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"Refinance"

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Abstract

This paper provides a micro-foundation for refinance. The micro-foundation relates the optimality of refinanceable contracts to the financing relationship being governed by a preference for robustness. As an application, I consider a canonical, dynamic financial contracting setting with asymmetric information, where the borrower privately observes the state of the world. Preference for robustness is sufficient and, in some sense, almost necessary for the optimal contract to be refinanceable debt.

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Refinance

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November 2, 2015

Abstract

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1 Introduction

Most financing contracts contain, either implicitly or explicitly, the option to refinance. That is, the financial contract itself represents only a preliminary financing and repayment plan, maturing at some preliminary date T. However, at any interim date $t \leq T$, the borrower can *refinance*. This means that the borrower can rewrite the remaining terms of the financial contract, including, possibly, extending the maturity date. The only restriction is that the lender's "continuation payoff" must remain the same. In practice, the definition of continuation payoff may change depending on the situation, and additional restrictions may apply to how the borrower can refinance. However, roughly speaking, this is what refinance means. Well known examples of refinanceable contracts include mortgages with explicit refinance clauses and short-term corporate debt with implicit rollover options.

This paper seeks to provide some theoretical foundations for refinance by considering a benchmark dynamic financial contracting framework and showing that optimal long-term contracts can be implemented as refinanceable contracts. The key novelty of the paper is that it assumes that the lender is not Bayesian, but, rather, has a preference for robustness. This means that, 1) the lender has an incomplete model of the future repayment capabilities of the borrower, 2) the further into the future the lender looks, the more incomplete is the model, and 3) the lender is aware it has an incomplete model and seeks a contract that will do well no matter what the true model is. After deriving the refinance principle, I then apply it to a canonical contracting setting with asymmetric information, where the borrower privately observes the contract relevant state of the world. I show that assuming the lender has a preference for robustness is sufficient for the optimal contract to be refinanceable debt. I then argue that in this canonical setting, preference for robustness is also, in some sense, necessary for the optimal contract to be refinanceable debt.

It may come as a surprise to the reader that a micro-foundation for refinance doesn't already exist. After all, refinanceable contracts are ubiquitous and they appear in many applied theory papers. However, there is a good reason for this omission - refinanceable contracts are not very compatible with the standard Bayesian approach to optimal dynamic contracting. With a Bayesian decision maker, it is well known that optimal dynamic contracts generically exhibit a strong form of "completeness." Loosely speaking, there can be a strong disconnect between what is ex-ante optimal and what is ex-post optimal, so that, in general, everything must be determined beforehand. Moreover, distant payments depend in a fine way on what happens before and this dependence is tightly linked to the Bayesian decision maker's ex-ante beliefs. These generic qualities of optimal dynamic contracts in the Bayesian setting go against the spirit of refinance. In refinanceable contracts, things are not set in stone beforehand; distant payments - if they even occur - are there to provide flexibility; and when refinance occurs, the terms are not tightly governed by ex-ante conditions but, rather, are informed by the current state of the world.

To be fair, one can capture some of the spirit of refinance by focusing on renegotiation-proof optimal contracts. And the literature sometimes does this. Imposing the renegotiation-proof constraint means that the contract must always remain on the Pareto-frontier. However, at the end of the day, even renegotiation-proof optimal contracts exhibit "completeness" and simply do not resemble anything that might reasonably be interpreted as a refinanceable contract.

All of this suggests that the Bayesian assumption should be replaced. My paper can be seen as essentially arguing that the replacement should be preference for robustness. To get a sense of how preference for robustness is linked to refinance, suppose the lender has some idea about the borrower's ability to pay one date into the future but has no idea two or more dates into the future. Then from the perspective of the initial date 0 when the financing contract is signed, contingent repayments by the borrower at date 2 and beyond are worthless: The borrower can promise the world at date 2, but the lender simply has no confidence the borrower can deliver on such promises. Does this then mean that the financing contract should end at date 1? Actually, the answer is no. When today is date 0, date 2 is far away, but when today becomes date 1, date 2 is no longer far away. That is, the lender understands that even though right now it has no idea what the borrower can do at date 2, tomorrow things might change. This means that date 2 payments can actually be of value. Again, these distant payments cannot be directly valuable to the lender in the sense that adding such a payment will not increase the lender's ex-ante utility. However, they can provide the borrower financial flexibility and it means that in an optimal contract, distant payments must comprise "refinancings" of earlier obligations.

This paper is related to a number of recent papers looking at contracting under various forms of preference for robustness. One difference between this paper and Chung and Ely (2007), Frankel (2014), Garrett (2014), and Carroll (2015) is that I work in a dynamic setting and I'm interested in the implications of preference for robustness for how to structure contracts optimally along the time dimension. The max-min structure of the preference for robustness also relates this paper to the ambiguity aversion literature (e.g. Gilboa and Schmeidler, 1989).

This paper's application of the refinance principle to contracting with asymmetric information and its derivation of refinanceable debt relates it to a long line of work trying to explain the central role of debt as a method of securing financing. See Townsend (1979), Gale and Hellwig (1985), Innes (1990), DeMarzo-Duffie (1999), Yang (2015), and Hebert (2015). One way this paper contributes to the literature is that it not only explains debt but also the implied refinance option.

The intuition for the debt result is simple: The lender demands a flat repayment because - and here is where preference for robustness comes in - it simply does not know enough about the borrower and about what the borrower knows to be able to pull off a more delicate, clever repayment plan that is both more efficient and incentive-compatible. When the borrower can't deliver the flat repayment, then the lender demands everything so that the borrower won't be tempted to report that it can't repay when in fact it can.

