until the situation of the firm improves, at time T_2, and the manager discloses good news.

Unlike the case without litigation costs analyzed in Sections 2, the manager strictly prefers not to disclose good news, unless the price experiences the most severe undervaluation (i.e., when $p_t = 0$). The reason why the manager becomes less prone to disclosing good news is that, in this setting, the absence of good news disclosures becomes a positive sign: since disclosures are expected to sometimes convey bad news, withholding information is often perceived as a good sign about the asset value. Though the price still drifts downward, it does it more slowly than without litigation costs; in fact the drift stops when the price reaches the threshold.

As previously mentioned, the equilibrium must entail randomization. If, at any point, the manager disclosed bad news with probability one, then the absence of such disclosures would be interpreted by investors as an indication that the asset value was high, which would cause an immediate upward jump in the stock price. This would destroy the manager's incentives to disclose bad news in the first place: witholding bad news would totally offset the risk of litigation. The manager's disclosure randomization allows the price not to jump upward when the price reaches the threshold, but either to remain constant, in the absence of disclosure, or to drop down to zero in the presence of a disclosure (see Figure 4). Of course, for randomization to be the manager's optimal response, he must be indifferent between disclosing low values (so as to avoid the risk of litigation) and not disclosing it (so as to enjoy inflated prices). This indifference condition allows us to pin down uniquely the disclosure threshold.

The threshold p_* characterizes an optimal disclosure strategy if the manager's payoff satisfy the following HJB equation. For $p_t > p_*$,

$$rU_1(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)] \tag{31}$$
$$rU_0(p) = p - \mu\theta + f(p)U_0'(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)]. \tag{32}$$

These equations are analogous to those encountered in previous settings, except that in the low state the manager's instant payoff is given by the price net of expected litigation costs.

The manager is exposed to two types of shocks. First, the asset value may experience a "real shock" which even when not observed by the market will affect the trajectory of prices and the expected litigation costs. Second, the manager may experience a "news shock": a news arrival may reveal the manager withheld information which would trigger both a drop in the stock price and potential litigation costs.

The manager's decision to disclose bad news has the flavor of the real options problem analyzed by Dixit (1989), where a firm has the option, at any point in time, to shut-down a project (i.e., disclose bad news) or restart it (i.e., disclose good news), based on the project's observed profitability. The difference is that the payoffs are endogenous here, because they are linked to the market's equilibrium belief about asset values. When the stock price is low (and the value of the asset is low), disclosing bad news becomes profitable for the same reason shutting down a project that is making losses is optimal in Dixit's model. Also, as in Dixit's problem, here the decision to disclose bad news today is linked to the option to disclose good news future: if the likelihood of disclosing good news in the future goes down (perhaps because λ_1 is smaller, or the proprietary costs are higher), then the manager's incentive to disclose bad news today weaken. Consequently, he further delays such disclosures. This speaks to a certain complementarity between disclosure of bad news and disclosure of good news: the higher the propensity of the manager to disclose good news, the higher will be his propensity to disclose bad news.

To complete the characterization of the equilibrium, we need to derive the boundary conditions. When $p_t = p_*$, we have

$$U_0(p_*) = E\left[\int_0^{\tau_N\wedge\tau_D\wedge\tau_1} e^{-rt}(p_* - \mu\theta)dt + e^{-r\tau_N\wedge\tau_D\wedge\tau_1}\left(U_0(0)\mathbf{1}_{\{\tau_N\wedge\tau_D<\tau_1\}} + U_1(p_*)\mathbf{1}_{\{\tau_N\wedge\tau_D>\tau_1\}}\right)\right],$$

where τ_N is the first arrival of public (bad) news, τ_D is the time at which the manager voluntarily discloses bad news, and τ_1 is the time at which the value of assets jump from 0 to 1. We can solve for the expected payoff of a low type manager, as given by

$$U_0(p_*) = \int_0^\infty e^{-(r+\mu+\zeta+\lambda_1)t}\big(p_* - \mu\theta + (\mu+\zeta)U_0(0) + \lambda_1 U_1(p_*)\big)dt$$
$$U_0(p_*) = \frac{p_* - \mu\theta}{r+\mu+\lambda_1+\zeta} + \frac{\mu+\zeta}{r+\mu+\lambda_1+\zeta}U_0(0) + \frac{\lambda_1}{r+\mu+\lambda_1+\zeta}U_1(p_*). \tag{33}$$

Following similar steps as the ones above we get the boundary condition for a high type manager as given by

$$U_1(p_*) = \frac{p_* + \lambda_0 U_0(p_*)}{r+\lambda_0}. \tag{34}$$

In addition, we have the following conditions when $p_t = 0$:

$$U_0(0) = \frac{\lambda_1}{r+\lambda_1}U_1(0) \tag{35}$$
$$U_1(0) = U_1(1) - c. \tag{36}$$

As the manager is using a mix strategy when $p_t = p_*$, he must be indifferent between disclosing negative information and not disclosing it, otherwise he would not be willing to randomize. Hence, we can determine the threshold p_* using the indifference condition for a mixed strategy:

$$U_0(p_*) = U_0(0). \tag{37}$$

We can solve for $U_0(p_*)$ by combining equations (33) and (37), which give us

$$U_0(p_*) = \frac{p_* - \mu\theta + \lambda_1 U_1(p_*)}{r+\lambda_1}. \tag{38}$$

Then combining (34) with (38) we get

$$U_0(p_*) = \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{r+\lambda_0}{r+\kappa} \tag{39}$$
$$U_1(p_*) = \frac{p_*}{r} - \frac{\mu\theta}{r}\frac{\lambda_0}{r+\kappa}. \tag{40}$$

The value of p_* can thus be obtained from

$$U_0(p_*) = \frac{\lambda_1}{r+\lambda_1}\left[U_1(1) - c\right]. \tag{41}$$

The strategies above constitute an equilibrium as long as the following conditions are satisfied

1. $U_1(1) - c \geq 0$.

2. $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$.

3. $U_0(p) \geq U_0(0)$ for $p > p_*$.

