No News is News: Volatility Speculation and Multidimensional Heterogeneous Beliefs

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Abstract

This paper develops a theoretical model to explore the asset pricing implications of investors' multi-dimensional belief heterogeneity, specifically distinguishing between disagreements over the frequency of news arrival and the content of news. Besides directional trades, investors could use derivatives to bet against each other and speculate on volatility: greater disagreement of this kind could give rise to more extreme derivative positions. When disagreement about news arrival frequency is low, volatility exhibits mean reversion because extreme optimists and pessimists incur substantial wealth losses amid intense market swings. In contrast, high disagreement about the news arrival rate leads to volatility persistence. If news is absent in such environments, volatility sellers dominate, and extreme payoffs are underweighted in the formation of market expectations, resulting in lower implied volatility—"no news" effectively becomes good news for risky asset valuations.

Key words: *News arrival, heterogeneous beliefs, derivatives, volatility*

JEL codes: *G11, G12, D83, D84*

1 Introduction

News is a central force in financial markets, acting as both an information update and a catalyst for trading, volatility, and price dynamics. Yet not all news is equal in its market impact—what matters is not only what the news says, but also how often it arrives. While much of the literature has focused on disagreement over content—capturing the classic divide between optimists and pessimists—less attention has been paid to how investors form beliefs about the frequency of information arrival. The frequency of news is often connected to market volatility, which is watched closely by both policy makers and practitioners. Most of the time those two move together with occasional divergence, as shown in Figure 1. During the 'dot-com' period between 1995 and 2001, the uncertainty index (based on news frequency) stays at a relatively low level while volatility index shifted up and stayed lifted; contrasting to that are the recent two episodes of 'US trade war' in 2017-18 and 2024-25 when the opposite occurred: the background uncertainty, in terms of news arrival intensity, is high while market volatility stayed at a rather calm level despite some brief spikes.

Motivated by those observations, we develop a theoretical model to investigate the asset pricing implications of investors' belief heterogeneity about news process. We work in a discrete-time setting where the world evolves along a trinomial tree, with each period yielding "good news," "no news," or "bad news" on risky asset's terminal payoffs. News arrival is random in each period: even scheduled events may convey "no news" if uninformative. There is a continuum of agents with logarithm utility of their terminal wealth who would agree to disagree about the probability distribution of the trinomial tree and trade stocks, derivatives and bonds against each other to express their market views about the frequency and content of fundamental news.[1] Agents' heterogeneous beliefs are parameterized by $(u, v) \in (0, 1) \times (0, 1)$, where v is news arrival probability (volatility expectation) and u is the conditional good-news probability (optimism).

When investors disagree on frequency of news, those expecting frequent arrivals could

[1] Andersen (1996) provided empirical evidence of how financial markets process information and showed it is consistent with theoretical models where agents agree to disagree.

long straddle—an option strategy that longs one unit of at-the-money call and one unit of at-the-money put options simultaneously—and those anticipating a calm market will go short straddle. When investors disagree on news content, i.e. whether good or bad news will occur given there would be news. This captures the classic optimist and pessimist divide: bullish agents could buy stock and call options while bearish agents could purchase put options for protection or short stocks. As agents trade, wealth shifts toward those with beliefs aligning with news outcomes. Those proven correct in hindsight (either by luck or skill) would accumulate wealth and their beliefs about asset prices will dominate. Prices reflect this evolving wealth-weighted belief distribution: the underlying price is a harmonic average of terminal payoffs over future trajectories, while risk-neutral return variance captures belief dispersion. Frequent news shapes wealth differently by disagreement source. Content disagreement concentrates wealth on accurate assessors, curbing implied volatility. Frequency disagreement broadens dispersion, boosting it. Since frequent news raises realized volatility, this implies mean reversion under content disagreement but persistence under frequency disagreement.

As the model will demonstrate, speculations about the frequency (and content) of news are closely related to agents' derivative trading behavior and the pricing of market volatility.[2] Those two types of disagreements exert opposite effects on market volatility persistence. Under content disagreement, frequent news induces mean reversion in volatility: high realized volatility concentrates wealth among accurate believers, reducing implied volatility and supporting higher prices. In contrast, frequency disagreement fosters persistence: frequent news sustains wealth dispersion, amplifying implied volatility and dampening prices. This dynamic explains why implied volatility often undershoots realized volatility during news droughts, as short-volatility investors dominate, making "no news" good news for risky assets. These insights extend beyond derivatives; through wealth reallocation, disagreement reshapes market risk perceptions, thereby affecting underlying prices and returns.

[2] Stein (1989) showed the pricing of implied volatility is irrational. More recently, Bryzgalova, Pavlova and Sikorskaya (2023) document a rapid increase in retail trading in options in the United States.

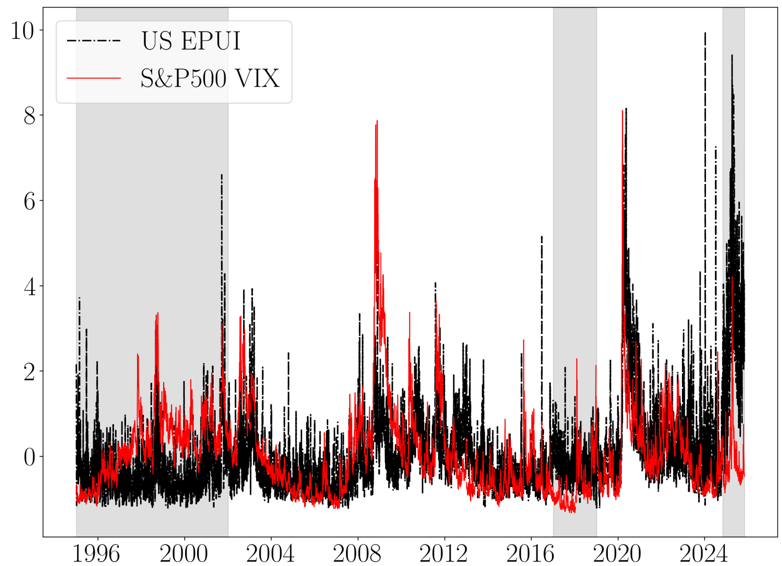


Figure 1: **News frequency based uncertainty index v.s. volatility index**

The figure plots standardized history of VIX of US stock market against the US economic uncertainty index. Uncertainty index is constructed by method from Baker, Bloom and Davis (2016). Both series are retrieved from FRED, Federal Reserve Bank of St. Louis' website.

In further analysis, we relate derivative positioning across investors to their return expectations: extreme optimists and pessimists take large volatility bets, anticipating frequent news arrivals, while moderates serve as their counterparties. Closed-form solutions confirm the U-shaped pattern, underscoring how heterogeneous beliefs drive speculative behavior. By taking continuous-time limits, we show that volatility speculation is driven more from discrete news arrivals than continuous diffusion—fundamental information may arrive gradually, as a Brownian motion, or in discrete jumps, captured by a Poisson process. In the Brownian limit, disagreement over diffusion volatility dissipates quickly: short-maturity bets rapidly reallocate wealth toward agents with more accurate beliefs, making residual disagreements irrelevant for prices. In the Poisson limit, disagreement over news frequency can persist because jumps occur sporadically and agents with incorrect beliefs lose wealth only gradually.

This framework is well-suited for periods during which the data-generating process is stable, with regime shifts occurring outside these periods potentially giving rise to heterogeneous beliefs among investors. The heterogeneous agent model's pricing outcome is equivalent to that of a representative log-utility agent whose prior reflects the initial wealth-weighted distribution of heterogeneous beliefs, updating over time via Bayes' rule—capturing how collective learning emerges from wealth redistribution in the market. Moreover, the framework can also be interpreted through the lens of investor speculation on return autocorrelation: realized momentum or reversal dynamics redistribute wealth among investors with heterogeneous beliefs about return autocorrelation, giving rise to divergent volatility perceptions path-dependent pricing outcomes.

Relevant literature. Heterogeneous beliefs shape financial markets and diverse expectations drive speculative trading and volatility (Harrison and Kreps (1978), Scheinkman and Xiong (2003)). Our work builds on the vast literature on heterogeneous beliefs, enriching the analysis by modeling belief heterogeneity in both news content (optimistic vs. pessimistic views) and news arrival frequency—a dimension offering new insights into asset pricing, volatility, and portfolio dynamics. Another possible dimension of heterogeneity is agents' preferences, and it has been studied by Dumas (1989), Chan and Kogan (2002), although agents' belief heterogeneity is not taken into account (Shiller (1987), Ben-David, Graham and Harvey (2013)). Martin and Papadimitriou (2022) highlight how sentiment-driven trading amid belief heterogeneity amplifies both volatility and trading volume, resonating with our content dimension. By modeling news frequency with a trinomial tree and gamma derivative, we enhance those works by revealing how news frequency disagreement alongside news content disagreement drives volatility and pricing dynamics.

The question of whether investors use options to trade on directional information has been examined in details by Stephan and Whaley (1990); Easley, O'hara and Srinivas (1998); Chan, Chung and Fong (2002); Chakravarty, Gulen and Mayhew (2004); Pan and Poteshman (2006). But the literature on volatility information trading is relatively thin. Ni, Pan and Poteshman (2008) investigated informed trading on stock volatility in the option market. Atmaz and Buffa (2023) connect volatility disagreement to variance risk premia and time-varying leverage effect. Our framework considers the above two motives of trading derivatives jointly.

Earlier works in asset pricing also studied the role of disagreement in market efficiency (Figlewski (1978)), in the evolution (Zapatero (1998), Jouini and Napp (2007), Bhamra and Uppal (2014)), in volatility and trading volume (Basak (2005), Banerjee and Kremer (2010), Atmaz and Basak (2018)), in the amplification of important but rare states (Kogan et al. (2006)), in the pricing of derivatives (Buraschi and Jiltsov (2006)), and in the reactions of prices to public information (Ottaviani and Sørensen (2015)). Our model extends these by incorporating news frequency and our gamma-exposed derivative isolates frequency beliefs, enhancing the understanding of derivative pricing. Cao and Ou-Yang (2008) model options trading driven by differences in signal precision. We extend that by incorporating wealth effects and analyzing how changes in wealth influence the dynamics of derivatives trading. We also show frequency disagreement spurs derivative speculation, notably among extreme optimists/pessimists.

Most prior literature is restricted to the diffusion setting. Within the papers mentioned above, one exception is Chen, Joslin and Tran (2012), where a model with heterogeneous beliefs about disaster risk is considered. The model is tractable, which allows us to study many issues analytically. The tractability partially benefits from the assumption of log-utility, which we view as a reasonable benchmark given the results of Martin (2017), Kremens and Martin (2019), Martin and Wagner (2019), Gao and Martin (2021) and Martin and Papadimitriou (2022)—representative investor's perceived risk premium is equivalent to the risk-neutral variance. It also reflects the fact that we work with a continuum of beliefs, as in Geanakoplos (2010), Atmaz and Basak (2018), Martin and Papadimitriou (2022) but unlike the two-agent models of Harrison and Kreps (1978), Scheinkman and Xiong (2003), Basak (2005), Buraschi and Jiltsov (2006), Kogan et al. (2006), Dumas, Kurshev and Uppal (2009), Banerjee and Kremer (2010), Simsek (2013), Bhamra and Uppal (2014), Borovička (2020), Chabakauri and Han (2020), and Atmaz and Buffa (2023). The continuum of beliefs structure allows us to separately identify the effects of wealth-weighted disagreement and the average belief, which are non-linearly related in two-agent models.

The paper proceeds as follows. Section 2 derives equilibrium prices and portfolio holdings. Section 3 examines how news frequency and belief heterogeneity shape prices, volatility, and allocations through expectations and wealth dynamics. Section 4 presents the continuous-time limits. Section 5 concludes. Proofs are provided in Appendix A.

2 Model

We consider a finite-horizon discrete-time economy with dates t ranging from 0 to T. The state of the world evolves along a trinomial tree, where each node branches into three potential outcomes for the subsequent period: 'good news', 'no news', or 'bad news' regarding the terminal payoff of the underlying asset. From any single investor's perspective, the probability distribution over these outcomes is identical and independent across periods.

Asset payoffs are structured to ensure dynamic market completeness. Agents trade three non-redundant assets: (1) a risk-free asset with one-period maturity, zero net supply, and constant gross return normalized to one; (2) a long-lived underlying asset representing the market, in unit supply; and (3) a derivative on the underlying, in zero net supply. All agents exhibit logarithmic preferences over terminal wealth.

The trinomial tree accommodates information arrival flexibly, capturing both anticipated and unanticipated events. Even scheduled announcements, such as FOMC policy statements, are classified by their informational content. A fully anticipated event conveying no deviation from prior communications qualifies as 'no news'. This emphasis highlights the model's focus on informational innovations rather than event occurrences per se.

2.1 Agents' beliefs

Agents hold heterogeneous, dogmatic beliefs about state evolution and agree to disagree. Each agent's belief is parameterized by a vector $(u, v) \in (0, 1) \times (0, 1)$, where v is the probability of news arrival (good or bad), and u is the conditional probability that arriving news is good. Thus, v captures perceived news frequency, while u reflects relative optimism. These yield probabilities for good news (g), no news (m), and bad news (b):

$$g = uv\,, \quad m = 1 - v\,, \quad b = (1-u)v\,, \quad \text{or conversely} \quad u = \frac{g}{g+b}\,, \quad v = g + b\,. \tag{1}$$

One could embed this trinomial setup in a recombining binary tree with heterogeneous beliefs (as in Martin and Papadimitriou (2022)) by mapping two binary periods to one trinomial period. However, this restricts beliefs to a one-dimensional subset of $(u, v) \in [0, 1] \times [0, 1]$, with $v \geq 0.5$.[3]

Throughout this paper, we use (u, v) to index agents according to their beliefs. Because of trading, the wealth distribution of agents across different beliefs is time varying. We use $f_t(u, v)$ to represent the wealth distribution at time t.

We index agents by (u, v). Trading induces time-varying wealth distributions $f_t(u, v)$ across beliefs. Our framework features two belief heterogeneity dimensions. The first concerns fundamental volatility, via differing news arrival rates (v): some agents expect stability (high m), others volatility (high $g + b$). The second involves news content disagreement, via bullishness (u), the conditional good-news probability upon arrival.

2.2 Asset payoffs

Let p_t denote the time-t price of the underlying market asset, and q_t the price of the one-period derivative with payoff x_{t+1} at $t+1$. At the subsequent nodes, underlying prices are $(p_{t+1,g}, p_{t+1,m}, p_{t+1,b})$ and derivative payoffs $(x_{t+1,g}, x_{t+1,m}, x_{t+1,b})$. Dynamic completeness implies derivative payoffs do not alter wealth allocations across states. For convenience, we assume payoffs uncorrelated with the underlying under the risk-neutral measure:

$$\mathrm{Cov}_t^*[x_{t+1}, p_{t+1}] = 0\,. \tag{2}$$

[3]For an agent with binary up-move probability $h \in [0, 1]$, good news corresponds to up-up ($g = h^2$), bad news to down-down ($b = (1-h)^2$), and no news to up-down or down-up ($m = 2h(1-h)$). Then, $u = h^2/[1 - 2h + 2h^2]$ and $v = h^2 + (1-h)^2 \geq 1/2$ imply a functional relation between u and v. We discuss this case further in Section 3.3, contrasting it with settings featuring only v-disagreement.

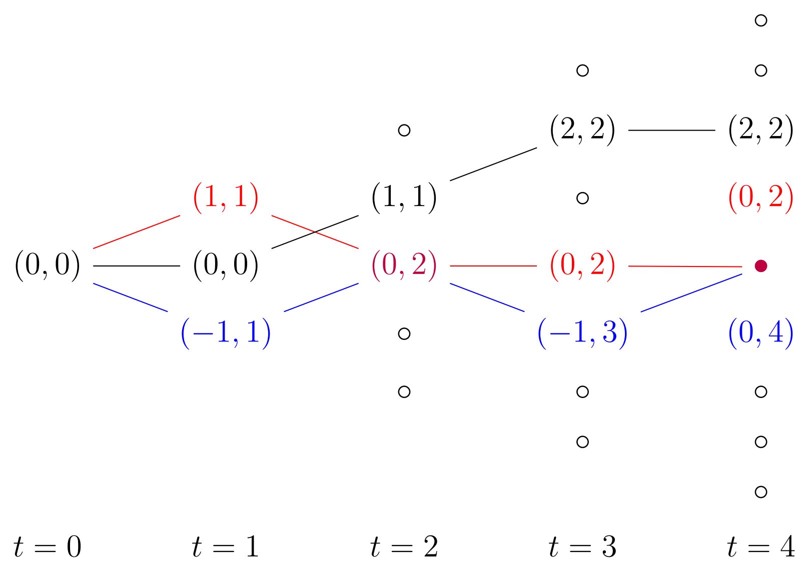


Figure 2: **State variable (n_t, ν_t) along sample paths between $t = 0$ and $t = 4$**

This focus is without loss of generality: any derivative can be transformed into a zero-delta version by delta-hedging with the underlying asset. The hedged payoff satisfies the above condition while preserving the span of attainable payoffs. Investors hold the same quantity of the derivative before and after this transformation.