The above intuition for debt is widely held and is basically a folk intuition. This paper is not the first to attempt to formalize it. The most well known previous example is the costly state verification (CSV) literature initiated by Townsend (1979) and Gale and Hellwig (1985). Indeed, I argue that there is a deep connection between the CSV approach to financial contracting under asymmetric information and this paper's approach despite the fact that this paper does not have costly state verification and those papers do not have preference for robustness.

The starting point is the Bayesian version of the asymmetric information model this paper uses to prove the optimality of refinanceable debt. Due to its canonical structure, and the fact that it has standard Bayesian decision makers, this setting or some enrichment of it would be what most people think of first when contemplating financial contracting under asymmetric information. However, in this setting, a combination of contracts being complete (i.e. contracts completely control both parties' consumption) and the parties being Bayesian leads to optimal contracts that are very state sensitive: Complete consumption control means the contract can be very state sensitive and Bayesian decision makes means that state sensitivity is valuable. Thus, if one wants a simpler optimal contract to emerge under asymmetric information, either complete consumption control or Bayesian decision makers has to go.

The connection between the CSV paper and this paper is that both want a simple optimal contract to emerge - debt - but they choose to discard different parts of the Bayesian asymmetric information model to get there. This paper chooses to get rid of the Bayesian assumption. The CSV papers chose to get rid of complete consumption control and have incomplete consumption control via costly-state-verification. Both routes lead to formalizations of the aforementioned folk intuition for debt. However, there are well known limitations of the route taken by the CSV literature - there must be bilateral risk-neutrality, the optimal contract is not renegotiation-proof, and random state verification must be disallowed. See Mookherjee and Png (1989) and Attar and Campioni (2003). In the preference for robustness, asymmetric information model considered in this paper, refinanceable debt is the optimal renegotiation-proof contract, the firm can be risk averse, and contracts can be random. In this sense, the preference for robustness approach provides a more robust theory of debt.

2 The Refinance Principle

In this section I derive the basic refinance principle in a benchmark, perfect information dynamic financial contracting setting with a bank and a firm. I first assume that the bank and the firm are both Bayesian. Optimal dynamic contracts do not exhibit any salient structures and cannot be interpreted as a refinanceable contracts. I then change the model by assuming the bank has a preference for robustness and show that optimal contracts are refinanceable contracts.

2.1 Bayesian, Perfect Information Model

The model contains three dates $t = 0, 1, 2$ and two players - a bank and a firm. I now describe the model starting from the end.

On date 2, the firm realizes a value $v_2 \geq 0$. Then the bank and the firm consume p_2 and c_2 respectively. The firm is protected by limited liability, so $c_2 \geq 0$. The bank has no such constraint. The budget constraint requires $p_2 + c_2 \leq v_2$. After consumption, the model ends.

The firm's date 2 value v_2 is realized according to a distribution that depends on the capital invested at date 1. Specifically, at date 1, the firm first realizes a value v_1. Then, the bank and the firm consume p_1 and c_1 subject to the same limited liability and budget

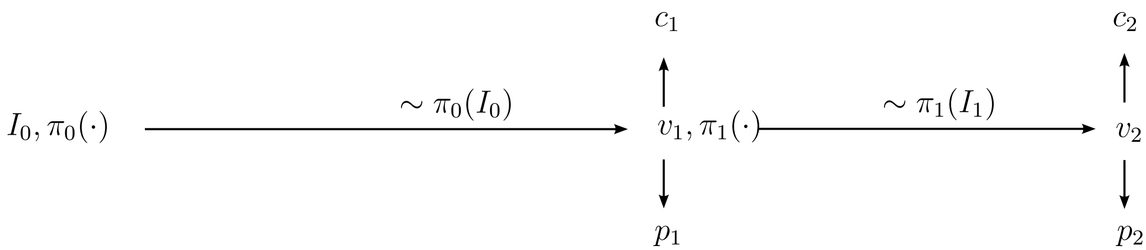

Figure 1

constraints as in date 2. Note, since p_1 can be negative, so can the quantity $p_1 + c_1$. A negative $p_1 + c_1$ means that there is net investment at date 1, with the bank injecting more capital into the firm. Let $I_1 := -p_1 - c_1$ denote the date 1 investment amount. How much investment I_1 occurs determines what kind of probability distribution over v_2 one can expect. Formally, there is a belief function $\pi_1(\cdot)$ with domain $[-v_1, \infty)$ that maps date 1 investment I_1 to full-support probability distributions over the firm's date 2 value v_2 with finite expectation.

The date 1 *state of the world* s_1 is a realization of $v_1 \geq 0$ and a realization of the belief function $\pi_1(\cdot)$. $\pi_1(\cdot)$ can be any function with the property that higher investment does not lead to a strictly first-order stochastic dominated distribution. The probability distribution of s_1 is $\pi_0(I_0)$ where $I_0 \geq 0$ is the investment made by the bank at date 0 and $\pi_0(\cdot)$ is a belief function mapping date 0 investment to full-support probability distributions over s_1 with finite expected v_1. See Figure 1 for a graphical representation of the model.

At date 0, the firm and the bank sign a long-term contract spanning the entire model. A contract stipulates a financing and repayment plan $\{I_0, p_1, c_1, p_2, c_2\}$. Here, p_1 and c_1 can depend on the date 1 state of the world s_1, and p_2 and c_2 can depend on v_2 as well as a s_1. All terms of the contract are allowed to be random.