A necessary condition for optimality of the disclosure strategy above is that $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$. If U_1 is increasing in p_*, then this condition is satisfied if and only if

$$U_1(p_*) \geq U_1(1) - c = \left(1 + \frac{r}{\lambda_1}\right) U_0(p_*), \tag{42}$$

where we have used the equilibrium condition (41). Combining (39), (40) and (42) we get the following upper bound for the disclosure threshold p_*

$$p_* \leq \mu\theta.$$

The disclosure threshold is lower than the myopic threshold (namely the threshold that a manager exclusively concerned with his instantaneous payoffs would select). This is natural: the more the manager cares about future prices, the weaker is his incentive to reveal information that will cause a price drop. When the price has reached the level of the litigation costs $\mu\theta$ the manager has an incentive to wait even further and "bet for resurrection": since the asset may recover its value, the manager has the option to wait and see if this event realizes thus avoiding the need to disclose bad new. Of course, this bet only makes sense if there is a positive probability of "resurrection" (i.e., $\lambda_1 > 0$). This idea is borne out by the survey evidence in Graham, Harvey, and Rajgopal (2005). Some CFOs claim that they delay bad news disclosures in the hope that they may never have to release the bad news if the firm's status improves. This is, in essence, Verrecchia (1983)'s alternative explation.[21]

The condition $U_1(1) - c \geq 0$ is satisfied if and only if $U_0(p_*) \geq 0$. Thus, using (39) we obtain a lower bound for p_* given by

$$p_* \geq \frac{r + \lambda_0}{r + \kappa}\mu\theta.$$

This lower bound reveals an intuitive feature of the model: if litigation costs are too high the bound will hit 1 which means that no asymmetry of information can ever be experienced

[21] "An alternative to my explanation for why a manager delays the reporting of 'bad news' is that he hopes that during the interim some 'good news' will occur to offset what he has to say.' The disadvantage of this explanation is that it ignores the fact that rational expectations traders will correctly infer 'bad news' as soon as it becomes apparent that the information is being withheld" Verrecchia (1983)

in equilibrium: negative information must be revealed immediately when litigation costs are prohibitively high.

Proposition 9. *For any $p_* \in [\mu\theta(r+\lambda_0)/(r+\kappa), \mu\theta]$, let $U_v(p)$ be the solution to equations* (31) *and* (32)*, with initial conditions* (39) *and* (40)*. Suppose that $U_v(p)$ are non-decreasing functions satisfying*

$$U_0(p_*) = \frac{\lambda_1}{r+\lambda_1}\left[U_1(1) - c\right],$$

where $U_0(p_*)$ is given by the initial condition (39)*. Then, there exist an equilibrium such that*

1. *If $V_t = 1$, then $d_t = \mathbf{1}_{\{p_t < p_*\}}$.*
2. *If $V_t = 0$, then:*
   - *$d_t = \emptyset$ for $p_t > p_*$.*
   - *If $p_t = p_*$, then the manager discloses with intensity*

$$\zeta = \kappa\frac{p_* - \bar{p}}{p_*(1-p_*)} - \mu.$$

   - *If $p_t < p_*$, then the manager discloses immediately with probability*

$$\frac{p_t}{1-p_t}\frac{1-p_*}{p_*}.$$

4.1.1 A tractable example: permanent shocks

A particularly tractable example arises when $\lambda_1 = 0$, namely when negative shocks are permanent. Since in that case the asset never recovers its value after an adverse shock, we must have

$$U_0(0) = 0.$$

Using this condition along with (37) and (39) we get

$$p_* = \mu\theta.$$

Hence, the optimal disclosure strategy is the myopic policy. This is natural: since the option to wait for the asset's recovery has no value, there is no point in further delaying disclosures of bad news when the current payoffs are non positive.

When the price reaches p_* the manager discloses low value of assets with intensity

$$\zeta = \max\left(\frac{\lambda_0}{1-\mu\theta} - \mu, 0\right).$$

Notice that $\zeta = 0$ if and only if $\mu \in \left[\frac{1-\sqrt{1-4\theta\lambda_0}}{2\theta}, \frac{1+\sqrt{1-4\theta\lambda_0}}{2\theta}\right]$. When μ is in this interval, there is no disclosue. The manager always prefers to bear the risk of litigation, instead of revealing that the asset value is low. This shows that the frequency of disclosure may be non-monotonic in the intensity of public news μ. This result is perhaps surprising because a higher μ leads to higher costs of litigation, other things equal. One would think that when μ is higher, the manager should become more inclined to disclosing his information so as to avoid the (more likely) litigation costs. There is nontheless another effect going in the opposite direction. Observe that

$$\hat{p} = 1 - \frac{\lambda_0}{\mu}$$

also increases in μ. Hence, the price tends to drift slower and stay at higher level, absent disclosure and news arrivals. This effect naturally reduces the incentive to disclose bad news, and may offset litigation costs effect, explaining the non-monotonicity of the frequency of disclosure with respect to the intensity of public news.

4.2 The Frequency of Disclosure

In order to find the frequency of disclosure we must compute the expected time at which the manager will disclose his information. As before, the Markov structure of the problem allows us to focus on the expected time of the first disclosure. Let $T = \inf\{t > 0 | d_t = 1\}$. We want to compute $\overline{T}_v(p) := E(T|p_t = p, V_t = v)$. By standard arguments, $\overline{T}_v$ satisfies:

$$-1 = f(p)\overline{T}_1'(p) + \lambda_0[\overline{T}_0(p) - \overline{T}_1(p)] \tag{43}$$

$$-1 = f(p)\overline{T}_0'(p) + \lambda_1[\overline{T}_1(p) - \overline{T}_0(p)] + \mu[\overline{T}_0(0) - \overline{T}_0(p)] \tag{44}$$

In order to find the right boundary conditions, we note that

$$\overline{T}_1(p_*) = \frac{1}{\lambda_0} + \overline{T}_0(p_*) \tag{45}$$

$$\overline{T}_0(p_*) = \frac{1}{\mu+\zeta+\lambda_1} + \frac{\lambda_1}{\mu+\zeta+\lambda_1}\overline{T}_1(p_*) + \frac{\mu}{\mu+\zeta+\lambda_1}\overline{T}_0(0). \tag{46}$$

A high type manager never disclose when $p_t = p_*$. Equation (45) simply says that the expected time until the next disclosure equals the expected time that it takes for the value of the firm to jump down to zero plus the expected time that it takes for a low type manager to disclose. Equation (46) has a similar interpretation. The expected time that it takes for a low type firm to disclose consider three possible events: 1) the firm disclose negative information, 2) the value of the assets jumps up to one, 3) there are negative public news that take beliefs down to zero. In addition, as in the case without litigation, we have that

$$\overline{T}_1(0) = 0$$
$$\overline{T}_0(0) = \frac{1}{\lambda_1}.$$

The solution to these differential equations characterize the frequency of disclosure as a function of market beliefs and asset values. Figure 5 studies the determinants of the frequency of disclosure. Intuitively, the frequency of disclosure decreases in the cost of disclosure c. A higher disclosure cost delays the disclosure of bad news by making the disclosure of good news more costly hence less attractive to the manager. If the manager anticipates that overcoming a possible undervaluation by disclosing good news will be too costly, he might as well delay the bad news in the first place.

By contrast, the frequency of disclosure increases in the cash flows' mean reversion κ. A higher mean reversion means the shocks are more transitory. This means that disclosing bad news will have a weaker impact on the trajectory of future stock prices.

5 Concluding Remarks

This paper studies a model of dynamic costly disclosure. We make the following contribution to the literature. To our knowledge, this is the first dynamic model of disclosure with the realistic feature that private and public information flows happen in an ongoing (continuous) basis. We characterize the dynamics of disclosure, and derive its implications for the time-series of stock returns. Our analysis is consistent with a number of stylized facts such as the clustering of announcements in bad times, the downward drift of stock prices prior to a disclosure, the negative market reaction to firm's breaking their disclosure (implicit) commitments, the higher volatility of prices given no disclosure.

