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"Non-Myopic Betas"

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Non-Myopic Betas[*]

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November 1, 2015

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JEL CLASSIFICATION: G01, G11, G12, G14, G15
KEYWORDS: Asset Prices, Beta, CAPM, Hedging

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1 Introduction

We introduce non-myopic mean-variance optimizing agents into the standard conditional CAPM. In equilibrium, the inter-temporal hedging demand of non-myopic investors leads to a two-factor CAPM in which risk premiums are determined both by the market beta and by the non-myopic beta of returns with respect to the *return on the future mean-variance efficient portfolio*. We show that the future risk factor is indeed priced in the cross-section of stock returns, and the relationship between expected returns and non-myopic betas is monotone and increasing. Furthermore, the gains from exploiting this risk factor for portfolio construction are economically significant. Using a cross-section of mutual fund returns, we find that non-myopic betas of mutual fund returns are highly significantly related to mutual fund alphas, suggesting that non-myopic mutual funds generate alpha relative to their myopic peers.

We show (both empirically and theoretically) that market- and non-myopic betas exhibit interesting dynamics and are strongly negatively correlated in the time series while they are positively correlated in the cross-section. Furthermore, the betas have a pronounced factor structure: The correlation of the five leading principal components of market- and non-myopic betas is between -30% and -50%. Furthermore, these principal components, as well as the non-myopic factor risk premium dynamics are strongly linked to empirical measures of liquidity. We argue that a part of these dynamics is driven by time variations in the risk-bearing capacities and funding liquidity needs of myopic and non-myopic traders. In particular, the future factor risk premium is positively related to the risk-bearing capacity of non-myopic traders, while the liquidity premiums are negatively related to non-myopic betas. As in Frazzini and Pedersen (2014), *betting against the non-myopic betas* provides exposure to these liquidity premiums and should deliver abnormal returns. Our empirical tests find strong support for these predictions.

In our model, equilibrium dynamics and the corresponding non-myopic behavior is driven by two effects: changing fundamentals (dividends) and fluctuating risk-bearing capacities of myopic and non-myopic traders. In order to isolate each of these effects, we explicitly solve two versions of the model: in the first one, we assume that dividends are i.i.d. over time, so that the only source of equilibrium dynamics are the fluctuations in the agents' risk-bearing capacities; in the second one, risk bearing capacities are fixed, but dividends follow a general autoregressive process. We show that fluctuations in the risk-bearing capacities are able to generate the empirically observed negative co-movement of market- and non-myopic betas as well as their positive cross-sectional relationship: Our theoretical results imply that these effects are driven respectively by the negative relationship between stock returns and volatility and negative aggregate stock market skewness. The second version of model exhibits an even more interesting behavior. Namely, we show that fundamental (dividends) fluctuations may cause non-myopic dynamics to emerge as a *self-fulfilling* phenomenon. Namely, we show that the model has multiple Markov perfect linear equilibria that co-exist with the standard myopic CAPM equilibrium. In stark contrast to the classical results on equilibrium multiplicity in overlapping generations (OLG) models (see Spiegel (1998)), our

equilibria are characterized by different sensitivities of equilibrium prices to dividend shocks and may generate a rich set of cross-sectional dynamics of betas. In particular, we show that all these equilibria exhibit a pronounced factor structure of non-myopic betas, with factors characterized through a new object that we call the "non-myopic subspace."

Our assumption of mean-variance preferences is made for analytical tractability and allows us to obtain simple, linear pricing formulas as in the standard CAPM. Introducing different preferences (such as, e.g., the constant relative risk aversion utility) would lead to complicated non-linear dynamics and the nature of the hedging demand would become highly intricate and dependent on higher moments. In this regard, our mean-variance preferences specification could be viewed a form of bounded rationality for agents who only care about the first two moments of returns. In fact, using the general framework of Gabaix (2014), Gabaix (2015) shows that the solution to a boundedly rational version of the Merton problem has the intertemporal hedging demand expressed in terms of the covariance of returns with expected future returns, which is similar to the behaviour of non-myopic mean-variance optimizing agents in our model.

We now discuss related literature.

The fact that inter-temporal considerations play an important role in optimal portfolio choice and should therefore have an impact on equilibrium asset prices has been known at least since the seminal work by Merton on the intertemporal capital asset pricing model (ICAPM) (see Merton (1973)). One of the main predictions of ICAPM is the emergence of a multi-factor model, with multiple factors that are linked to equilibrium dynamics of fundamentals and prices through the hedging demand expressed in terms of the derivatives of the (endogenous) agents' value functions. This makes ICAMP notoriously difficult to test empirically: In most cases, value functions cannot be computed explicitly, and the nature of equilibrium hedging demand depends on the exact parametric nature of the underlying fundamental shocks in a non-trivial manner. Maio and Santa-Clara (2012) propose a test of ICAPM based on the above-mentioned intuition: If a state variable forecasts positive changes in *future expected market returns,* its innovation should earn a positive price of risk in the cross-sectional test. Maio and Santa-Clara (2012) apply these ICAPM criteria to several popular multifactor models and show that most models do not satisfy the ICAPM restrictions, expect for the Fama and French (1993) and Carhart (1997) do consistently meet the ICAPM restrictions. In our version of ICAPM, a state variable's innovation earn a positive cross-sectional price of risk if it forecasts positive changes in the *future risk factor.* The latter can be quite different from the market portfolio, and hence a variable that fails the Maio and Santa-Clara (2012) test may still pass the test implied by our model.

Initiated in the seminal paper by Campbell (1993), a large strand of cross-sectional asset pricing literature utilizes the approximation of Campbell and Shiller (1988) combined with a representative investor equipped with Epstein and Zin (1989) preferences to obtain approximate expressions for the ICAPM equilibrium pricing factors. These solutions can be implemented empirically if they are combined with vector autoregressive (VAR) estimates of asset return dynamics (see Campbell (1996)). Campbell and Vuolteenaho (2004), Campbell, Polk, and Vuolteenaho (2010), and Camp-

bell, Giglio, and Polk (2013) use this approach. We follow a different approach: Due a simple preference specification, we are able to solve the model non-parametrically, without relying on approximations.[1] This allows us to express the ICAPM-type risk factor explicitly in terms of the return on the efficient portfolio. Since efficient portfolio itself is not directly observable, we construct and test several observable proxies for this portfolio. In addition to the market portfolio and two (conditionally) efficient portfolio proxies constructed from Fama-French-Carhart factors, we introduce a model-consistent (MC) efficient portfolio that agrees with our equilibrium equations, and show that the performance of the MC portfolio in cross-sectional tests is superior to that of other proxies. Interestingly enough, we find that the MC portfolio is most of the time quite close to the market portfolio, in agreement with the results of Levy and Roll (2010), who find that market portfolio is close to being *unconditionally* efficient.

We test our two-factor CAPM using a large cross-section of stocks (following Ang, Liu, and Schwarz (2008)) rather than using only portfolios sorted by characteristics: This makes our results immune to the critiques by Daniel and Titman (1997) and Lewellen, Nagel, and Shanken (2010). In agreement with existing results, we also find that sorts on past CAPM betas do not lead to a significant positive relationship between CAPM betas and future returns. By contrast, sorts on non-myopic betas generate a significant monotonic relationship. This holds for all efficient portfolio proxies that we use, including the market portfolio. This is particularly surprising because the *standard (contemporaneous) market betas are only mildly significant* in the cross-section, while *betas computed with future market returns are highly significant.*

Our model also has an interesting relationship with the Roll (1977) "mean-variance tautology:" the fact that any mean-variance efficient portfolio satisfies the (myopic) CAPM relationship exactly and hence the validity of CAPM is equivalent to the efficiency of the market portfolio. In our model, the fact that non-myopic betas are priced in the cross-section is not a tautology: Whether they are priced or not depends on the presence of non-myopic agents. Furthermore, market portfolio is not efficient in most cases, and the efficient portfolio is an endogenous equilibrium object.

Several papers study the role of different investment horizons for investor behavior and for asset pricing. For example, Brennan and Zhang (2012) develop and test an extended version of CAPM in which agents have stochastic investment horizons, but do not rebalance. They find that this "stochastic horizon CAPM" is better able to explain the cross-section of stock returns. Beber, Driessen, and Tuijp (2014) develop version of the liquidity CAPM with stochastic transaction costs and investors with heterogenous horizons. They show that their model fits average returns substantially better than a standard liquidity CAPM. Our approach is different from that of Brennan and Zhang (2012) and Beber, Driessen, and Tuijp (2014): While both of these papers assume that longer-term investors do not rebalance, we let the non-myopic investors trade optimally, taking into account their future rebalancing needs. This leads to the emergence of a hedging component in their demand and a corresponding risk premium that is absent in the above mentioned models.

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[1]Of course, mean-variance preferences specification could itself be viewed as an approximation to the "true," non-quadratic agents' utility.

Kondor and Vayanos (2014) develop a dynamic equilibrium model with two classes of agents that they name hedgers (who have either myopic mean-variance preferences or non-myopic CARA preferences) and arbitrageurs (who have constant relative risk aversion preferences). In their model, the wealth of arbitrageurs is a key state variables that is responsible for equilibrium dynamics. They study how illiquidity (defined as the sensitivity of prices to supply shocks) co-moves with arbitrage capital and derive closed form solutions. In particular, in their model the risk-bearing capacity of non-myopic agents is endogenous and depends of their capital.

Our model predicts that one could use the exposures to non-myopic betas in order to distinguish between myopic and non-myopic investors' behavior. This links our paper to the literature on mutual fund behavior and performance. For example, Cremers and Petajisto (2009) and Pástor, Stambaugh, and Taylor (2014) find that more active mutual fund managers generate superior returns, while Lan, Moneta, and Wermers (2014) find that mutual funds with longer horizon deliver superior returns because they possess longer-term fundamental information. Our results imply that non-myopic mutual funds should naturally have a lower non-myopic beta due to their hedging demand. We test this prediction by performing cross-sectional regressions of mutual fund alpha on the non-myopic betas of fund returns. We show that differences in non-myopic betas account for a significant part of the variation in mutual fund performance and risk exposure (with up to 10% $\bar{R}$ from cross-sectional regressions of alpha on non-myopic beta), and the relationship is indeed negative. This result suggests that non-myopic mutual funds tend to deviate more from the myopic efficient portfolio by loading less on high non-myopic beta stocks, and, as a result, tend to deliver higher alphas.

Our paper proposes a modified version of CAPM that does a better job in pricing the cross-section of stock returns. However, due to the symmetric mean-variance preferences specification, our model misses the important asymmetries in the behavior of asset returns across the business cycle. These asymmetries have been shown to play a major role in the cross-sectional behavior of asset returns. See, for example, Lettau, Maggiori, and Weber (2014) who find that a "downside CAPM" that uses betas conditional on up- and down- markets does a very good job in pricing the cross-section of stock returns. Our model can be modified to incorporate such asymmetries and study the behavior of non-myopic betas over the business cycle.

In an extension of the basic model, we introduce funding liquidity frictions using margin constraints, as in Frazzini and Pedersen (2014), who show that constrained investors naturally prefer holding high beta securities due to their embedded leverage. This pushes their prices up and reduces returns, implying that a portfolio that is long levered low beta stocks and short de-levered high beta stocks should deliver positive abnormal returns. Frazzini and Pedersen call this strategy "Betting Against the Beta." We show that the non-myopic non-myopic betas have a similar effect on investor behavior in the presence of constraints. All else equal, stocks with low non-myopic beta are less risky for non-myopic investors, and hence the margin constraint is more likely to bind for these stocks, implying a higher liquidity premium. We test this "Betting Against the Non-myopic Beta" strategy and find that its returns resemble that of a myopic market portfolio with adjust-

ments similar to those used by non-myopic agents to eliminate the future exposure to future risk factor. Our "Betting Against the Non-myopic Beta" strategy delivers significant abnormal returns over and above a number of factor models, including the one with market BAB factor.

The paper is organized as follows. Section 2 describes the model setup; Section 3 introduces the preparation of data that we will be using to test model predictions; Section 4 derives the Non-Myopic CAPM with a number of predictions for asset pricing and portfolio behavior, and tests these predictions empirically. Section 5 introduces the link between risk-bearing capacities of myopic and non-myopic agents and dynamics of betas and risk premiums, and documents some empirical observations about such joint dynamics; Section 6 extends the theoretical results to explain the dynamics of non-myopic betas. Section 7 introduces a version of the model with portfolio constraints, and provides empirical support for it, including the construction of a "Betting Against the Non-myopic Beta" strategy. Finally, Section 8 concludes. Technical proofs of the propositions are collected in the Appendix.

2 Model Setup

As is common in the literature on conditional CAPM, we consider an overlapping generations (OLG) economy. New agents are born each period $t = 1, 2, \cdots$, and can be of two types: *S*hort-term and *L*ong-term agents. Agents can trade risky securities $s = 1, \cdots, S$ that pay dividends $d_t = (d_{s,t})_{s=1}^S$. The market prices of these securities are denoted by $P_t = (P_{s,t})_{s=1}^S$, There is also a riskless security with an interest rate r. The total supply of risky securities can be time-varying and is denoted by $x_t = (x_{s,t})_{s=1}^S$. Agents of each type S, L choose their portfolio π_t^S, π_t^L (the amount of wealth invested into risky assets) that maximize their respective objective functions. Namely, we assume that short term agents choose their portfolio π_t^S to maximize the simple mean-variance objective

$$(\pi_t^S)^T \left(E_t[R_{t+1}] - e^r \right) \; - \; \frac{\gamma_t^S}{2} (\pi_t^S)^T \Sigma_t \pi_t^S \,.$$

Here, π^T denotes the transposed (row) vector π; γ_t^S is the time t risk aversion of myopic agents; and Σ_t is the time t conditional covariance matrix of returns $R_{t+1} = \frac{P_{t+1} + d_{t+1}}{P_t}$. The optimal portfolio is given by

$$\pi_t^S \; = \; \frac{1}{\gamma_t^S} \Sigma_t^{-1} \mu_t \tag{1}$$

where

$$\mu_t \; = \; E_t[R_{t+1}] - e^r$$

is the risk premium. The quantity

$$\Psi_{t+1} \; \equiv \; (R_{t+1} - e^r)^T \Sigma_t^{-1} \mu_t \; = \; \gamma_t^S (\pi_t^S)^T \left(R_{t+1} - e^r \right)$$

will play an important role in our subsequent analysis. This is the *future excess return on the mean-variance efficient portfolio.*

We now come to discussing the behavior of non-myopic agents. We assume that they are also mean-variance optimizers, but care about the risk-return tradeoff over two periods instead of one. Namely, we assume that the non-myopic investors' objective is to maximize

$$J_{t,T}^L(w_t^L) = E_t[w_T^L] - \frac{\gamma_t^L}{2}\mathrm{Var}_t[w_T^L]$$

with $T = t + 2$.[2] Here, w_T is the terminal wealth of a type L-agent. As is well known, the multi-period mean-variance problem is time-inconsistent. In order to deal with this time-inconsistency, we follow the approach of Basak and Chabakauri (2010) and assume that non-myopic agents solve the time-consistent version of this dynamic problem. As Basak and Chabakauri (2010) show, the time-consistent value function satisfies the following backward recursion:

$$J_{t,T}^L(w_t^L) = E_t[J_{t+1,T}^L(w_{t+1}^L)] - \frac{\gamma_t^L}{2}\mathrm{Var}_t[E_{t+1}[w_T^L]]$$

where $J_{t+1,T}^L$ is the one period (myopic) mean-variance objective function, and w_t is the agent's wealth at time t. The solution to this optimization problem is provided in the following lemma.