Given $\pi_0(\cdot)$ and some bank outside option function $OC(\cdot)$, the optimal contracting problem is

$$\begin{aligned} \max_{\{I_0,p_1,c_1,p_2,c_2\}} \quad & \mathbf{E}_{\pi_0(I_0)}\left[u(c_1) + \mathbf{E}_{\pi_1(I_1)} u(c_2)\right] \\ s.t. \quad & \mathbf{E}_{\pi_0(I_0)}\left[p_1 + \mathbf{E}_{\pi_1(I_1)} p_2\right] \geq OC(I_0) \end{aligned}$$

where $u(\cdot)$ is a weakly concave function.

The optimal contract is exceedingly simple. Risk-sharing implies that the firm consumes a constant $c_1 = c_2 = c$ and the bank becomes the full residual claimant. There are three things worth noting here. The contract cannot be interpreted as a refinanceable contract; the contract is not a debt contract; and the simple structure is not robust - if both the bank and the firm had risk-averse preferences, then risk-sharing would imply fully state sensitive consumption for both parties. Overall, there is really not much to say about optimal financial contracting in the Bayesian, perfect information model.

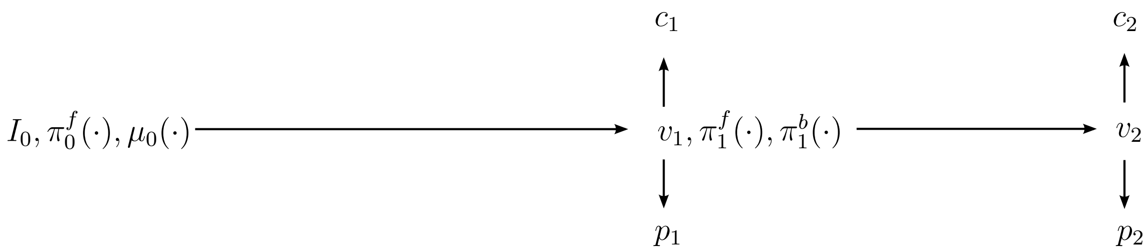

Figure 2

2.2 Preference for Robustness, Perfect Information Model

I now replace the Bayesian bank with a bank that has a preference for robustness. The model is changed in the following way:

First, nothing is different at date 2. However, at date 1, the bank and the firm may have differing belief functions, so that given investment $I_1 = -p_1 - c_1$, the firm believes that v_2 will be distributed according to some $\pi_1^f(I_1)$ and the bank believes that v_2 will be distributed according to some $\pi_1^b(I_1)$. At date 1, a state of the world $s_1 := (v_1, \pi_1^f(\cdot), \pi_1^b(\cdot))$ consists of a realization of firm value $v_1 \geq 0$ along with the realized subjective belief functions for the bank and the firm.

At date 0, the firm has a belief function $\pi_0^f(\cdot)$, so that, given date 0 investment I_0, the firm believes s_1 is distributed according to some full-support $\pi_0^f(I_0)$. Unlike the firm, the bank, given I_0, does not have a single belief about s_1. Rather, the bank entertains numerous beliefs similar to an ambiguity-averse decision maker. The set of beliefs the bank entertains is defined as follows. The bank has a *confidence belief* function $\mu_0(\cdot)$. Given I_0, the confidence belief $\mu_0(I_0)$ is a full-support probability distribution over v_1, not s_1. The confidence belief $\mu_0(I_0)$ represents a lower bound distribution for the firm's date 1 value that the bank is confident the firm can achieve. As a result, the bank is willing to entertain any belief π_0 about the distribution over the date 1 state of the world so along as the implied distribution over firm value weakly first-order-stochastic-dominates its confidence belief $\mu_0(I_0)$. See Figure 2.

A contract is exactly the same object as before: $\{I_0, p_1, c_1, p_2, c_2\}$. The only difference is how the firm and the bank evaluate contracts. Given a belief function $\pi_0^f(\cdot)$, confidence belief function $\mu_0(\cdot)$ and some bank outside option function $OC(I_0)$, the optimal contracting problem is,

$$\max_{\{I_0, p_1, c_1, p_2, c_2\}} \mathbf{E}_{\pi_0^f(I_0)} \left[u(c_1) + \mathbf{E}_{\pi_1^f(I_1)} u(c_2) \right]$$

$$s.t. \quad \min_{\{\pi_0 \ | \ \pi_0|_{v_1} \geq \mu_0(I_0)\}} \mathbf{E}_{\pi_0} \left[p_1 + \mathbf{E}_{\pi_1^b(I_1)} p_2 \right] \geq OC(I_0)$$

Notice, the bank evaluates contracts based on their worst-case performance where the scenarios the bank is willing to consider are determined by its confidence belief $\mu_0(I_0)$.

For each date 1 state of the world s_1, define $B_1(s_1) := \mathbf{E}[p_1(s_1) + \mathbf{E}_{\pi_1^b(I_1)} p_2(s_1, \cdot)]$ to be the bank's date 1 continuation payoff. The outer expectation captures the fact that given state s_1, $p_1(s_1)$ may still be random.

Lemma 1. *If* $\{I_0, p_1, c_1, p_2, c_2\}$ *is an optimal contract, then* $B_1(s_1) = B_1(v_1)$.

Proof. Pick two date 1 states of the world s_1' and s_1'' that share the same v_1. Suppose $B_1(s_1') > B_1(s_1'')$. Then consider the alternate contract that changes the continuation contract - $\{p_1(s_1'), c_1(s_1'), p_2(s_1', \cdot), c_2(s_1', \cdot)\}$ - following s_1' into the one that maximizes the firm's continuation payoff subject to delivering continuation payoff $B_1(s_1'')$ to the bank. Given the bank's preference for robustness, it is indifferent between the two contracts. However, the firm is strictly better off. □

Lemma 1 is the formal statement of the refinance principle under perfect information. It implies that an optimal contract admits a natural implementation as a refinanceable contract.