Our model has several limitations. First, the state of nature is binary. One could consider the possibility of asset values that are continuously distributed. This is not just for the sake of elegance but because some properties of the equilibrium —such as the *blackout* period where no disclosure is observed– are purely an artifact of the binary setting. Moreover the multiplicity of equilibria is also driven by the discontinuous nature of the distribution of the

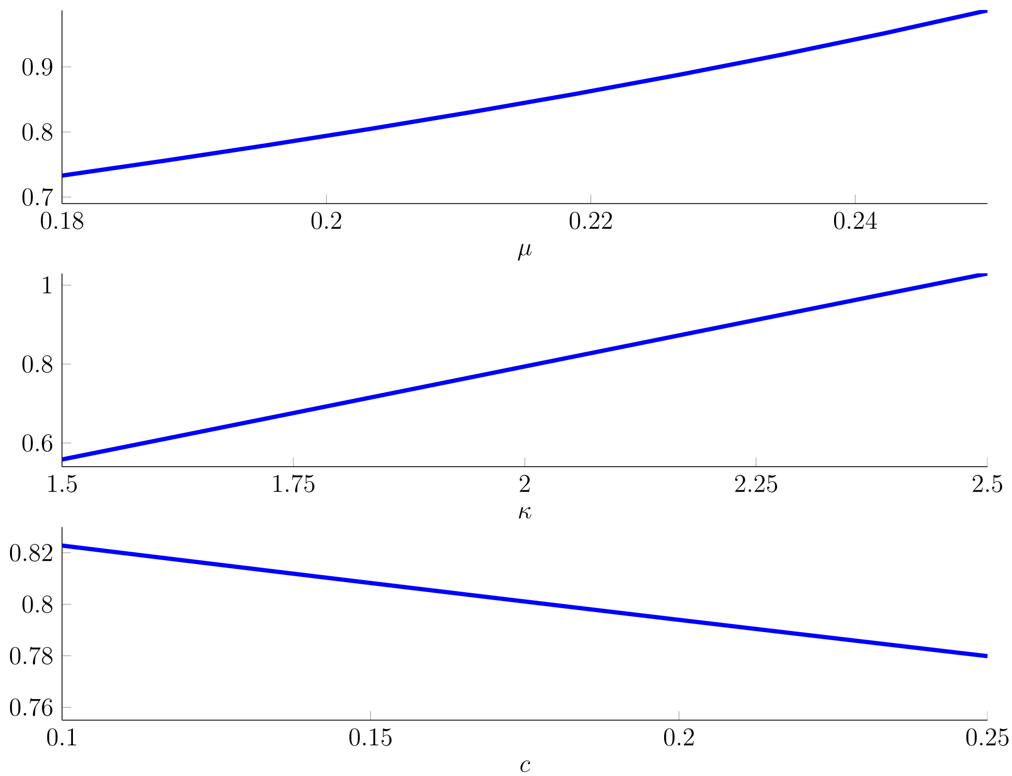


Figure 5: The Frequency of Disclosure ($1/\overline{T}_1(1)$)

asset value process.

Second, we have modeled the public information process as a Poisson process. An interesting but difficult extension is to consider the idea that the public information process follows a Brownian motion whose drift depends on the state of nature, along the lines of the information structure considered by Daley and Green (2012). Again, this would allow for a more realistic characterization of stock returns.

References

Acharya, Viral V., Peter DeMarzo, and Ilan Kremer. 2011. "Endogenous Information Flows and the Clustering of Announcements." *American Economic Review*, 101(7): 2955–79.

Asmussen, Soren. 2003. *Applied Probability and Queues.* . 2 ed., New York:Springer.

Bar-Isaac, Heski. 2003. "Reputation and Survival: learning in a dynamic signaling model." *The Review of Economic Studies*, 70(2): 231–251.

Benmelech, Efraim, Eugene Kandel, and Pietro Veronesi. 2010. "Stock-based compensation and CEO (dis) incentives." *The Quarterly Journal of Economics*, 125(4): 1769–1820.

Beyer, Anne, and RonaldA. Dye. 2012. "Reputation management and the disclosure of earnings forecasts." *Review of Accounting Studies*, 17: 877–912.

Chen, Shuping, Dawn Matsumoto, and Shiva Rajgopal. 2011. "Is silence golden? An empirical analysis of firms that stop giving quarterly earnings guidance." *Journal of Accounting and Economics*, 51(1â2): 134 – 150.

Daley, Brendan, and Brett Green. 2012. "Waiting for News in the Market for Lemons." *Econometrica*, 80(4): 1433–1504.

Davis, Mark HA. 1993. *Markov Models and Optimization.* 2-6 Boundary Row, London:Chapman & Hall.

Dixit, Avinash. 1989. "Entry and Exit Decisions under Uncertainty." *Journal of Political Economy*, 97(3): pp. 620–638.

Dye, Ronald A. 1985. "Disclosure of Nonproprietary Information." *Journal of Accounting Research*, 23(1): pp. 123–145.

Dye, Ronald A. 2010. "Disclosure Bunching." *Journal of Accounting Research*, 48(3): 489–530.

Graham, John R., Campbell R. Harvey, and Shiva Rajgopal. 2005. "The economic implications of corporate financial reporting." *Journal of Accounting and Economics*, 40(1â3): 3 – 73.

Grossman, Sanford J. 1981. "The Informational Role of Warranties and Private Disclosure about Product Quality." *Journal of Law and Economics*, 24(3): pp. 461–483.

Gul, Faruk, and Wolfgang Pesendorfer. 2012. "The war of information." *The Review of Economic Studies*, 79(2): 707–734.

Jovanovic, Boyan. 1982. "Truthful Disclosure of Information." *The Bell Journal of Economics*, 13(1): pp. 36–44.

Karlin, Samuel, and Howard M. Taylor. 1981. *A Second Course in Stochastic Processes.* Vol. 2, 111 Fifth Avenue, New York:Academic Press.

Kothari, S. P., Susan Shu, and Peter D. Wysocki. 2009. "Do Managers Withhold Bad News?" *Journal of Accounting Research*, 47(1): 241–276.

Kremer, Ilan, and Andrzej Skrzypacz. 2007. "Dynamic signaling and market breakdown." *Journal of Economic Theory*, 133(1): 58–82.

Kremer, Ilan, Ilan Guttman, and Andrzej Skrypacz. 2012. "Not Only What But also When: A Theory of Dynamic Voluntary Disclosure." Working Paper, Stanford University.

Milgrom, Paul R. 1981. "Good News and Bad News: Representation Theorems and Applications." *The Bell Journal of Economics*, 12(2): pp. 380–391.

Skinner, Douglas J. 1997. "Earnings disclosures and stockholder lawsuits." *Journal of Accounting and Economics*, 23(3): 249 – 282.

Verrecchia, Robert E. 1983. "Discretionary disclosure." *Journal of Accounting and Economics*, 5(0): 179 – 194.

Verrecchia, Robert E. 1990. "Information quality and discretionary disclosure." *Journal of Accounting and Economics*, 12(4): 365 – 380.