Let n_{gt}, n_{mt}, n_{bt} count good, no, and bad news from 0 to t. Since $t = n_{gt} + n_{mt} + n_{bt}$, two variables suffice: fundamental level $n_t = n_{gt} - n_{bt}$ and news count $\nu_t = n_{gt} + n_{bt}$.

$$n_t = n_{gt} - n_{bt}\,, \qquad \nu_t = n_{gt} + n_{bt}\,. \tag{3}$$

The underlying's terminal payoff is $p_T(n_T)$. The news count ν_t proxies realized fundamental volatility, as each news event shifts n_t by ± 1. Figure 2 illustrates (n_t, ν_t) along some sample paths. At $t = 4$, red and black paths share $n_4 = 0$ but black has higher ν_4; blue and red share $n_4 = 0$ but blue has higher ν_4.

2.3 Equilibrium

Log utility implies myopic optimization: an agent with belief (u, v) allocates wealth to $\theta_t^{(u,v)}$ units of underlying, $\phi_t^{(u,v)}$ of derivative, and the rest in risk-free bond, solving

$$\max_{\theta_t,\phi_t} \mathbb{E}_t^{(u,v)} \log \left[w_t^{(u,v)} + \theta_t^{(u,v)}(p_{t+1} - p_t) + \phi_t^{(u,v)}(x_{t+1} - q_t) \right] , \tag{4}$$

where w_t represents the time t wealth of the individual agent with beliefs (u, v).

Dynamic completeness allows Arrow-Debreu representations. In equilibrium, subjective probability-weighted stochastic discount factors equal Arrow-Debreu prices, i.e., risk-neutral probabilities (given unit risk-free rate):

$$g_t^* = g \cdot \frac{w_t^{(u,v)}}{w_{t+1,g}^{(u,v)}} , \quad m_t^* = m \cdot \frac{w_t^{(u,v)}}{w_{t+1,m}^{(u,v)}} , \quad b_t^* = b \cdot \frac{w_t^{(u,v)}}{w_{t+1,b}^{(u,v)}} , \tag{5}$$

where (g_t^*, m_t^*, b_t^*) are risk-neutral probabilities; $w_t^{(u,v)}$ is the wealth of the agent with beliefs (u, v) at time t; $w_{t+1,h}^{(u,v)}$, $w_{t+1,m}^{(u,v)}$, and $w_{t+1,l}^{(u,v)}$ denote this agent's wealth at the respective nodes at time $t + 1$.

Optimal allocations align Arrow-Debreu weights with subjective beliefs (g, m, b). One-period wealth growth is belief over risk-neutral measure: in the good state, wealth grows by g/g_t^*; in the medium state, by m/m_t^*; and in the bad state, by b/b_t^*. Aggregate wealth growth equals market portfolio change, so one-period state dependent returns satisfy

$$\frac{p_{t+1,g}}{p_t} = \frac{G_t}{g_t^*} \quad \frac{p_{t+1,m}}{p_t} = \frac{M_t}{m_t^*} \quad \frac{p_{t+1,b}}{p_t} = \frac{B_t}{b_t^*} . \tag{6}$$

Here, (G_t, M_t, B_t) are wealth weighted average beliefs.[4] Equation (6) also implies p_t^{-1} is a sentiment-weighted harmonic mean of next-period prices

$$p_t^{-1} = G_t p_{t+1,g}^{-1} + M_t p_{t+1,m}^{-1} + B_t p_{t+1,b}^{-1} . \tag{7}$$

[4]If the distribution of (u, v) are uncorrelated, i.e. $f(u, v) = f(u)f(v)$, we have $G_t = U_t V_t$, $M_t = 1 - V_t$,$B_t = V_t(1 - U_t)$.

Thus, sentiment—wealth-weighted (g, m, b)—drives pricing.

We call investors allocating fully to the underlying *Mr. Market*; their identity shifts with sentiment. Imposing initial beliefs as a separable 2D beta distribution

$$f_0(u, v) = f_0(u) f_0(v) = \frac{u^{\alpha_{u0}-1}(1-u)^{\beta_{u0}-1}}{B(\alpha_{u0}, \beta_{u0})} \frac{v^{\alpha_{v0}-1}(1-v)^{\beta_{v0}-1}}{B(\alpha_{v0}, \beta_{v0})}, \tag{8}$$

yields tractable Mr. Market beliefs:

$$U_t(n_t, \nu_t) = \frac{n_{gt} + \alpha_{u0}}{\nu_t + \alpha_{u0} + \beta_{u0}}, \quad V_t(n_t, \nu_t) = \frac{\nu_t + \alpha_{v0}}{t + \alpha_{v0} + \beta_{v0}}. \tag{9}$$

Note that $n_{gt} = \frac{1}{2}(n_t + \nu_t)$. U_t reflects the proportion of good news among all the arrived news, implying less bullishness for fixed t, n_t on volatile paths. V_t depends solely on ν_t / t: higher realized frequency elevates wealth-weighted v.

Example.—Figure 3 illustrates a $T = 3$ case with uniformly distributed u, v. At $t = 1$, good or bad news raises V from 0.5 to 0.67, as high-v agents outperform low-v ones; good news also shifts wealth to high-u agents, raising U, while bad news lowers it. No news preserves U (no content update) but depresses V, elevating prices—"no news is good news'. At $t = 2$, both the black path and red path's no news further depresses V, as volatility sellers gain more. The blue path's good news offsets $t = 1$ bad news ($n_2 = 0$, like red), but higher ν_2 on blue path yields $V_2 = 0.75$ (vs. red's 0.25), depressing prices relative to red despite the positive shock from its own history from $t = 1$.

2.4 Asset positions

Although the choice of derivative payoff structure does not impact the equilibrium portfolio allocation or wealth distribution, specifying it could simplify the interpretation of asset holdings. We show in internet appendix I.A.1 that any derivative security payoff satisfying the zero-delta assumption (2) must be spanned by the risk-free payoff $(1, 1, 1)$

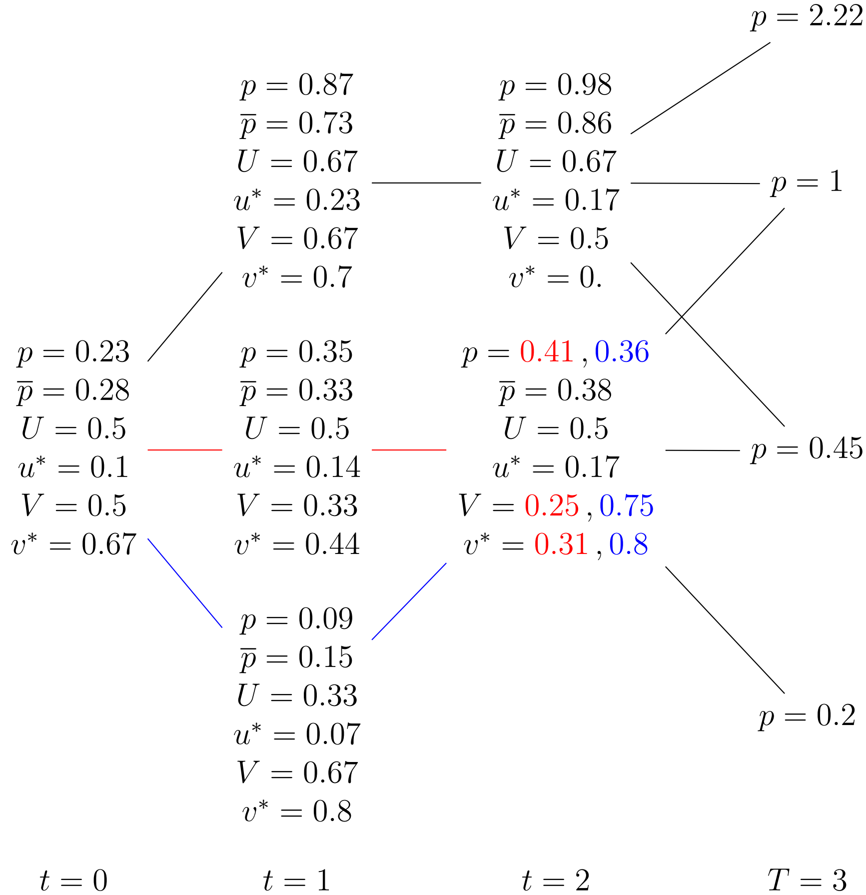


Figure 3: **A numerical example with $T = 3$**

In this example agents' beliefs u and v are both uniformly distributed. p is price of market, $\overline{p}$ is price in homogeneous economy where everyone agrees with the median agent $(u, v) = (1/2, 1/2)$. (U, V) represents Mr. Market's beliefs and (u^*, v^*) is the risk-neutral measure. There are three nodes at $T = 3$ omitted from the figure to make it compact: top node $p = 4.95$ and the bottom two $p = 0.09$ and $p = 0.04$.

and the vector $(b_t^* - B_t, 0, G_t - g_t^*)$

$$(x_{t+1,g}, x_{t+1,m}, x_{t+1,b}) \in \operatorname{span}\left\{(1,1,1), (b_t^* - B_t, 0, G_t - g_t^*)\right\}. \tag{10}$$

Since the risk-free asset is tradable, any component of the derivative payoff proportional to $(1, 1, 1)$ can be replicated independently and thus does not affect the demand for the derivative itself. Consequently, without loss of generality, we may restrict attention to the remaining component and represent the derivative as

$$x_{t+1,g} = p_t(b_t^* - B_t)\,, \quad x_{t+1,m} = 0\,, \quad x_{t+1,b} = p_t(G_t - g_t^*)\,. \tag{11}$$

The return of this derivative is uncorrelated with the market under the risk-neutral measure: in the context of options trading, the delta of the derivative is equal to zero. Therefore, the derivative can be interpreted as a delta-neutral combination of call and put options on the market portfolio, with a strike price at the "no news" realization $p_{t+1,m}$. Since $(b_t^* - B_t, 0, G_t - g_t^*) = (1, 0, 1)\,(b_t^* - B) + (0, 0, 1)\,(v_t^* - V_t)$, trading of x_{t+1} at t reflects agents' two motives in trading options: speculations of news frequency and news content.

Because of market completeness, agents' holdings in Arrow-Debreu securities maps into positions in the underlying asset and the derivative security. For an agent with beliefs (u, v), the optimal positions in stock (delta exposure) and derivative (gamma exposure) are given by:

$$\omega_{\Delta,t}^{(u,v)} = \frac{\theta_t^{(u,v)} p_t}{\omega_t^{(u,v)}} = \frac{\mathbb{E}_t^{(u,v)}[p_{t+1}/p_t] - 1}{\mathbb{E}_t^{(U_t,V_t)}[p_{t+1}/p_t] - 1}\,, \quad \omega_{\Gamma,t}^{(u,v)} = \frac{\phi_t^{(u,v)} q_t}{w_t^{(u,v)}} = \frac{\mathbb{E}_t^{(u,v)}[x_{t+1}/q_t] - 1}{\mathrm{Var}_t^*[x_{t+1}/q_t]}\,. \tag{12}$$

Investors with beliefs equal to the risk-neutral measure (u_t^*, v_t^*) would not hold any risky asset at all, and Mr. Market would stay out of bond and derivative markets.

Since the derivative is *delta-neutral* by construction, investors' positions in the underlying asset are purely driven by their bullishness (or lack thereof) in our model.[5] Agents' portfolio weight of gamma exposure could also be expressed in terms of their beliefs of news intensity and delta exposure

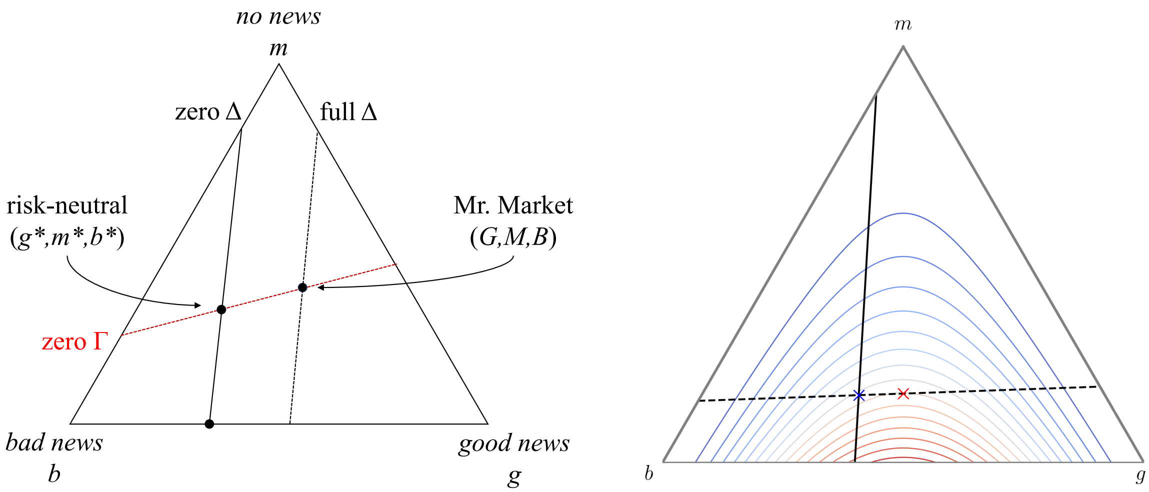
$$\omega_{\Gamma,t}^{(u,v)} = \frac{v - v_t^*}{1 - v_t^*} + \frac{v_t^* - V_t}{1 - v_t^*}\,\omega_{\Delta,t}^{(u,v)}\,. \tag{13}$$

The first component of the gamma portfolio weight comes from agent's speculation about

[5]Agents with Gamma-neutral position would have belief that satisfy

$$(g, m, b) = \kappa(G_t, M_t, B_t) + (1 - \kappa)(g_t^*, m_t^*, b_t^*)\,,$$

where κ is a scalar. It could be verified by taking the subjective beliefs back to (12).



(a) Graphic illustration

(b) An example with $T = 6$ at $t = 4$

Figure 4: **Mapping investor beliefs to portfolio allocations**

The left panel is a graphic illustration about the positions of agents with different beliefs on the plane of (g, m, b). The right panel is a numerical example with $T = 6$ and terminal payoff set as $e^{0.05\, n_T}$. Agents initial beliefs are assumed to be uniformly distributed on the domain $u, v \in [0, 1]$. The portfolio allocation is at $t = 4$ after the blue sample path in Figure 2.

news frequency and the second component reflects agent's speculation of news content through option trading. Agent (u, v) has cash position

$$1 - \omega_{\Delta,t}^{(u,v)} - \omega_{\Gamma,t}^{(u,v)} = \frac{1 - v}{1 - v_t^*} - \frac{1 - V_t}{1 - v_t^*}\, \omega_{\Delta,t}^{(u,v)} \,. \tag{14}$$

Figure 4a demonstrates agents' portfolio allocations in the (g, m, b) space. Mr. Market's belief is marked with red cross and the risk-neutral measure is marked with blue cross. For other investors, according to (12), their holdings in the underlying asset are determined by their relative bullishness to Mr. Market. Investors with beliefs located on the right side of the solid line are long in market and those go short are on the left side. All agents on the 'zero Δ' line would stay out of stock market, including the agent whose belief is identical to risk-neutral measure. Around the intersect of 'zero Δ' line and bottom line of the triangular are agents who holds almost 100% wealth in Gamma exposure with very little cash or stock positions.

Investors who speculate the fundamental is going to be more volatile than sentiment thinks could take a positive position in the derivative against those who think otherwise and go short. The dashed red 'zero Γ' line separates these two groups and it passes through both Mr. Market's belief (G_t, M_t, B_t) and the risk-neutral belief (g_t^*, m_t^*, b_t^*) —all agents on this line are out of the derivative market. Investors taking a long position in the derivative, i.e. those who believe relatively lower m, are located below this line while those with short gamma exposures are above.

Figure 4b shows a numerical example at $t = 4$ with $T = 6$ and uniformly distributed beliefs in (u, v). The news history overlaps the blue path in figure 2. Because the net news is $\nu_4 = 0$, Mr.Market's belief U_4 stayed at 0.5 while the volatile history depressed the value $M_4 = 1 - V_4$ from 0.5.

2.5 Asset prices

Although aggregate beliefs are path-dependent, (7) yields tractable pricing: time-t market value is the wealth-weighted harmonic mean of $p_T(n_T)$. The same weights allows us to compute the risk-neutral variance of returns.

Given current fundamentals n_t and the prevailing wealth distribution f_t, the probability $\Pr(n_T|n_t, f_t)$ encapsulates the collective expectation—weighted by investor wealth—regarding the likelihood of reaching a terminal state with fundamental value n_T. It is computed by aggregating over all feasible news trajectories between t and T that lead to a change of $n_T - n_t$ for the fundamental. Ultimately, the market price and risk-neutral variance reflect this distribution, capturing the wealth-weighted consensus over all possible terminal outcomes.