In the implementation, there is an *initial contract* $\{I_0, B_1(\cdot)\}$ that specifies only how much the bank invests at date 0 and how much the firm has to repay at date 1 where the repayment schedule $B_1(\cdot)$ depends only on the firm's value v_1. The initial contract also grants the firm the *option to refinance.* This means that at date 1, the firm can change the realized repayment obligation $B_1(v_1)$ into another renegotiated contract $\{p_1(s_1), p_2(s_1, \cdot)\}$ so long as the bank receives a total expected payoff of at least $B_1(v_1)$. It is worth emphasizing that $p_1(s_1)$ may be negative, so that the renegotiated contract can incorporate situations where the obligation is rolled over or the bank injects even more capital into the firm.

The contrast between an optimal complete contract and its implementation as a refinanceable contract highlights the fact that refinanceable contracts are essentially incomplete contracts. The initial contract only specifies an obligation to the bank but leaves out the details of how the obligation should be fulfilled. The firm is granted control rights over how to specify those details in the future. Moreover, neither the initial contract nor the renegotiated contract specify the firm's consumption schedule, only what the bank is owed. Thus, Lemma 1 can also be interpreted as providing solid theoretical foundations for a major class of incomplete contracts.

3 Financing under Asymmetric Information

As an application of the refinance principle and the ideas introduced in the previous section, I now replace the perfect information assumption in the financial contracting model considered in the previous section with a canonical asymmetric information problem. I show that the refinance principle continues to apply and that the optimal contract is debt that can be refinanced. I connect this work with the costly state verification literature and show how my result can be thought of as a robust justification of debt.

3.1 Preference for Robustness, Asymmetric Information Model

The model is based on the preference for robustness model considered in Section 2.2, except I now assume that the firm first privately observes the state of the world at date 1 and can strategically reveal information about the state of the world to the bank. Specifically, after the firm privately observes $s_1 := (v_1, \pi_1^f(\cdot), \pi_1^b(\cdot))$, it publicly and credibly reports that it has value $\hat{v}_1 \in [0, v_1]$ and reports a message m_1 regarding its own belief function $\pi_1^f(\cdot)$. Here the

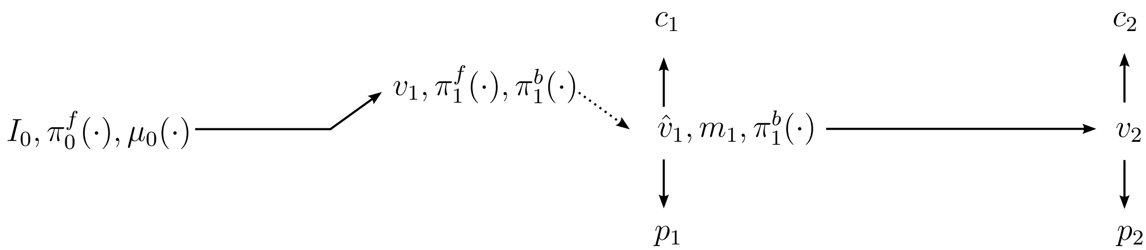

Figure 3

reported firm belief function must have a domain that matches the reported $\hat{v}_1$. I assume that a report of $\hat{v}_1$ leads to a publicly observable bank belief function which is the true belief function $\pi_1^b(\cdot)$ except restricted to the domain of $[-\hat{v}_1, \infty)$. As an abuse of notation, I will say the bank has belief function $\pi_1^b(\cdot)$. Thus, while the firm knows the true state s_1, the bank observes the reported state of the world $\hat{s}_1 = (\hat{v}_1, m_1, \pi_1^b(\cdot))$ which is also the state the contract is written over. For simplicity, I continue to assume that v_2 is publicly observable. See Figure 3.

A contract stipulates $\{I_0, p_1, c_1, p_2, c_2, \hat{v}_1, m_1\}$ where p_1 and c_1 depend on the reported date 1 state of the world $\hat{s}_1$, p_2 and c_2 depend on $\hat{s}_1$ and v_2, and the report strategy $(\hat{v}_1, m_1)$ depends on the privately observed true state of the world s_1. Given true state s_1 and reported state $\hat{s}_1$, the date 1 investment is $I_1(\hat{s}_1) = -p_1(\hat{s}_1) - c_1(\hat{s}_1)$ and the firm and the bank have beliefs $\pi_1^f(I(\hat{s}_1))$ and $\pi_1^b(I(\hat{s}_1))$ about v_2. The budget constraint at date 1 is $p_1 + c_1 \leq \hat{v}_1$. I focus on truth-telling contracts. In a truth-telling contract, the firm truthfully reports $\hat{v}_1 = v_1$ and $m_1 = \pi_1^f(\cdot)$. Given a true state of the world s_1, for every possible reported state of the world $\hat{s}_1$, the continuation payoff of the firm following $\hat{s}_1$ is $\mathbf{E}[u(c_1(\hat{s}_1)) + \mathbf{E}_{\pi_1^f(I(\hat{s}_1))} u(c_2(\hat{s}_1, \cdot))]$. Here, just like in the definition of the bank's continuation payoff in the previous section, the outer expectation captures the fact that after the reported state is realized, $c_1(\hat{s}_1)$ may still be random.

Definition 1. *A truth-telling contract is incentive-compatible if truth-telling maximizes the firm's continuation payoff given any realization of the date 1 state of the world.*

From now on, all contracts are assumed to be incentive-compatible truth-telling contracts.