Lemma 2.1 *The optimal portfolio of non-myopic agents satisfies*

$$\pi_t^L = \frac{1}{\gamma_t^L} e^{-r} \Sigma_t^{-1} (\mu_t - \mathrm{Cov}_t[R_{t+1}, \Psi_{t+2}]) \tag{2}$$

Lemma 2.1 shows that the non-myopic optimal portfolio differs from the simple myopic optimal portfolio only through the presence of a single additional term: The hedge $\mathrm{Cov}_t[R_{t+1}, \Psi_{t+2}]$ against future fluctuations in the return on the efficient portfolio that the agent anticipates to hold next period. This is intuitive: if an asset's return is positively correlated with the future return on the efficient portfolio, it contributes too much to the total volatility of multi-period returns, and this discourages investment.

3 Data and Methodology

While each theoretical section provides a number of predictions, we make an attempt to formulate and test the respective hypotheses in each section using the following data. We collect stock and company data for the US market from CRSP and Compustat. We use daily and monthly returns for all common stocks (codes 10 and 11) for all available time span until December 2013, and we compute most statistics for the period from January 1980 until December 2013. We use returns in excess of the US Treasury Bill rate, and we take CRSP value-weighted index as a proxy for the

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[2]The assumption that non-myopic agents have a two-period horizon is not essential. All of our results can be easily extended to the case of any $T > t + 1$ and multiple cohorts of agents with different horizons. We chose the simplest specification that allows us to distill the basic message about the nature of the non-myopic demand and future fluctuations in the return on the efficient portfolio.

market return. Stock characteristics are computed from data in both databases: market equity combines prices from CRSP with number of shares outstanding from Compustat (when available) or CRSP, book-to-market computation follows the procedure outlined in Fama and French (1992), and momentum, as defined by Jegadeesh and Titman (1993), is the cumulative return from month $t-12$ until $t-1$. In addition, we use a number of market-wide liquidity factors and liquidity proxies: Pastor and Stambaugh (2003) factor (PS), level of aggregate liquidity (PS-LVL) and its innovations (PS-INNOV) are from Wharton Research Data Service, TED spread (spread between 3-Month LIBOR and 3-Month Treasury Bill) and its monthly innovations (TED-INNOV) starting in January 1986 are from Federal Reserve Bank of St. Louis, funding liquidity factor (FLIQ) by Fontaine and Garcia (2012)[3] and its monthly innovations (FLIQ-INNOV) from January 1986. We also compute Amihud (2002) illiquidity measure for the US stock market using exact filters and procedures from Fontaine, Garcia, and Gungor (2013).[4] We also download mutual fund monthly returns from CRSP (from January 1991), and institutional (13f) holdings (FRC) at quarterly frequency from the Thomson-Reuters database. We convert institutional holdings into the total proportion of market equity held by institutions by using market prices of holdings at each reporting date. We also use market (MKT) factor returns as well as factor returns based on size (SMB), book-to-market (HML), momentum (up minus down, UMD), which are available in Kenneth French's data library; we also use the BAB factor by Frazzini and Pedersen available from AQR data library.

4   Non-Myopic CAPM

In this section we derive CAPM in the presence of non-myopic agents. Suppose that at each date there is a mass m_t^S of myopic and a mass m_t^L of non-myopic agents present in the market.[5] Let $\delta_t^S \equiv m_t^S/\gamma_t^S$, $\delta_t^L \equiv e^{-r}m_t^L/\gamma_t^L$ be the total *risk-bearing capacities* of classes S and L respectively, and let

$$R_{t+1}^M \equiv \sum_{i=1}^{S} R_{i,t+1}\, y_{i,t}$$

be the return on the market portfolio. Here, $y_t = x_t P_t$ is the vector of market capitalizations of risky assets. Let also

$$\beta_{i,t}^M \equiv \frac{\mathrm{Cov}_t[R_{i,t+1}, R_{t+1}^M]}{\mathrm{Var}_t[R_{t+1}^M]} \;,\; \beta_{i,t}^\Psi \equiv \frac{\mathrm{Cov}_t[R_{i,t+1}, \Psi_{t+2}]}{\mathrm{Var}_t[\Psi_{t+2}]}$$

be respectively the market beta and the non-myopic beta of security i. In the sequel, we will always refer to non-myopic betas as "Ψ-betas." Note a very important distinction between the two betas:

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[3]Provided by the authors on the personal web-site www.jean-sebastienfontaine.com.

[4]We compute two versions of the Amihud measure—value-weighted and equal-weighted one, so that the earlier is adjusted for the relative size of the assets and hence quantifies market liquidity, and the latter provide average level of illiquidity across assets.

[5]Note that these masses naturally need to satisfy the inequality $m_t^S \geq m_{t-1}^L$ for all t because non-myopic agents turn into myopic agents next period.

While the standard CAPM market beta is a conditional covariance with the *contemporaneous* market return, a Ψ-beta is a conditional covariance with the *future* return on the efficient portfolio.

Substituting (1) and (2) into the market clearing condition

$$m_t^S \pi_t^S + m_t^L \pi_t^L = y_t \,,$$

we arrive at the following result.

Theorem 4.1 *Equilibrium risk premium on security i is given by*

$$E_t[R_{i,t+1}] - e^r \;=\; \lambda_t^M \, \beta_{i,t}^M \;+\; \lambda_t^\Psi \, \beta_{i,t}^\Psi \tag{3}$$

with

$$\lambda_t^M \;=\; \frac{\mathrm{Var}_t[R_{t+1}^M]}{\delta_t^S + \delta_t^L} \;,\; \lambda_t^\Psi \;=\; \frac{\delta_t^L \mathrm{Var}_t[\Psi_{t+2}]}{\delta_t^S + \delta_t^L}$$

Theorem 4.1 is the main result of this section. It shows how non-myopic behavior leads to a two-factor ICAPM, with the second factor being the expected future return on the efficient portfolio. Assets that positively covary with future risk factor command a positive risk premium because they are risky from the point of view of non-myopic investors: Holding them increases the total variance of multi-period (longer-term) returns and therefore non-myopic investors need to be compensated for taking on this additional risk. Naturally, the size of this risk premium λ_t^Ψ is positively related to the risk-bearing capacity of non-myopic investors δ_t^L. In fact, both the market risk premium λ_t^M and the future factor risk premium are closely related to the relative risk-bearing capacity δ_t^L/δ_t^S of the two investor groups, with the market risk premium (respectively, the future factor risk premium) being negatively (respectively, positively) related to this relative capacity.

In order to test the predictions of Theorem 4.1, we first need to find an empirically observable counterpart of the (conditionally) efficient portfolio. The first question that naturally arises is whether the market portfolio itself can be efficient as in the standard CAPM. If market portfolio is efficient, then short term agents hold a fraction of the market and therefore, in equilibrium, so do the non-myopic agents. This means that the hedging demand $\Sigma_t^{-1} \mathrm{Cov}_t(R_{t+1}, \Psi_{t+2})$ has to be proportional to the market portfolio. Since Ψ_{t+2} is also proportional to the return on the market portfolio, we arrive at the following result.

Corollary 4.1 *Market portfolio is conditionally efficient if and only if there exists a random process $\alpha_t \in \mathbb{R}$ such that $\mathrm{Cov}_t(R_{i,t+1}, R_{t+1}^M) \;=\; \alpha_t \, \mathrm{Cov}_t(R_{i,t+1}, R_{t+2}^M)$ for all i.*

While this condition is restrictive and is rejected by the data, we will still use market portfolio as one of the proxies for Ψ in our empirical analysis and denote it by $\Psi_{t+2}^M \;\equiv\; R_{t+2}^M$. This is particularly important, given the evidence that market portfolio might be quite close to being (unconditionally) efficient. See, Levy and Roll (2010).

Another important special case in which the efficient portfolio can be explicitly computed corresponds to an economy in which the weights of the efficient portfolio do not move (too much) over time. Let $B_t = (B_{t,i,j})$ be the autocovariance matrix of securities' returns:

$$B_{t,i,j} = \text{Cov}_t(R_{i,t+1}, R_{j,t+2}).$$

Denoting the efficient portfolio weights by $\pi^{\text{eff}} = \Sigma_t^{-1}\mu_t$, we get that (3) can be rewritten as

$$\mu_t = \hat{\lambda}_t^M \Sigma_t y_t + \hat{\lambda}_t^\Psi B_t \pi^{\text{eff}}, \tag{4}$$

where $\hat{\lambda}_t^M = \text{Var}_t[R_{t+1}^M]\lambda_t^M$, $\hat{\lambda}_t^\Psi = \lambda_t^\Psi \text{Var}_t[\Psi_{t+2}]$. Multiplying this equation by Σ_t^{-1}, we get the *consistency equation:*

$$\pi^{\text{eff}} = \hat{\lambda}_t^M y_t + \hat{\lambda}_t^\Psi \Sigma_t^{-1} B_t \pi^{\text{eff}},$$

implying that

$$\pi_t^{\text{eff}} = (\text{Id} - \hat{\lambda}_t^\Psi \Sigma_t^{-1} B_t)^{-1} \hat{\lambda}_t^M y_t. \tag{5}$$

While the right-hand side of (5) can only be constant under strong restrictions on the return dynamics, one can view (5) as an approximation to the true equilibrium efficient portfolio assuming that the weights of the efficient portfolio do not move too much over time. It also has an intuitive structure: when the risk premium $\hat{\lambda}_t^\Psi$ is small or B_t is close to a zero matrix, π_t^{eff} is very close to the market portfolio, and can therefore be viewed as a correction to the market portfolio that accounts for auto-covariances in returns. We will call portfolio (5) the Model-Consistent efficient (MC) portfolio, and use the return on this portfolio as another proxy for Ψ, and define $\Psi_{t+2}^{MC} = R_{t+2}^{\pi_t^{\text{eff}}}$.

4.1 Efficient Portfolio Implementations

In order to proceed with computing empirical proxies for the efficient portfolio, we need to select the asset universe used in the portfolio construction. While one would ideally like to construct the efficient portfolio using the whole stock universe, it is technically not feasible and inefficient due to the known problems with degenerate large-dimensional covariance matrices. A more robust and more easily implementable approach is to select several portfolios that span a large part of the asset space: For example, the commonly used Fama-French-Carhart factors or the principal components of stock returns, as recently proposed by Kozak, Nagel, and Santosh (2014). In this paper we follow the more conventional factor approach.[6] We use four different proxies for efficient portfolio: the mentioned above future market portfolio with return Ψ^M, which has an advantage of not being estimated from any model, and three optimal portfolios, which are built and estimated from the four Fama and French (1993) and Carhart (1997) factors, namely, the Model-Consistent portfolio Ψ^{MC}

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[6]In a recent paper, Adrian, Etula, and Muir (2013) use shocks to the leverage of securities broker-dealers to construct an intermediary stochastic discount factor. They show that this factor is highly correlated with the mean-variance efficient portfolio constructed from three Fama-French factors. This finding suggests that the efficient portfolio constructed from Fama-French factors may indeed be close to the "true" efficient portfolio.

in (5), the mean-variance efficient portfolio with the return Ψ^{FFMV}, and the minimum-variance efficient portfolio with the return Ψ^{FFMinV}.

We estimate portfolio weights π^{eff} for each method at the end of each month, and use them to compute the return of the respective portfolio over the next month. In terms of the inputs used for time-t portfolio construction, we need the variance-covariance matrix Σ_t, which we proxy on each day t by the realized variance-covariance matrix $\hat{\Sigma}_t$ computed from three years of past daily factor realizations; the autocovariance matrix B_t, which we also estimate as the sample autocovariance matrix $\hat{B}_t$ from monthly factor realizations over the past three years; and the vector of mean returns μ_t. If order to avoid dealing with return predictability, we do not exploit any sophisticated models for μ_t and use either the historical mean returns on the assets in the portfolio $\hat{\mu}$ or simply a unit vector $\mathbf{1}$. The latter corresponds to the "minimum-variance" portfolio studied by Jagannathan and Ma (2003), who show that the minimum-variance portfolio is ex post "more efficient" in the mean-variance sense than the more theoretically sound mean-variance efficient portfolio with sample means used as expected return proxy. The weights for the optimal mean- and minimum-variance efficient portfolios are computed in a standard way:

$$
\pi_t^{FFMV} = \frac{\hat{\Sigma}_t^{-1}\hat{\mu}_t}{\mathbf{1}^T \hat{\Sigma}_t^{-1}\hat{\mu}_t},
$$

$$
\pi_t^{FFMinV} = \frac{\hat{\Sigma}_t^{-1}\mathbf{1}}{\mathbf{1}^T \hat{\Sigma}_t^{-1}\mathbf{1}}.
$$

To compute the Model-Consistent portfolio weights we also need to know the risk premiums $\hat{\lambda}_t^M$ and $\hat{\lambda}_t^\Psi$ for the market and the future efficient portfolio risk, respectively. These quantities are not readily available, and we estimate them jointly with the portfolio weights from the consistency equation and restriction (4) used to estimate the expected returns μ_t of the factors used for the efficient portfolio construction:

$$
\pi_t^{MC} = \pi_t^{\text{eff}}(\hat{\lambda}_t^M, \hat{\lambda}_t^\Psi),
$$

$$
\text{s.t.}
$$

$$
(\hat{\lambda}_t^M, \hat{\lambda}_t^\Psi) = \arg\min_{\hat{\lambda}_t^M, \hat{\lambda}_t^\Psi} \|\hat{\mu}_t - \mu_t\|
$$

$$
\mu_t = \hat{\lambda}_t^M \Sigma_t y_t + \hat{\lambda}_t^\Psi B_t \pi^{\text{eff}},
$$

$$
\pi_t^{\text{eff}} = (\text{Id} - \hat{\lambda}_t^\Psi \Sigma_t^{-1} B_t)^{-1} \hat{\lambda}_t^M y_t,
$$

where y_t is the vector of market-clearing weights with elements $[1,0,0,0]'$—the first element is the weight of the market factor, and the other three are the long-short Fama-French-Carhart factor portfolios, and $\hat{\mu}_t$ is the vector of sample mean returns for these factors estimated over the 3-year rolling window.

