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Constrained deformations of positive scalar curvature metrics

What manifolds support metrics of positive scalar curvature? What can one say about the associated moduli space, when not empty? These are two fundamental problems in Riemannian Geometry, for which great progress has been made over the last fifty years, but that are nevertheless highly elusive and far from being fully resolved.
Partly motivated by the study of initial data sets for the Einstein equations in General Relativity, I will present some results that aim at moving one step further, studying the interplay between two different curvature conditions, given by pointwise conditions on the scalar curvature of a manifold and the mean curvature of its boundary.
In particular, after a broad contextualization, I will focus on recent joint work with Chao Li (Princeton University), where we give a complete topological characterization of those compact 3-manifolds that support Riemannian metrics of positive scalar curvature and mean-convex boundary and, in any such case, we prove that the associated moduli space of metrics is path-connected. We can also refine our methods so to construct continuous paths of non-negative scalar curvature metrics with *minimal* boundary, and to obtain analogous conclusions in that context as well. In particular, we thereby derive the path-connectedness of asymptotically flat scalar flat Riemannian 3-manifolds with minimal boundary, which goes in the direction of investigating (from a global perspective) the space of vacuum black-hole solutions to the Einstein field equations.
Our work relies on a combination of earlier fundamental contributions by Gromov-Lawson and Schoen-Yau, on the smoothing procedure designed by Miao to handle singular interfaces, and on the interplay of Perelman's Ricci flow with surgery and conformal deformation techniques introduced by Codá Marques in dealing with closed manifolds.