Fix a contract and any state $s_1 = (v_1, \pi_1^f(\cdot), \pi_1^b(\cdot))$. Consider a (possibly off-equilibrium) revealed state $\hat{s}_1 = (\hat{v}_1, \hat{m}_1, \pi_1^b(\cdot))$. The continuation contract given $\hat{s}_1$ is $\{p_1(\hat{s}_1), c_1(\hat{s}_1), p_2(\hat{s}_1, \cdot), c_2(\hat{s}_1 \cdot)\}$. A *renegotiation* of this continuation contract is some alternate continuation contract $\{p_1', c_1', p_2'(\cdot), c_2'(\cdot)\}$. The budget constraint at date 1 is still $p_1' + c_1' \leq \hat{v}_1$.

Under the renegotiation, the continuation payoffs of the firm and the bank

$$\mathbf{E}[u(c_1') + \mathbf{E}_{\pi_1^f(-p_1'-c_1')} u(c_2'(\cdot))] \qquad \mathbf{E}[p_1' + \mathbf{E}_{\pi_1^b(-p_1'-c_1')} p_2'(\cdot)]$$

The firm is strictly better off under the renegotiation if

$$\mathbf{E}[u(c_1') + \mathbf{E}_{\pi_1^f(-p_1'-c_1')} u(c_2'(\cdot))] > \mathbf{E}[u(c_1(s_1)) + \mathbf{E}_{\pi_1^f(I_1(s_1))} u(c_2(s_1, \cdot))]$$

The bank *thinks* it is strictly better off under the renegotiation if

$$\mathbf{E}[p_1' + \mathbf{E}_{\pi_1^b(-p_1'-c_1')} p_2'(\cdot)] > \mathbf{E}[p_1(\hat{s}_1) + \mathbf{E}_{\pi_1^b(I_1(\hat{s}_1))} p_2(\hat{s}_1, \cdot)]$$

Definition 2. *A contract is renegotiation-proof if there does not exist a state s_1 and a renegotiation of the continuation contract $\{p_1(\hat{s}_1), c_1(\hat{s}_1), p_2(\hat{s}_1, \cdot), c_2(\hat{s}_1 \cdot)\}$ following some reported state $\hat{s}_1$ that makes the firm strictly better off and makes the bank think it is strictly better off.*

There is a natural equivalence between renegotiation-proof contracts and a subset $\mathscr{C}$ of the set of contracts in the perfect information setting satisfying incentive-constraints implied by Definitions 1 and 2. Thus, the optimal contracting problem can be stated as a perfect information contracting problem where the contract space is constrained to be $\mathscr{C}$. Given $\pi_0^f(\cdot)$, $\mu_0(\cdot)$ and $OC(\cdot)$, the optimal contracting problem is

$$\max_{\{I_0, p_1, c_1, p_2, c_2\} \in \mathscr{C}} \mathbf{E}_{\pi_0^f(I_0)} \left[u(c_1) + \mathbf{E}_{\pi_1^f(I_1)} u(c_2) \right]$$
$$s.t. \quad \min_{\{\pi_0 | \ \pi_0|_{v_1} \geq \mu_0(I_0)\}} \mathbf{E}_{\pi_0} \left[p_1 + \mathbf{E}_{\pi_1^b(I_1)} p_2 \right] \geq OC(I_0)$$

For every state of the world $s_1 = (v_1, \pi_1^f(\cdot), \pi_1^b(\cdot))$, define the bank's maximal continuation payoff to be:

$$\overline{B}_1(s_1) = \overline{B}_1(v_1, \pi_1^b(\cdot)) := \max_{\{p_1', c_1', p_2'(\cdot), c_2'(\cdot)\}} \mathbf{E}[p_1' + \mathbf{E}_{\pi_1^b(-p_1'-c_1')} p_2'(\cdot)]$$

Definition 3. *A refinanceable debt contract has the property that $B_1(s_1) = \overline{B}_1(s_1) \wedge F_1$ for some constant F_1. Specifically, a contract is called refinanceable debt if there exists an F_1 such that, for every state of the world s_1, if $\overline{B}_1(s_1) = \overline{B}_1(v_1, \pi_1^b(\cdot)) > F_1$, then*

$$\{p_1(s_1), c_1(s_1), p_2(s_1, \cdot), c_2(s_1, \cdot)\} = \underset{\{p_1', c_1', p_2'(\cdot), c_2'(\cdot)\}}{\arg\max} \mathbf{E}[u(c_1') + \mathbf{E}_{\pi_1^f(-p_1'-c_1')} u(c_2'(\cdot))]$$
$$s.t. \quad \mathbf{E}[p_1' + \mathbf{E}_{\pi_1^b(-p_1'-c_1')} p_2'(\cdot)] \geq F_1$$

and if $\overline{B}_1(s_1) = \overline{B}_1(v_1, \pi_1^b(\cdot)) \leq F_1$, then

$$\{p_1(s_1), c_1(s_1), p_2(s_1, \cdot), c_2(s_1, \cdot)\} = \underset{\{p_1', c_1', p_2'(\cdot), c_2'(\cdot)\}}{\arg\max} \mathbf{E}[p_1' + \mathbf{E}_{\pi_1^b(-p_1'-c_1')} p_2'(\cdot)]$$

It is straightforward to verify that refinanceable debt contracts are members of $\mathscr{C}$.