A Proofs of Section 2

Proof of Proposition 1

Proof. We divide the proof of Proposition 1 in two steps. In step 1, we show that the functions in the proposition solve the HJB equation with the required boundary conditions. In step 2, we show that the solution constitutes an equilibrium.

Step 1:

In the absence of any disclosure, the beliefs at time t are given by

$$\phi_t(p_0) = \bar{p} + e^{-\kappa t}(p_0 - \bar{p}).$$

Let's define $T(p;p_*)$ as the time that it takes the beliefs to reach p_* give that current beliefs are p. That is,

$$T(p;p_*) = -\frac{1}{\kappa}\log\left(\frac{p_* - \bar{p}}{p - \bar{p}}\right),$$

where $\frac{\partial T(p;p_*)}{\partial p} > 0$ and $\frac{\partial T(p;p_*)}{\partial p_*} < 0$. The results in Davis (1993, pp. 92-93) imply that the solution to the HJB equation (6)-(7) satisfies

$$\begin{aligned}
U_0(p|p_*) &= \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\Big[\Pr(V_{T(p;p_*)} = 0|V_0 = 0)U_0(p_*|p_*) \\
&+ \Pr(V_{T(p;p_*)} = 1|V_0 = 0)U_1(p_*|p_*)\Big] \\
U_1(p|p_*) &= \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\Big[\Pr(V_{T(p;p_*)} = 0|V_0 = 1)U_0(p_*|p_*) \\
&+ \Pr(V_{T(p;p_*)} = 1|V_0 = 1)U_1(p_*|p_*)\Big].
\end{aligned}$$

Replacing $\Pr(V_{T(p;p_*)} = j|V_0 = i)$ for $i,j \in \{0,1\}$, and using the boundary conditions, we can write the manager's expected payoff as

$$U_0(p|p_*) = \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} - \frac{r\bar{p}}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1|p_*) - c\Big) \tag{47}$$

$$U_1(p|p_*) = \int_0^{T(p;p_*)} e^{-rt}\phi_t(p)dt + e^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1|p_*) - c\Big). \tag{48}$$

Using equation (48) we can write $U_1(1|p_*)$ as

$$U_1(1|p_*,\kappa) = \frac{\int_0^{T(1;p_*)} e^{-rt}\phi_t(1)dt}{1-\delta(1)} - \frac{\delta(1)}{1-\delta(1)}c, \tag{49}$$

where

$$\delta(1) = e^{-rT(1;p_*)}\Big[\frac{r\bar{p}+\kappa\bar{p}}{r+\kappa\bar{p}} + \underbrace{\frac{r(p_*-\bar{p})}{r+\kappa\bar{p}}}_{\frac{r(1-\bar{p})}{r+\kappa\bar{p}}e^{-\kappa T(1;p_*)}}\Big].$$

The first term in (49) can be written as

$$U^{ND}(1)\frac{\overbrace{1-e^{-rT(1;p_*)}\frac{U^{ND}(p_*)}{U^{ND}(1)}}^{1-\delta(1)}}{1-\delta(1)} = U^{ND}(1).$$

Hence,

$$U_1(1|p_*) = U^{ND}(1) - \frac{\delta(1)}{1-\delta(1)}c, \tag{50}$$

Step 2:

The only step left is to show that (10) and (11) imply $U_1(p) \geq U_1(1) - c$ for all $p > p_*$ so a threshold policy is optimal. We first show that (10) and (11) imply $U_1'(p) \geq 0$ for all $p > p_*$. The derivative of U_1 is given by

$$U_1'(p) = e^{-rT(p;p_*)}\Phi(p)\frac{\partial T(p;p_*)}{\partial p} + \int_0^{T(p;p_*)} e^{-(r+\kappa)t}dt \tag{51}$$

where

$$\Phi(p) := p_* - re^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{(1-\bar{p})(r+\kappa)}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\Big(U_1(1) - c\Big).$$

From here we get that $U_1'(p_*) \geq 0$ if and only if $\Phi(p_*) \geq 0$. Moreover, $U_1(1) - c > 0$ implies $\Phi'(p) > 0$, which means that $\Phi(p) \geq 0$ for all $p > p_*$. Accordingly, $U_1'(p) \geq 0$ for all $p > p_*$, and

$$U_1(p) = U_1(p_*) + \int_{p_*}^{p} U_1'(y)dy = U_1(1) - c + \int_{p_*}^{p} U_1'(y)dy > U_1(1) - c.$$

∎

Proof of Proposition 2

We begin proving two lemmas.

Lemma 1. *Suppose that conditions* (10) *and* (11) *are satisfied, then* $\frac{\partial}{\partial p_*}U_1(p|p_*) < 0$.

Proof. Differentiating (48) with respect to p_* we get

$$\frac{\partial}{\partial p_*}U_1(p|p_*) = e^{-rT(p;p_*)}\Phi(p;p_*)\frac{\partial T(p;p_*)}{\partial p_*} + e^{-rT(p;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1}e^{-\kappa T(p;p_*)}\right]\frac{\partial}{\partial p_*}U_1(1|p_*). \tag{52}$$

From here we get

$$\frac{\partial}{\partial p_*}U_1(1|p_*) = \frac{e^{-rT(1;p_*)}\Phi(1;p_*)}{1 - e^{-rT(1;p_*)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1}e^{-\kappa T(1;p_*)}\right]}\left.\frac{\partial T(p;p_*)}{\partial p_*}\right|_{p=1} < 0,$$

so $\frac{\partial}{\partial p_*}U_1(p|p_*) \leq 0$ as $\Phi(p;p_*) \geq 0$ (see proof of Proposition 1). ∎

Lemma 2. *Suppose that conditions* (10) *and* (11) *are satisfied, then* $U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*}U_1'(p_*|p_*) > 0$.

Proof. Rearranging the HJB equation (6) we can write

$$U_1'(p|p_*) = \frac{rU_1(p|p_*) - p - \lambda_0[U_0(p|p_*) - U_1(p|p_*)]}{\kappa(\overline{p} - p)}$$

Evaluating at $p = p_*$ and using the boundary conditions, equations (8) and (9), yields

$$\begin{aligned} U_1'(p_*|p_*) &= \frac{rU_1(p_*|p_*) - p_* + U_1(p_*|p_*)\frac{r\lambda_0}{r+\lambda_1}}{\kappa(\overline{p} - p_*)} \\ &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}U_1(p_*|p_*) - p_*}{\kappa(\overline{p} - p_*)} \end{aligned} \tag{53}$$

Now, we can show that

$$U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*}U_1'(p_*|p_*) > 0.$$

Differentiating equation (53) with respect to p_* yields

$$\begin{aligned}
\frac{\partial}{\partial p_*}U_1'(p_*|p_*) &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}\frac{\partial U_1(p|p_*)}{\partial p_*}\Big|_{p=p_*} - 1}{\kappa(\overline{p}-p_*)} + \frac{\kappa}{\kappa(\overline{p}-p_*)}U_1'(p_*|p_*) \\
&= \frac{\frac{r(r+\kappa)}{r+\lambda_1}\frac{\partial U_1(p|p_*)}{\partial p_*}\Big|_{p=p_*} - 1}{\kappa(\overline{p}-p_*)} > 0
\end{aligned}$$