Result 1 (Market price and risk-neutral variance)**.** *Assuming the trinomial tree has T periods and the payoffs are $p_T(n_T)$ with $n_T \in [-T, T]$, the value of the market at time t*

is given by

$$p_t^{-1} = \mathbb{E}_t\left[p_T^{-1}(n_T)\right] \,, \tag{15}$$

and the risk-neutral variance of the return from time t to T is given by:

$$\mathrm{Var}_t^*[p_T/p_t - 1] = \mathbb{E}_t\left[p_T(n_T)\right]\mathbb{E}_t\left[p_T^{-1}(n_T)\right] - 1\,, \tag{16}$$

where the expectation is taken under the wealth-weighted belief distribution over terminal states n_T, given by:

$$\Pr(n_T|n_t, f_t) = \sum_{\substack{\forall\, n_g - n_b = n_T - n_t,\\ n_g + n_m + n_b = T - t}} \binom{T-t}{n_g, n_m, n_b} \int u^{n_g}(1-u)^{n_b} v^{T-t-n_m}(1-v)^{n_m} f_t(u,v)\,\mathrm{d}u\,\mathrm{d}v\,, \tag{17}$$

conditional on the current fundamental n_t and the wealth distribution f_t.

In limits with no u- or v-disagreement, markets ignore news content (n_g, n_b) or arrivals (n_m), reducing $f(u,v)$ to a marginal. Result 1 holds generally; for tractability, we specialize to 2D beta (8).[6] Then, comparing (5)–(6), time-t wealth distribution is

$$f_t(u,v) = \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut},\beta_{ut})}\,\frac{v^{\alpha_{vt}-1}(1-v)^{\beta_{vt}-1}}{B(\alpha_{vt},\beta_{vt})}\,, \tag{18}$$

where $t = n_{gt} + n_{mt} + n_{bt}$ and $(\alpha_{ut}, \beta_{ut}, \alpha_{vt}, \beta_{vt})$ evolve according to

$$\alpha_{ut} = \alpha_{u0} + \frac{n_t + \nu_t}{2}, \quad \beta_{ut} = \beta_{u0} + \frac{\nu_t - n_t}{2}, \quad \alpha_{vt} = \alpha_{v0} + \nu_t, \quad \beta_{vt} = \beta_{v0} + t - \nu_t\,. \tag{19}$$

The derivations of (18) and (19) are provided in Appendix A. The 2D beta specification

[6] This nests the Dirichlet distribution on (g, m, b) when $\alpha_{v0} = \alpha_{u0} + \beta_{u0}$, yielding parameters $(\alpha_{u0}, \beta_{v0}, \beta_{u0})$. Dirichlet distribution's rotational symmetry implies weak linkage between content and arrival disagreement; our framework relaxes this.

in (18) would give explicit expression (17) and make the model tractable

$$\Pr(n_T|n_t, f_t) = \sum_{\substack{\forall n_g - n_b = n_T - n_t, \\ n_g + n_m + n_b = T - t}} \binom{T-t}{n_g, n_m, n_b} \frac{B(\alpha_{ut} + n_g, \beta_{ut} + n_b)}{B(\alpha_{ut}, \beta_{ut})} \frac{B(\alpha_{vt} + T - t - n_m, \beta_{vt} + n_m)}{B(\alpha_{vt}, \beta_{vt})} . \tag{20}$$

Individual Bayesian Learning.—The model can be extended to allow heterogeneous individuals to update their beliefs over time according to Bayes' rule. In this setting, (u, v) serve merely as indices of agents, while the actual news content and arrival probabilities are characterized by $(\tilde{u}, \tilde{v})$. The initial wealth distribution is given by (8), and agent (u, v) holds prior beliefs $\tilde{u} \sim \text{Beta}(\zeta u, \zeta(1-u))$ and $\tilde{v} \sim \text{Beta}(\zeta v, \zeta(1-v))$, where ζ determines the degree of uncertainty perceived by the individual. The value of the market at time 0 is given by (15), where the expectation is taken under the distribution

$$\Pr(n_T) = \sum_{\substack{n_g - n_b = n_T \\ n_g + n_m + n_b = T}} \binom{T}{n_g, n_m, n_b} \frac{B(\zeta u + n_g, \zeta(1-u) + n_b)}{B(\zeta u, \zeta(1-u))} \frac{B(\zeta v + T - n_m, \zeta(1-v) + n_m)}{B(\zeta v, \zeta(1-v))} f_0(u, v) \mathrm{d}u\mathrm{d}v\,. \tag{21}$$

In the limit as $\zeta \to \infty$, agents hold dogmatic beliefs, the terminal states distribution converges to (20), and the value of the market converges accordingly. We focus on this limiting case for the remainder of the paper.

The 2D beta specification in (20) equates heterogeneous agents pricing to a representative log agent with initial (8) prior, updating his belief about (u, v) via Bayes rule. While such a model would not be consistent with our model since heterogeneous beliefs are absent, hence no trade is happening, the comparison between those two highlights a feature of our model: markets' collective learning via wealth redistribution—hindsight winners amplify their beliefs in pricing.

Result 2 (Wisdom of the crowd)**.** *Pricing in the heterogeneous-agent economy is identical to pricing in an economy with a representative agent with log utility whose prior belief at time* 0 *about probability of future news process has the same distribution as in* (8)*, and* *the agent updates his or her beliefs over time following Bayes' rule.*

The crowd's wisdom (9) never surpasses its most informed members. If beliefs include the true (u, v), prolonged data would enable Mr. Market converge to truth. In contrast, uniform bias in u or v precludes learning.[7]

3 Analysis of equilibrium

3.1 Derivative positioning across return expectations

In this section, we examine the relationship between investors' positions in the derivative security and their subjective return expectations for the underlying asset. As shown in (12), investors' positions in the underlying asset are linear with respect to these subjective expectations. Consequently, this relationship also reveals the connection between investors' holdings in the derivative and those in the underlying asset. This observation motivates our decision to group investors according to their subjective expectations of the underlying asset's returns, thereby offering an interpretable perspective on derivative positioning.

Result 3 (Wealth share across beliefs in expected returns)**.** *Define the variable $z(r)$, k and γ,*

$$z(r) = \frac{(1+r)g_t^* m_t^* - g_t^* M_t}{G_t m_t^* - g_t^* M_t}\,, \quad k = \frac{G_t m_t^* - g_t^* M_t}{g_t^* M_t - \frac{B_t}{b_t^*} g_t^* m_t^*}\,, \quad \gamma = \frac{g_t^* m_t^*}{G_t^* m_t^* - g_t^* M_t^*}\,, \tag{22}$$

the wealth share of agents who believe expected return of underlying is r satisfies

$$f_t(r) = \frac{\gamma k^{\beta_{ut}} z^{\alpha_{vt}-1}(1-z)^{\beta_{ut}+\beta_{vt}-1} B(\beta_{ut}, \beta_{vt})}{(k+1)^{\beta_{ut}} B(\alpha_{ut}, \beta_{ut}) B(\alpha_{vt}, \beta_{vt})} F_1\left(\beta_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\alpha_{ut}; \beta_{ut}+\beta_{vt}; 1-z, \frac{k(1-z)}{1+k}\right),$$

[7] The above result has been established in Martin and Papadimitriou (2022) in a binary tree setting with only disagreement on news content.

for $z > 0$*, and*

$$f_t(r) = \frac{\gamma k(-kz)^{\alpha_{vt}-1}(1+kz)^{\alpha_{ut}} B(\alpha_{ut}, \beta_{vt})}{(1+k)^{\alpha_{ut}+\beta_{vt}-1} B(\alpha_{ut}, \beta_{ut}) B(\alpha_{vt}, \beta_{vt})} F_1 \left(\alpha_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt}; 1 + kz, \frac{1+kz}{1+k} \right),$$

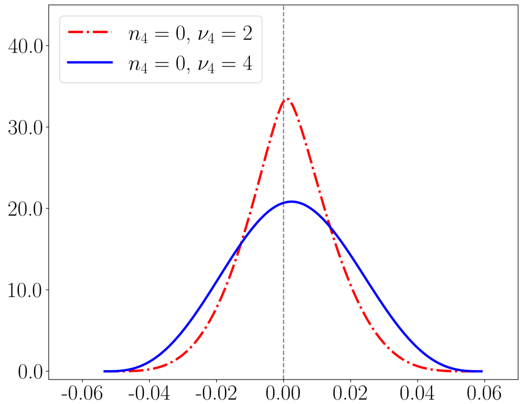
for $z < 0$*. The function* $F_1(\dots)$ *represents the Appell hypergeometric function, and* $(\alpha_{ut}, \beta_{ut}, \alpha_{vt}, \beta_{vt})$ *follows the same definition as in* (19)*.*

Figure 5 illustrates the wealth distribution across investor cohorts under conditions of low and high disagreement regarding the news arrival rate. The three colored lines correspond to the sample paths depicted in Figure 2. In the left panel (5a), under high disagreement, wealth is more dispersed along the blue path ($\nu_4 = 4$), which features more frequent news arrivals, than along the red path ($\nu_4 = 2$). Conversely, in the right panel (5b), under low disagreement, the blue path yields a more concentrated wealth distribution relative to the red path. Overall, greater disagreement about the news arrival rate leads to a more dispersed distribution of return beliefs, as illustrated by comparing the same-colored bell-shaped curves in the left and right panels of Figure 5.

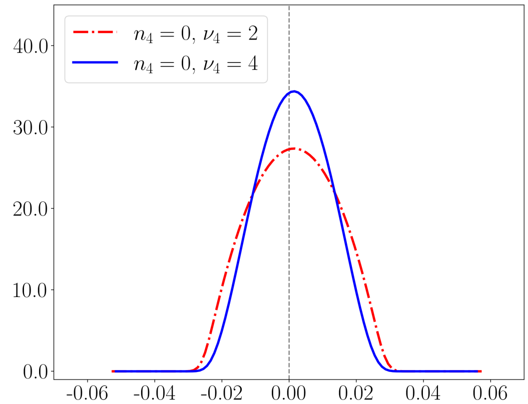
Since investors' positions reflect their beliefs regarding the underlying asset, optimists who expect positive returns adopt long positions, whereas pessimists who anticipate negative returns take short positions. Extending this reasoning, investors with extremely bullish views will have portfolio allocations that resemble long call options, benefiting disproportionately from significant upward price movements. Conversely, those with extremely bearish sentiments will hold portfolios akin to long put options, profiting most from substantial price declines.

More broadly, investors' market outlooks correlate with their derivative positions. The following result examines the average derivative position for investor groups segmented according to their subjective expectations of market returns.

Result 4 (Aggregated gamma positioning across same beliefs of return)**.** *For cohort of investors with same subjective expectation* r *for the market return, the wealth-weighted*



(a) High v-disagreement



(b) Low v-disagreement

Figure 5: **Wealth distribution across investor cohorts by expected return** r
The figure illustrates the wealth distribution across investor cohorts as a function of the expected return r. The left panel (5a) assumes a uniform initial wealth distribution across agents, with $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. In contrast, the right panel (5b) corresponds to low disagreement regarding the news arrival rate, with parameters $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. The terminal payoff at $T = 6$ is given by the exponential function $e^{0.05 n_T}$. See Figure 2 for an illustration of the possible sample paths leading to the corresponding wealth distributions and gamma positions at $t = 4$.

'average portfolio weight' in the derivative satisfies

$$\mathbb{E}\left[\left.\frac{\phi_t^{(u,v)} q_t}{w_t^{(u,v)}}\right| r\right] = \frac{1}{1 - v_t^*}\left(\mathbb{E}[v|r] - v_t^* + \frac{(v_t^* - V_t)}{\mathbb{E}^{(U_t,V_t)}[r]}\, r\right), \tag{23}$$

where $\mathbb{E}[v|r]$ *denotes the wealth-weighted average news arrival rate* v *among investors with expectation* r *with the following close form expression: when* $z \geqslant 0$,

$$\mathbb{E}[v|r] = z + z(1-z) \cdot \frac{\beta_{ut}}{\beta_{ut} + \beta_{vt}} \cdot \frac{F_1\left(\beta_{ut}+1; \alpha_{vt}+\beta_{vt}, 1-\alpha_{ut}; \beta_{ut}+\beta_{vt}+1; 1-z, \frac{k(1-z)}{1+k}\right)}{F_1\left(\beta_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\alpha_{ut}; \beta_{ut}+\beta_{vt}; 1-z, \frac{k(1-z)}{1+k}\right)}$$

and when $z < 0$, *we have*

$$\mathbb{E}[v|r] = -kz + (-kz)(1+kz) \cdot \frac{\alpha_{ut}}{\alpha_{ut} + \beta_{vt}} \frac{F_1\left(\alpha_{ut}+1; \alpha_{vt}+\beta_{vt}, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}+1; 1+kz, \frac{1+kz}{1+k}\right)}{F_1\left(\alpha_{ut}; \alpha_{vt}+\beta_{vt}-1, 1-\beta_{ut}; \alpha_{ut}+\beta_{vt}; 1+kz, \frac{1+kz}{1+k}\right)}$$

The variable $z(r)$ and k follow the same definition in (22) and $F_1(\dots)$ represents the Appell hypergeometric function.

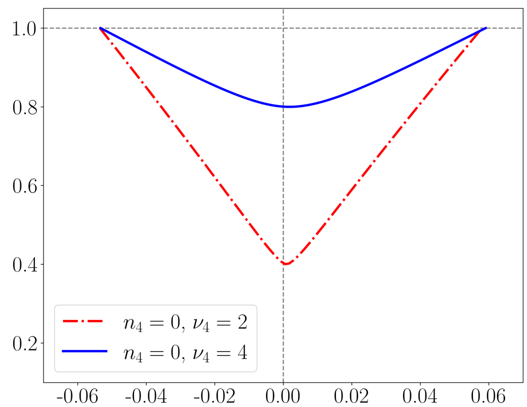
Utilizing this closed-form solution, we examine how investors' varying outlooks on market performance translate into their derivative positions.

The wealth-weighted average news arrival rate, $\mathbb{E}[v|r]$, exhibits a U-shaped relationship with the subjective expected return r. Investors with highly optimistic or pessimistic expectations also tend to hold extreme views about the news arrival rate, resulting in $\mathbb{E}[v|r] = 1$ at the boundaries of possible returns, $r = G_t/g_t^* - 1$ and $r = B_t/b_t^* - 1$. In contrast, those with more moderate expectations typically assign lower average news arrival rates. Figures 6a and 6b illustrate this relationship under high and low disagreement in news arrival rates, using $(\alpha_v, \beta_v) = (1, 1)$ and $(100, 100)$, respectively. While both exhibit the U-shape, the low-disagreement case produces a noticeably flatter curve near the center, reflecting stronger consensus among investors. Differences in news arrival rate expectations naturally lead to variation in derivative positions. Figures 6c and 6d show the derivative holdings of investor cohorts, expressed as fractions of total wealth in the economy. These holdings are obtained by multiplying the wealth distribution in Result 3 with the corresponding portfolio weights from (23).

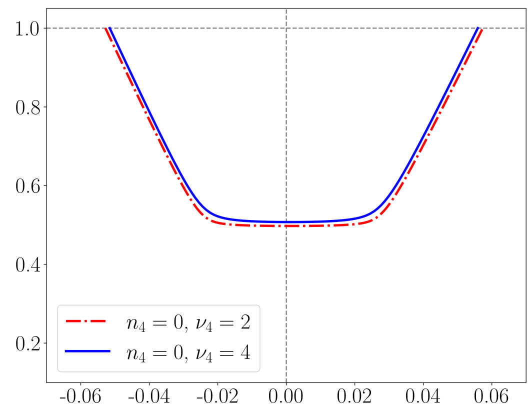
Since every derivative contract has both a buyer and a seller, aggregate net positions must sum to zero. Market clearing thus requires that the extreme positions of highly optimistic or pessimistic investors be offset by counterparties. This role is primarily filled by investors with more moderate beliefs, who act as derivative sellers. These participants, with less extreme views on market returns, provide essential liquidity and facilitate trade execution. The equilibrium in the derivatives market emerges from the interaction between investors expressing strong views on the news arrival rate and those accommodating these trades through more tempered expectations. [8]

Derivative positions offer an empirically observable metric to distinguish between cases of high and low disagreement about the news arrival rate, as we discuss in Section 3.2.

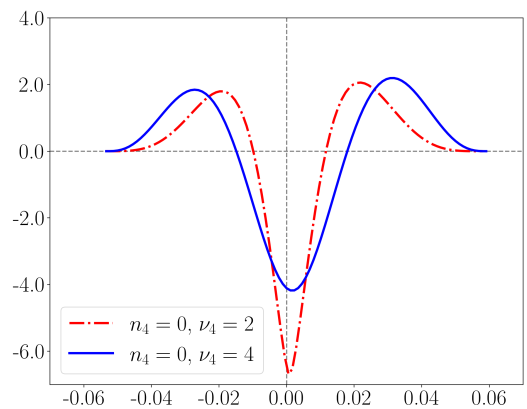
[8] In the online appendix I.A.3, we plot the time variation of all figures at $t = 1, 2, 3, 4$ to demonstrate how different sample paths affect the wealth distribution and gamma positioning across agents' market return expectations.