Note that, in contrast to all other efficient portfolio proxies, the Model-Consistent portfolio is constructed using our equilibrium equations. Therefore, it is naturally our "preferred portfolio";

furthermore, as we explain above, the MC portfolio is often quite close to the market portfolio because $\hat{\lambda}_t^\Psi B_t$ is typically small. This leads to a desirable and stable behavior with the MC portfolio significantly deviating from the market only when the non-myopic effects are sufficiently large. Yet, it is important to compare the behavior of the MC portfolio with other proxies because this may shed a different light on the nature of risk premium dynamics.

4.2   Computing Factor Betas

We compute factor betas for all stocks in our sample adopting a rolling window approach with adjustments adopted from Frazzini and Pedersen (2014), i.e., the estimated factor beta is computed as ratio of stock and factor return volatility multiplied by the correlation between the two. The correlation and volatilities are estimated separately from log returns, – for volatilities we use one-year historical window of daily returns, and for correlation – a five-year window of overlapping three-day returns. We require at least 120 trading days of non-missing data to estimate volatilities and at least 750 trading days of non-missing return data for correlations. To reduce the effect of regression tendency of market betas we follow Vasicek (1973) and shrink them on each estimation date towards the gross mean of one. Based on the discussion in Frazzini and Pedersen (2014), we use constant shrinkage intensity of 0.6.

To compute the conditional non-myopic beta, i.e., the beta with respect to the future return on the efficient portfolio, $\beta_{i,t}^\Psi \equiv \frac{\mathrm{Cov}_t[R_{i,t+1}, \Psi_{t+2}]}{\mathrm{Var}_t[\Psi_{t+2}]}$, we use five years of monthly log returns for each stock and an efficient portfolio, and compute sample equivalents of covariances for the formula. Note that while for estimating the market and other factor betas we use contemporaneous returns, for the non-myopic beta we have to use stock return for a given month and efficient portfolio return for the next month, i.e., we look at the auto-covariance-type risk, and the selection of the sampling frequency of returns will affect the results. For simplicity, we assume here that non-myopic agents have the forward planning horizon of one month, and their hedging portfolio has the same duration.

To estimate the conditional non-myopic beta at the end of month t we will use stock returns up to month $t-1$ and efficient portfolio returns up to month t. Because the market beta $\beta_{i,t}^M \equiv \frac{\mathrm{Cov}_t[R_{i,t+1}, R_{t+1}^M]}{\mathrm{Var}_t[R_{t+1}^M]}$ is "displaced" by one period with respect to non-myopic beta (note that in numerator of the latter we have efficient portfolio returns at $t+2$, while in the market beta expression we have returns on only up to $t+1$), for the predictor of the conditional market (factor) beta at the end of month t we will use the beta estimated using returns up to month $t-1$.

4.3   Testing the Non-myopic CAPM

To test Theorem 4.1, we follow two commonly used procedures: (i) forming Ψ-beta-sorted portfolios, and (ii) performing the two-stage Fama and MacBeth (1973) regression. We select all the stocks from the CRSP universe, with 8695 stocks in the sample; to mitigate the effect of microstructure biases stemming from the noise in reported returns, we use not only the equal-weighted (EW),

but also prior gross return-weighted (RW) portfolios and the two-stage regressions as suggested in Asparouhova, Bessembinder, and Kalcheva (2010, 2012). As the prior return weight, we use the gross return on the stock from a previous month normalized by sum of all returns in a given portfolio or in a regression.

We construct the portfolios each month by sorting the predicted Ψ-betas into quintiles and computing both the next month return and the average predicted beta using either the EW or the RW weights. The average dependency between quintile portfolio betas and realized returns is depicted in Figure 1. Two observations are worth noting here: First, the range of betas depends on the choice of the efficient portfolio, and for all portfolios with the exception of the future market the values of betas are asymmetric, with a tendency to be positive in most cases. Second, Figure 1 shows that the link between expected returns and Ψ-betas is increasing for all four FMVP-proxies. To test the significance of this monotonicity, we perform a formal monotonicity relation test of Patton and Timmermann (2010). The results of this test are reported in Table 1: As we can see, the relationship is indeed monotone increasing and significant. Note also in Figure 1 that MC, FFMinV, and M portfolios are visually strongly monotone for the whole range of betas, while FFMV portfolio is almost flat for negative betas. Testing monotonicity for top four quintile portfolios of FFMV and MC betas confirms a strict positive relation under all statistical metrics.[7]

To test economic significance of Ψ-betas, we construct long-short portfolios from extreme Ψ-beta quintiles. As Table 1 shows, equal-weighted Ψ-betas-sorted portfolios deliver positive annualized returns ranging from 2.43% p.a. for MC betas to 3.69% p.a. for FFMinV betas, and all returns are significant at 4% level or better. RW portfolios show slightly lower returns, but they are also positive and significant (at 7% level of better). Furthermore, the relationship between non-myopic beta and expected returns is positive and significant at 6% level or better for all betas.

To estimate the premium for bearing the risk of conditional covariance with the FMVP we proceed in the standard way using two-stage Fama and MacBeth (1973) regressions. In the first stage, at the end of each month we regress cross-sectionally the excess returns $r_{t+1} - r_t^f$ for all stocks in month $t + 1$ on a vector X_t consisting of a constant and the conditional market- and Ψ-betas, $X_t = [\mathbf{1}\ \beta_t^M\ \beta_t^\Psi]$, observed in month t, and then estimate the resulting regression coefficients. Note that, importantly, we use individual stocks, instead of using portfolios in the cross-sectional regression, following Ang, Liu, and Schwarz (2008). This makes our results immune to the critiques by Daniel and Titman (1997) and Lewellen, Nagel, and Shanken (2010); moreover, we plan to use the time series of conditional risk premiums in further analysis, and according to Gagliardinia, Ossola, and Scaillet (2014) we should use individual stocks with large T and large n ($n >> T$ with potentially unbalanced panel). We weight the first-stage regression either by equal weights, or by prior gross-return weights in the portfolio consisting of all assets used in the regression; then, we use weighted least squares with the corresponding weighting matrix W_t for each month t, so that

---

[7]Our construction of FFMV portfolio depends on estimation of expected factor returns (we use sample means), and thus we expect this efficient portfolio proxy to have higher errors relative to the "true," unobservable future risk factor.

the vector of premiums λ_t^{WLS} is estimated in the usual way:

$$\lambda_t^{WLS} = (X_t^T W_t X_t)^{-1}(X_t^T W_t (r_{t+1} - r_t^f)),$$

where the weighting matrix W_t is the diagonal matrix with $\text{diag}(W_t) = w_t^J$, $J \in \{EW, RW\}$, that is, with the diagonal consisting of equal, or prior gross return weights of all assets in month t.[8] If $J = EW$, the cross-sectional regression reduces to OLS.

Then, in the second stage, for each of the two weighting rules, we compute the time-series averages of the risk premiums λ^{WLS} for market- and Ψ-betas and test their significance. Table 2 reports the results of the estimation of the FMVP risk premium in different settings: Namely, we include FMVP into the one-factor (MKT), three factor, and four-factor models to see if the "non-myopic risk" captured by the FMVP risk premium is spanned by other commonly used factors. Table 2 shows that the risk premium is always positive, and ranges from 1.06% p.a. to 2.62% p.a. if we include the FMVP in addition to the MKT factor, and from 0.57% p.a. to 2.14% p.a. if we use FMVP together with the four Fama-French-Carhart factors. The MC portfolio gives the highest estimate of the risk premium per unit of risk (2.62% for the original one-factor model and 2.14% for the four-factor one), and the estimates are highly significant with t-stat around 3.30. Correcting for microstructure biases using RW regressions (reported in Panel B of the table) does not change either the magnitude or the significance of the risk premiums.

The reported evidence suggests that the "non-myopic risk" captured by the covariance with the future efficient portfolio (FMVP) is indeed priced in the cross-section of stock returns, and the risk premium is positive, in agreement with Theorem 4.1.

4.4   Equilibrium portfolio behavior

Denote by R_{t+1}^L and R_{t+1}^S the returns on the non-myopic and the myopic portfolios $\gamma_t^L \pi_t^L$ and $\gamma_t^S \pi_t^S$, respectively. By Lemma 2.1, we have that

$$\text{Cov}_t[R_{t+1}, R_{t+1}^L] \;=\; \mu_t - \text{Cov}_t[R_{t+1}, \Psi_{t+2}] \;,\; \text{Cov}_t[R_{t+1}, R_{t+1}^S] \;=\; \mu_t.$$

Thus, we arrive at the following result.

Proposition 4.1 *Returns on the non-myopic portfolio covary more with returns on securities that covary less with Ψ_{t+2}.*

---

[8]Ferson and Harvey (1999) suggest using generalized least squares for the Fama-MacBeth regressions to improve the efficiency of the estimator; Asparouhova, Bessembinder, and Kalcheva (2010) show that using prior-gross returns of the assets to form the weighting matrix is effective for correcting the biases arising from microstructure effects. A number of studies use value-weighted Fama-MacBeth regressions along with a more standard equal-weighted regressions as a robustness check for establishing the relation between the return and characteristics (see, for example, Ang, Hodrick, Xing, and Zhang (2009)).

Proposition 4.1 is a formalization of the simple observation that non-myopic agents tend to reduce holdings of securities that have a high Ψ-beta. As a result, we should expect that returns on portfolios of non-myopic agents should themselves have a lower Ψ-beta than those for portfolio of myopic agents. This predicted *negative relationship between Ψ-beta and non-myopic behavior* naturally leads to the question of whether we can use it to study non-myopic behavior of mutual funds using their observed portfolio returns.[9]

To this end we proceed as follows: we estimate market and non-myopic betas for mutual funds from CRSP database from January 1991 until December 2013.[10] We select funds having at least 100 monthly returns observations, and use rolling window procedure with the 5-year window to estimate a standard time-series regression at the end of each month t for each fund i:

$$r_{i,t-59:t} - r^f_{t-59:t} = \alpha_{i,t} + \beta^{MKT}_{i,t}(r^{MKT}_{t-59:t} - r^f_{t-59:t}) + \beta^{\Psi}_{i,t}\Psi^{FMVP}_{t-58:t+1}, \qquad (6)$$

where subscript $t - n : t$ indicates time series of a given variable from period $t - n$ until t. We also estimate standard market, three- and four-factor models on a rolling window basis, and save the time series of alphas for each model as $\alpha^{1f}_{i,t}$, $\alpha^{3f}_{i,t}$, and $\alpha^{4f}_{i,t}$. On the next stage we regress cross-sectionally at each point in time (starting at $t = 60$) the saved alphas on a number of regressors estimated from model (6), namely, on non-myopic beta β^{Ψ}, and on the absolute market beta $|\beta^{MKT}|$ or, alternatively, on the on the market beta β^{MKT} itself. To concentrate on funds driven mostly by equity risk factors, we restrict our sample at each point in time to funds having market betas in the range from 0.75 to 1.25—this filter gives us on average 1769 funds in the cross-section.

Table 3 provides a summary of our main findings, relating mutual fund factor alphas to Ψ-betas in the cross-section. The numbers in Table 3 convey several interesting messages:

- *Mutual fund one-factor (MKT) alphas.* The most striking result is the value of the average cross-sectional $\bar{R}$ of 9.8% reported in Panel A: It means that the Ψ-beta (computed with the MC proxy) accounts for almost 10% of the cross-sectional variation in one-factor alphas. Moreover, the explanatory power is significantly driven by the β^{Ψ}, and there is a *significant negative relationship between Ψ-betas and one-factor alphas.* Panels B-D show that this negative relationship is robust and is independent on the choice of FMVP proxy. Combined with Proposition 4.1, this negative relationship suggests that non-myopic mutual fund behavior serves an an important component of alpha generation and is responsible for a significant fraction of alpha variation. Importantly, adding market beta to the regression, which takes into account the deviation from the pure market portfolio exposure, explains only about 3.2% more of the variation and a negative coefficient is in general consistent with low beta anomaly,

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[9]Potentially one can test the hypothesized relations by looking at differences in the behavior of non-myopic and myopic investors, such as pension funds and mutual funds. However, anecdotal evidence suggests that many of the conservative institutional investors are actually more myopic than retail ones, and thus our hypotheses face an endogenous identification problem: on the one hand, we can identify non-myopic agents by looking at the magnitude of Ψ-betas, and on the other hand, we should test differences in the magnitude of the non-myopic Ψ-betas for short- and non-myopic investors.

[10]Before 1991 many funds report only quarterly, and we need monthly returns for non-myopic beta estimation.

where low beta stocks demonstrate highest one-factor alphas (e.g., Black, Jensen, and Scholes (1972)).

- *Mutual fund three- and four-factor alphas.* The results multi-factor alphas are similar: While the magnitude of $\bar{R}$ drops a lot, we still see a highly significant negative relationship between Ψ-betas estimated for MC efficient portfolio and alphas. Other efficient portfolio proxies in Panels B-D deliver similar results with a few exceptions.

- Overall, MC efficient portfolio demonstrates the most significant and consistent results.