But from Lemma 1 we know that $\frac{\partial U_1(p|p_*)}{\partial p_*} < 0$. This along with $\kappa(\overline{p}-p_*) < 0$ proves the lemma. ∎

Proof of Proposition 2. Suppose there exist $p_*^- < p_*^+$ such that

$$\begin{aligned}
U_1'\left(p_*^-|p_*^-\right) &= 0 \\
U_1\left(1|p_*^+\right) - c &= 0
\end{aligned}$$

A direct consequence of Lemma 2 is that $U_1'(p_*|p_*)$ crosses 0 only once. Thus, $U_1'(p_*|p_*) \geq 0$ for $p_* \geq p_*^-$, and $U_1'(p_*|p_*) < 0$ for $p_* < p_*^-$. Moreover, from Lemma 1 we have that $U_1(p_*|p_*) - c \geq 0$ for all $p_* \leq p_*^+$. Hence, p_* satisfies conditions (10) and (11) if and only if $p_* \in [p_*^-, p_*^+]$.

The only step left is to show that if the cost of disclosure satisfy the conditions in the proposition then exist $p_*^-, p_*^+ \in (\bar{p}, 1)$ with the required properties.

Claim 1: If $c < \frac{r+\lambda_1}{r(r+\kappa)}$, then there is a threshold $p_*^+ \in (\bar{p}, 1)$ such that $U_1(1|p_*^+) - c = 0$.

First, from equation (48) we have that $U(1|\bar{p}) = U^{ND}(1)$. Hence, $U(1|\bar{p}) - c > 0$ if and only if

$$c < \frac{\lambda_1 + r}{r(r+\kappa)}.$$

Second, $U(1|1-\epsilon) - c < 0$ for ϵ close to zero. Let

$$\beta(\epsilon) := e^{-rT(1;1-\epsilon)}\left[\frac{r\bar{p}+\lambda_1}{r+\lambda_1} + \frac{r(1-\bar{p})}{r+\lambda_1}e^{-\kappa T(1;1-\epsilon)}\right].$$

Using equation (48) we get that

$$(1-\beta(\epsilon))\big[U(1|1-\epsilon) - c\big] < T(1;1-\epsilon) - \beta(\epsilon)c,$$

where $T(1;1-\epsilon) - \beta(\epsilon)c <$ for ϵ close to zero. Hence, by continuity there exist $p_*^+ \in (\bar{p}, 1)$ such that $U_1(1|p_*^+) - c = 0$. Moreover, equation (51) implies $U_1'(p_*^+|p_*^+) > 0$.

Claim 2: If $c < \frac{r+\lambda_1}{r(r+\kappa)}(1-\bar{p})$, then there there is $p_*^- < p_*^+$ such that $U'(p_*^-|p_*^-) = 0$.

First, we verify that $\lim_{p_* \downarrow \bar{p}} U_1'(p_*|p_*) < 0$. Using the HJB equation

$$U_1'(p_*|p_*) = \frac{\frac{r(r+\kappa)}{r+\lambda_1} U_1(p_*|p_*) - p_*}{\kappa(\overline{p} - p_*)}$$

Noting that $\lim_{p_* \downarrow \bar{p}} U_1(p_*|p_*) = U_1^{ND}(1) - c$, it suffices to show that

$$\frac{r(r+\kappa)}{r+\lambda_1}\left(U_1^{ND}(1) - c\right) - \bar{p} > 0,$$

which, after straightforward algebra, is satisfied if and only if $c < \frac{r+\lambda_1}{r(r+\kappa)}(1-\bar{p})$. Second, we verify that $\lim_{p_* \uparrow 1} U_1'(p_*|p_*) > 0$. When $p_* \uparrow 1$ the firm starts disclosing infinitely often. Hence, the cost of disclosure grows without bound. Moreover, the benefit of disclosing is bounded. Accordingly

$$\lim_{p_* \uparrow 1} \frac{r(r+\kappa)}{r+\lambda_1} U_1(p_*|p_*) - p_* < 0$$

so, from the HJB equation, $\lim_{p_* \uparrow 1} U_1'(p_*|p_*) > 0$. By continuity there is $p_*^- \in (\bar{p}, 1)$ with the required properties. Moreover, $U_1'(p_*^+|p_*^+) > 0$ implies that $p_*^- < p_*^+$.

∎

Proof of Proposition 3

Proof. From Lemma 1, $\partial U_1(p|p_*)/\partial p_* < 0$. Hence, it only remains to show that $\partial U_0(p|p_*)/\partial p_* < 0$. Differentiating (47) with respect to p_*, we get

$$\frac{\partial}{\partial p_*} U_0(p|p_*) = e^{-rT(p;p_*)} \Gamma(p;p_*) \frac{\partial T(p;p_*)}{\partial p_*} + e^{-rT(p;p_*)} \left[\frac{r\bar{p} + \lambda_1}{r+\lambda_1} - \frac{r\bar{p}}{r+\lambda_1} e^{-\kappa T(p;p_*)} \right] \frac{\partial}{\partial p_*} U_1(1|p_*), \tag{54}$$

where

$$\begin{aligned}
\Gamma(p;p_*) &= p_* - re^{-rT(p;p_*)} \left[\frac{r\bar{p} + \lambda_1}{r+\lambda_1} - \frac{\bar{p}(r+\kappa)}{r+\lambda_1} e^{-\kappa T(p;p_*)} \right] \left(U_1(1) - c \right) \\
&= p_* - re^{-rT(p;p_*)} \left[\frac{r\bar{p} + \lambda_1}{r+\lambda_1} + \frac{(1-\bar{p})(r+\kappa)}{r+\lambda_1} e^{-\kappa T(p;p_*)} - \frac{(r+\kappa)}{r+\lambda_1} e^{-\kappa T(p;p_*)} \right] \left(U_1(1) - c \right) \\
&= \Phi(p;p_*) + \frac{r(r+\kappa)}{r+\lambda_1} e^{-(r+\kappa)T(p;p_*)} \left(U_1(1) - c \right) \\
&\geq 0.
\end{aligned}$$

Thus, $\partial U_0(p|p_*)/\partial p_* < 0$ as both $\partial U_1(1|p_*)/\partial p_* < 0$ and $\partial T(p;p_*)/\partial p_* < 0$. ∎

Proof of Proposition 5

We are interested in $p_*'(\kappa)$ for p_* solving $U_1'(p_*) = 0$. Using the HJB equation and the boundary conditions we have that

$$r(r+\kappa)U_1(p_*) = (r+\lambda_1)p_*.$$

We we want to change κ keeping $\bar{p}$ constant. Noting that $\lambda_1 = \kappa\bar{p}$ we have that p_* solves

$$r(r+\kappa)U_1(p_*) = (r+\kappa\bar{p})p_*.$$

The proof is going to be by contradiction. We are going to assume that $p_*'(\kappa) > 0$ and then arrive to a contradiction.