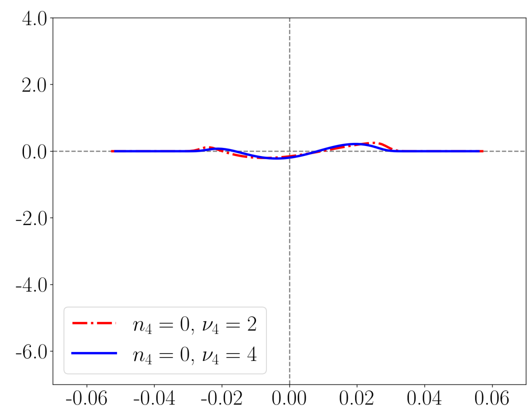
(a) Wealth averaged beliefs of v across r
High v-disagreement



(b) Wealth averaged beliefs of v across r
Low v-disagreement



(c) Gamma exposure across r
High v-disagreement



(d) Gamma exposure across r
Low v-disagreement

Figure 6: **Beliefs of news intensity, market return and gamma positioning**

The figure reports wealth-weighted average news arrival rate v and Gamma exposures for the cohort of investors with expected return r. The left panels (6a and 6c) assume a uniform initial wealth distribution across agents, with $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. In contrast, in the right panels (6b and 6d), the initial wealth distribution corresponds to low disagreement in the news arrival rate dimension, characterized by $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$. See Figure 2 for an illustration for the possible sample paths that leads to the corresponding wealth distribution and gamma position at $t = 4$.

When disagreement about the news arrival rate is high, investors take significantly larger derivative positions, as shown in Figure 6c. These sizable derivative investments, together with the resulting wealth redistribution, are the central drivers of volatility persistence in Result 5. In contrast, when disagreement is low, $\mathbb{E}[v|r]$ remains close to the market average V_t across most investor cohorts. Derivative portfolio weights are therefore small, leading to limited activity in derivative markets, as illustrated in Figure 6d.

3.2 Volatility Dynamics: Persistence vs. Mean Reversion

Asset prices are influenced by both the occurrence and absence of news events. The content of news is salient: positive news signals an improvement in fundamentals and shifts the wealth distribution in favor of optimistic investors. The frequency of news arrival also plays a crucial role. Frequent news redistributes wealth among investors with varying degrees of optimism and additionally empowers those who anticipate high news arrival rate. Consequently, the number of news events ν_t shapes market disagreement and aggregate beliefs, which in turn determine volatility expectations and asset prices.

News arrival exerts two distinct effects on the market. First, a high historical count of news increases the market's average belief about future news arrival rate, V_t. This upward shift in expectations places downward pressure on asset prices. Second, frequent news arrivals lead to a concentration of wealth among agents whose beliefs are correct ex-post, and reduce disagreement over news content. The level of "agreement" among investors, $\alpha_{ut} + \beta_{ut} = \alpha_{u0} + \beta_{u0} + \nu_t$, increases with ν_t. As demonstrated in Section 2.5, this reduction in disagreement over news content raises asset prices.

The overall impact depends on the relative strength of these two channels, which is, in turn, determined by the degree of disagreement among investors regarding news arrival rate. We consider a specification in which each piece of news has a multiplicative effect:

$$p_T(n_T) = e^{\sigma n_T} , \tag{24}$$

so that the terminal payoff is exponential in the fundamental. [9]

When disagreement about news arrival rate is strong, the first channel dominates. A comparison between disagreement in news arrival rate and disagreement in news content can be made by comparing α_v with the degree of "agreement" in news content $\alpha_u + \beta_u$. If, in the initial wealth distribution, α_{v0} is lower than $\alpha_{u0} + \beta_{u0}$, this inequality persists over time. Specifically, for any t:

$$\alpha_{vt} = \alpha_{v0} + \nu_t \leqslant \alpha_{u0} + \beta_{u0} + \nu_t = \alpha_{ut} + \beta_{ut} \tag{25}$$

Disagreement about news arrival rate remains persistently high at any point in time. The following result shows that a higher frequency of news arrival raises implied volatility and lowers asset prices.

Result 5 (News count and asset prices, prominent news arrival rate disagreement)**.** *Suppose $\alpha_{v0} \leqslant \alpha_{u0} + \beta_{u0}$ in the initial wealth distribution. Holding constant the average expectation of the fundamental growth, $G_t - B_t$, a higher number of news events ν_t increases the risk-neutral variance $\mathrm{Var}_t^*[p_T/p_t - 1]$ and lowers the ratio of the current asset price to the current fundamental, $p_t/e^{\sigma n_t}$.*

Frequent news events, even when neutral on average, can lead to significant fluctuations in investor beliefs and substantial wealth redistribution. Derivative sellers are particularly vulnerable, as their positions are exposed to sharp market movements. These shifts often transfer wealth to investors with extreme views on the underlying market's returns.

This mechanism is illustrated in our $T = 6$ period example with an initial wealth distribution in which both u and v are uniformly distributed. At $t = 4$, the wealth-weighted distribution of return under the blue path becomes more dispersed than under the red path, as shown in Figure 5a. Consistent with Result 5, when disagreement is high,

[9] This specification lends itself naturally to a continuous-time limit, where the interval between trading opportunities shrinks to zero. Unlike an arithmetic payoff, the exponential (or geometric) form guarantees that the payoff remains strictly positive and cannot fall below zero.

the frequent news arrivals along the blue path ($\nu_4 = 4$) increases the wealth of agents with strong beliefs in v, resulting in a more dispersed distribution.

As a result, the likelihood of extreme outcomes in the wealth-weighted distribution of beliefs rises, increasing implied volatility and depressing current asset prices.Consequently, markets fare worse during periods of volatile but offsetting fundamental swings than during periods of relative informational silence or "no news."

Result 5 highlights a critical link between volatility dynamics and the disagreement in news arrival rate. When investors disagree about the frequency of news arrivals, volatility becomes more persistent. These disagreements fuel sustained derivative trading, as opposing beliefs generate ongoing speculative demand. A history of volatile price movements signals elevated future option prices, as market participants anticipating continued turbulence gains more influence. In essence, high realized volatility today translates into high implied volatility going forward, making derivatives more expensive and reflecting the market's expectation of continued instability.

In contrast to Result 5, when there is no disagreement over news arrival rate, the second channel dominates. To isolate the effects of belief heterogeneity along specific dimensions, we examine a benchmark and limit scenario in which all investors share a common view about the frequency of news arrivals. In this setting, belief heterogeneity arises solely from differences in investors' assessments of the likelihood of receiving "good news" versus "bad news". The following result shows that frequent news arrivals lower expected volatility and increase asset prices.

Result 6 (News count and asset prices, no news arrival rate disagreement)**.** *Suppose all agents share a common belief about news arrival rate. Holding constant the average expectation of the fundamental growth, $G_t - B_t$, a higher number of news events ν_t reduces the risk-neutral variance $\mathrm{Var}_t^*[p_T/p_t - 1]$ and increases the ratio of the current asset price to the current fundamental, $p_t/e^{\sigma n_t}$.*

When investors hold similar expectations about the frequency of news arrivals, derivative trading becomes insignificant, as one key source of speculative demand—heterogeneous views on news arrival rate—is absent. In this environment, frequent occurrence of news events spur more active trading of the underlying asset, concentrating wealth among agents whose beliefs are more closely aligned with realized fundamentals.

For instance, frequent but offsetting fundamental shocks, neutral on average, benefits moderate investors, while those with more extreme views are more likely to incur losses. Consider the initial wealth distribution where both u is uniformly distributed but the cross-section of beliefs of v tightly centered near 1/2. The blue path in Figure 2 features more frequent news arrivals than the red path. By time $t = 4$, the wealth-weighted distribution along the blue path becomes more concentrated around 'the center' relative to the red path. This contrast is visualized in Figure 5b, which is in line with the analysis underlying Result 6—under low disagreement, wealth is more concentrated along the blue path ($\nu_4 = 4$), which features more frequent news arrivals, than along the red path ($\nu_4 = 2$). The convergence of the wealth-weighted distribution lowers implied volatility and raises asset prices.

Notably, the lack of disagreement in news arrival rate results in mean reversion of volatility. Frequent news arrivals lead to high realized volatility in both fundamentals and asset prices, reflecting the market's continuous adjustment to new information. However, news also sustains active trading of the underlying asset, which diminishes disagreement and consequently lowers implied volatility.

3.3 Speculation on return autocorrelation

Our trinomial tree model could be related to a periodic recombining binary tree model, where agents' beliefs about the binary distribution of news (being good or bad) switch periodically. To be precise, we consider the following parametrization of subjective beliefs on the trinomial tree (g, m, b) using (h, ε):

$$g = h(h+\varepsilon), \quad m = 2h(1-h) - \varepsilon, \quad b = (1-h)(1-h+\varepsilon)\,, \tag{26}$$

where $h \in (0, 1)$ and ε is small number around zero.[10] Although equivalent to some of the trinomial tree settings, the recombining binary tree model has a rather different interpretation: h captures agents' overall bullishness about market going up; ε captures agents belief about inter-temporal correlation of news between two sub-periods ($t \to t+0.5$ and $t + 0.5 \to t + 1$).

The expression of m from (26) indicates two channels that could drive volatility disagreement. The first channel is through disagreement in ε: agents who believe a positive correlation between sub-periods would perceives a higher volatility of fundamental between t and $t + 1$. Figure 3 shows a numerical example with three periods where $h = 0.5$ across all agents and $\varepsilon \in [-1/2, 1/2]$ follows a uniform distribution.[11] The red path's sub-period history on the binary tree is (up, down, up, down) and the blue path history is (down, down, up, up). The disagreement of ε breaks the permutation symmetry of the binary tree model and results different pricing and sentiment about volatility. On the red path, those who beliefs negative ε would become hindsight winners and Mr. Market's belief of ε is negative at $t = 2$. On the contrary, the blue path results a positive sentiment $\mathcal{E}$ because the positive correlations of news between $0 \to 0.5$ and $0.5 \to 1$ and between $1 \to 1.5$ and $1.5 \to 2$. The lower $\mathcal{E}$ (recall that $\mathcal{E} = V - 0.5$) means a lower sentiment of news intensity which would push price higher: 'no news is good news'.

The second channel is through disagreement in h, agents with extreme bullishness (in terms of h) or bearishness (in terms of $1 - h$) would think market is more volatile across multiple sub-periods. In the internet appendix (Figure I.A.5), we show another numerical

[10] The technical condition for (g, m, b) to be probability measure is when ε has to satisfy

$$\max(-h, h - 1, -h^2 - (1 - h)^2) \leq \varepsilon \leq \min(h, 1 - h, 2h(1 - h)).$$

The more extreme h is, the narrower the range of ε is around 0.

[11] Under this setting, individual agents beliefs in (26) become

$$g = 1/4 + 1/2\,\varepsilon, \quad m = 1/2 - \varepsilon, \quad b = 1/4 + 1/2\,\varepsilon\,.$$

Let $g = 1/2\,v$, $m = 1 - v$, and $b = 1/2\,v$ by replacing $v = 1/2 + \varepsilon$, this is equivalent to a trinomial tree model when disagreement about news content is absent and $u = 0.5$ for all agents.

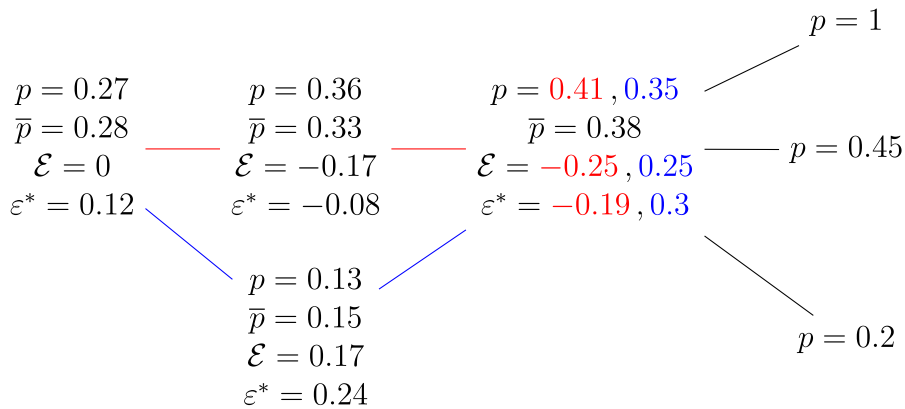


Figure 7: **A numerical example of ε-disagreement**

Agents' beliefs ε are uniformly distributed and they agree on $h = 1/2$. We only report pricing at $t = 0, 1, 2, 3$.

example with $\varepsilon = 0$ and h varies across agent with uniform distribution.[12] In this setting, the permutation symmetry of the binary tree would be preserved and pricing would not be different after experience the red or blue news sample paths.

4 Continuous time limits

We investigate two continuous time limits motivated by that fact that information about fundamental may either arrives continuously in small increments or comes discretely in lumps. The former can be formally modeled as a Brownian motion, while the latter is represented by a Poisson process. The trinomial tree model has closed form formulas on price and volatility in continuous-time limits which enable us to make comparative studies conveniently.

[12] Under this setting, (26) becomes

$$g = h^2, \quad m = 2h(1-h), \quad b = (1-h)^2 \,,$$

and this is essentially a recombining binary tree where agents disagree about the probability h of an up-move. As we pointed out in section 2.1, the pricing of market asset in this case is equivalent to the model in Martin and Papadimitriou (2022) if we recombine the odd and even periods into one.

4.1 Brownian limit

To consider a Brownian limit, the interval $[0, T]$ is partitioned into $N = T/\delta$ periods, each with length δ. The terminal payoff is defined as $p_T(n_T) = e^{\sigma\sqrt{\frac{T}{N}}n_T}$, reflecting that each piece of news has a smaller effect on the fundamental, which is scaled as $\sqrt{T/N}n_T$. This is the same setting as in Cox, Ross and Rubinstein (1979). As the number of steps increases, the wealth distribution parameters $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0})$ may also be adjusted to ensure meaningful limiting behavior.

As an illustration, we set the parameters as following

$$\alpha_{u0} = \frac{1}{2}\lambda\theta N + \frac{1}{2}\eta\sqrt{N}\,, \quad \beta_{u0} = \frac{1}{2}\lambda\theta N - \frac{1}{2}\eta\sqrt{N}\,, \quad \alpha_{v0} = \lambda\theta N\,, \quad \beta_{v0} = \alpha_{v0}(\lambda^{-1} - 1)\,,$$

where $\alpha_{u0} + \beta_{u0} = \alpha_{v0}$ always hold and the 2D-Beta distribution in (8) would reduce to Dirichlet distribution $B(\alpha_{u0}, \beta_{u0})B(\alpha_{v0}, \beta_{v0}) = B(\alpha_{u0}, \beta_{v0}, \beta_{u0})$. The cross-sectional average of (u, v) would be

$$\mathbb{E}\, u = \frac{1}{2} + \frac{\eta\lambda}{2\theta}N^{-\frac{1}{2}}, \quad \mathbb{E}\, v = \lambda,$$

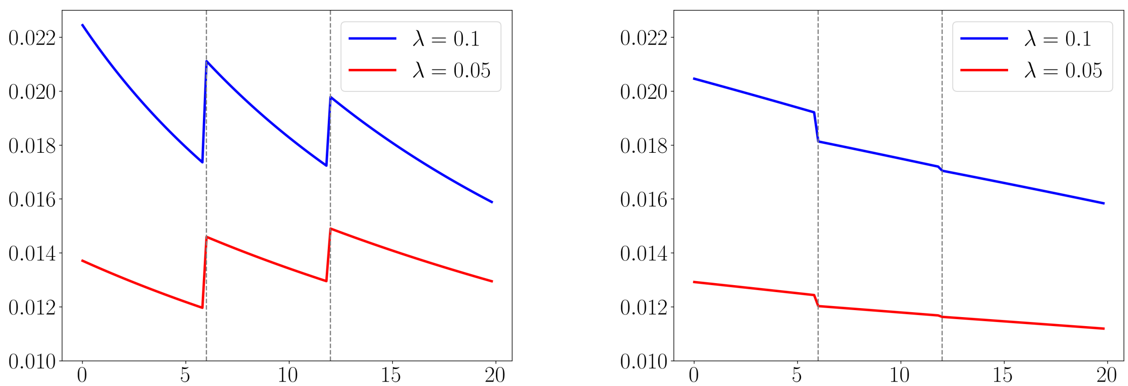
where θ captures the disagreement of news content and the model reduce to homogeneous belief case when $\theta \to \infty$. Note that the disagreement of news arrival rate does not affect $\widetilde{\mathbb{E}}v$ and those with inaccurate beliefs of v would become irrelevant in the diffusion limit—parameter θ only effectively capture the disagreement of news content.

Applying results 1 and equation (16) in the large-N limit, we obtain closed-form expressions for the prices of the market and risk-neutral variances.

Result 7 (Brownian limit)**.** *In the Brownian limit, the market price at time t is*

$$p_t = \exp\left[\frac{1-\phi}{\theta+\phi}\eta\sigma\sqrt{T} + \left(1 + \frac{1-\phi}{\theta+\phi}\right)\eta_t\sigma\sqrt{\phi T} - \frac{1}{2}\frac{\theta+1}{\theta+\phi}(1-\phi)\lambda\sigma^2 T\right]. \qquad (27)$$

where $\phi = t/T$ and $\eta_t = n_t/\sqrt{\phi N}$.



(a) High v-disagreement

(b) Low v-disagreement

Figure 8: **Implied volatility trajectories**

The figure plots the implied volatility under from t to T under high disagreement ($\alpha_{v0} = 2$) and low disagreement ($\alpha_{v0} = 1000$) about news arrival rate, following a "good news" event at $t = 6$ and a "bad news" event at $t = 12$. The parameters are $\alpha_{u0} = \beta_{u0} = 1$, and the terminal payoff at $T = 20$ is $e^{0.05n_T}$. Blue lines correspond to a higher expected news arrival rate $\lambda = 0.1$, while red lines correspond to a lower rate $\lambda = 0.05$. We 'annualized' by computing $\sqrt{\text{Var}^*[p_T/p_t - 1]/(T-t)}$.