Our findings suggest that differences in Ψ-betas account for a significant part of the variation in mutual fund performance. Moreover, funds with lower Ψ-betas (that we identify as "more non-myopic") tend to deliver higher one- and multi-factor alphas. It would be interesting to link our findings to the results of Pástor, Stambaugh, and Taylor (2014) who show that more active funds (that is, the funds that trade more) deliver superior performance. We can use the time-series volatility of a fund's market beta estimated from equation (6) as a simple proxy for fund trading activity.[11] We observe that higher volatility of market betas is negatively related (with a p-value less than 0.01) to average Ψ-betas of mutual funds. Speculating a bit, our results suggest that a part of fund's alpha is driven by the fact that more active funds behave in a more non-myopic fashion, deviate from market beta more often, and load more on stocks with lower Ψ-betas.

5 Risk-Bearing Capacity and Beta Dynamics

In our model, fluctuations in the relative risk bearing capacities of the two classes of agents (myopic and non-myopic) may be responsible for an important part of risk premiums- and beta-dynamics. In order to gain a better understanding of these dynamics, in this section we consider a parametric version of our model in which dividends are i.i.d., and changing risk-bearing capacity is the only source of fluctuations. More precisely, we assume that securities' dividends are normally distributed with mean $\bar{d}$ and a covariance matrix Σ^D, while the pair of risk bearing capacities (δ_t^S, δ_t^L) follows a K-state Markov chain with values (δ_k^S, δ_k^L), $k = 1, \cdots, K$ and a transition probability matrix $\rho = (\rho_{k_1,k_2})_{k_1,k_2=1}^K$. The vector of asset supplies is fixed and is given by $x = (x_s)_{s=1}^S$. We will use the notation $\hat{\lambda}_k^M = \frac{1}{\delta_k^S + \delta_k^L}$, $\hat{\lambda}_k^\Psi = \frac{\delta_k^L}{\delta_k^S + \delta_k^L}$. Recall that the risk premiums $\hat{\lambda}_k^M$ are empirically observable in the sense that they can be estimated using cross-sectional regressions as described above.

To study the effects of purely non-fundamental shocks, we assume that the Markov state does not co-vary with dividends. We only consider Markov perfect equilibria in which asset prices are function of the Markov state k and we denote the vector of prices in state k by $P^k = (P_s^k)_{s=1}^S$. In a Markov perfect equilibrium, the expected return on the efficient portfolio will also be a function of the Markov state, and we denote it by $\bar{\Psi}_k = E_{t+1}[\Psi_{t+2}]$ where k is the state at time $t+1$. When the risk bearing capacities δ^S, δ^L are constant, returns do not move over time and therefore there

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[11] We appreciate useful discussions of this idea with Luke Taylor.

is no need for hedging. Hence, both groups of agents behave identically, and a direct calculation implies that equilibrium prices are given by

$$P = \frac{\bar{d}}{e^r - 1} - \frac{1}{(e^r - 1)\delta^S + \delta^L}\Sigma^D x .$$

This is a classical CAPM result: Prices are equal to the present value of dividends net of a price discount that is proportional to the fundamental covariance with the market portfolio. If risk bearing capacities move over time, a similar formula still holds and equilibrium prices behave analogously to those in a myopic OLG economy with a time-varying risk aversion. Namely, the following is true.

Proposition 5.1 *In any Markov perfect equilibrium, there exists a tuple of price discounts* $(\Gamma_k)_{k=1}^K$ *such that*

$$P^k = \frac{\bar{d}}{e^r - 1} - \Gamma_k \Sigma^D x .$$

In particular, equilibrium betas are proportional to fundamental betas,

$$\beta_{i,t}^M = c_t^M \frac{(\Sigma^D x)_i}{P_i^k} , \quad \beta_{i,t}^{\Psi} = c_t^{\Psi} \frac{(\Sigma^D x)_i}{P_i^k} ,$$

where the common factors c_t^M, c_t^{Ψ} *are given by*

$$c_t^M = \frac{x^T \bar{d}(e^r - 1)^{-1}}{x^T \Sigma^D x} - \Gamma_t ,$$

$$c_t^{\Psi} = -\frac{\text{Cov}_t[\Gamma_{t+1}, \psi_{t+1}]x^T \Sigma^D x}{\text{Var}_t[\Psi_{t+2}]}$$

with

$$\psi_{t+1} = \frac{(e^r \Gamma_{t+1} - E_{t+1}[\Gamma_{t+2}])^2}{1 + \text{Var}_{t+1}[\Gamma_{t+2}]x^T \Sigma^D x} .$$

Furthermore, a tuple Γ_k *is an equilibrium if and only if it solves the fixed point system*

$$e^r \Gamma_k - E_t[\Gamma_{t+1}] = \hat{\lambda}_k^M + \text{Cov}_t[\Gamma_{t+1}, (\hat{\lambda}_k^M \Gamma_{t+1} - \hat{\lambda}_k^{\Psi} \psi_{t+1})]x^T \Sigma^D x .$$

In the sequel we will frequently use the words "risk premium" to refer to the common price discount Γ_t. Note that, by Corollary 4.1, Proposition 5.1 implies that market portfolio is efficient, and the dynamics of risk sharing between myopic and non-myopic agents reduces to the allocation of the market portfolio. The analysis of this section can be directly extended to the case when $\delta_t^{S,L}$ co-move with dividends, in which case market portfolio will not be efficient anymore.

There are several important implications of Proposition 5.1. First, the dynamic behavior of risk bearing capacities leads to a *joint dynamic behavior of risk premiums and betas:* The two risk premiums and the two betas co-move in equilibrium, and the nature of this co-movement may be

16

non-trivial. Second, the magnitude of Ψ-betas is directly linked to the *variability of risk premiums.* In particular, if Γ_t does not move, investment opportunity set does not change either, and hence Ψ-betas are all zero. If the variability of $\hat{\lambda}_k^M$ and $\hat{\lambda}_k^\Psi$ is sufficiently small, it is possible to show that $\mathrm{Cov}_t[\Gamma_{t+1}, \psi_{t+1}] \approx 2E[\Gamma_{t+1}](e^r - 1)\,\mathrm{Cov}_t[\Gamma_{t+1}, \Gamma_{t+1} - E_{t+1}[\Gamma_{t+2}]]$. In particular, if we interpret Γ_{t+1} as a proxy for the average return on the market,[12] the well known leverage effect (the fact that returns and volatility negatively co-move; see Black (1976)) implies that the following is true.

Proposition 5.2 *Suppose that Γ_t and $\mathrm{Var}_t[\Gamma_{t+1}]$ are negatively correlated, and that $\mathrm{Var}_t[\Gamma_{t+1}]$ is sufficiently small. Then, average market betas and average Ψ-betas are negatively correlated in the time series.*

To get a deeper understanding of equilibrium dynamics, consider the case when the there is no persistence in risk bearing capacities. In this case, expected risk premiums and their higher moments are constant over time, and we will use the notation

$$\Gamma^* \equiv E_t[\Gamma_{t+1}], \ V^* \equiv \mathrm{Var}_t[\Gamma_{t+1}], \ C^* \equiv \mathrm{Cov}_t[\Gamma_{t+1}, \psi_{t+1}].$$

Let also

$$\nu_I^* = E[\nu_t^I], \ \nu_{I,J}^* = \mathrm{Cov}[\nu_t^I, \nu_t^J]$$

for $I, J = S, L$. Let also $\sigma^* \equiv x^T \Sigma^D x$. Then, equilibrium equations take the form

$$e^r \Gamma_k - \Gamma^* = \hat{\lambda}_k^M (1 + V^* \sigma^*) - \hat{\lambda}_k^\Psi C^* \sigma^*$$

and hence a direct calculation implies the following fixed point system for these three objects:

$$
\begin{aligned}
(e^r - 1)\Gamma^* &= \nu_S^*(1 + V^*\sigma^*) - \nu_L^* C^*\sigma^* \\
e^{2r} V^* &= \nu_{S,S}^*(1 + V^*\sigma^*)^2 + \nu_{L,L}(C^*\sigma^*)^2 - 2\nu_{S,L}(1 + V^*\sigma^*)C^*\sigma^* \\
(1 + V^*\sigma^*)C^* &= e^{2r}\,\mathrm{Cov}_t[\Gamma_{t+1}, \Gamma_{t+1}^2] - 2\Gamma^* e^r V^*.
\end{aligned}
$$

Recalling that Γ_t is a proxy for aggregate stock market returns and using the fact that aggregate stock market returns are negatively skewed (e.g., Black (1976), Albuquerque (2012)), we get that C^* is negative. This immediately implies the following result.

Proposition 5.3 *Suppose that aggregate stock market returns are negatively skewed. Then, market betas and Ψ-betas are positively correlated in the cross-section.*

We now formulate and test main empirical predictions following from our findings.

Predictions. There is a common factor in the dynamics of market betas, Ψ-betas, market risk premiums, and Ψ risk premiums. This factor is linked to the dynamics of risk bearing capacities

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[12]We have $E_t[R_{t+1}^M] = \dfrac{\frac{e^r x^T \bar{d}}{e^r - 1} - E_t[\Gamma_{t+1}]x^T \Sigma^D x}{\frac{x^T \bar{d}}{e^r - 1} - \Gamma_t x^T \Sigma^D x} \approx e^r + \frac{e^r - 1}{x^T \bar{d}}(e^r \Gamma_t - E_t[\Gamma_{t+1}])$.

of the two classes of investors. Furthermore, market betas and Ψ-betas negatively co-move in the time series, while they are positively correlated in the cross-section.

We test these predictions in the next section.

5.1 Empirical Properties of Risk Premiums and Beta Dynamics

It is well-known that getting a precise beta estimate is hard, and that getting a usable estimate of a mean return is a truly daunting task. In a typical two-stage identification procedure of a risk premium any systematic error in the magnitude of betas (especially in the distribution of their magnitudes across assets) causes a compensating error in the risk premium estimate. We mitigate this issue by using full sample of assets to estimate time series of market and FMVP risk premiums, and using a subsample of betas for the most liquid and large stocks that have no missing market and Ψ beta estimates during our sample period (we end up with 392 stocks). To reduce dimensionality we extract from each set of betas (market betas and Ψ-betas for each FMVP proxy) several leading principal components: the first five components typically explain more than 70% of the total variance (see Figure 2), and the cumulative variance explained with the same number of PCs is typically highest for market betas, and Model-Consistent Ψ betas. In order to be independent of the rotation of the factors, we average the five leading principal components for each set of betas, and regard such an average as a composite factor driving a given set of betas.

One of the most important predictions from the above states that there is a common factor in the dynamics of market and Ψ betas. Indeed, we find that the composite factors for market and non-myopic betas are highly correlated. The time series correlation between these factors for market betas and Ψ betas is negative, at -0.33 for FFMV, -0.49 for FFMinV, and -0.28 for MC betas; it is however positive for M betas (0.17), which is to be expected given the high persistence of market variance. By contrast, we find that the risk premiums for the market and a non-myopic FMVP risk are positively correlated, and their correlations are in the range from 0.10 (for M and FFMinV) to 0.19 (for MC portfolio). Furthermore, our prediction that market and Ψ-betas are positively correlated in the cross-section is also confirmed by the data (it ranges from slightly positive 0.01 for M to 0.09 for MC, and to 0.24 for FFMinV portfolio). That is, assets with higher market risk typically bear a higher non-myopic (FMVP) risk.

Thus, we observe a non-trivial dynamics of betas and risk premiums, and their factor structure has important implications for asset pricing and asset allocation: aggregate exposure to current and future risks tend to move in opposite direction, i.e., risk is somewhat mean-reverting, but cross-sectionally assets with highest exposure now tend to be exposed to future risk at a higher degree, and hence both current and future risks are compensated by higher risk premium simultaneously.

6 Fundamental Shocks and Equilibrium Dynamics: Self-Fulfilling Non-Myopic Behavior

In the previous section, we study how non-fundamental shocks may lead to a rich set of dynamic behavior of betas and risk premiums. In this section, we investigate the nature of non-myopic beta dynamics that may arise purely due to fundamental (dividend) shocks in an economy with constant risk bearing capacities. We show that *non-myopic behavior is self-fulfilling:* The presence of non-myopic investors may lead to the emergence of equilibria with time varying expected risk premiums that co-exist with the simple, constant risk-premium equilibria.

We assume that dividends follow a Gaussian process

$$d_{t+1} = A(\bar{d} - d_t) + \varepsilon_{t+1},$$

where $\varepsilon_t \sim N(0, \Sigma^D)$ are i.i.d. normal. We always impose the no-bubble condition $\max(|\text{eig}(A)|) < e^r$ and solve for stationary linear equilibria of the form

$$P_t = \bar{P} + A_P d_t$$

with some constant vector $\bar{P}$ and a matrix A_P describing the sensitivity of equilibrium prices to dividend shocks. As in the previous section, the vector of asset supplies is fixed and is given by $x = (x_s)_{s=1}^S$. We will only consider non-degenerate equilibria such that the matrix $A_P + \text{Id}$ is invertible. Let us first solve the model with only myopic agents present. In this case, a direct calculation implies that

$$A_P = -A(e^r \text{Id} + A)^{-1}, \quad \bar{P} = e^r A(e^r + A)^{-1}\bar{d} - \hat{\lambda}^M e^{2r}(e^r + A)^{-1}\Sigma^D(e^r + A)^{-1}x,$$

where we use Id to denote the identity matrix. In particular, the risk premium in this model is constant, given by $\hat{\lambda}^M e^{2r}(e^r + A)^{-1}\Sigma^D(e^r + A)^{-1}x$, conditional variance of payoffs $P_{t+1} + d_{t+1}$ is also constant, and hence the expected return of the efficient portfolio (the market) is also constant. This immediately implies that even in the presence of non-myopic agents such a constant risk premium equilibrium will still exist: If non-myopic agents believe that risk premium is constant, they behave myopically, and the equilibrium coincides with that in the model without non-myopic agents. This naturally raises the question of whether there can be other equilibria with more interesting dynamics. The following proposition shows that this is indeed the case.