Lemma 3. *Suppose that $p_*'(k) > 0$, then $dU_1(1|p_*(\kappa),\kappa)/d\kappa < 0$.*

Proof. Let $U_1(1|p_*,\kappa)$ be the manager's expected utility given an equilibrium p_* and mean reversion κ. Then

$$\frac{d}{d\kappa}U_1(1|p_*(\kappa),\kappa) = \frac{\partial}{\partial p_*}U_1(1|p_*(\kappa),\kappa)p_*'(\kappa) + \frac{\partial}{\partial \kappa}U_1(1|p_*(\kappa),\kappa).$$

Lemma 1 and $p_*'(\kappa) > 0$ imply that the first term is negative. With some abuse of notation let's define $\delta(\kappa,p_*) := \delta(1)$ for $\delta(1)$ in Proposition 1 as a function of κ and p_*. Thus, we have from Proposition 1 that

$$U_1(1|p_*,\kappa) = U^{ND}(1) - \frac{\delta(\kappa,p_*)}{1-\delta(\kappa,p_*)}c. \tag{55}$$

Finally,

$$\delta_\kappa(\kappa,p_*) = \underbrace{-rT_\kappa(1;p_*)}_{>0}\delta(\kappa,p_*) + \frac{re^{-rT(1;p_*)}\bar{p}(1-p_*)}{(r+\kappa\bar{p})^2} > 0$$

implies that $\frac{\partial}{\partial\kappa}U_1(1|p_*,\kappa) < 0$ completing the proof of the Lemma. ∎

Lemma 4. *Suppose that $p_*'(k) > 0$, then $\partial\big(\kappa U_1(1)\big)/\partial\kappa > 0$.*

Proof. Using the HJB equation and the boundary conditions we get

$$\kappa U_1(1) = (r+\kappa)c + (1+\kappa\bar{p}/r)p_*(\kappa) - rU_1(1)$$

By Lemma 3, given the hypothesis $p_*'(\kappa) > 0$, we have that $U_1(1)$ is decreasing in κ. Hence, $\kappa U_1(1)$ is increasing in κ. ∎

Proof of Proposition 5. We prove the proposition by contradiction. Take $\tilde{\kappa} > \kappa$ and let $\tilde{U}_\theta$ and U_θ be the respective solutions. Suppose that $p_*'(\kappa) > 0$ so $\tilde{p}_* > p_*$. Using the HJB equation and the boundary conditions we get

$$\begin{aligned} r(r+\kappa)U_1(p_*) - r(r+\tilde{\kappa})\tilde{U}_1(\tilde{p}_*) &= (r+\kappa\bar{p})p_* - (r+\tilde{\kappa}\bar{p})\tilde{p}_* &(56)\\ &= (r+\tilde{\kappa}\bar{p})(p_* - \tilde{p}_*) + \bar{p}(\kappa - \tilde{\kappa})p_* < 0. &(57)\end{aligned}$$

In order to establish the contradiction we need to show that

$$r(r+\kappa)U_1(p_*) - r(r+\tilde{\kappa})\tilde{U}_1(\tilde{p}_*) > 0 \tag{58}$$

We can rewrite the right hand side in (56) as

$$r(r+\kappa)(U_1(1)-c) - r(r+\tilde{\kappa})(\tilde{U}_1(1)-c) = r^2(U_1(1)-\tilde{U}_1(1)) + r(\kappa U_1(1) - \tilde{\kappa}\tilde{U}_1(1)) + r(\tilde{\kappa}-\kappa)c. \tag{59}$$

By Lemma 4 we have that

$$\tilde{\kappa}(U_1(1) - \tilde{U}_1(1)) \geq \kappa U_1(1) - \tilde{\kappa}\tilde{U}_1(1) \geq 0. \tag{60}$$

Equation (60) together with (59) give us (58) and yields the desired contradiction. ∎

Proof of Proposition 6

Proof. $\{p_t\}_{t\geq 0}$ is a regenerative process with right continuos sample paths and nonlattice cycle length distribution.[22] By applying Theorem 1.2 in Asmussen (2003, p. 170) we get that p_t has a limiting distribution which is given by

$$F(p) = \frac{E\left[\int_0^\tau \mathbf{1}_{\{p_t \leq p\}} dt\right]}{E[\tau]}. \tag{61}$$

Let $S(p) = 1 - F(p)$, then we have that

$$S(p) = \frac{E\left[\int_0^\tau \mathbf{1}_{\{p_t > p\}} dt\right]}{E[\tau]}. \tag{62}$$

[22] A distribution is lattice if it is concentrated on a set of the form $\{\delta, 2\delta, \ldots\}$ (Asmussen, 2003, p. 153)

For $p \geq p_*$ we have that

$$S(p) = \frac{T(1;p)}{\overline{T}_1(1)} = \frac{\bar{p}\left(\log(1-\bar{p}) - \log(p-\bar{p})\right)}{1 - p_* + \bar{p}\left(\log(1-\bar{p}) - \log(p_*-\bar{p})\right)},$$

where $T(1;p)$ and $\overline{T}_1(1)$ are given by equations (16) and (17), respectively. Accordingly, for all $p \geq p_*$ we have

$$F(p) = \frac{1 - p_* + \bar{p}\left(\log(p-\bar{p}) - \log(p_*-\bar{p})\right)}{1 - p_* + \bar{p}\left(\log(1-\bar{p}) - \log(p_*-\bar{p})\right)}.$$

Finally, for $p < p_*$ we have $\Pr\left(p_t \in (0, p_*)\right) = 0$ for all t, and F has an atom at $p = 0$ given by

$$F(0) = \frac{\frac{1-p_*}{\lambda_1}}{\overline{T}_1(1)} = \frac{1-p_*}{1 - p_* + \bar{p}\left(\log(1-\bar{p}) - \log(p_*-\bar{p})\right)}.$$

∎

B Proofs of Section 3

Proof of Proposition 7

Let $\Delta(p) := U_1(p) - U_0(p)$, which satisfies

$$(r+\kappa)\Delta(p) = f(p)\Delta'(p) + \mu[U_0(p) - U_0(0)]. \tag{63}$$

Differentiating the HJB equation we get

$$rU_0'(p) = 1 + f'(p)U_0'(p) + f(p)U_0''(p) + \lambda_1\Delta'(p) - \mu U_0'(p) \tag{64}$$

$$rU_1'(p) = 1 + f'(p)U_1'(p) + f(p)U_1''(p) - \lambda_0\Delta'(p). \tag{65}$$

The proof is a direct consequence of the following two lemmas.