The risk-neutral variance of the return from t to T is given by:

$$\text{Var}^*[p_T/p_t - 1] = \exp\left(\frac{\theta+1}{\theta+\phi}(1-\phi)\lambda\sigma^2 T\right) - 1\,. \tag{28}$$

The above Brownian limit corresponds to that of a binary tree model with only heterogeneous beliefs about news content: everyone agrees that news always come with a fixed probability λ and everyone knows it's the truth. [13] The risk-neutral variance between t and T is deterministic and unaffected by news realizations.

4.2 Poisson limit

In the Poisson limit, we partition the interval $[0, T]$ into $N = T/\delta$ periods. Each piece of news is assumed to have a substantial impact on fundamentals, and the terminal payoff remains identical to the discrete-time case: $p_T = e^{\sigma n_T}$. With α_{u0}, β_{u0}, and α_{v0} held constant and independent of N, and β_{v0} defined as

$$\beta_{v0} = \alpha_{v0} \left(\lambda^{-1} \frac{N}{T} - 1 \right) ,$$

the cross-sectional averages of (u, v) are given by $\mathbb{E}\, u = \alpha_{u0}/(\alpha_{u0} + \beta_{u0})$ and $\mathbb{E}\, v = \lambda\delta$. In the limit as $N \to \infty$ and $\delta \to 0$, the counts of "good news" and "bad news" events follow Poisson distributions.

Taking the limit $N \to \infty$, we obtain market price and risk-neutral variance.

Result 8 (Poisson limit)**.** *In the Poisson limit, the market price at time t is given by*

$$p_t = e^{\sigma n_t} \left[1 - \Lambda_t (e^{\sigma} - 1)\right]^{\alpha_{vt}} {}_2F_1 \left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^{\sigma} - e^{-\sigma}}{e^{\sigma} - 1 - \Lambda_t^{-1}} \right)^{-1} , \tag{29}$$

where $\Lambda_t = (T - t)/(\alpha_{v0}\lambda^{-1} + t)$, ${}_2F_1(\ldots)$ denotes the Gauss hypergeometric function, and the expressions for α_{vt}, α_{ut}, and β_{ut} are provided in (19).

The risk-neutral variance of the return from t to T is:

$$\mathrm{Var}^*[p_T/p_t - 1] = p_t^{-1} e^{\sigma n_t} \left[1 + \Lambda_t (1 - e^{-\sigma})\right]^{-\alpha_{vt}} {}_2F_1 \left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^{\sigma} - e^{-\sigma}}{1 + \Lambda_t^{-1} - e^{-\sigma}} \right) - 1 . \tag{30}$$

We consider an example with $T = 20$, with $\alpha_{u0} = \beta_{u0} = 1$, and a terminal payoff given by $e^{0.05 n_T}$. Figure 8 presents trajectories for the implied volatility $\sqrt{\mathrm{Var}^*[p_T/p_t - 1]/(T - t)}$ following a "good news" event at $t = 6$ and a "bad news" event at $t = 12$, under high and low disagreement about the news arrival rate v. It also represents the volatility strike

[13] The case when $\lambda = 1$ has been studied extensively in Martin and Papadimitriou (2022). At time $t = 0$, equation (27) has a simple expression $p_0 = \exp\left[\frac{\eta}{\theta}\sigma\sqrt{T} - \frac{1+\theta}{2\theta}\lambda\sigma^2 T\right]$.

of the simple variance swap, which is introduced by Martin (2011). [14] In the left panel, implied volatility rises after news arrivals, following result 5. In contrast, the right panel shows implied volatility falling after news events, reflecting result 6.

4.3 Survival of agents with inaccurate beliefs

Since the market reflects wealth-weighted beliefs, as shown in result 2, a natural question arises regarding the long-run survival of agents with incorrect beliefs. Examining survival behavior as N grows large, when the trading interval shortens and agents trade more frequently, yields important insights. It turns out the Kullback-Leibler divergence—defined as $KL(a, b) = a\log(a/b) + (1-a)\log(1-a)/(1-b)$ for Bernoulli distributions with parameters a and b—between any agent's belief and the truth governs the rate at which an agent with incorrect beliefs becomes irrelevant in the long run. This fundamental insight is a key theme in the literature on belief selection and wealth dynamics (e.g., Blume and Easley (2006)).

Result 9 (Survival Index)**.** *Assume there is truth (u_{true}, v_{true}) that guarantees the data generating process of fundamental news, the wealth share of agent with beliefs (u, v) will decay exponentially at the rate of $KL(v_{true}, v) + v_{true}\, KL(u_{true}, u)$ per trading period.*

Result 9 indicates the true news arrival rate affects survival negatively—the rarer the news is, the longer it takes for agents with wrong beliefs to be extinct. Moreover, when the true news arrival rate is low, having an accurate understanding of the flow of information is more crucial than knowing whether the news will be positive or negative.

A dichotomy about existence of volatility disagreement emerges. In a Brownian limit, persistent disagreement about the frequency of news arrivals cannot be sustained. By Result 9, agents with beliefs $v \neq v_{\text{true}}$ lose wealth each period at a strictly positive rate.

[14] A well-known derivative related to the VIX index is the *variance swap*. In this contract, one party pays the other the difference between the actual variance of an asset's returns and a preset strike variance. Martin (2011) noted that the usual log-normal assumption may not hold during crises times, and introduced a jump-robust version called the *simple variance swap*. In our model, the contract payoff is $\mathcal{V}_t/p_t^2$ where $\mathcal{V}_t$ is $(p_{t+1} - p_t)^2 + (p_{t+2} - p_{t+1})^2 + \cdots + (p_T - p_{T-1})^2$. The time t price of this payoff is equal to the risk-neutral variance $\text{Var}^*[p_T/p_t - 1]$.

As the number of trading periods grows, their repeated mistakes drive their wealth share to zero. Short-maturity volatility derivatives allow investors to make repeated bets on continuous variance, quickly reallocating wealth toward those with more accurate beliefs and rendering residual disagreements largely irrelevant for prices.

In contrast, because jump events arise only occasionally, the Poisson limit supports enduring disagreement over news arrival frequency, which can materially affect asset prices. The impact of such disagreement on expected wealth growth is captured by:

$$KL(v_{true}, v) = v_{true} \log\left(\frac{v_{true}}{v}\right) + (1 - v_{true}) \log\left(\frac{1 - v_{true}}{1 - v}\right). \tag{31}$$

As $N \to \infty$, both v_{true} and v approach zero. The second term, representing the expected effect during no news periods, remains bounded. Moreover, since news arrives infrequently, the cumulative impact of news events—captured by the first term, $v_{\text{true}} \log(v_{\text{true}}/v)$—is also limited. These bounds ensure that agents with incorrect beliefs survive. Overall, this contrast indicates that volatility speculation tends to reflect disagreements over discrete news arrivals rather than over continuous diffusion risk.

5 Conclusion

Our study underscores the critical role of multidimensional belief heterogeneity in shaping financial market dynamics, revealing how disagreements over news content and arrival frequency drive asset pricing, volatility, and portfolio decisions. By introducing a trinomial tree framework that incorporates a gamma-exposed derivative, our model isolates the distinct effects of those two belief dimensions, demonstrating that frequent news can both mitigate content disagreement by redistributing wealth to accurate believers and amplify volatility when news frequency expectations diverge. The model is tractable and produces an U-shaped pattern of Gamma exposure among optimists and pessimists of market return, which highlights the speculative nature of trading driven by these heterogeneous beliefs.

These findings deepen our understanding of how complex belief structures influence behaviors of underlying and derivative markets, providing new insights on volatility persistence and asset valuation. Future research could explore empirical validations of these dynamics or extend the model to incorporate additional belief dimensions, further enriching the analysis of financial markets under heterogeneous expectations.

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A Mathematical Appendix

Proof for equations (18) and (19). Comparing the individual wealth growth and aggregate wealth growth equations in (5) and (6), we could derive the following equation for change of wealth distribution

$$\frac{f_{t+1}(g,m,b)}{f_t(g,m,b)} = \begin{cases} \frac{g}{G_t}, & \text{`good news' at } t+1 \\ \frac{m}{M_t}, & \text{`no news' at } t+1 \\ \frac{b}{B_t}, & \text{`bad news' at } t+1 \end{cases}$$

Solving this recursively back from t to 0, the wealth distribution $f_t(g,m,b)$ should satisfy

$$f_t(g,m,b) = \lambda_t \, f_0(g,m,b) \, g^{n_{gt}} m^{n_{mt}} b^{n_{bt}}$$

where the constant λ_t is path-dependent. Take integrals of both sides, we have

$$1 = \int \lambda_t \, f_0(g,m,b) \, g^{n_{gt}} m^{n_{mt}} b^{n_{bt}} \, dg \, dm \, db,$$

and it implies

$$\lambda_t = \frac{B(\alpha_{u0},\beta_{u0})B(\alpha_{v0},\beta_{v0})}{B(\alpha_{ut},\beta_{ut})B(\alpha_{vt},\beta_{vt})},$$

where $\alpha_{ut} = \alpha_{u0} + n_{gt}$, $\beta_{ut} = \beta_{u0} + n_{bt}$, $\alpha_{vt} = \alpha_{v0} + n_{gt} + n_{bt}$, and $\beta_{vt} = \beta_{v0} + n_{mt}$. Converting the notation of (n_{gt}, n_{mt}, n_{bt}) into (n_t, ν_t), we obtain result in equations (18) and (19).

□

Proof for Result 1. Due to (7), we could solve from T backwards to t the value $1/p_t$. It is merely the linear combination of wealth-weighted aggregate beliefs of all possible future sample paths between t and T multiplying with the one over terminal payoff $1/p_T(\mathbf{s}_t)$. To formally prove this is statement, let (a,b,c) be the number of moves that correspond to (high, middle, low) of a sample path from t to T. They must satisfy

$$a - c = n_t, \quad a + b + c = T - t\,.$$

The above could be solved in terms of b as

$$a = \frac{1}{2}(T - t + n - b), \quad c = \frac{1}{2}(T - t - n - b).$$

and (a, c) are integers only when b and $T-t-n$ are both odd or even numbers ($T-t+n$ is always the same as $T-t-n$). Since the arguments for two cases when n is positive and negative are symmetric, we focus on $n > 0$. If $T-t-n$ is odd number, b starts from t+1, otherwise it starts from t. The size of b can not exceeds $T-t-n$ for c stays non-negative. Once b is fixed, the value of a and c are chosen. There are $\frac{(T-t)!}{a!b!c!}$ possible paths that satisfies this characteristic, i.e. a times high, b times middle and c times low.

We now prove that those paths could be merged since they make equal contribution to the coefficient c_n. To show this, it suffices to show the three basic cases: 1) two trips of 'middle-low' and 'low-middle' would give the same risk-neutral probability given the same initial condition; 2) two trips of 'middle-high' and 'high-middle' are equivalent; 3) two trips of 'low-high' and 'high-low' are equivalent.

The risk-neutral variance of the return from t to T, $p_T/p_t - 1$, is given by:

$$\frac{1}{p_t^2}\,\mathbb{E}_t^*[p_T^2 - p_t^2] = \mathbb{E}_t[p_T(n_T)]/p_t - 1 \tag{A1}$$

Substituting in (15), we obtain (16).

$\square$

Proof for Result 2. Let $f_0(u, v)$ denote the prior distribution of (u, v) at time 0. Given n_{gt}, n_{mt}, and n_{bt} realizations of good, no, and bad news up to time t, the posterior is

$$f_t(u, v) = \frac{\big(uv\big)^{n_{gt}}\big(1-v\big)^{n_{mt}}\big((1-u)v\big)^{n_{bt}} f_0(u, v)}{\int_0^1\int_0^1 \big(uv\big)^{n_{gt}}\big(1-v\big)^{n_{mt}}\big((1-u)v\big)^{n_{bt}} f_0(u, v)\,\mathrm{d}u\,\mathrm{d}v} \tag{A2}$$

When the prior distribution takes the 2D Beta form in (8), the corresponding time-t posterior distribution simplifies to (18).

In general, the representative agent's belief distribution over terminal states n_T follows (17). With log utility, the inverse price is a martingale,

$$p_t^{-1} = \sum_{n_T} \Pr(n_T | n_t, f_t) p_T^{-1}(n_T) \,, \tag{A3}$$

consistent with (15).

□

Proof for Results 3 and 4. Consider the group of agents whose subjective expectation of the market return equals r:

$$\mathbb{E}_t^{(u,v)}[p_{t+1}/p_t - 1] = r \,. \tag{A4}$$

We can express u as a function of v and r:

$$u = \frac{p_{t+1,m} - p_{t+1,b}}{p_{t+1,g} - p_{t+1,b}} + \frac{(1+r)p_t - p_{t+1,m}}{(p_{t+1,g} - p_{t+1,b})v} \tag{A5}$$

When $r < p_{t+1,m}/p_t - 1$, u stays in $[0,1]$ if and only if $v \in [(p_{t+1,m} - (1+r)p_t)/(p_{t+1,m} - p_{t+1,b}), 1]$. When $r > p_{t+1,m}/p_t - 1$, u stays in $[0,1]$ if and only if $v \in [((1+r)p_t - p_{t+1,m})/(p_{t+1,g} - p_{t+1,m}), 1]$.

Substituting (A5) into (8), the conditional distribution of v given r satisfies

$$f_t(v|r) \propto (v + kz)^{\alpha_{ut}-1} \, (v - z)^{\beta_{ut}-1} \, v^{\alpha_{vt}-\alpha_{ut}-\beta_{ut}} (1-v)^{\beta_{vt}-1} \,, \tag{A6}$$

where z as a function of r represents the scaled return and k is a constant

$$z = \frac{(1+r)p_t - p_{t+1,m}}{p_{t+1,g} - p_{t+1,m}} = \frac{1 + r - M_t/m_t^*}{G_t/g_t^* - M_t/m_t^*} \,, \tag{A7}$$

$$k = \frac{p_{t+1,g} - p_{t+1,m}}{p_{t+1,m} - p_{t+1,b}} = \frac{G_t/g_t^* - M_t/m_t^*}{M_t/m_t^* - B_t/b_t^*} \,. \tag{A8}$$

Integrating the joint distribution $f_t(v,z)$ over v yields the marginal distribution of z:

$$
\begin{aligned}
f_t(z) = &\left(\frac{k}{k+1}\right)^{\beta_{ut}} \frac{B(\beta_{ut},\beta_{vt})}{B(\alpha_{ut},\beta_{ut})B(\alpha_{vt},\beta_{vt})} z^{\alpha_{vt}-1}(1-z)^{\beta_{ut}+\beta_{vt}-1} \\
&F_1\left(\beta_{ut};\alpha_{vt}+\beta_{vt}-1, 1-\alpha_{ut}, \beta_{ut}+\beta_{vt}; 1-z, \frac{k(1-z)}{1+k}\right)
\end{aligned}
$$

for $z>0$ and

$$
\begin{aligned}
f_t(z) = &\frac{k}{(k+1)^{\alpha_{ut}}} \frac{B(\alpha_{ut},\beta_{vt})}{B(\alpha_{ut},\beta_{ut})B(\alpha_{vt},\beta_{vt})} (-kz)^{\alpha_{vt}-1}(1+kz)^{\alpha_{ut}+\beta_{vt}-1} \\
&F_1\left(\alpha_{ut};\alpha_{vt}+\beta_{vt}-1, 1-\beta_{ut};\alpha_{ut}+\beta_{vt}; 1+kz, \frac{1+kz}{1+k}\right)
\end{aligned}
$$

for $z<0$. Substituting into $f_t(r) = p_t/(p_{t+1,g}-p_{t+1,m})f_t(z)$, we obtain the two equations in Result 3.