Proposition 6.1 *There is a one-to-one correspondence between linear stationary equilibria and solutions B to the matrix equation*

$$(\Sigma^D)^{-1}B = 2\hat{\lambda}^\Psi B^T(\Sigma^D)^{-1}BA \tag{7}$$

satisfying the no-bubble condition $\max(|\operatorname{eig}(B-A)|) < e^r$. *Namely, for each such B there exists an equilibrium with*

$$A_P = (B-A)(e^r - (B-A))^{-1}$$

and

$$\begin{aligned}
\bar{P} &= (e^r - 1)^{-1}\Big(2\hat{\lambda}^\Psi (A_P + \operatorname{Id})\Sigma^D B^T (\Sigma^D)^{-1}(A_P + \operatorname{Id})^{-1} + \operatorname{Id}\Big)^{-1}\\
&\quad \times \Big((A_P + \operatorname{Id})A\bar{d} - \hat{\lambda}^M (A_P + \operatorname{Id})\Sigma^D (A_P + \operatorname{Id})^T x\\
&\quad + 2\hat{\lambda}^\Psi (A_P + \operatorname{Id})\Sigma^D B^T (\Sigma^D)^{-1}(\operatorname{Id} - B)A\bar{d}\Big)
\end{aligned}$$

In this equilibrium, the vector of Ψ-betas is given by

$$\beta_{i,t}^\Psi = \frac{1}{\operatorname{Var}_t[\Psi_{t+2}]P_{i,t}\hat{\lambda}^\Psi}\left((A_P + \operatorname{Id})A\bar{d} + (1 - e^r)\bar{P} - (A_P + \operatorname{Id})Bd_t\right)_i$$

while the market betas are given by

$$\beta_{i,t}^M = \frac{P_t^M}{P_{i,t} \cdot x^T (A_P + \operatorname{Id})\Sigma^D (A_P + \operatorname{Id})^T x}(A_P + \operatorname{Id})\left(\Sigma^D (A_P + \operatorname{Id})^T\right)x.$$

One obvious solution to (7) is $B = 0$, corresponding to the "purely myopic" equilibrium discussed above. However, as we will now show, there can be other equilibria. Since in general the structure of solutions to (7) can be complicated, in this section we only consider the case when $A = a\operatorname{Id}$. To describe the set of equilibria, recall that a matrix C on $\mathbb{R}^S$ is called a projection if there exist two disjoint subspaces, $Y, Z \subset \mathbb{R}^S$ called respectively the image ($\operatorname{Im}C$) and the kernel ($\ker C$) of C such that (1) Y, Z span $\mathbb{R}^S$; (2) $Cy = y$ for all $y \in Y$ and $Cy = 0$ for all $y \in Z$. In this case, C is usually called the projection onto Y along Z. It is possible to show that a matrix C is a projection if and only if $C^2 = C$. The following is true.

Proposition 6.2 *Suppose that $A = a\operatorname{Id}$ for some $a \in \mathbb{R}$. Then a matrix B solves (7) if and only if $B = C/(2\hat{\lambda}^\Psi a)$ where C is an arbitrary projection. This matrix corresponds to an equilibrium if and only if $\max(|a|\mathbf{1}_{\operatorname{rank}(C)<S}, |-a + 1/(2\hat{\lambda}^\Psi a)|) < e^r$.*

Several important properties of equilibrium dynamics are worth mentioning. First, in contrast to the case of purely non-fundamental shocks from the previous section, Corollary 4.1 implies that market portfolio is efficient if and only if $B \equiv 0$. In particular, there may be a significant dispersion in Ψ-betas, and this dispersion is stochastic and moves over time. Furthermore, the weights in the efficient portfolio are also stochastic, and this leads to a non-trivial dynamics in betas: While market betas are always positive, Ψ-betas may be changing sign over time. The image of the projection C is the "non-myopic subspace." It has an important meaning: Since Ψ-betas only depend on dividend shocks through Bd_t, only fundamental shocks to returns on portfolio in the "non-myopic subspace" will driving Ψ-beta dynamics. In particular, this implies that Ψ-betas will have a factor structure with the number of factors equal to rank of B. For example, if B is of rank

one, there will be a single factor driving the whole cross-section of Ψ-beta dynamics, and this factor may have nothing to do with the market portfolio, unless $x \in \text{Im}(B)$.

It is instructive to compare the multiplicity of equilibria that we find in our OLG model with the multiplicity of equilibria in myopic OLG models, discovered by Spiegel (1998). He shows that, in the presence of supply shocks a myopic OLG model has multiple equilibria. Importantly, all equilibria in Spiegel (1998) have identical sensitivities of prices to dividend shocks, and only differ in price sensitivities to supply shocks. By contrast, there are no supply shocks in our model, and equilibria differ in price sensitivities to dividend shocks. This is a very different channel for multiplicity that cannot arise in myopic model a-la Spiegel (1998). Interestingly enough, if, as in Spiegel (1998), we assume that dividends follow a random walk (i.e., $a = -1$), multiple equilibria always exist if only if $\hat{\lambda}^\Psi$ is sufficiently large.

7   Portfolio Constraints and Betting Against Ψ-Betas

In this section, we follow Frazzini and Pedersen (2014) and introduce funding liquidity frictions into the model by assuming that agents (both myopic and non-myopic) are subject to portfolio constraints. For simplicity, we assume that all agents within a given class $i = L, S$ face the same constraints

$$\nu_t^i \sum_j \pi_t^s \ \leq \ W_t^i, \ i = L, S,$$

where W_t^i is agent i's wealth. As Frazzini and Pedersen (2014) explain, a $\nu_t^i \geq 1$ means that agents cannot use leverage, while $\nu_t^i \leq 1$ corresponds to the case when agents can use leverage but are facing margin requirements.

The first order condition for the myopic agent implies

$$\pi_t^S \ = \ \frac{1}{\gamma_t^S} \Sigma_t^{-1} (\mu_t - \xi_t^S \mathbf{1})$$

where ξ_t^S is the Lagrange multiplier, while the optimal portfolio of a non-myopic agent is given by

$$\pi_t^L \ = \ \frac{1}{\gamma_t^L} e^{-r} \Sigma_t^{-1} \left(\mu_t - \text{Cov}_t[R_{t+1}, \Psi_{t+2} - \xi_{t+1}^S R_{t+2}^{\min V}] - \xi_t^L \mathbf{1} \right)$$

where ξ_t^L is the corresponding Lagrange multiplier and

$$R_{t+2}^{\min V} \ = \ R_{t+2}^T \Sigma_{t+1}^{-1} \mathbf{1}$$

is the return on the *minimal variance portfolio*. Substituting these expressions into the market clearing condition, we arrive at the following result.

Proposition 7.1 *Equilibrium risk premium on security i is given by*

$$E_t[R_{i,t+1}] \; - \; e^r \; = \; \alpha_t \; + \; \lambda_t^M \beta_{i,t}^M \; + \; \lambda_t^\Psi \beta_{i,t}^\Psi \; - \; \lambda^\Phi \beta_{i,t}^\Phi \tag{8}$$

with

$$\alpha_t \; = \; \frac{\delta_t^S \xi_t^S + \delta_t^L \xi_t^L}{\delta_t^S + \delta_t^L}, \; \lambda_t^M \; = \; \frac{\text{Var}_t[R_{t+1}^M]}{\delta_t^S + \delta_t^L} \; , \; \lambda_t^\Psi \; = \; \frac{\delta_t^L \text{Var}_t[\Psi_{t+2}]}{\delta_t^S + \delta_t^L}, \; \lambda_t^\Phi = \frac{\delta_t^L \text{Var}_t[\Phi_{t+2}]}{\delta_t^S + \delta_t^L}$$

and

$$\beta_{i,t}^\Phi \; \equiv \; \frac{\text{Cov}_t[R_{i,t+1}, \xi_{t+1}^S R_{t+2}^{\text{minV}}]}{\text{Var}_t[R_{t+2}^{\text{minV}}]}$$

is the future liquidity beta.

Proposition 7.1 shows that, just as in Frazzini and Pedersen (2014), portfolio constraints (funding liquidity) lead to the emergence of alpha that can be expressed as a weighted combination of agent-specific Lagrange multipliers; the latter are linked to severity of portfolio constraints. But, most importantly, funding liquidity combined with non-myopic behaviour leads to the emergence of a new *liquidity risk factor.* Note that this risk factor is different from that in Acharya and Pedersen (2005): While in their model liquidity risk arises due to time-varying exogenous transaction costs, in our model it arises due to non-myopic behavior because agents hedge against future funding constraints.

Following the methodology similar to the one of Frazzini and Pedersen (2014), we can construct a Ψ-beta neutral portfolio for any pair i, j of securities with $\beta_{i,t}^\Psi > \beta_{j,t}^\Psi$,

$$R_{i,j,t+1}^\Psi \; \equiv \; \beta_{i,t}^\Psi R_{j,t+1} - \beta_{j,t}^\Psi R_{i,t+1} \,.$$

By construction, $\text{Cov}_t(R_{i,j,t+1}^\Psi, \Psi_{t+2}) = 0$, and by (8), we have

$$E_t[R_{i,j,t+1}^\Psi] \; - \; e^r \; = \; (\beta_{i,t}^\Psi \alpha_{jt} - \beta_{j,t}^\Psi \alpha_{it}) \; + \; \lambda_t^M \beta_{i,t}^M \beta_{j,t}^M \left(\frac{\beta_{i,t}^\Psi}{\beta_{i,t}^M} - \frac{\beta_{j,t}^\Psi}{\beta_{j,t}^M} \right) \,,$$

where we have defined the liquidity-adjusted alpha as

$$\alpha_{it} \; \equiv \; \alpha_t \; - \; \lambda^\Phi \beta_{i,t}^\Phi \,. \tag{9}$$

We immediately see that the Ψ-beta neutral portfolio has a natural positive exposure to the common alpha and a residual exposure to the market portfolio. Taking an average over all such pairs, we arrive at a portfolio that we call "betting against Ψ-beta," or Ψ-BAB. The weight of each asset in this portfolio is determined from all pairwise beta comparisons (normalized so that the long side

weights sum up to one):

$$w_{i,t} = \frac{\sum_{i \neq j} \beta_{jt}^{\Psi} \, \mathrm{sign}(\beta_{jt}^{\Psi} - \beta_{it}^{\Psi})}{\sum_{i:w_{it}>0} w_{it}}, \tag{10}$$

and the expected return on this portfolio, that we denote by $R_{t+1}^{\Psi-BAB}$ satisfies

$$E_t[R_{t+1}^{\Psi-BAB}] - e^r \;=\; \alpha_t^{\Psi-BAB} \;+\; \lambda_t^M \beta_t^{\Psi-BAB} \tag{11}$$

where $\alpha_t^{\Psi-BAB} > 0$, while market beta of the strategy is

$$\beta_t^{\Psi-BAB} \;\propto\; \sum_{i,j:\ \beta_{it}^{\Psi} > \beta_{jt}^{\Psi}} \beta_{i,t}^M \beta_{j,t}^M \left(\frac{\beta_{i,t}^{\Psi}}{\beta_{i,t}^M} - \frac{\beta_{j,t}^{\Psi}}{\beta_{j,t}^M} \right)$$

should be positive because $\beta_{i,t}^{\Psi}$ and $\frac{\beta_{i,t}^{\Psi}}{\beta_{i,t}^M}$ are strongly positively correlated in the cross-section (empirical correlation is between 0.26 and 0.64 for different Ψ portfolios), implying that, on average, $\beta_{i,t}^{\Psi} > \beta_{j,t}^{\Psi}$ leads to $\frac{\beta_{i,t}^{\Psi}}{\beta_{i,t}^M} > \frac{\beta_{j,t}^{\Psi}}{\beta_{j,t}^M}$. These results imply that the Ψ-BAB portfolio should have an alpha that is proportional to the alpha of a non-myopic CAPM and a positive exposure to the market. *A priori* it is not clear how sensitive the Ψ-BAB return will be to portfolio constraints, because even if alpha $\alpha_t^{\Psi-BAB}$ should increase in constraint severity, market return will typically drop at the same time, thus both might well compensate each other, and being immune to liquidity shocks would be a very desirable portfolio property.

7.1 Empirical Link: Constraints and Betting Against Ψ Betas

Theoretically, both risk premiums and factor loadings on FMVP are affected by the severity of portfolio constraints and liquidity frictions. In this section we use several common measures of liquidity to study the dynamics of Ψ-betas and the corresponding risk premiums.

We start our analysis by investigating the link between the non-myopic betas and market-wide liquidity measures. To this end, we compute time-series correlations between these liquidity measures and the sum of the five leading principal components of non-myopic betas, constructed in Section 5.1.[13]

We include the following four liquidity measures in our study: the Pastor and Stambaugh (2003) liquidity measure and liquidity factor that are based on the notion of stock market reversal; the Amihud (2002) measure that is determined by the average price impact per unit of trading volume[14]; the TED spread (the difference between LIBOR and T-Bill rates), which is commonly viewed as a measure of "flight-to-quality incentives," and is supposed to reflect both the current degree of risk aversion and the degree of funding constraints in the financial sector; and the FLIQ

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[13]Using a different number of principal components would affect the magnitude of these correlations, but should generally not affect their sign.

[14]This measure is also naturally related to price reversals.

measure, which is a true measure funding liquidity, designed to capture funding scarcity, and the ability of agents to maintain their long positions.

As we can see from Table 4, contemporaneous correlations between Ψ betas and all Pastor and Stambaugh liquidity indicators are very low, with mixed signs across FMVP methods. It tells us that the average strength of the reversal effect has little in common with the effect of changing investment opportunity set.

The correlations with the TED spread, however, are non-trivial and positive for all FMVPs, going up to 0.45 for the Model-Consistent portfolio. The fact that an increase in the TED spread generally corresponds to an increase in Ψ-betas may potentially be driven by one of the following effects: A an increase in risk expectations; an increase in risk aversion that dries up the demand; or an increase in the severity of portfolio constraints. TED spread is also responsible for a large fraction of negative co-movement between market and Ψ-betas —the correlation of TED with market beta is strongly negative at -0.25. At the same time, the Ψ beta correlations with TED-INNOV are *negative*, though quite small in magnitude, while the market beta correlation to TED-INNOV is positive.

The correlation with the level of funding liquidity (FLIQ) is on average positive across different Ψ, but is much smaller in magnitude, and even slightly negative for FFMV (-0.10) and FFMinV (-0.07). Innovations to FLIQ are negatively correlated with Ψ betas, and positively—with market betas.