Theorem 1. *If a truth-telling, renegotiation-proof contract is optimal, then it is refinanceable debt.*

Proof. See appendix. □

To get an intuition for the result, consider an alternative option, say, one where $B_1(s_1) = v_1$. Depending on the realization of $(\pi_1^f(\cdot), \pi_1^b(\cdot))$, the optimal way to deliver continuation payoff v_1 to the bank may not involve setting $p_1 = v_1$. For example, if both parties believe that the firm is going to be sufficiently productive at date 2, the optimal thing to do may be for both parties to consume zero at date 1 and only consume at date 2. Suppose this is the case. Let $s_1' = (v_1', \pi_1^{f\prime}(\cdot), \pi_1^{b\prime}(\cdot))$ be such a state and let $p_2(s_1', \cdot)$ be the bank's payment at date 2. By assumption then, $\mathbf{E}_{\pi_1^{b\prime}(0)} p_2(s_1', v_2) = v_1'$. But now consider an alternate state $s_1'' = (v_1'', \pi_1^{f\prime\prime}(\cdot), \pi_1^{b\prime\prime}(\cdot))$ where $v_1'' < v_1'$, $\pi_1^{f\prime\prime} = \pi_1^{f\prime}|_{[-v_1'',\infty)}$, and $\pi_1^{b\prime\prime} = \pi_1^{b\prime}|_{[-v_1'',\infty)}$.

If state s_1' is realized, then, in fact, the firm is strictly better off misreporting that the state is s_1''. To see why, note that reporting s_1'' means that the firm only needs to deliver continuation payoff v_1'' to the bank. This fact, by itself, does not imply the firm is better off reporting s_1'' because by reporting that the firm has lower value $v_1'' < v_1'$, the budget constraint for $c_1 + p_1$ is further tightened. This may be unattractive for the firm if it wants to consume a lot at date 1. But notice, in our situation, this tightening of the budget constraint at date 1 is of no concern: If the firm had reported the truth, they would not have wanted to consume anything at date 1 anyways. Thus, by misreporting the state as s_1'', the firm can guarantee that it will be better off, by, say renegotiating the continuation contract so that $c_1 = p_1 = 0$ and choosing a $p_2(s_1'', \cdot)$ that is strictly less than $p_2(s_1', \cdot)$ for every value of v_2. That the latter is possible is because, by misreporting, the firm is only obligated to choose a $p_2(s_1'', \cdot)$ that delivers continuation payoff $v_1'' < v_1'$. The lesson learned by studying state s' is:

Remark. *Trying to tie the bank's payoff to how valuable the firm is may backfire.*

Of course, this entire analysis was based on examining a state with a particular realization of belief functions that makes the firm tempted to misreport. There may be other realizations of belief functions such that when paired with the same firm value, would make the firm unwilling to misreport to a state with lower firm value. However, recall the bank has preference for robustness. So while the worst case outcome of misreporting may not be always happen, it does matter at the margin.

The proof of Theorem 1 is basically a more fleshed-out version of the above arguments.

3.2 Bayesian, Asymmetric Information Model

The lessons learned from the previous subsection evoke the following intuition for (refinanceable) debt: The bank demands a flat obligation from the firm - it doesn't care how the firm fulfills the obligation, just as long as the obligation is fulfilled. The bank acts this way because, frankly, it doesn't know enough about the firm to be able to pull off a more clever, state-dependent repayment scheme that is both more efficient and also incentive-compatible. If, however, the firm is unable to fulfill its flat obligation, then the bank demands everything so as to discourage the firm from claiming it can't pay up when in fact it can.

This intuition for debt is of course not novel. It is, more or less, the folk intuition for debt. The challenge of explaining debt has never been about coming up with the correct intuition, but, rather, coming up with the right formalization of the folk intuition that everyone already knows is correct. The application, in Section 3.1, of the refinance principle to contracting under asymmetric information can be seen as an attempt to take on this challenge.

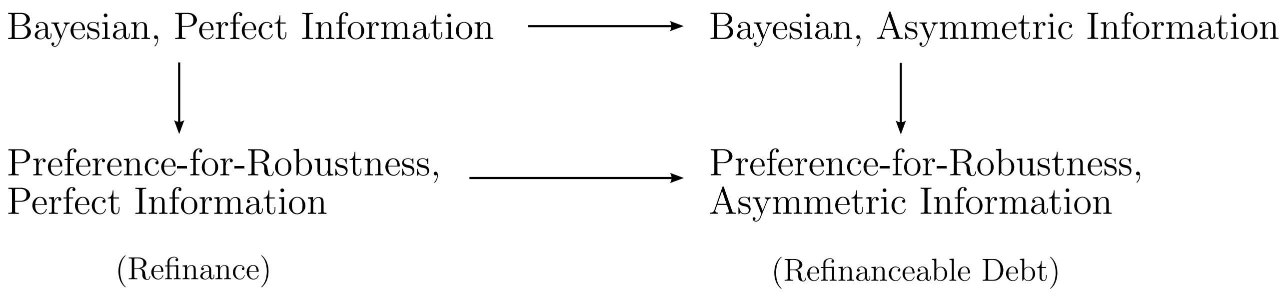

Figure 4

The financial contracting setting I considered in Section 3.1 is standard and general, and the asymmetric information problem I introduce is canonical. Theorem 1 then shows that in this standard setting, all one needs to do is add preference for robustness, and the optimality of (refinanceable) debt results. What I would like to do now is to argue that preference for robustness is not only sufficient but is, in some sense, also necessary for the optimal contract to be refinanceable debt.

Let us start by recalling the sequence of models considered that finally led to refinanceable debt. I began with the benchmark Bayesian, perfect information model. Solving the optimal contracting problem did not lead to any meaningful insights. I then moved to the preference for robustness, perfect information model. This led to the refinance principle. Finally, I considered the preference for robustness, asymmetric information model, which led to refinanceable debt. See Figure 4 for a graphical summary. The diagram in Figure 4 implies there is one more model that I have, thus far, overlooked - the Bayesian, asymmetric information model. Moreover, the diagram suggests that the Bayesian, asymmetric information model might be able to formalize the folk intuition for debt (without refinance). This, if true, would of course weaken my argument that preference for robustness is, in some sense, necessary for the optimality of debt.