From the distribution $f_t(v|r)$, the conditional expectation $\mathbb{E}[v|r]$ can be expressed in terms of ratios of Appell hypergeometric functions. This expression allows us to compute the wealth-weighted average derivative position for this group of agents. Specifically, when $z \geqslant 0$, we obtain:

$$
\mathbb{E}[v|r] = z + z(1-z)\cdot\frac{\beta_{ut}}{\alpha_{ut}+\beta_{vt}}\cdot\frac{F_1\left(\beta_{ut}+1;\alpha_{vt}+\beta_{vt}, 1-\alpha_{ut};\beta_{ut}+\beta_{vt}+1; 1-z, \frac{k(1-z)}{1+k}\right)}{F_1\left(\beta_{ut};\alpha_{vt}+\beta_{vt}-1, 1-\alpha_{ut};\beta_{ut}+\beta_{vt}; 1-z, \frac{k(1-z)}{1+k}\right)}.
$$

When $z<0$, we obtain:

$$
\mathbb{E}[v|r] = -kz + (-kz)(1+kz)\cdot\frac{\alpha_{ut}}{\beta_{ut}+\beta_{vt}}\frac{F_1\left(\alpha_{ut}+1;\alpha_{vt}+\beta_{vt}, 1-\beta_{ut};\alpha_{ut}+\beta_{vt}+1; 1+kz, \frac{1+kz}{1+k}\right)}{F_1\left(\alpha_{ut};\alpha_{vt}+\beta_{vt}-1, 1-\beta_{ut};\alpha_{ut}+\beta_{vt}; 1+kz, \frac{1+kz}{1+k}\right)}.
$$

For agent with belief (u,v), the expected excess payoff of the derivative is given by:

$$
\begin{aligned}
\mathbb{E}_t^{(u,v)}[x_{t+1}-q_t] &= p_t(b_t^*-B_t)uv + p_t(G_t-g_t^*)(1-u)v - p_t(b_t^*G_t - g_t^*B_t) \\
&= \frac{\mathrm{Var}_t^*[p_{t+1}]}{p_{t+1,g}-p_{t+1,b}}\left(v - v_t^* + \frac{(v_t^*-V_t)}{\mathbb{E}^{(U_t,V_t)}[r]}r\right) = \frac{\mathrm{Var}_t^*[x_{t+1}-q_t]}{m_t^* q_t}\left(v - v_t^* + \frac{(v_t^*-V_t)}{\mathbb{E}^{(U_t,V_t)}[r]}r\right).
\end{aligned}
\tag{A9}
$$

Substituting this expression into (12), we obtain (23). □

Proof for Result 5. Suppose news arrival rate disagreement is stronger than the Dirichlet benchmark $\alpha_{v0} \leqslant \alpha_{u0} + \beta_{u0}$. Holding the average expectation of the fundamental growth $G_t - B_t$ constant, a higher historical news count $\nu_t \geqslant \nu'_t$ suggests that

1. $g + b = v$ first-order stochastically dominates $g' + b' = v'$

2. $g - b = (2u - 1)v$ second-order stochastically dominates $g' - b' = (2u' - 1)v'$

Thus, $n_T - n_t$ is a mean-preserving spread of $n'_T - n'_t$. As a result,

$$\begin{aligned} p_t^{-1} &= \sum_{n_T - n_t} \Pr(n_T - n_t | n_t, f_t)\, e^{-\sigma(n_T - n_t)}\, e^{-\sigma n_t} \\ &\geqslant \sum_{n'_T - n'_t} \Pr(n'_T - n'_t | n'_t, f'_t)\, e^{-\sigma(n'_T - n'_t)}\, e^{-\sigma n_t} = (p'_t)^{-1}\, e^{-\sigma(n_t - n'_t)} \end{aligned} \tag{A10}$$

confirming $p_t/e^{\sigma n_t} \leqslant p'_t/e^{\sigma n'_t}$. Similarly,

$$\begin{aligned} \mathbb{E}_t\left[p_T | n_t, f_t\right] &= \sum_{n_T - n_t} \Pr(n_T - n_t | n_t, f_t)\, e^{\sigma(n_T - n_t)}\, e^{\sigma n_t} \\ &\geqslant \sum_{n'_T - n'_t} \Pr(n'_T - n'_t | n'_t, f'_t)\, e^{\sigma(n'_T - n'_t)}\, e^{\sigma n_t} = \mathbb{E}_t\left[p'_T | n'_t, f'_t\right] e^{\sigma(n_t - n'_t)} \end{aligned} \tag{A11}$$

Multiplying the above two inequalities, we obtain

$$\mathrm{Var}_t^*\left[p_T/p_t - 1\right] \geqslant \mathbb{E}_t\left[p'_T | n'_t, f'_t\right]/p'_t - 1 = \mathrm{Var}_t^*\left[p'_T/p'_t - 1\right] \tag{A12}$$

□

Proof for Result 6. The risk-neutral variance of the market return satisfies

$$\mathrm{Var}_t^*\left[p_T/p_t - 1\right] = \mathbb{E}_t^*\left[p_T^2/p_t^2\right] - 1 = \mathbb{E}_t\left[p_T | n_t, f_t\right]/p_t - 1$$

Holding news arrival rate v and the average expectation of the fundamental growth $G_t - B_t$ constant, a higher historical news count $\nu_t \geqslant \nu'_t$ reduces disagreement in bullishness u and resulting in a more concentrated distribution of fundamental growth $n_T - n_t$, such that $n'_T - n'_t$ is a mean-preserving spread of $n_T - n_t$.

The rest follows from the proof for Result 5. $\square$

Proof for Result 7. By construction, from Mr. Market's point of view at time 0, the variables (n_{gT}, n_{mT}, n_{bT}) follows a Dirichlet distribution $B(\alpha, \beta, \gamma) = B(\frac{1}{2}\theta\lambda N + \frac{1}{2}\eta\sqrt{N}, \theta N(1-\lambda), \frac{1}{2}\theta\lambda N - \frac{1}{2}\eta\sqrt{N})$. It is then straight forward to compute the reciprocal of initial price using result 1. To get a close form formula, we use Paul and Plackett (1978). In the limit when $\mathbb{N} \to \infty$, the variable $n_T = n_{gT} - n_{bT}$ follows normal distribution with the mean $\mathbb{E}\, n_T = N\left(\frac{\alpha-\gamma}{\alpha+\beta+\gamma}\right) = \frac{\eta}{\theta}\sqrt{N}$ and the variance $\mathrm{var}[n_T] = N\left(\lambda - \frac{\eta^2}{\theta^2}N^{-1}\right)\frac{1+\theta}{\theta} \to \lambda\frac{1+\theta}{\theta}N$. Thus the following random variable $n_T/\sqrt{N}$ follows a normal distribution in large N limit $\frac{n_T}{\sqrt{N}} \sim \mathcal{N}\left(\frac{\eta}{\theta}, \frac{\lambda(1+\theta)}{\theta}\right)$. Consequently, the reciprocal of price is given by $p_0^{-1} = \mathbb{E}\exp\left(-\sigma\sqrt{T}\frac{n_T}{\sqrt{N}}\right)$ and it implies $p_0 = \exp\left(\frac{\eta}{\theta}\sigma\sqrt{T} - \frac{\lambda(1+\theta)}{2\theta}\sigma^2 T\right)$.

For market price at time t, we need to adjust the wealth distribution and also the number of periods. The number of periods left is $N(1-t/T) = N(1-\phi)$, where $\phi = t/T$. The random variables $(n_{gT} - n_{gt}, n_{mT} - n_{mt}, n_{bT} - n_{bt})$ follows a Dirichlet distribution at time t $B_t(\alpha', \beta', \gamma') = B_t(\frac{1}{2}\lambda(\theta+\phi)N + \frac{1}{2}\eta\sqrt{N} + \frac{1}{2}\eta_t\sqrt{\phi N}, (\theta+\phi)N(1-\lambda), \frac{1}{2}\theta\lambda(1+\phi)N - \frac{1}{2}\eta\sqrt{N} - \frac{1}{2}\eta_t\sqrt{\phi N})$. The parameter η_t captures the wealth transfer between 0 and t among agents because of heterogeneous beliefs about news content $\eta_t = \frac{n_t}{\sqrt{\phi N}}$. The price at time t can then be written as $p_t^{-1} = \mathbb{E}_t\, p_T^{-1} = e^{-\eta_t\sigma\sqrt{\phi T}}\,\mathbb{E}_t\left(\exp\sigma\sqrt{T/N}(n_t - n_T)\right)$.

The large N limits for mean and variance of $n_T - n_t$ (which would be Gaussian distributed), according to Paul and Plackett (1978), shall be $\mathbb{E}_t[n_T - n_t] = (1-\phi)N\frac{\alpha'-\gamma'}{\alpha'+\beta'+\gamma'} = \frac{1-\phi}{\theta+\phi}(\eta+\eta_t\sqrt{\phi})\sqrt{N}$ and $\mathrm{var}_t[n_T - n_t] = (1-\phi)N\left(\lambda - \left(\frac{\eta+\eta_t\sqrt{\phi}}{\theta}\frac{1-\phi}{1+\phi}\right)^2 N^{-1}\right)\left((1-\phi) + \frac{1}{\theta+\phi}\right) \to \lambda\frac{\theta+1}{\theta+\phi}(1-\phi)N$. We can then compute p_t as

$$p_t = \exp\left(\frac{1-\phi}{\theta+\phi}\eta\sigma\sqrt{T} + \left(1 + \frac{1-\phi}{\theta+\phi}\right)\eta_t\sigma\sqrt{\phi T} - \frac{\lambda(\theta+1)}{2(\theta+\phi)}(1-\phi)\sigma^2 T\right)$$

The risk-neutral variance. Similar argument gives us

$$\mathbb{E}\, p_T(n_T) = \lim_{N\to\infty}\mathbb{E}\exp\left(\sigma\sqrt{T}\frac{n_T}{\sqrt{N}}\right) = \exp\left(\frac{\eta}{\theta}\sigma\sqrt{T} + \frac{\lambda(1+\theta)}{2\theta}\sigma^2 T\right).$$

Thus, we have $\mathcal{V}_{0\to T} = \exp\left(\frac{\lambda(1+\theta)}{\theta}\sigma^2 T\right)$. $\square$

Proof for Result 8. Let $\xi = v/\delta$ denote the continuous-time Poisson arrival rate of news. As $\delta \to 0$, the distribution of ξ at time t converges to a Gamma distribution $\xi \sim \Gamma(\alpha_{vt}\,, \alpha_{v0}\lambda^{-1} + t)$.

For an agent with belief $(u, \xi\delta)$, the arrival rate of "good news" is ξu and bad news $\xi(1-u)$. The counts of "good news" and "bad news" for the time interval $[t, T]$ respectively follow Poisson distributions $n_{gT} - n_{gt} \sim \text{Poisson}(\,\xi u(T-t)\,)$, $n_{bT} - n_{bt} \sim \text{Poisson}(\,\xi(1-u)(T-t)\,)$.

Using the moment generating functions for $n_{gT} - n_{gt}$, $n_{bT} - n_{bt}$ and ξ, we find that

$$
\begin{aligned}
p_t^{-1} &= e^{-\sigma n_t}\,\mathbb{E}[e^{-\sigma(n_T - n_t)}|f_t] = e^{-\sigma n_t}\int e^{-\sigma(n_{gT}-n_{gt})-\sigma(n_{bT}-n_{bt})} f_t(u,\xi)d\xi du \\
&= e^{-\sigma n_t}\int \exp\big(\xi u(T-t)(e^{-\sigma}-1) + \xi(1-u)(T-t)(e^{\sigma}-1)\big) f_t(u,\xi)d\xi du \\
&= e^{-\sigma n_t}\int\left(1 - \frac{u(T-t)(e^{-\sigma}-1) + (1-u)(T-t)(e^{\sigma}-1)}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut},\beta_{ut})}du \\
&= e^{-\sigma n_t}\left(1 - \frac{(T-t)(e^{\sigma}-1)}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut}+\beta_{ut}; -\frac{e^{\sigma}-e^{-\sigma}}{(\alpha_{v0}\lambda^{-1}+t)/(T-t) - (e^{\sigma}-1)}\right).
\end{aligned}
$$

The final step uses Euler's integral representation for the hypergeometric function ${}_2F_1$. Similarly,

$$
\begin{aligned}
\mathbb{E}[p_T] &= e^{\sigma n_t}\,\mathbb{E}[e^{\sigma(n_T-n_t)}|f_t] \\
&= e^{\sigma n_t}\int\left(1 - \frac{u(T-t)(e^{\sigma}-1) + (1-u)(T-t)(e^{-\sigma}-1)}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut},\beta_{ut})}du \\
&= e^{\sigma n_t}\left(1 + \frac{(T-t)(1-e^{-\sigma})}{\alpha_{v0}\lambda^{-1}+t}\right)^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut}+\beta_{ut}; \frac{e^{\sigma}-e^{-\sigma}}{(\alpha_{v0}\lambda^{-1}+t)/(T-t) + (1-e^{-\sigma})}\right).
\end{aligned}
$$

Substituting the above expression into (16), we obtain (30). $\square$

Proof for Result 9. For the discussion in this section, we assume there is true probability measure (u_{true}, v_{true}) that guarantees the data generating process of fundamental news. We also assume there is no learning, so agents would stick to their dogmatic beliefs.

We define the wealth share of agent (u, v) at time T with sample history (n_T, ν_T) as

$$\Omega(u, v; n_T, \nu_T) = \frac{u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T}(1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} v^{\nu_T}(1-v)^{N-\nu_T} f_0(u,v)}{\int u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T}(1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} v^{\nu_T}(1-v)^{N-\nu_T} f_0(u,v)\,\mathrm{d}u\,\mathrm{d}v}$$

We want to compute the asymptotic decay rate (exponentially) of the above quantity, which is the following limit

$$\lim_{N\to\infty} -\frac{\log \Omega(u, v; n_T, \nu_T)}{N} = \lim_{N\to\infty} -\frac{1}{N} \log \frac{u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T}(1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} v^{\nu_T}(1-v)^{N-\nu_T} f_0(u,v)}{\int u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T}(1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} v^{\nu_T}(1-v)^{N-\nu_T} f_0(u,v)\,\mathrm{d}u\,\mathrm{d}v}$$

Note that in large N limit, we could write

$$\nu_T = N v_{true}, \quad n_T = n_{gT} - n_{bT} = N(u_{true}v_{true} - (1-u_{true})v_{true}) = N(2u_{true}v_{true} - v_{true})$$

and this means we can write

$$\begin{aligned}
&-\frac{1}{N}\log\left(u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T}(1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} v^{\nu_T}(1-v)^{N-\nu_T}\right)\\
&= \underbrace{-v_{true}\log v - (1-v_{true})\log(1-v) - v_{true}\left(u_{true}\log u + (1-u_{true})\log(1-u)\right)}_{-h(u_{true}, v_{true}; u, v)}
\end{aligned}$$

and also compute

$$\begin{aligned}
&\lim_{N\to\infty} \frac{1}{N} \log \int u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T}(1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} v^{\nu_T}(1-v)^{N-\nu_T} f_0(u,v)\,\mathrm{d}u\,\mathrm{d}v\\
&= \lim_{N\to\infty} \frac{1}{N} \log \int \exp(N\,h(u_{true}, v_{true}; u, v) + \log f_0(u,v))\,\mathrm{d}u\,\mathrm{d}v\\
&= sup_{u,v}\{h(u_{true}, v_{true}; u, v)\}
\end{aligned}$$

Note that the maximum of $h(u_{true}, v_{true}; u, v)$ is achieved at (u_{true}, v_{true}) because the following quantity is always positive when $(u, v) \neq (u_{true}, v_{true})$ (this is the so called *Gibbs's inequality* in information theory, which guarantees the relative entropy is always positive between two probability measures)

$$
\begin{aligned}
&h(u_{true}, v_{true}; u_{true}, v_{true}) - h(u_{true}, v_{true}; u, v) \\
&= v_{true} \log \frac{v_{true}}{v} + (1 - v_{true}) \log \frac{1 - v_{true}}{1 - v} - v_{true} \left(u_{true} \log \frac{u_{true}}{u} + (1 - u_{true}) \log \frac{1 - u_{true}}{1 - u} \right) \\
&> 0
\end{aligned}
$$

Thus we could conclude

$$
\begin{aligned}
&\lim_{t \to \infty} -\frac{\log \Omega(u, v; n_t, \nu_t)}{t} \\
&= v_{true} \log \frac{v_{true}}{v} + (1 - v_{true}) \log \frac{1 - v_{true}}{1 - v} + v_{true} \left(u_{true} \log \frac{u_{true}}{u} + (1 - u_{true}) \log \frac{1 - u_{true}}{1 - u} \right) \\
&= \mathrm{KL}(v_{true}, v) + v_{true} \mathrm{KL}(u_{true}, u)
\end{aligned}
$$

□

Internet Appendix to

No News is News: Volatility Speculation and Multidimensional Heterogeneous Beliefs

(not for publication)

Abstract

This Internet Appendix presents supplementary material and results not included in the main body of the paper.

I.A.1 Individual's optimal portfolio choices

I.A.1.1 Agent's optimal position

Recall that we stated for an agent with beliefs (u, v), the optimal positions are given by (same as (12) in the paper)

$$\theta_t^{(u,v)} = w_t^{(u,v)} \frac{\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t]}{\mathrm{Var}_t^*[p_{t+1} - p_t]}, \quad \phi_t^{(u,v)} = w_t^{(u,v)} \frac{\mathbb{E}_t^{(u,v)}[x_{t+1} - q_t]}{\mathrm{Var}_t^*[x_{t+1} - q_t]}.$$

Derivation for Equation (12). The subjective expectation of the price change $p_{t+1} - p_t$, for an agent characterized by belief parameters (u, v) and represented by probabilities (g, m, b), is given by:

$$\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t] = g(p_{t+1,g} - p_t) + m(p_{t+1,m} - p_t) + b(p_{t+1,b} - p_t)\,. \tag{I.A.1}$$

Using equation (5),

$$g = g_t^* \cdot \frac{w_{t+1,g}^{(u,v)}}{w_t^{(u,v)}}, \quad m = m_t^* \cdot \frac{w_{t+1,m}^{(u,v)}}{w_t^{(u,v)}}, \quad b = b_t^* \cdot \frac{w_{t+1,b}^{(u,v)}}{w_t^{(u,v)}}\,. \tag{I.A.2}$$

Substituting into (I.A.1) and simplifying,

$$\begin{aligned}
&\mathbb{E}_t^{(u,v)}[p_{t+1} - p_t] \\
=&\frac{1}{w_t^{(u,v)}} \left[g_t^* w_{t+1,g}^{(u,v)}(p_{t+1,g} - p_t) + m_t^* w_{t+1,m}^{(u,v)}(p_{t+1,m} - p_t) + b_t^* w_{t+1,b}^{(u,v)}(p_{t+1,b} - p_t) \right] \\
=&\frac{1}{w_t^{(u,v)}} \left[g_t^* \theta_t^{(u,v)}(p_{t+1,g} - p_t)^2 + m_t^* \theta_t^{(u,v)}(p_{t+1,m} - p_t)^2 + b_t^* \theta_t^{(u,v)}(p_{t+1,b} - p_t)^2 \right] \\
=&\frac{1}{w_t^{(u,v)}} \theta_t^{(u,v)}\, \mathrm{Var}_t^*[p_{t+1} - p_t]\,,
\end{aligned} \tag{I.A.3}$$

where the second equality follows from the fact that the risk-neutral covariance between the derivative payoff and the market price is zero.