Lemma 5. *Suppose there is $p^1 \geq p_*$ such that $U_1'(p^1) = 0$, then $U_0'(p^1) > 0$.*

Proof. Evaluating (63) at p_* we get that $U_0'(p_*) > 0$. If U_0 is nondecreasing for $p > p_*$ we are done. Suppose that $U_0'(p)$ is decreasing for some $p > p_*$, then there must be some $p > p_*$ such that $U_0'(p) = 0$. Let $p^0 = \inf\{p \geq p_* : U_0'(p) < 0\}$. We have two possibilities, $p^0 \geq p^1$ or $p^0 < p^1$. Suppose that $p^0 < p^1$, if this is the case, using equation (64), we get $-f(p^0)U_0''(p^0) = 1 + \lambda_1 U_1'(p^0) > 0$. This means that $U_0''(p^0) > 0$ which contradicts the fact that $p^0 = \inf\{p > p_* : U_0'(p) < 0\}$ so it must be the case that $p^0 \geq p^1$. But then, by definition of p^0, we have $U_0'(p^1) > 0$. ∎

Lemma 6. *Suppose that $U_1'(p_*) \geq 0$, then U_1 is nondecreasing for all $p \geq p_*$.*

Proof. Suppose that $U_1(p)$ is decreasing in some interval, then there is p such that $U_1'(p) = 0$. Let's define $p^1 = \inf\{p > p_* : U_1'(p) < 0\}$. Then, by (65) we have that

$$-f(p^1)U_1''(p^1) = 1 + \lambda_0 U_0'(p^1).$$

By lemma 5 we have $U_0'(p^1) > 0$. This means that $U_1''(p^1) > 0$ which is a contradiction with $p^1 = \inf\{p > p_* : U_1'(p) < 0\}$. ∎

Proof Proposition 7. From the boundary condition we have $U_1(p) = U_1(1) - c$; moreover, U_1 is nondecreasing by lemma 6. Hence, $U_1(p) \geq U_1(1) - c$ for all $p > p_*$. ∎

Proof of Proposition 8

Lemma 7. *Let $p_*^1 < p_*^2$ be two equilibrium thresholds, then $U_1(1|p_*^1) > U_1(1|p_*^2)$.*

Proof. The solution to the HJB equation satisfies (Davis, 1993, Theorem 32.10, p. 94)

$$\begin{aligned} U_1(1) &= E\left[\int_0^\infty e^{-rt} p_t dt - c d_t \sum_{t \geq 0} e^{-rt}\right] \\ &= E\left[\int_0^\infty e^{-rt} E(V_t \big| \mathcal{F}_t) dt - c d_t \sum_{t \geq 0} e^{-rt}\right] \\ &= \int_0^\infty e^{-rt} E(V_t) dt - cE\left[d_t \sum_{t \geq 0} e^{-rt}\right] \\ &= \int_0^\infty e^{-rt} E(V_t) dt - \frac{\delta}{1-\delta} c, \end{aligned}$$

where $\delta := E(e^{-r\tau_d})$ and $\tau_d := \inf\{t \geq 0 : d_t = 1\}$. Let τ_d^1 and τ_d^2 be the first disclosure times for p_*^1 and p_*^2, respectively. To show that $U_1(1|p_*^1) > U_1(1|p_*^2)$ it is sufficient to show that $\tau_d^1 \geq \tau_d^2$ and that $\tau_d^1(\omega) > \tau_d^2(\omega)$ for a positive measure set of states ω.

Let $\tau_N := \inf\{t \geq 0 : dN_t = 1\}$ and let T_*^i, $i = 1, 2$ be given by $\phi_{T_*^i} = p_*^i$ where ϕ_t is the solution to the differential equation

$$\frac{dp_t}{dt} = \kappa(\bar{p} - p_t) + \mu p_t(1 - p_t), \ p_0 = 1.$$

By construction we have $T_*^2 < T_*^1$. We consider several cases:

1. If $\tau_N < T_*^2$ then $\tau_d^2 = \tau_d^1$.

2. If $\tau_N > T_*^2$ and $V_{T_*^2} = 1$ then $\tau_d^2 = T_*^2 < \tau_d^1$.

3. If $\tau_N > T_*^2$ and $V_{T_*^2} = 0$ we have several sub-cases. Let $\sigma = \inf\{t > T_*^2 : V_t = 1\}$.

   (a) If $\tau_N < T_*^1$ then $\tau_d^2 = \tau_d^1 = \inf\{t \geq \tau_N : V_t = 1\}$.

   (b) If $\tau_N > T_*^1$ and $\sigma < T_*^1$ then $\tau_d^2 = \sigma < T_*^1 \leq \tau_d^1$.

   (c) If $\tau_N > T_*^1$ and $\sigma > T_*^1$ then $\tau_d^2 = \tau_d^1 = \sigma$.

According, $\tau_d^1 \geq \tau_d^2$ a.s. and $\Pr(\tau_d^1 > \tau_d^2) > 0$ which means that $E(e^{-r\tau_d^1}) < E(e^{-r\tau_d^2})$ and $U_1(1|p_*^1) > U_1(1|p_*^2)$. ∎

Lemma 8. *Suppose that $U_1'(p_*|p_*) \geq 0$ and $U_1(1|p_*) - c \geq 0$,, then $U_1(p|p_*)$ is non increasing in p_*.*

Proof. Following the same computation as in (Davis, 1993, Theorem 32.10, p. 94) we can integrate the HJB equation to get

$$U_0(p_t) = \int_t^{T_*} e^{-(r+\lambda_1+\mu)(s-t)}\Big(p_s + \lambda_1 U_1(p_s) + \mu U_0(0)\Big)ds + e^{-(r+\lambda_1+\mu)(T_*-t)}U_0(0)$$
$$U_1(p_t) = \int_t^{T_*} e^{-(r+\lambda_0)(s-t)}\Big(p_s + \lambda_0 U_0(p_s)\Big)ds + e^{-(r+\lambda_0)(T_*-t)}[U_1(1) - c].$$

where T_* is the time it gets for beliefs to reach p_* in absence of any shock. Differentiating with respect to p_* we get

$$\begin{aligned}
\frac{\partial}{\partial p_*}U_0(p_t) &= \int_t^{T_*} e^{-(r+\lambda_1+\mu)(s-t)}\Big(\lambda_1 \frac{\partial}{\partial p_*}U_1(p_s) + \mu\frac{\partial}{\partial p_*}U_0(0)\Big)ds + e^{-(r+\lambda_1+\mu)(T_*-t)}\frac{\partial}{\partial p_*}U_0(0)\\
&\quad + \Big[e^{-(r+\lambda_1+\mu)(T_*-t)}\Big(p_* + \lambda_1 U_1(p_*) + \mu U_0(0)\Big) - (r+\lambda_1+\mu)e^{-(r+\lambda_1+\mu)(T_*-t)}U_0(p_*)\Big]\frac{\partial T_*}{\partial p_*}\\
\frac{\partial}{\partial p_*}U_1(p_t) &= \int_t^{T_*} e^{-(r+\lambda_0)(s-t)}\lambda_0\frac{\partial}{\partial p_*}U_0(p_s)ds + e^{-(r+\lambda_0)(T_*-t)}\frac{\partial}{\partial p_*}U_1(1)\\
&\quad + \Big[e^{-(r+\lambda_0)(T_*-t)}\Big(p_* + \lambda_0 U_0(p_*)\Big) - (r+\lambda_0)e^{-(r+\lambda_0)(T_*-t)}U_1(p_*)\Big]\frac{\partial T_*}{\partial p_*}
\end{aligned}$$