I will now argue that this suggestion *is*, to some extent, true, and that moreover, one can view the costly state verification (CSV) approach to financial contracting as a manifestation of this suggestion. However, I will also argue that the link between the Bayesian, asymmetric information model and the optimality of debt is not robust. Ultimately, I will conclude that the right way to explain debt is to actually explain debt and its implied refinance option together, and that the only way to do this is through the preference for robustness, asymmetric information model.

Formally, in the Bayesian, asymmetric information model, at date 1 the firm first privately observes the state of the world at date 1, which is now $s_1 = (v_1, \pi_1(\cdot))$. It then reveals $\hat{v}_1$ and the bank has belief $\pi_1(\cdot)$ restricted to the domain $[-\hat{v}_1, \infty)$. At date 0, the bank and the firm share a belief function $\pi_0(\cdot)$ taking I_0 to distributions over the date 1 state of the world instead of the bank having a confidence belief function taking I_0 to distributions over the date 1 value of the firm. Once again, the set of incentive-compatible truth-telling, renegotiation-proof contracts can be identified with a certain subset $\mathscr{C}$ of perfect information contracts. Given $\pi_0(\cdot)$ and $OC(I_0)$, the optimal contracting problem is

$$\max_{\{I_0,p_1,c_1,p_2,c_2\}\in\mathscr{C}} \mathbf{E}_{\pi_0(I_0)}\left[u(c_1)+\mathbf{E}_{\pi_1(I_1)}u(c_2)\right]$$
$$s.t.\ \mathbf{E}_{\pi_0(I_0)}\left[p_1+\mathbf{E}_{\pi_1(I_1)}p_2\right]\geq OC(I_0)$$

First, consider the risk neutral case where $u(c_i)=c_i$. Risk-neutrality means that there is a multiplicity of optimal contracts in the Bayesian, perfect information world. Many of these first-best contracts lie in $\mathscr{C}$. Therefore, first-best can be achieved in the Bayesian, asymmetric information model. However, the model lacks predictive power since there is a multitude of very different looking optimal, renegotiation-proof, truth-telling contracts. For example, one optimal contract has the property that $p_i/(c_i+p_i)=\alpha$ for a constant α and can be interpreted as an equity contract. Another optimal contract has the property that $p_1=0$, $p_2=v_2\wedge F$ for some constant F and can be interpreted as debt. And yet others have more delicate structures and lack any real-life interpretations.

In order to narrow the set of optimal contracts, one could re-introduce risk-aversion. However, in this case, little can be said about the structure of the optimal contract just like in the Bayesian, perfect information case. In particular, the optimal contract will have a delicate structure, exhibiting full state-dependence and will generally not look like debt or any other recognizable simple contract.

This complexity problem comes from a combination of the contract being able to control everyone's consumption (i.e. complete contracts) and everyone being Bayesian. Complete consumption control means that the contract can be very fine-tuned. Being Bayesian means that fine-tuning contracts is valuable. Thus, if the goal is to get a relatively simple optimal contract, either complete consumption control or the Bayesian assumption must go.

The approach taken by the CSV literature is to essentially get rid of the complete consumption control assumption. That is to say, a unified way to view this paper and the CSV papers is to realize that both encounter the same complexity problem and both are trying to arrive at a simple optimal contract - debt. The difference is that the two take different routes to get there. This paper chooses to get rid of the Bayesian assumption. The CSV papers choose to get rid of complete consumption control.

The CSV literature assumes that the contract can only control the bank's consumption but gives the bank the ability to make costly state verifications. Costly state verification represents an incomplete way for the contract to control the firm's consumption. When the bank verifies the state, it can indirectly control how much the firm consumes by consuming whatever it does not want the firm to consume. However, the bank cannot do this when it does not verify the state because then it is constrained to only consume a portion of what is revealed by the firm. Under this incomplete consumption control plus Bayesian decision makers approach, the CSV literature shows that debt is the optimal contract but only if both players are risk neutral, there is no renegotiation-proof constraint, random state verifications are disallowed, and the model is static. These restrictions have been shown to be quite serious.

In contrast, this paper shows that by replacing the Bayesian decision maker with one who has a preference for robustness, not only can debt be explained, but so can the implicit refinance option. Moreover, refinanceable debt is renegotiation-proof, and it is optimal even allowing for random contracts and non risk-neutral preferences.

4 Conclusion

This paper provided a robust micro-foundation for refinanceable contracts based on assuming that the bank has a preference for robustness. I considered two versions of a standard dynamic financial contracting setting - a perfect information benchmark and a canonical asymmetric information version where the firm privately observes the contract relevant state of the world. I first showed that optimal contracts in the perfect information model can be implemented as refinanceable contracts. I argued that refinanceable contracts admit a natural interpretation as incomplete contracts with delegation of control rights and details left to be negotiated later. Thus, the optimality of refinance result provides solid theoretical foundations for an important class of incomplete contracts in the financing setting.

I then moved to the asymmetric information setting and showed that the refinance principle continues to hold and that the optimal contract is refinanceable debt. The result represents a robust formalization of a widely accepted folk intuition.

5 Appendix

Proof of Theorem 1. I begin with a characterization of contracts in $\mathscr{C}$ before focusing on the optimal ones.

Claim: If a contract is in $\mathscr{C}$, then $B_1(s_1) = B_1(v_1, \pi_1^b(\cdot))$.