This concludes the proof for the market demand $\theta_t^{(u,v)}$. The proof for the derivative demand $\phi_t^{(u,v)}$ follows analogously. $\square$

I.A.1.2 Zero-delta derivative payoff

Any derivative security payoff satisfying the zero-delta assumption (2) must be spanned by the risk-free payoff $(1, 1, 1)$ and the vector $(b_t^* - B_t, 0, G_t - g_t^*)$

$$(x_{t+1,g}, x_{t+1,m}, x_{t+1,b}) \in \text{span}\left\{(1,1,1), (b_t^* - B_t, 0, G_t - g_t^*)\right\}. \tag{I.A.4}$$

Derivation for Equation (10). Assumption (2) is mathematically equivalent to

$$g_t^* x_{t+1,g} + m_t^* x_{t+1,m} + b_t^* x_{t+1,b} = G_t x_{t+1,g} + M_t x_{t+1,m} + B_t x_{t+1,b}\,. \tag{I.A.5}$$

which can be rewritten as can be rewritten as

$$(g_t^* - G_t) x_{t+1,g} + (m_t^* - M_t) x_{t+1,m} + (b_t^* - B_t) x_{t+1,b} = 0\,. \tag{I.A.6}$$

Noting that $(g_t^* - G_t) + (m_t^* - M_t) + (b_t^* - B_t) = 0$, this implies that any vector $(x_{t+1,g}, x_{t+1,m}, x_{t+1,b})$ satisfying the zero-delta condition must lie in the span of the risk-free vector $(1, 1, 1)$, and a second vector that is linearly independent it, which we may take as $(b_t^* - B_t, 0, G_t - g_t^*)$. $\square$

I.A.1.3 Gamma exposure for individual agent

$$\Gamma^{(u,v)} = \frac{2 g_t^* b_t^*}{p_t\left(G_t b_t^* - g_t^* B_t\right)} \left[\frac{g-m}{G_t - M_t} - \frac{m-b}{M_t - B_t}\right] f_t(u,v) \tag{I.A.7}$$

Recall the variables (g, m, b) in (I.A.7) can be expressed in terms of (u, v) using (1).

Derivation for (I.A.7). The expression is derived by constructing a Lagrange interpolating polynomial $w(p)$ that expresses the agent's wealth as a function of price, based on the wealth–price pairs from three scenarios: $(p_{t+1,g}, w_{t+1,g}^{(u,v)})$, $(p_{t+1,m}, w_{t+1,m}^{(u,v)})$, and $(p_{t+1,b}, w_{t+1,b}^{(u,v)})$.

$$\begin{aligned} w(p) =& w_{t+1,g}^{(u,v)} \cdot \frac{(p - p_{t+1,m})(p - p_{t+1,b})}{(p_{t+1,g} - p_{t+1,m})(p_{t+1,g} - p_{t+1,b})} + w_{t+1,m}^{(u,v)} \cdot \frac{(p - p_{t+1,g})(p - p_{t+1,b})}{(p_{t+1,m} - p_{t+1,g})(p_{t+1,m} - p_{t+1,b})} \\ &+ w_{t+1,b}^{(u,v)} \cdot \frac{(p - p_{t+1,g})(p - p_{t+1,m})}{(p_{t+1,b} - p_{t+1,g})(p_{t+1,b} - p_{t+1,m})} . \end{aligned} \tag{I.A.8}$$

The gamma exposure corresponds to the second derivative of wealth $w(p)$ with respect to the price. Since $w(p)$ is quadratic, $\Gamma^{(u,v)}$ is equal to twice the quadratic coefficient. We compute the following

$$\begin{aligned} \Gamma^{(u,v)} &= \frac{2}{p_{t+1,g} - p_{t+1,b}} \left[\frac{w_{t+1,g}^{(u,v)} - w_{t+1,m}^{(u,v)}}{p_{t+1,g} - p_{t+1,m}} - \frac{w_{t+1,m}^{(u,v)} - w_{t+1,b}^{(u,v)}}{p_{t+1,m} - p_{t+1,b}} \right] \\ &= \frac{2}{p_{t+1,g} - p_{t+1,b}} \left[\frac{\frac{w_{t+1,g}^{(u,v)}}{w_t^{(u,v)}} \frac{w_t^{(u,v)}}{p_t} - \frac{w_{t+1,m}^{(u,v)}}{w_t^{(u,v)}} \frac{w_t^{(u,v)}}{p_t}}{\frac{p_{t+1,g}}{p_t} - \frac{p_{t+1,m}}{p_t}} - \frac{\frac{w_{t+1,m}^{(u,v)}}{w_t^{(u,v)}} \frac{w_t^{(u,v)}}{p_t} - \frac{w_{t+1,b}^{(u,v)}}{w_t^{(u,v)}} \frac{w_t^{(u,v)}}{p_t}}{\frac{p_{t+1,m}}{p_t} - \frac{p_{t+1,b}}{p_t}} \right] \\ &= \frac{2 f_t(u,v)}{p_{t+1,g} - p_{t+1,b}} \left[\frac{g - m}{G_t - M_t} - \frac{m - b}{M_t - B_t} \right] \end{aligned}$$

□

I.A.2 Examples with $T = 3$

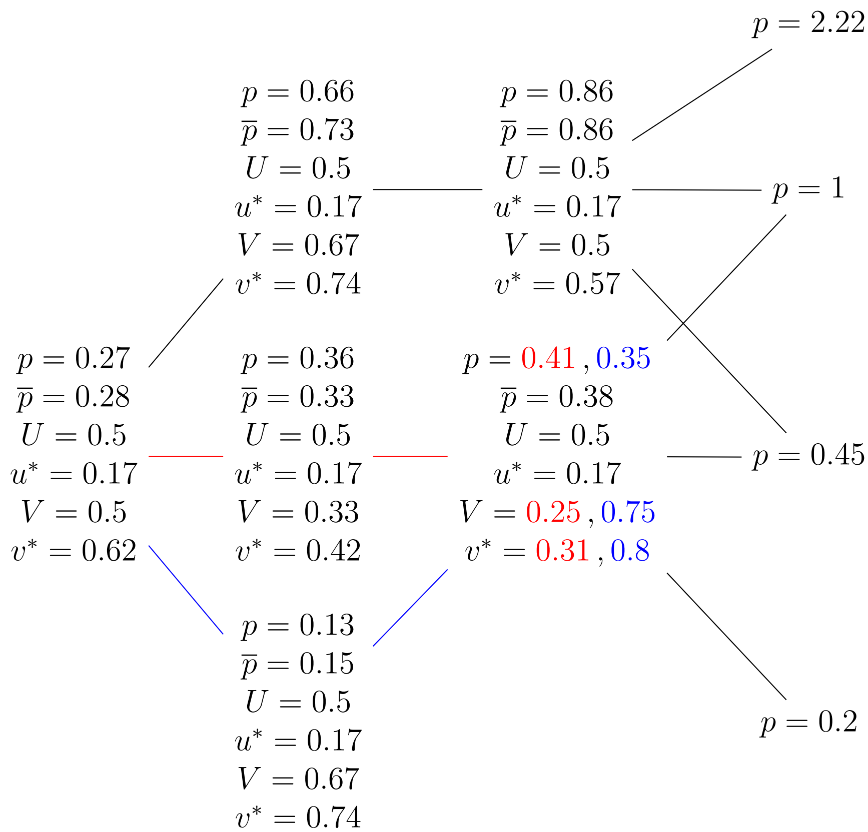


Figure I.A.1: v-**disagreement**

In this example agents' beliefs v are uniformly distributed and they agree on $u = 1/2$. p is price of market, $\overline{p}$ is price in homogeneous economy where everyone agrees with the median agent $(u, v) = (1/2, 1/2)$. (U, V) represents Mr. Market's beliefs and (u^*, v^*) is the risk-neutral measure.

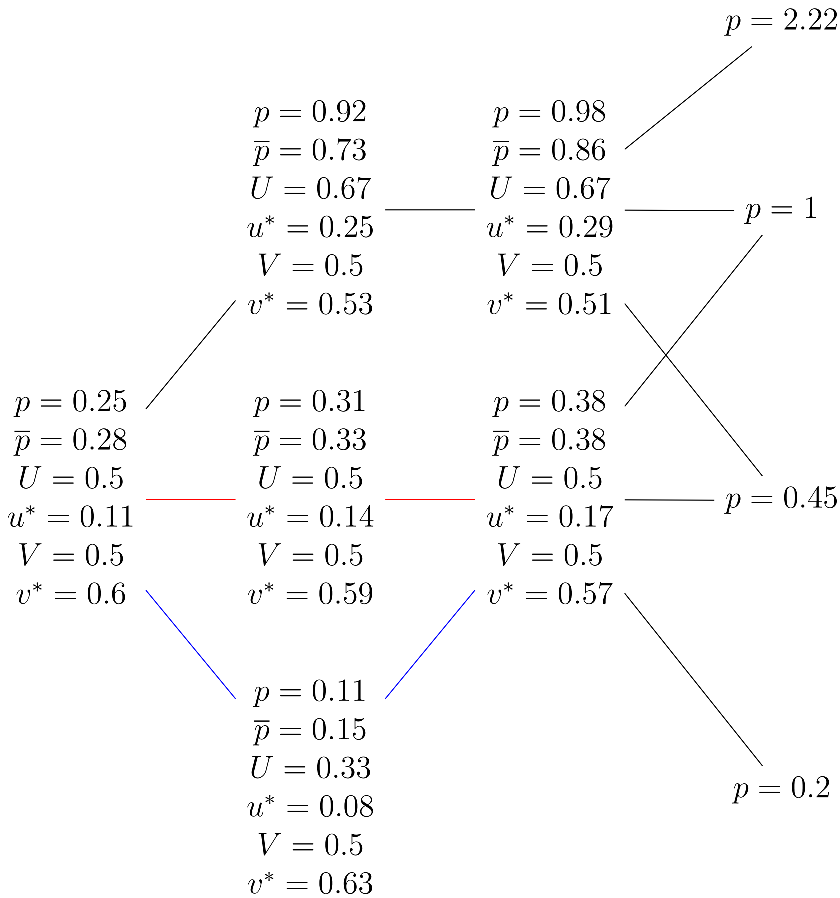


Figure I.A.2: u**-disagreement, neutral initial sentiment**

In this example agents' beliefs u are uniformly distributed and they agree on $v = 1/2$. p is price of market, $\overline{p}$ is price in homogeneous economy where everyone agrees with the median agent $(u, v) = (1/2, 1/2)$. (U, V) represents Mr. Market's beliefs and (u^*, v^*) is the risk-neutral measure.

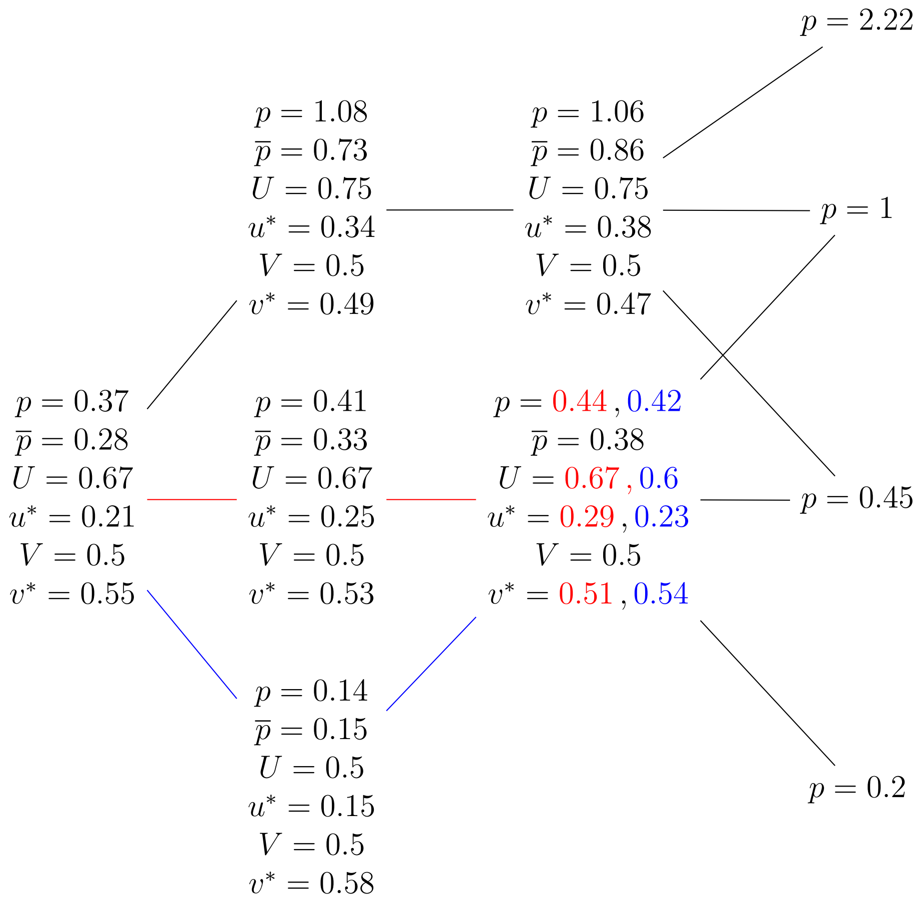


Figure I.A.3: u**-disagreement, bullish initial sentiment**

In this example agents' beliefs u follow $Beta\,(2,1)$ distribution with mean $2/3$, and they agree on $v = 1/2$. p is price of market, $\overline{p}$ is price in homogeneous economy where everyone agrees with the agent $(u,v) = (1/2,1/2)$. (U,V) represents Mr. Market's beliefs and (u^*,v^*) is the risk-neutral measure.

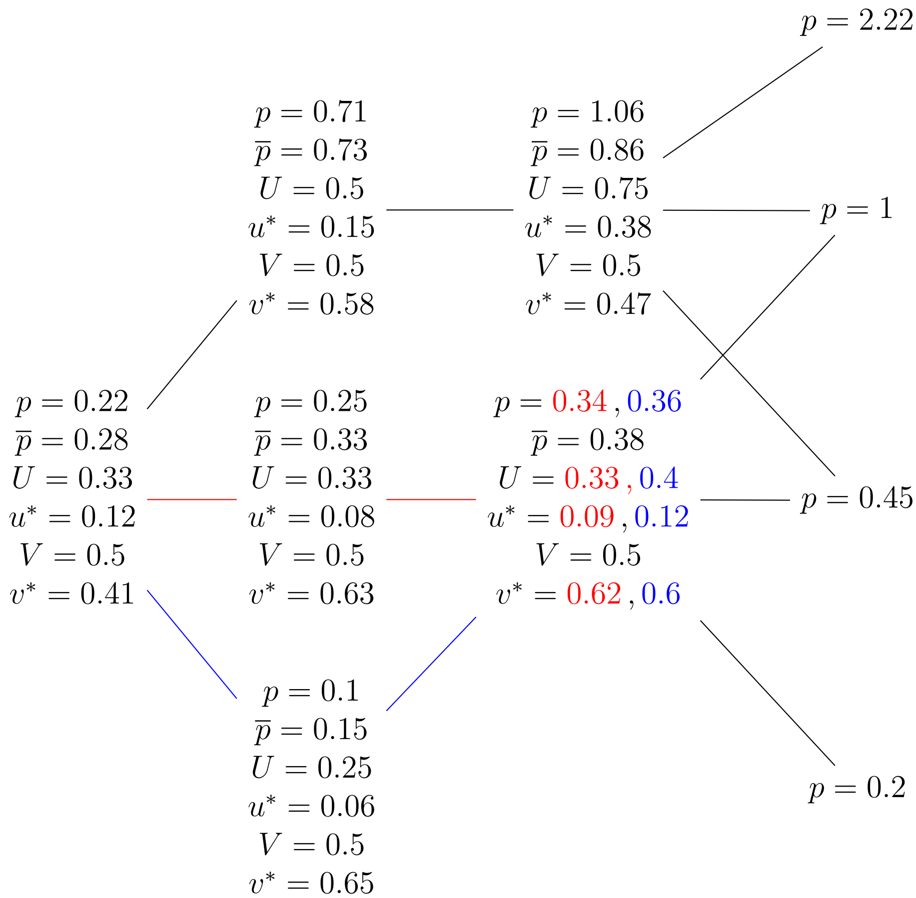


Figure I.A.4: u**-disagreement, bearish initial sentiment**

In this example agents' beliefs u follow $Beta\,(1, 2)$ distribution with mean $2/3$, and they agree on $v = 1/2$. p is price of market, $\overline{p}$ is price in homogeneous economy where everyone agrees with the agent $(u, v) = (1/2, 1/2)$. (U, V) represents Mr. Market's beliefs and (u^*, v^*) is the risk-neutral measure.