Summarizing, our findings suggest that the common factor responsible for the strong negative co-movement between market and Ψ-betas is closely related to TED. This is intuitive if we interpret TED as a common measure of market-wide risk-bearing capacity (i.e., risk aversion of both myopic and non-myopic agents). However, at the same time some Ψ betas are also highly correlated with the Amihud illiquidity measure. Namely, MC and FFMinV exhibit very high positive correlations with Amihud's price reaction measure (0.49 and 0.62, respectively), while the correlation between market beta and Amihud measure is large and negative (-0.57). The link to M portfolio is weaker (correlation is 0.17), and it is basically nil for FFMV portfolio. This drastic difference in the behavior of different Ψ betas is surprising, and we leave it for future research.

Table 5 shows the time-series correlations between risk premiums and the same liquidity measures as above. Risk premiums are less correlated with liquidity than betas, and the only liquidity measure that delivers somewhat stable correlations for all Ψ risk premiums is TED-INNOV with an average correlation of 0.07, and PS-LVL and PS-INNOV with negative correlations ranging from almost zero for MC to -0.24 for M portfolio. In these three cases the correlation with MKT is of opposite sign. Note that none of these correlations are significantly different from zero but two—between the risk premium on Ψ^M and PS-LVL and PS-INNOV.

Proposition 7.1 implies that the pricing error (alpha) should be directly linked to the severity of portfolio constraints and, as a result, to market-wide liquidity measures. We test this prediction by regressing alpha on the same liquidity measures as above. Table 6 shows that PS-LVL, TED-LVL

and FLIQ-LVL are the most significant determinants of two-factor alphas with t-stats around 3 for all future portfolios. Moreover, alphas are positively related to contemporaneous *liquidity*, i.e., model pricing errors are higher when liquidity is high. Lagged liquidity measures switch sign, and PS-LVL has now the most significant effect on alphas, while TED-LVL also changes sign, but loses most of its significance (t-stats of 1.3 to 1.8). Interestingly enough, Frazzini and Pedersen (2014) find a *negative relationship between one-factor alpha and lagged TED-LVL* and mention that this finding is inconsistent with their model. By contrast, our two-factor alpha is positively related to lagged TED-LVL and several other lagged illiquidity measures. This suggests that non-myopic behavior might be important in reinforcing the link between alpha and illiquidity.

Motivated by this evidence, we formally test model (8) using the Fama and MacBeth (1973) two-stage approach. As the future mean-variance efficient portfolio we take the Model-Consistent portfolio (MC), and as the minimum-variance portfolio we use FFMinV; moreover, assume that we can proxy the Lagrange multiplier on the funding constraint of myopic investors using the positive deviation of the short-term mean TED spread from its longer-term mean as follows:

$$\xi^S(t) = \max(EWMA_{TED,short}(t) - EWMA_{TED,long}(t), 0),$$

where $EWMA_{TED,\lambda}(t) = \lambda \times EWMA_{TED,\lambda}(t-1) + (1-\lambda) \times TED(t-1)$. For the short-term EWMA we use $\lambda = 0.995$, while for the long-term one it is $\lambda = 0.999$; EWMA is initiated at the level of TED for the beginning of the series, i.e., for January 1986. The Figure 3 depicts the identified constrained states—they typically start several months before a market crash, and end sometime before the end of recovery. We compute the liquidity beta $\beta_{i,t}^{\Phi} \equiv \frac{\text{Cov}_t[R_{i,t+1}, \xi_{t+1}^S R_{t+2}^{\min V}]}{\text{Var}_t[R_{t+2}^{\min V}]}$ from monthly overlapping observations of stock returns and of the FFMinV returns scaled by the respective values of the Lagrange multiplier, using the same length of estimation window as for all other $\Psi-$betas. Note that the identification procedure leads to $\Phi-$betas being absent for all stocks for a number of periods. We assume that during these periods the funding liquidity constraints are not binding even in expectation, and hence at the cross-sectional stage we only include original market and $\Psi-$betas in the regressions during these periods, thus assuming that the λ^{Φ} is missing. The resulting liquidity risk premium λ^{Phi} is positive at 0.2230 per year and significant with p-value of 0.06. Thus, stocks correlated positively with future optimal minimum-variance portfolio in states when myopic investor constraint is binding reaps a lower risk premium.[15]

Finally, we construct the "betting against Ψ-beta" portfolio using the portfolio weights (10) and investigate its performance. In Table 7 we can see the total return, volatility, and factor alphas of portfolios constructed each non-myopic beta. Two observations worth mentioning: First, Ψ-BAB portfolios are long the market factor, with a market beta ranging from 0.74 to 0.86. Indeed, as formula (11) shows, eliminating FMVP exposure does not remove exposure to market risk, in contrast to the market-BAB strategy of Frazzini and Pedersen. Second, despite retaining a

---

[15]We have discovered that risk premium significance is especially sensitive to identification of the beginning of the constrained period; for example, if we miss first six months of the third constrained period in Figure 3, the risk premium will maintain its sign, but will not be significant.

significant exposure to the market, our Ψ-BAB strategy generates a significant alpha (from 1.6% to 3.5% p.a.) even after controlling for five + BAB factors. To explain the intuition behind this result, recall that non-myopic agents prefer to hold stocks with low Ψ exposure and they do so by reducing exposure to high FMVP-beta stocks. This means that non-myopic agents generate abnormal returns by effectively betting against Ψ-beta. In addition, these strategies also deliver a decent Sharpe ratio (around 0.80 p.a.) with a well-controlled level of volatility (around 0.14 p.a.).

In Table 8 we document the link between the dynamics of the Ψ-BAB portfolio and liquidity on the market, which we measure using the lagged TED spread value and the change in TED spread (TED-INNOV). In the standalone regression both variables are negatively related to the portfolio performance, which looks similar to the empirical sensitivity of market BAB to severity of funding constraints. When TED variables are added to the four-factor model, they turn out insignificant, though again with the negative sign. This effect is probably related to the omitted liquidity risk factor (see (9)). Understanding these phenomena is an interesting direction for future research.

8   Conclusions

We introduce simple extension of the classical CAPM in which the economy is populated by two classes of agents, myopic and non-myopic. Our model predicts the emergence of a new priced risk factor: the future return on the mean-variance efficient portfolio (FMVP). We show how efficient portfolio can be constructed in a model-consistent fashion and provide strong evidence that our new risk factor is priced in the cross-section of US stock returns. We use non-myopic betas to test for non-myopic behavior of mutual funds and find that non-myopic betas are able to explain a significant variation in mutual fund alphas, suggesting that no-myopic behavior plays an important role in alpha generation. Our model predicts that the dynamics of risk bearing capacities of non-myopic traders combined with leverage effect and the negative aggregate skewness should generate negative co-movement in market and non-myopic betas in the time series and positive correlation between these betas in the cross-section. Both predictions are confirmed by the data. We also show that, when economic fundamentals are autocorrelated, non-myopic behavior may become self-fulfilling and equilibria with non-trivial risk premium dynamics co-exist with standard, constant risk premium equilibrium.

A Proofs

Proof of Proposition 5.1. For analytical convenience, we work directly with prices instead of returns. We have

$$\Sigma_k \; \equiv \; \text{Cov}_t(P_{t+1} + D_{t+1}, P_{t+1} + D_{t+1}) \; = \; \Sigma^D \; + \; \text{Var}_t[\Gamma_{t+1}](\Sigma^D x)(\Sigma^D x)^T$$

where k is the time-t state. Hence

$$\Sigma_k^{-1} \; = \; (\Sigma^D)^{-1} \; - \; \zeta_k x x^T$$

where

$$\zeta_k \; = \; \frac{\text{Var}_t[\Gamma_{t+1}]}{1 + \text{Var}_t[\Gamma_{t+1}] x^T \Sigma^D x}$$

and where k is the state at time t. Then, equilibrium risk premium is given by

$$\mu^k \; = \; E_t[P_{t+1} + D_{t+1}] - e^r P_t \; = \; \alpha_k \Sigma^D x$$

where

$$\alpha_k = (e^r \Gamma_k - E_t[\Gamma_{t+1}]) \,.$$

Hence,

$$\begin{aligned}
\psi_k \; &= \; \mu_k^T \Sigma_k^{-1} \mu_k \\
&= \; (\alpha_k \Sigma^D x)^T ((\Sigma^D)^{-1} \; - \; \zeta_k x x^T)(\alpha_k \Sigma^D x) \; = \; \alpha_k x^T \Sigma^D x (1 - \zeta_k x^T \Sigma^D x)
\end{aligned}$$

Then, the equilibrium equation takes the form

$$\alpha_k \Sigma^D x \; = \; \mu^k \; = \; (\nu_k^S (1 + \text{Var}_t[\Gamma_{t+1}] x^T \Sigma^D x) - \nu_k^L \, \text{Cov}_t[\Gamma_{t+1}, \psi_{t+1}]) \Sigma^D x$$

and we get the required. ∎

Proof of Proposition 6.1. Conditional covariance matrix of payoffs is given by

$$\Sigma \; = \; \text{Cov}_t[d_{t+1} + P_{t+1}] \; = \; \text{Cov}_t[(A_P + \text{Id})(A(\bar{d} - d_t) \; + \; \varepsilon_{t+1})] \; = \; (A_P + \text{Id})\Sigma^D (A_P + \text{Id})^T$$

whereas the expected risk premiums are given by

$$E_t[d_{t+1} + P_{t+1}] - e^r P_t \; = \; (A_P + \text{Id})A(\bar{d} - d_t) + \bar{P} - e^r(\bar{P} + A_P d_t) = (A_P + \text{Id})A\bar{d} + (1 - e^r)\bar{P} - ((A_P + \text{Id})A + e^r A_P)d_t,$$

and hence the expected payoff of the efficient portfolio is given by

$$(A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} - (A + e^r(A_P + \text{Id})^{-1}A_P)d_{t+1})^T(\Sigma^D)^{-1}$$
$$\times (A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} - (A + e^r(A_P + \text{Id})^{-1}A_P)d_{t+1})$$
$$= (A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} - (A + e^r(A_P + \text{Id})^{-1}A_P)(A(\bar{d} - d_t) + \varepsilon_{t+1}))^T(\Sigma^D)^{-1}$$
$$\times (A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} - (A + e^r(A_P + \text{Id})^{-1}A_P)(A(\bar{d} - d_t) + \varepsilon_{t+1}))$$
$$= a_t^* - \xi_t^T\varepsilon_{t+1} + \varepsilon_{t+1}^T\Theta\varepsilon_{t+1}$$

where $\Theta = A^T(\Sigma^D)^{-1}A$, while a_t^* is an $\mathcal{F}_t$-measurable random variable, and

$$\xi_t = 2(A + e^r(A_P + \text{Id})^{-1}A_P)^T(\Sigma^D)^{-1}(A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} - (A + e^r(A_P + \text{Id})^{-1}A_P)A(\bar{d} - d_t)).$$

Since all third order moments of a centered Gaussian vector are identically zero, we have

$$\text{Cov}_t[(d_{t+1} + P_{t+1}), \Psi_{t+2}] = -\text{Cov}_t\left[(A_P + \text{Id})\varepsilon_{t+1}, \xi_t^T\varepsilon_{t+1}\right].$$

Hence,

$$\text{Cov}_t[(d_{t+1} + P_{t+1}), \Psi_{t+2}] = -2(A_P + \text{Id})\Sigma^D\xi_t$$
$$= -2(A_P + \text{Id})\Sigma^D(A + e^r(A_P + \text{Id})^{-1}A_P)^T(\Sigma^D)^{-1}$$
$$\times ((\text{Id} - (A + e^r(A_P + \text{Id})^{-1}A_P))A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} + (A + e^r(A_P + \text{Id})^{-1}A_P)Ad_t)$$

Thus, equilibrium condition takes the form

$$(A_P + \text{Id})A\bar{d} + (1 - e^r)\bar{P} - ((A_P + \text{Id})A + e^rA_P)d_t = E_t[P_{t+1} + d_{t+1}] - e^rP_t$$
$$= \hat{\lambda}^M(A_P + \text{Id})\Sigma^D(A_P + \text{Id})^Tx$$
$$- 2\hat{\lambda}^\Psi(A_P + \text{Id})\Sigma^D(A + e^r(A_P + \text{Id})^{-1}A_P)^T(\Sigma^D)^{-1}$$
$$\times ((\text{Id} - (A + e^r(A_P + \text{Id})^{-1}A_P))A\bar{d} + (1 - e^r)(A_P + \text{Id})^{-1}\bar{P} + (A + e^r(A_P + \text{Id})^{-1}A_P)Ad_t)$$

Equating the coefficient on d_t, we get

$$-((A_P + \text{Id})A + e^rA_P) = -2\hat{\lambda}^\Psi(A_P + \text{Id})\Sigma^D(A + e^r(A_P + \text{Id})^{-1}A_P)^T(\Sigma^D)^{-1}(A + e^r(A_P + \text{Id})^{-1}A_P)A.$$

Multiplying this identity by $(\Sigma^D)^{-1}(A_P + \text{Id})^{-1}$ and denoting $B \equiv (A + e^r(A_P + \text{Id})^{-1}A_P)$, we get

$$(\Sigma^D)^{-1}B = 2\hat{\lambda}^\Psi B^T(\Sigma^D)^{-1}BA.$$

Then, solving the equation

$$B = A + e^r(A_P + \text{Id})^{-1}A_P,$$

we get

$$A_P = (e^r - (B - A))^{-1}(B - A).$$

Finally, for $\bar{P}$ we get

$$
\begin{aligned}
(A_P + \mathrm{Id})A\bar{d} \;=\;& \hat{\lambda}^M (A_P + \mathrm{Id})\Sigma^D (A_P + \mathrm{Id})^T x \\
& - 2\hat{\lambda}^{\Psi}(A_P + \mathrm{Id})\Sigma^D (A + e^r (A_P + \mathrm{Id})^{-1}A_P)^T (\Sigma^D)^{-1}(\mathrm{Id} - (A + e^r (A_P + \mathrm{Id})^{-1}A_P))A\bar{d} \\
& + (e^r - 1)\Big(2\hat{\lambda}^{\Psi}(A_P + \mathrm{Id})\Sigma^D (A + e^r (A_P + \mathrm{Id})^{-1}A_P)^T (\Sigma^D)^{-1}(A_P + \mathrm{Id})^{-1} + \mathrm{Id}\Big)\bar{P}
\end{aligned}
$$

which gives the required. ∎

Proof of Proposition 6.2. Since A is scalar, equation for B takes the form

$$
(\Sigma^D)^{-1}B \;=\; 2\hat{\lambda}^{\Psi} a B^T (\Sigma^D)^{-1}B
$$

This means that $B^T y = (1/2\hat{\lambda}^{\Psi}a)y$ for all $y \in \mathrm{Im}((\Sigma^D)^{-1}B)$. In particular, since $B^T y = 0$ for all $y \in (\mathrm{Im}(B))^{\perp} = \ker B^T$, this means that we ought to have $\mathrm{Im}((\Sigma^D)^{-1}B) \cap \ker(B^T) = \{0\}$. This uniquely determines B^T given that we fix the subspace $\mathrm{Im}(B)$. In order to prove that $\mathrm{Im}((\Sigma^D)^{-1}B) \cap \ker(B^T) = \{0\}$, consider a vector y such that $B^T (\Sigma^D)^{-1}By = 0$. But, since $(\Sigma^D)^{-1}$ is positive definite, this can only happen if $By = 0$. The proof is complete. ∎