Suppose the claim is false. Then consider a let $s_1' = (v_1, \pi_1^{f\prime}(\cdot), \pi_1^b(\cdot))$ and $s_1'' = (v_1, \pi_1^{f\prime\prime}(\cdot), \pi_1^b(\cdot))$ be two states with $B_1(s_1') > B_1(s_1'')$. Then, the firm can always report v_1 truthfully but send message $\hat{m}_1 = \pi_1^{f\prime\prime}(\cdot)$ and then renegotiate the continuation contract into the optimal one for the firm subject to delivering $B_1(s_1'') + \varepsilon$ to the bank. If $\varepsilon > 0$ is sufficiently small, then both the firm and the bank are strictly better off. This violates the assumption that the contract is renegotiation-proof and the claim is proved.

Now, fix a contract in $\mathscr{C}$. Consider a pair $(v_1', \pi_1^{b\prime}(\cdot))$ and $(v_1'', \pi_1^{b\prime\prime}(\cdot))$ such that $v_1' > v_1''$ and $\pi_1^{b\prime}(\cdot) \equiv \pi_1^{b\prime\prime}(\cdot)$ on the domain $[-v_1'', \infty)$.

Claim: Suppose $B_1(v_1'', \pi_1^{b\prime\prime}(\cdot)) < \overline{B}_1(v_1'', \pi_1^{b\prime\prime}(\cdot))$. Then $B_1(v_1', \pi_1^{b\prime}(\cdot)) \leq B_1(v_1'', \pi_1^{b\prime\prime}(\cdot))$.

Suppose not. There exists a belief function $\pi_1^{f\prime}(\cdot)$ such that the continuation contract following state $s_1' = (v_1', \pi_1^{b\prime}(\cdot), \pi_1^{f\prime}(\cdot))$ involves date investment $I_1 \geq -v_1''$. In this case, I show that the firm is strictly better misreporting the state. Suppose instead the firm reported $v_1'' < v_1'$. Then by assumption, the bank would only demand continuation payoff $B_1(v_1'', \pi_1^{b\prime\prime}(\cdot)) < B_1(v_1', \pi_1^{b\prime}(\cdot))$. One continuation contract that achieves this is the following: The continuation contract keeps p_1 and c_1 the same. This is possible since by assumption $I_1 \geq -v_1''$. This leads to the same date 1 investment which generates the same distribution over v_2. Since the continuation contract needs to deliver a smaller continuation payoff to the bank, it can design p_2 in a way so that, for every single realization of v_2, it is strictly smaller than the $p_2(s_1', \cdot)$ that would've been enacted had the firm told the truth. In this new continuation contract, the firm's date 1 consumption is unchanged, but its date 2 consumption strictly increases, so the firm is strictly better off. This contradicts the truth-telling, renegotiation-proof assumption.

Lastly, fix an optimal contract and define the following constant:

$$F_1 := \inf_{\{(v_1, \pi_1^b(\cdot)) \ \mid \ B_1(v_1, \pi_1^b(\cdot)) < \overline{B}_1(v_1, \pi_1^b(\cdot))\}} B_1(v_1, \pi_1^b(\cdot))$$

Claim: The bank weakly prefers the refinanceable debt contract with face value F_1 to the optimal contract.

Fix an $\varepsilon > 0$ and pick a $(v_1'', \pi_1^{b\prime\prime}(\cdot))$ such that $B_1(v_1'', \pi_1^{b\prime\prime}(\cdot)) < \overline{B}_1(v_1'', \pi_1^{b\prime\prime}(\cdot)) \wedge (F_1 + \varepsilon)$. The previous claim implies that for every $v_1' > v_1''$, there exist states s_1' with firm value v_1' such that $B_1(s_1') \leq F_1 + \varepsilon$.

Now, let $\tilde{s}_1 = (\tilde{v}_1, \tilde{\pi}_1^f(\cdot), \tilde{\pi}_1^b(\cdot))$ denote any state such that $B_1(\tilde{s}_1) > F_1 + \varepsilon$ and let $\tilde{S}$ denote the set of all such states. Then for any belief π_0 satisfying $\pi_0|_{v_1} \geq \mu_0(I_0)$, there is another belief π_0' also satisfying $\pi_0'|_{v_1} \geq \mu_0(I_0)$ that is identical to π_0 except that for each state $\tilde{s}_1 \in \tilde{S}$, the weight on $\tilde{s}_1$ is now moved to another state s_1' with firm value $v_1' \geq \tilde{v}_1$ and $B_1(s_1') \leq F_1 + \varepsilon$. That this is possible comes from the first paragraph of the proof for this claim.

The expected payoff of the debt contract to the bank is the same under both π_0 and π_0'. The expected payoff of the optimal contract to the bank weakly decreases going from from π_0 to π_0'. For the bank, the expected payoff of the optimal contract is weakly smaller than the expected payoff of the debt contract under π_0'. Thus, the bank weakly prefers the refinanceable debt contract with face value $F_1 + \varepsilon$ to the optimal contract. Finally, since ε was arbitrary, the claim is proved.

Claim: Unless the optimal contract is refinanceable debt with face value F_1, the firm strictly prefers the refinanceable debt contract.

Pick a $(v_1, \pi_1(\cdot))$ such that $B_1(v_1, \pi_1(\cdot)) < \overline{B}_1(v_1, \pi_1(\cdot))$. By definition of F_1, $B_1(v_1, \pi_1(\cdot)) \geq F_1$. If $B_1(v_1, \pi_1(\cdot)) > F_1$, then the bank is strictly better off moving from the optimal contract to the refinanceable debt contract with face value F_1. Contradiction. If for every such $(v_1, \pi_1(\cdot))$, $B_1(v_1, \pi_1(\cdot)) = F_1$, then the optimal contract is refinanceable debt with face value F_1. □

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