Noting that $U_1(p_*) = U_1(1) - c$ and $U_0(p_*) = U_0(0) = \lambda_1[U_1(1) - c]/(r + \lambda_1)$

$$\begin{aligned}\frac{\partial}{\partial p_*}U_0(p_t) = &\int_t^{T_*} e^{-(r+\lambda_1+\mu)(s-t)}\Big(\lambda_1\frac{\partial}{\partial p_*}U_1(p_s) + \mu\frac{\partial}{\partial p_*}U_0(0)\Big)ds + e^{-(r+\lambda_1+\mu)(T_*-t)}\frac{\partial}{\partial p_*}U_0(0)\\ &+ e^{-(r+\lambda_1+\mu)(T_*-t)}p_*\frac{\partial T_*}{\partial p_*} \qquad (66)\\ \frac{\partial}{\partial p_*}U_1(p_t) = &\int_t^{T_*} e^{-(r+\lambda_0)(s-t)}\lambda_0\frac{\partial}{\partial p_*}U_0(p_s)ds + e^{-(r+\lambda_0)(T_*-t)}\frac{\partial}{\partial p_*}U_1(1)\\ &+ e^{-(r+\lambda_0)(T_*-t)}\left[p_* - \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1}U_1(p_*)\right]\frac{\partial T_*}{\partial p_*}. \qquad (67)\end{aligned}$$

Evaluating the HJB equation at p_* we get

$$p_* - \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1}U_1(p_*) = -f(p_*)U_1'(p_*)$$

which is greater or equal than zero if $U_1'(p_*) \geq 0$. Evaluating (66) and (67) at T_* we get

$$\begin{aligned}\frac{\partial}{\partial p_*}U_0(p_*) &= \frac{\lambda_1}{r+\lambda_1}\frac{\partial}{\partial p_*}U_1(1) + p_*\frac{\partial T_*}{\partial p_*}\\ \frac{\partial}{\partial p_*}U_1(p_*) &= \frac{\partial}{\partial p_*}U_1(1) + \left[p_* - \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1}U_1(p_*)\right]\frac{\partial T_*}{\partial p_*}\end{aligned}$$

Hence, using that $U_1(1)$ is decreasing in p_* (Lemma 7) and $\partial T_*/\partial p_* < 0$, we get that $U_0(p_*) = U_0(0)$ and $U_1(p_*)$ are also decreasing in p_*. Then, by working backward from $t = T_*$, it is straightforward that (66) and (67) must be negative for all $t \leq T_*$ and hence for all $p \geq p_*$. ∎

Lemma 9. *Suppose that $U_1'(p_*|p_*) \geq 0$ and $U_1(1|p_*) - c \geq 0$, then $U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*}U_1'(p_*|p_*) > 0$.*

Proof. Rearranging the HJB equation we can write

$$U_1'(p|p_*) = \frac{rU_1(p|p_*) - p - \lambda_0[U_0(p|p_*) - U_1(p|p_*)]}{f(p)}$$

Evaluating at $p = p_*$ and using the boundary conditions, equations (24) and (25), yields

$$\begin{aligned}U_1'(p_*|p_*) &= \frac{rU_1(p_*|p_*) - p_* + U_1(p_*|p_*)\frac{r\lambda_0}{r+\lambda_1}}{f(p_*)}\\ &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}U_1(p_*|p_*) - p_*}{f(p_*)} \qquad (68)\end{aligned}$$

Now, we can show that

$$U_1'(p_*|p_*) = 0 \Rightarrow \frac{\partial}{\partial p_*}U_1'(p_*|p_*) > 0.$$

Differentiating equation (68) with respect to p_* yields

$$\begin{aligned}
\frac{\partial}{\partial p_*}U_1'(p_*|p_*) &= \frac{\frac{r(r+\kappa)}{r+\lambda_1}\frac{\partial U_1(p|p_*)}{\partial p_*}\Big|_{p=p_*} - 1}{f(p_*)} + \frac{f'(p_*)}{f(p_*)}U_1'(p_*|p_*) \\
&= \frac{\frac{r(r+\kappa)}{r+\lambda_1}\frac{\partial U_1(p|p_*)}{\partial p_*}\Big|_{p=p_*} - 1}{f(p_*)} > 0
\end{aligned}$$

But from Lemma 8 we know that $\frac{\partial U_1(p|p_*)}{\partial p_*} < 0$. This along with $f(p_*) < 0$ proves the lemma. ∎

Proof of Proposition 8. Suppose there exist $p_*^- < p_*^+$ such that

$$\begin{aligned}
U_1'\left(p_*^-|p_*^-\right) &= 0 \\
U_1\left(1|p_*^+\right) - c &= 0
\end{aligned}$$

The proof is identical to Proposition 2. A direct consequence of Lemma 9 is that $U_1'\left(p_*|p_*\right)$ crosses 0 only once. Thus, $U_1'\left(p_*|p_*\right) \geq 0$ for $p_* \geq p_*^-$, and $U_1'\left(p_*|p_*\right) < 0$ for $p_* < p_*^-$. Moreover, from Lemma 8 we have that $U_1\left(p_*|p_*\right) - c \geq 0$ for all $p_* \leq p_*^+$. Hence, p_* satisfies conditions (26) and (27) if and only if $p_* \in [p_*^-, p_*^+]$. ∎

C Proofs of Section 4

Proof of Proposition 9

Proof. First, we verify that the disclosure strategy is optimal whenever $V_t = 0$. By construction $U_0(p_*) = U_0(0)$ so the manager is indifferent between disclosing negative information or not when $p_t = p_*$. Moreover, given that $U_0(p)$ is non-decreasing, the manager does not have incentives to deviate and disclose if $p_t > p_*$.

Next, we verify that the disclosure strategy is also optimal when $V_t = 1$. The manager disclosure strategy is optimal if the following two conditions are satisfied

(1) $U_1(1) \geq c$.

(2) $U_1(p) \geq U_1(1) - c$ for $p \geq p_*$.

For (1) note that, by construction (equation (41)),

$$U_1(1) - c = \left(1 + \frac{r}{\lambda_1}\right) U_0(p_*) = \left(1 + \frac{r}{\lambda_1}\right) \left(\frac{p_*}{r} - \frac{\mu\theta}{r} \frac{r + \lambda_0}{r + \kappa}\right),$$

which is always positive given the assumption that

$$p_* \geq \mu\theta \frac{r + \lambda_0}{r + \kappa}.$$

For (2), note that as U_1 is increasing (2) is satisfied if and only if $U_1(p_*) \geq U_1(1) - c$. This happens if and only if

$$\left(1 + \frac{r}{\lambda_1}\right) \left(\frac{p_*}{r} - \frac{\mu\theta}{r} \frac{r + \lambda_0}{r + \kappa}\right) \leq \frac{p_*}{r} - \frac{\mu\theta}{r} \frac{\lambda_0}{r + \kappa},$$

which is true for all $p_* \leq \mu\theta$. ∎