Tables

Table 1: FMVP beta-sorted quintile portfolios

This table shows the test statistics for quintile portfolios created from sorting by future-mean-variance-efficient portfolio (future risk factor) betas as described in Section 4.3. MC is the model-consistent portfolio from four factors, FFMinV and FFMV are the minimum- and mean-variance portfolios constructed from four factors, and M stands for future market portfolio (see Section 4.1 for details). Panel A uses equal weights (EW) in each quintile, and Panel B uses prior gross return-based weights (RW), where each stock is weighted inside a quintile by the gross return in the previous month. Column "Top-Bottom" shows the annualized return from long-short portfolio constructed from extreme quintiles, and "pval" gives its significance. The remaining columns provide the p-values for the monotonicity tests described in Patton and Timmermann (2010): "Decreasing' is the p-value for the null that returns are *monotonically decreasing*, and "Increasing" is the p-value for the null hypothesis that the returns are *monotonically increasing*. Simultaneously rejecting "Decreasing" and failing to reject "Increasing" indicates that we observe a monotonic increasing relationship. The p-values for monotonicity tests are computed with bootstrap using 100,000 simulated samples.

Panel A: Equal-weighted portfolios

Efficient portfolio	Top-Bottom	pval	Decreasing	Increasing
MC	2.43	0.04	0.06	0.94
FFMinV	3.69	0.01	0.01	0.91
FFMV	2.70	0.04	0.04	0.84
M	2.59	0.04	0.06	0.93

Panel B: Prior-gross-return weighted portfolios

Efficient portfolio	Top-Bottom	pval	Decreasing	Increasing
MC	2.20	0.06	0.09	0.94
FFMinV	3.29	0.03	0.02	0.91
FFMV	2.36	0.07	0.05	0.86
M	2.28	0.06	0.08	0.94

Table 2: Risk premium estimation

This table shows the annualized risk premiums on the covariance with the future-mean-variance-efficient portfolio (FMVP) as described in Section 4.3. MC is the model-consistent portfolio from four factors, FFMinV and FFMV are the minimum- and mean-variance portfolios constructed from four factors, and M stands for future market portfolio (see Section 4.1 for details). Panel A uses equal weights (EW) in the first regression stage, and Panel B uses prior gross return-based weighted least squares (RW), where each stock is weighted in a regression by the gross return in the previous month. Columns denote number of factors used in model beyond the FMVP–one factor, three Fama and French (1993) factors, or four Carhart (1997) factors. Below each premium is the t-statistics for the null that the risk premium is not different from zero.

Panel A: Equal-weighted first stage

Efficient portfolio	1f	3f	4f
MC	0.0262	0.0233	0.0214
	3.2849	3.3084	3.3628
FFMinV	0.0138	0.0106	0.0083
	2.9297	2.4177	2.3426
FFMV	0.0121	0.0086	0.0067
	2.5003	1.8999	1.8482
M	0.0106	0.0078	0.0057
	2.0611	1.7329	1.5911

Panel A: Prior-gross-return-weighted first stage

Efficient portfolio	1f	3f	4f
MC	0.0263	0.0234	0.0214
	3.3023	3.3017	3.3256
FFMinV	0.0129	0.0098	0.0076
	2.7864	2.2838	2.2276
FFMV	0.0114	0.0080	0.0062
	2.3984	1.8118	1.7849
M	0.0102	0.0076	0.0057
	2.0034	1.7118	1.6208

Table 3: Non-Myopic betas and mutual fund returns

This table shows the dependency between non-myopic exposure of mutual funds (β^{Ψ}), and their market exposure (β^{MKT}) and annualized factor alphas, as described in Section 4.4. MC is the model-consistent portfolio from four factors, FFMinV and FFMV are the minimum- and mean-variance portfolios constructed from four factors, and M stands for future market portfolio (see Section 4.1 for details). We first regress at each point in time fund factor alphas on a number of regressors, save the coefficients, and then compute their mean and test its significance. Under each coefficient value is the two-sided p-value. The $\bar{R}$ contains the average adjusted R-squared from the respective first-stage cross-sectional regression. Each cross-sectional regression is on average based on 1769 funds.

Panel A: Efficient Portfolio: MC

	One-factor alpha		Three-factor alpha		Four-factor alpha	
Const	0.000	0.002	-0.009	-0.003	-0.009	0.015
	0.005	0.000	0.000	0.742	0.000	0.000
β^{PSI}	-0.008	-0.008	-0.023	-0.019	-0.014	-0.012
	0.000	0.000	0.000	0.000	0.008	0.019
β^{MKT}		-0.002		-0.006		-0.024
		0.000		0.170		0.000
$\bar{R}$	9.8%	13.0%	3.2%	5.3%	2.4%	4.8%

Panel B: Efficient Portfolio: FFMinV

	One-factor alpha		Three-factor alpha		Four-factor alpha	
Const	0.000	0.004	-0.009	0.008	-0.010	0.019
	0.794	0.000	0.000	0.033	0.000	0.000
β^{PSI}	-0.002	-0.002	-0.020	-0.028	-0.006	-0.015
	0.000	0.000	0.000	0.000	0.050	0.000
β^{MKT}		-0.004		-0.017		-0.030
		0.000		0.000		0.000
$\bar{R}$	6.0%	10.6%	5.9%	8.9%	4.5%	7.8%

Panel C: Efficient Portfolio: FFMV

	One-factor alpha		Three-factor alpha		Four-factor alpha	
Const	0.000	0.004	-0.010	0.008	-0.011	0.022
	0.450	0.000	0.000	0.062	0.000	0.000
β^{PSI}	0.000	-0.001	-0.015	-0.022	-0.003	-0.012
	0.346	0.009	0.000	0.000	0.635	0.003
β^{MKT}		-0.004		-0.017		-0.033
		0.000		0.000		0.000
$\bar{R}$	8.3%	12.7%	6.6%	9.8%	5.6%	9.1%

Panel D: Efficient Portfolio: M

	One-factor alpha		Three-factor alpha		Four-factor alpha	
Const	0.000	0.003	-0.009	0.001	-0.010	0.013
	0.654	0.000	0.000	0.572	0.000	0.000
β^{PSI}	-0.007	-0.006	0.006	0.002	-0.018	-0.020
	0.000	0.000	0.917	0.342	0.048	0.040
β^{MKT}		-0.003		-0.010		-0.023
		0.000		0.004		0.000
$\bar{R}$	8.3%	12.0%	5.4%	7.6%	4.5%	7.4%

Table 4: Marketwide liquidity and betas

This table shows the contemporaneous time-series correlations between the betas (market β^{MKT} and non-myopic betas β^{Ψ}), and market-wide liquidity measures. The dynamics of betas is approximated by the sum of the five leading principal components of large and liquid stocks as described in Section 5.1. MKT denotes market beta, M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors. The liquidity measures are discussed in the Data and Methodology Section 3.

Illiquidity measure	M	FFMV	FFMinV	MC	MKT
PS-LVL	0.098	-0.047	0.080	0.006	0.074
PS-INNOV	0.038	-0.059	0.021	-0.017	0.036
PS-FACT	-0.074	-0.006	0.003	-0.067	-0.058
TED-LVL	0.159	0.066	0.202	0.447	-0.252
TED-INNOV	-0.070	-0.045	-0.011	-0.019	0.037
FLIQ-LVL	0.170	-0.104	-0.069	0.186	0.028
FLIQ-INNOV	-0.087	-0.072	0.002	-0.049	0.026
Amihud Illiq (EW)	0.169	-0.013	0.616	0.490	-0.570
Amihud Illiq (EW) INNOV	-0.005	0.005	-0.006	0.024	0.014
Amihud Illiq (VW)	-0.146	-0.147	0.518	0.227	-0.666
Amihud Illiq (VW) INNOV	0.004	0.012	-0.017	0.012	0.040

Table 5: Marketwide liquidity and risk premiums

This table shows the contemporaneous time-series correlations between the risk premiums for market factor and FMVP Ψ, and market-wide liquidity measures. MKT denotes market factor, M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors. The liquidity measures are discussed in the Data and Methodology Section 3.

Illiquidity measure	M	FFMV	FFMinV	MC	MKT
PS-LVL	-0.23	-0.09	-0.07	-0.01	0.07
PS-INNOV	-0.24	-0.06	-0.06	-0.02	0.14
PS-FACT	-0.09	-0.08	-0.06	0.09	-0.10
TED-LVL	0.10	0.04	0.04	-0.08	-0.02
TED-INNOV	0.09	0.08	0.04	0.08	-0.12
FLIQ-LVL	0.05	-0.03	0.01	-0.08	0.08
FLIQ-INNOV	0.07	-0.02	0.00	0.01	-0.01
Amihud Illiq (EW)	0.06	-0.03	0.01	-0.08	-0.08
Amihud Illiq (EW) INNOV	0.08	0.04	0.01	-0.13	-0.23
Amihud Illiq (VW)	0.03	-0.05	0.01	-0.03	-0.11
Amihud Illiq (VW) INNOV	0.06	-0.04	-0.05	-0.12	-0.23

Table 6: Marketwide liquidity and alpha

This table shows the t-stats from regressing alphas estimated in the first stage of the Fama and MacBeth (1973) regression of excess stock returns on market factor and FMVP Ψ betas as described in Section 4.3, and a number of contemporaneous (*Panel A*) and lagged (*Panel B*) liquidity measures. M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors. The liquidity measures are discussed in the Data and Methodology Section 3.

Panel A: Contemporaneous Illiquidity Measures

Illiquidity measure	M	FFMV	FFMinV	MC
PS-LVL	3.03	3.05	2.93	2.93
PS-INNOV	2.07	2.08	2.00	2.00
PS-FACT	1.96	1.94	1.89	2.06
TED-LVL	-3.82	-3.89	-3.93	-4.02
TED-INNOV	-0.12	-0.16	-0.12	-0.19
FLIQ-LVL	-3.10	-3.16	-3.29	-3.29
FLIQ-INNOV	-2.71	-2.76	-2.85	-2.91
Amihud Illiq (EW)	-0.49	-0.51	-0.54	-0.59
Amihud Illiq (EW) INNOV	-1.25	-1.25	-1.26	-1.31
Amihud Illiq (VW)	0.76	0.74	0.72	0.68
Amihud Illiq (VW) INNOV	-1.56	-1.56	-1.57	-1.58

Panel B: Lagged Illiquidity Measures

Illiquidity measure	M	FFMV	FFMinV	MC
PS-LVL	-5.34	-5.37	-5.29	-3.46
PS-INNOV	-4.73	-4.80	-4.72	-2.09
PS-FACT	0.92	0.86	0.94	-0.13
TED-LVL	1.33	1.38	1.40	1.75
TED-INNOV	-0.48	-0.50	-0.56	0.33
FLIQ-LVL	-0.53	-0.52	-0.52	-0.27
FLIQ-INNOV	-0.92	-0.88	-0.88	0.77
Amihud Illiq (EW)	-1.34	-1.31	-1.21	-1.38
Amihud Illiq (EW) INNOV	0.45	0.45	0.33	0.37
Amihud Illiq (VW)	-0.15	-0.17	-0.07	-0.18
Amihud Illiq (VW) INNOV	0.41	0.40	0.39	-0.33

Table 7: Performance of the Ψ-BAB portfolios

This table shows the performance of the Ψ-BAB portfolios, absolute and relative to several factor models. Each factor alpha, mean return, volatility, and Sharpe Ratio are annualized; three-factor model uses Fama and French (1993) factors, four-factor model adds the Carhart (1997) momentum factor, five-factor model adds the Pastor and Stambaugh (2003) liquidity factor, and five-factor + BAB adds the Frazzini and Pedersen (2014) betting-against-beta factor. M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors. Under each α is the p-value for its significance.

		M	FFMV	FFMinV	MC
No model	Mean Return	0.1197	0.1126	0.1054	0.1176
	Volatility	0.1479	0.1383	0.1289	0.1425
	Sharpe Ratio	0.8096	0.8146	0.8182	0.8253
CAPM	β	0.8680	0.8037	0.7390	0.8316
	α	0.0548	0.0483	0.0463	0.0554
	p-val	0.00	0.00	0.00	0.00
	$\bar{R}$	83.97%	82.49%	80.24%	82.97%
Three-factor	α	0.0342	0.0279	0.0258	0.0331
	p-val	0.00	0.00	0.00	0.00
	$\bar{R}$	94.19%	91.53%	89.99%	93.56%
Four-factor	α	0.0413	0.0304	0.0291	0.0402
	p-val	0.00	0.00	0.00	0.00
	$\bar{R}$	94.69%	91.58%	90.11%	94.09%
Five-factor	α	0.0424	0.0308	0.0287	0.0386
	p-val	0.00	0.00	0.00	0.00
	$\bar{R}$	94.70%	91.56%	90.09%	94.13%
Five-factor + BAB	α	0.0349	0.0202	0.0185	0.0319
	p-val	0.00	0.00	0.01	0.00
	$\bar{R}$	95.42%	93.09%	91.69%	94.77%

Table 8: Performance of the Ψ-BAB portfolios and liquidity

This table shows the link between performance of the Ψ-BAB portfolios and TED spread, as lagged value relative to portfolio return and as innovation in spread TED-INNOV, used as standalone variables (with a constant) and in addition to CAPM and the four-factor model (Fama and French (1993) and Carhart (1997)). M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors. Under each coefficient is the p-value for its significance, and $\bar{R}$ denotes the adjusted R-squared from the regression.

		M	FFMV	FFMinV	MC
No model	Lagged TED	-0.0198	-0.0196	-0.0192	-0.0215
	p-val	0.00	0.00	0.00	0.00
	TED-INNOV	-0.0421	-0.0400	-0.0361	-0.0426
	p-val	0.00	0.00	0.00	0.00
	$\bar{R}$	6.87%	7.33%	7.39%	7.64%
CAPM +	Lagged TED	-0.0062	-0.0070	-0.0076	-0.0081
	p-val	0.01	0.00	0.00	0.00
	TED-INNOV	-0.0003	-0.0014	-0.0006	-0.0014
	p-val	0.12	0.53	0.23	0.53
	$\bar{R}$	83.39%	81.83%	79.60%	83.02%
Four-factor +	Lagged TED	-0.0004	-0.0015	-0.0022	-0.0024
	p-val	0.54	0.64	0.31	0.15
	TED-INNOV	-0.0011	-0.0013	-0.0007	-0.0022
	p-val	0.74	0.72	0.37	0.77
	$\bar{R}$	94.94%	91.37%	90.13%	94.02%

Figures

Figure 1: Quintile returns for various efficient portfolio methods

We depict in four panels below the quintile portfolio returns based on Ψ-beta sorting of individual stocks. Stocks are sorted by respective betas each month, and the equal-weighted return (in text we also discuss prior-gross-return weighting) for the next month is computed. The y-axis shows average annualized return, while x-axis gives the average beta of each quintile. M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors (see Section 4.1 for details).
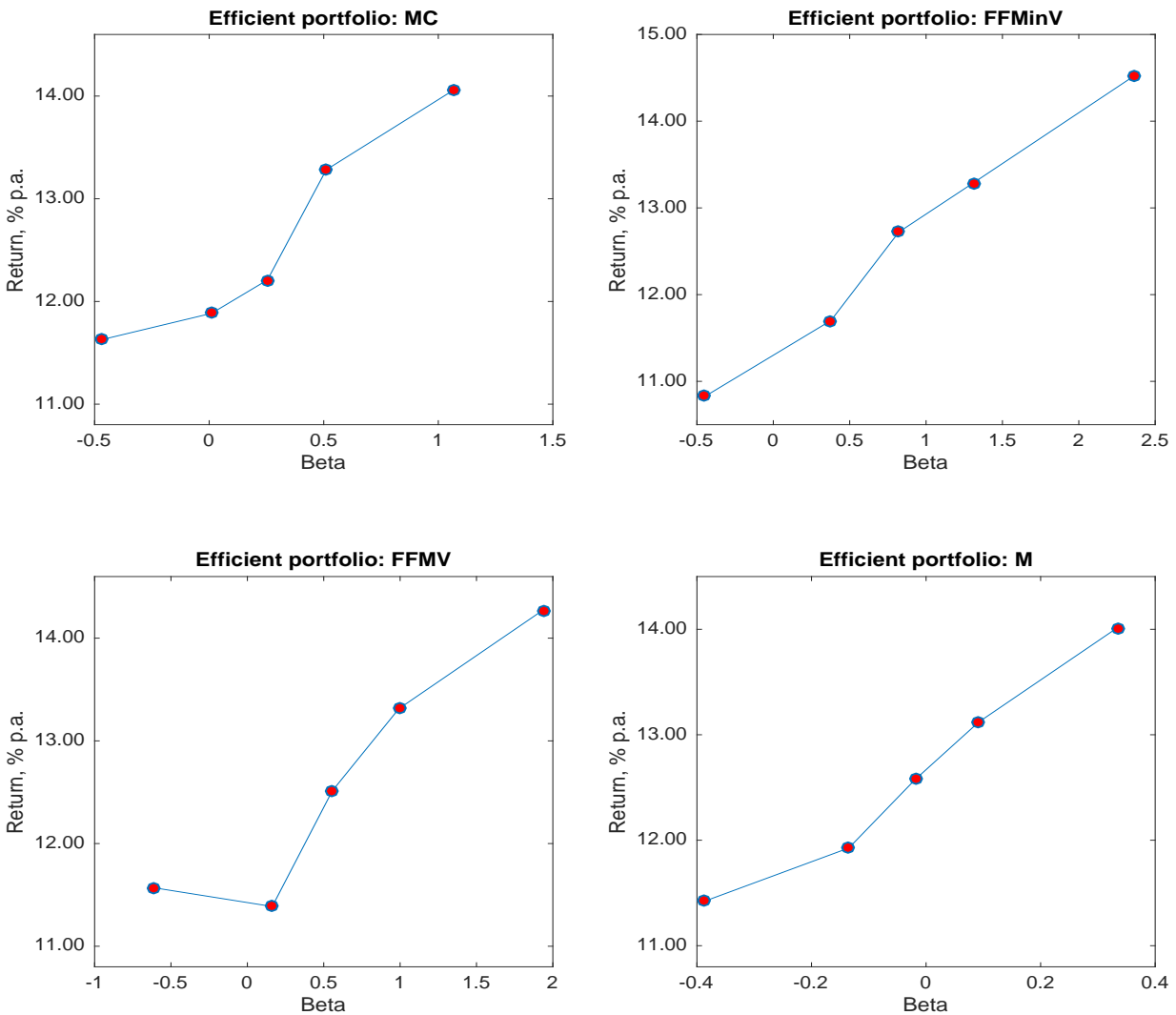
Figure 2: Variance in betas explained by leading principal components.

We depict the cumulative variance in various expected betas explained by five leading principal components, extracted from a panel of respective betas. MKT denotes market betas, and all others are Ψ betas for a given choice of FMVP: M stands for future market portfolio, FFMV and FFMinV are the mean- and minimum-variance portfolios constructed from four factors, and MC is the model-consistent portfolio from four factors (see Section 4.1 for details).
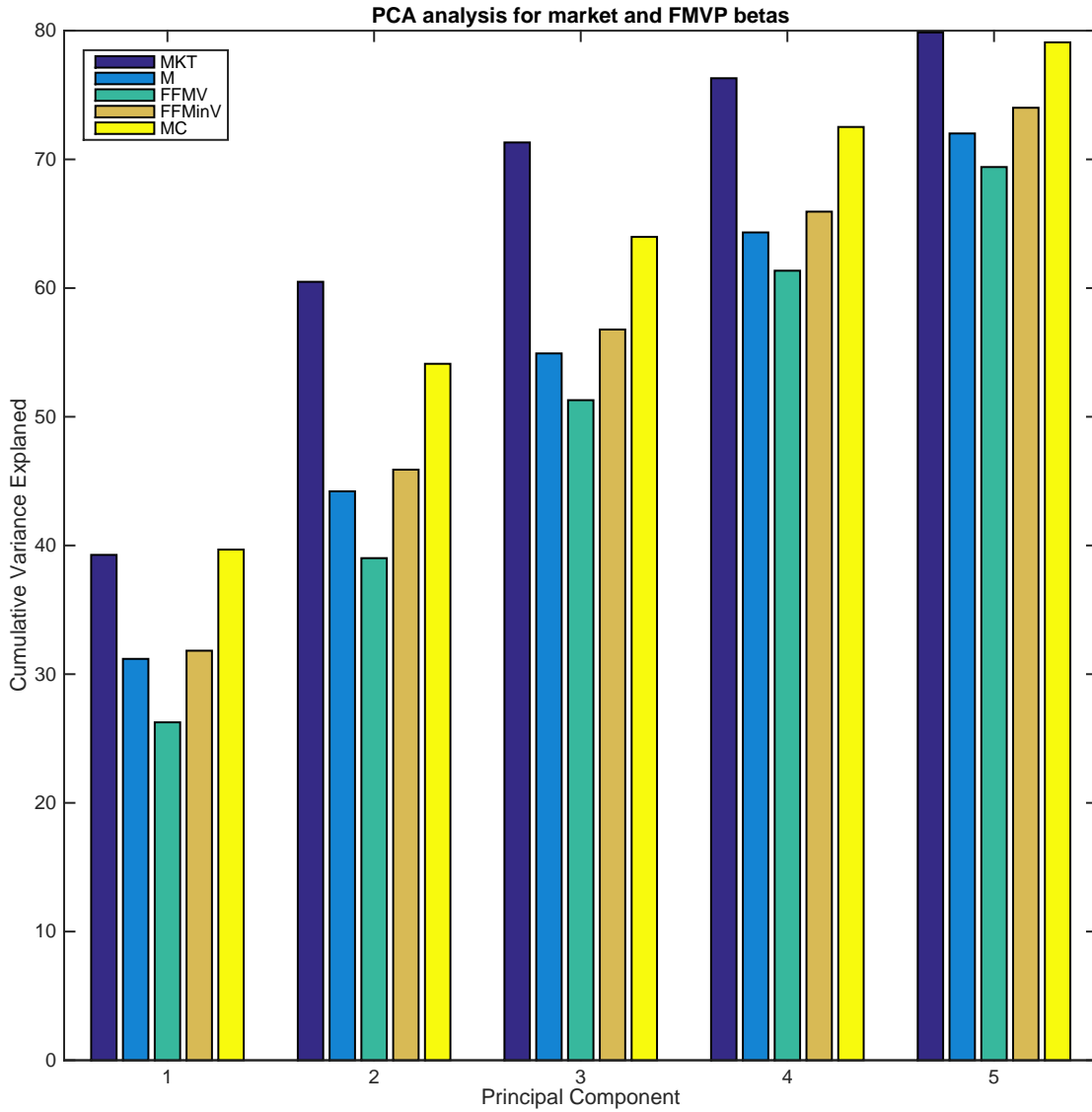
Figure 3: Identification of funding liquidity-constrained periods.

We depict the states, where myopic investors face binding funding liquidity constraints. Such states are shaded in grey, and they are identified by comparing short- and long-term EWMA of TED spread. Where short-term spread is higher, the constraint is assumed to be binding. The EWMA is computed by formula $EWMA_{TED,\lambda}(t) = \lambda \times EWMA_{TED,\lambda}(t-1) + (1-\lambda) \times TED(t-1)$, where for the short-term EWMA we use $\lambda = 0.995$, while for the long-term one it is $\lambda = 0.999$. EWMA is initiated at the level of TED for the beginning of the series, i.e., for January 1986.
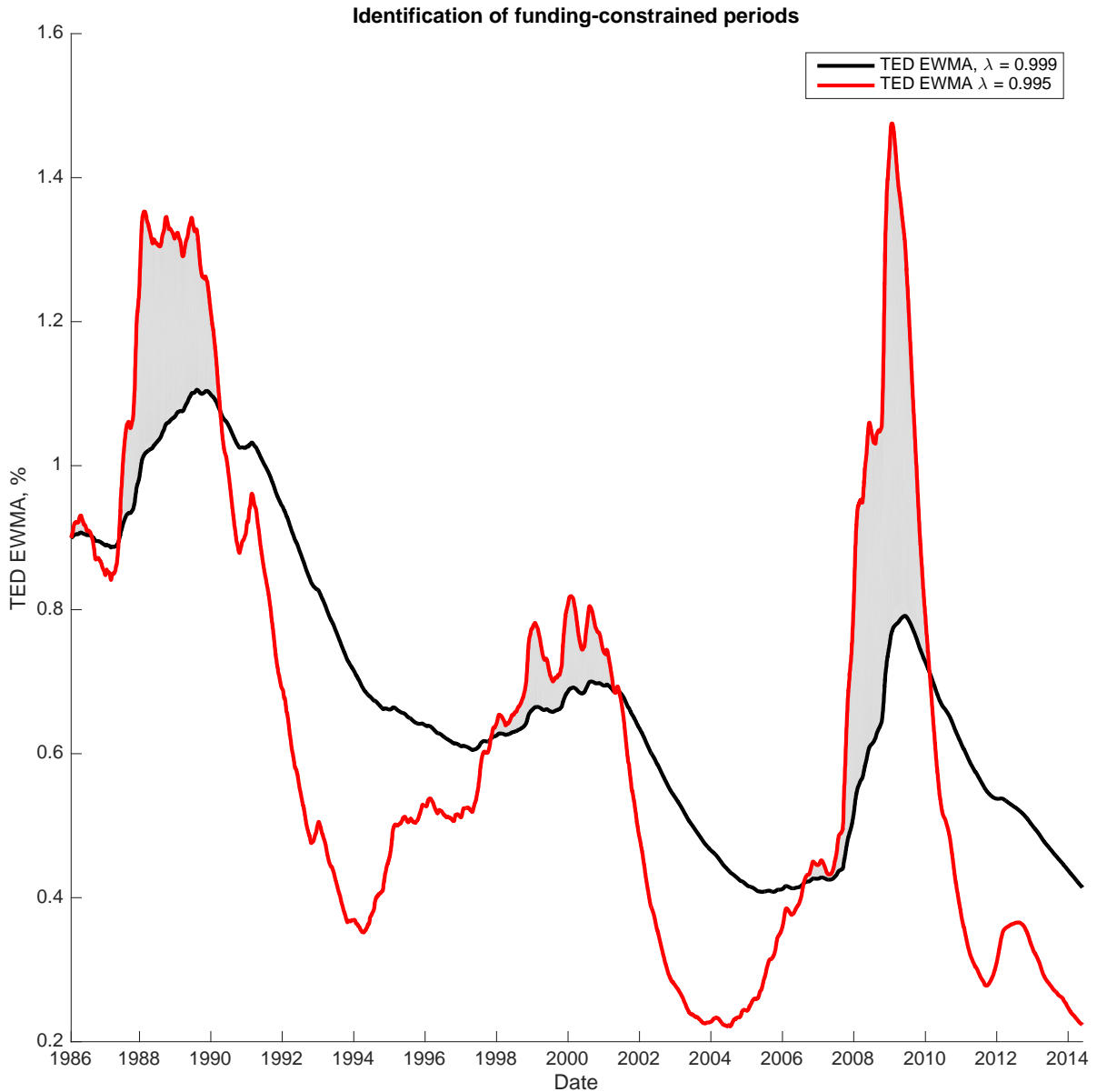
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