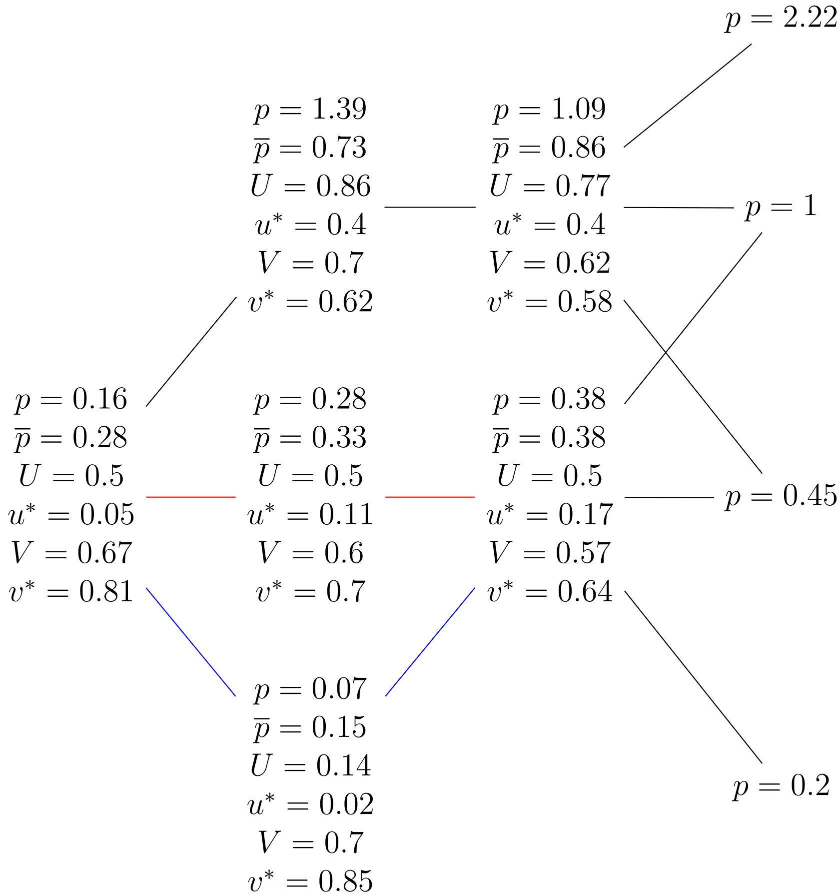
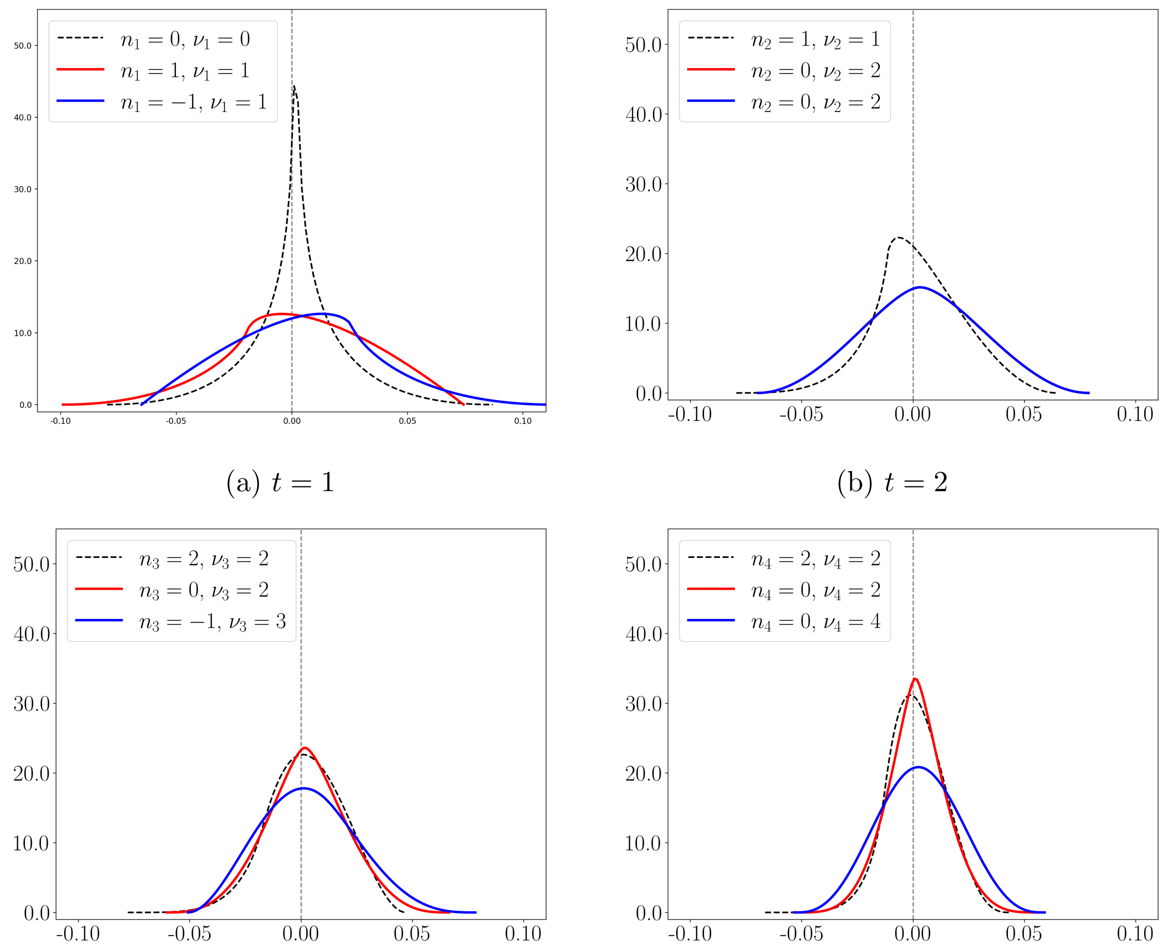


Figure I.A.5: **Recombining binary tree: h-disagreement**

This example uses the binary tree from Martin and Papadimitriou (2022) where h is uniformly distributed. We only report pricing at $t = 0, 1, 2, 3$ (while binary tree also have nodes at $t = 0.5, 1.5, 2.5$). p is price of market, $\overline{p}$ is price in homogeneous economy where everyone agrees with the median agent $h = 1/2$. (U, V) represents Mr. Market's beliefs and (u^*, v^*) is the risk-neutral measure.

I.A.3 Figure 5 and 6 at different periods

I.A.3.1 High disagreement on v

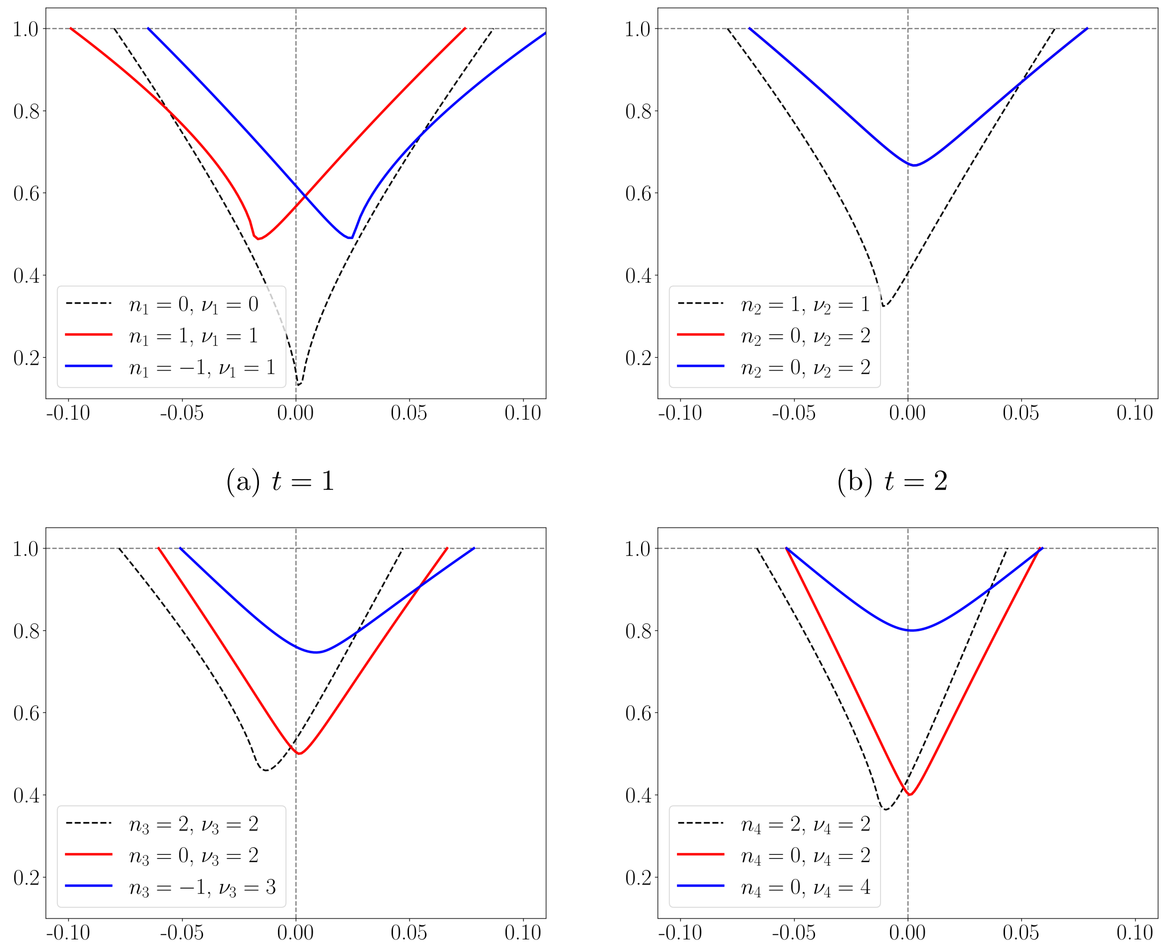


(a) $t = 1$ (b) $t = 2$

(c) $t = 3$ (d) $t = 4$

Figure I.A.6: **Wealth distribution across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.

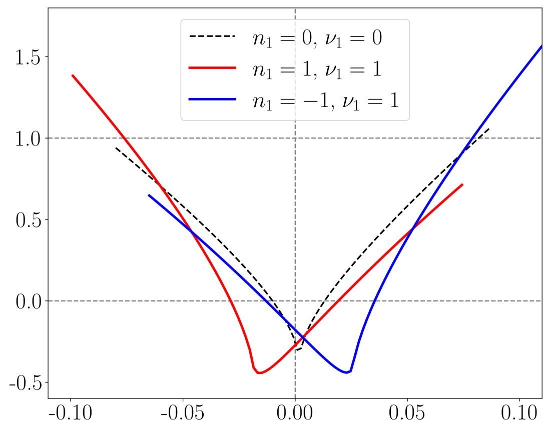


(a) $t = 1$ (b) $t = 2$

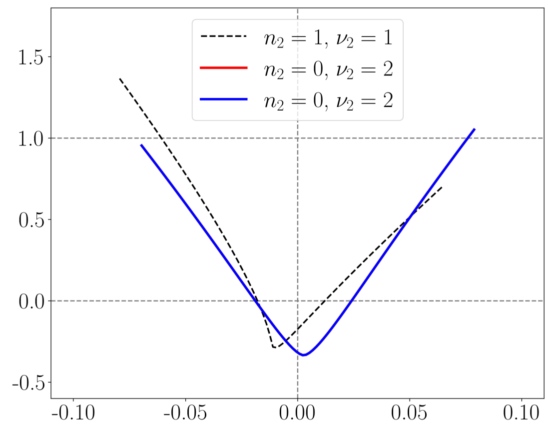
(c) $t = 3$ (d) $t = 4$

Figure I.A.7: **Wealth weighted beliefs of v across r**

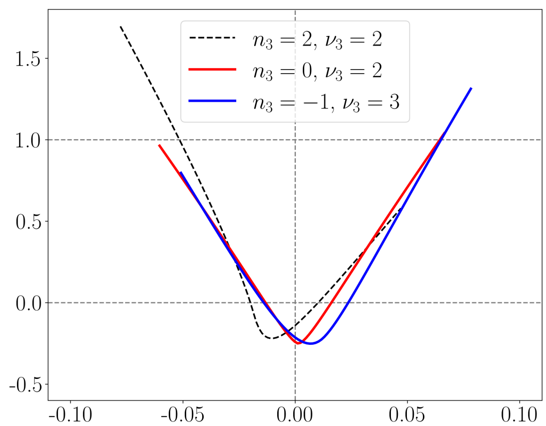
The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



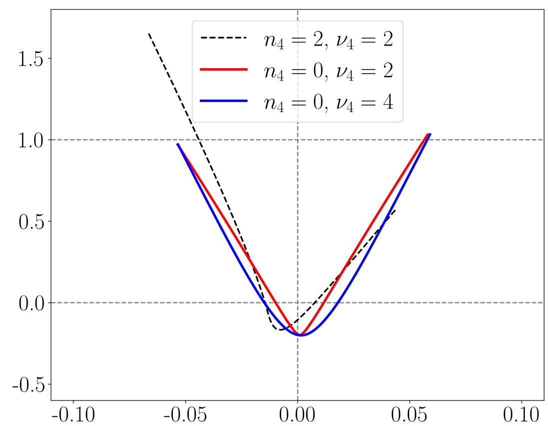
(a) $t = 1$



(b) $t = 2$



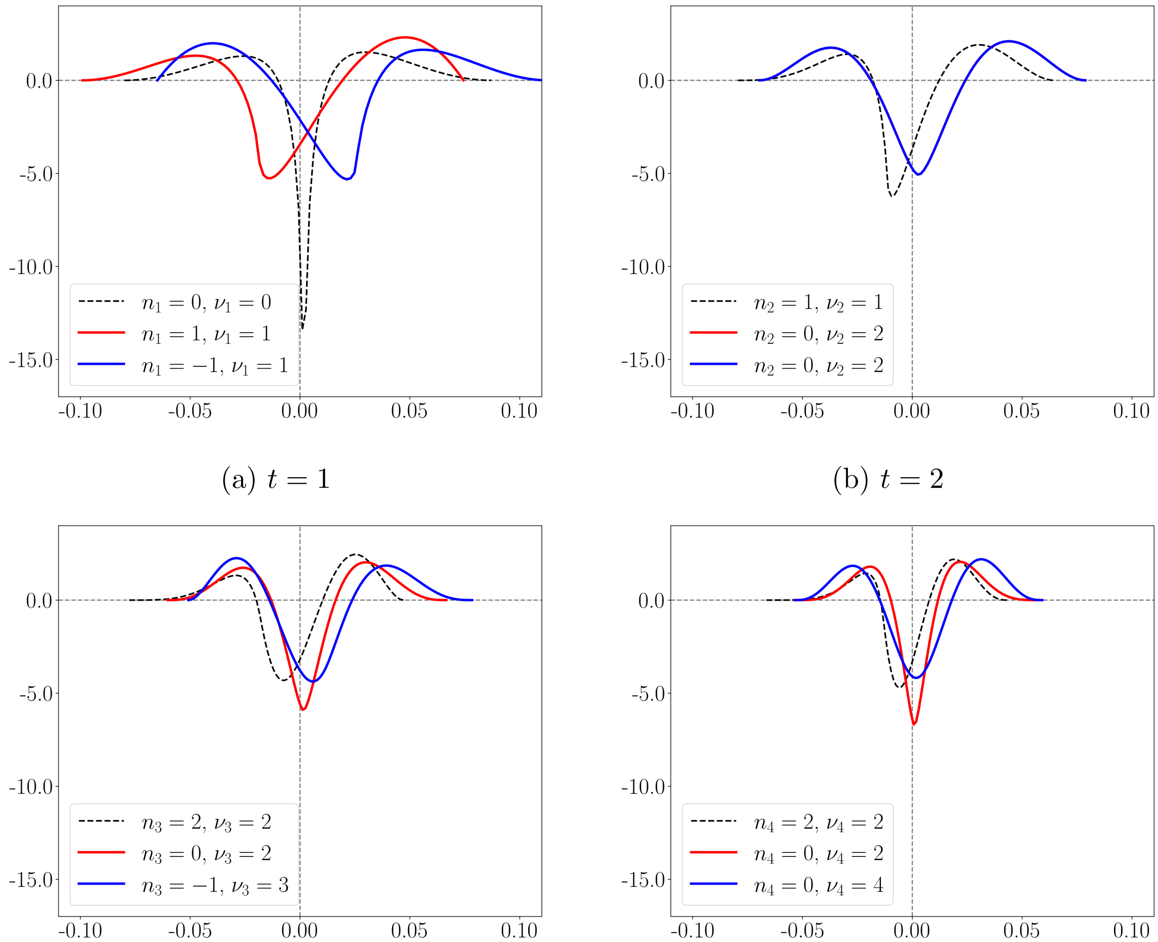
(c) $t = 3$



(d) $t = 4$

Figure I.A.8: **Portfolio weights in 'straddle' across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



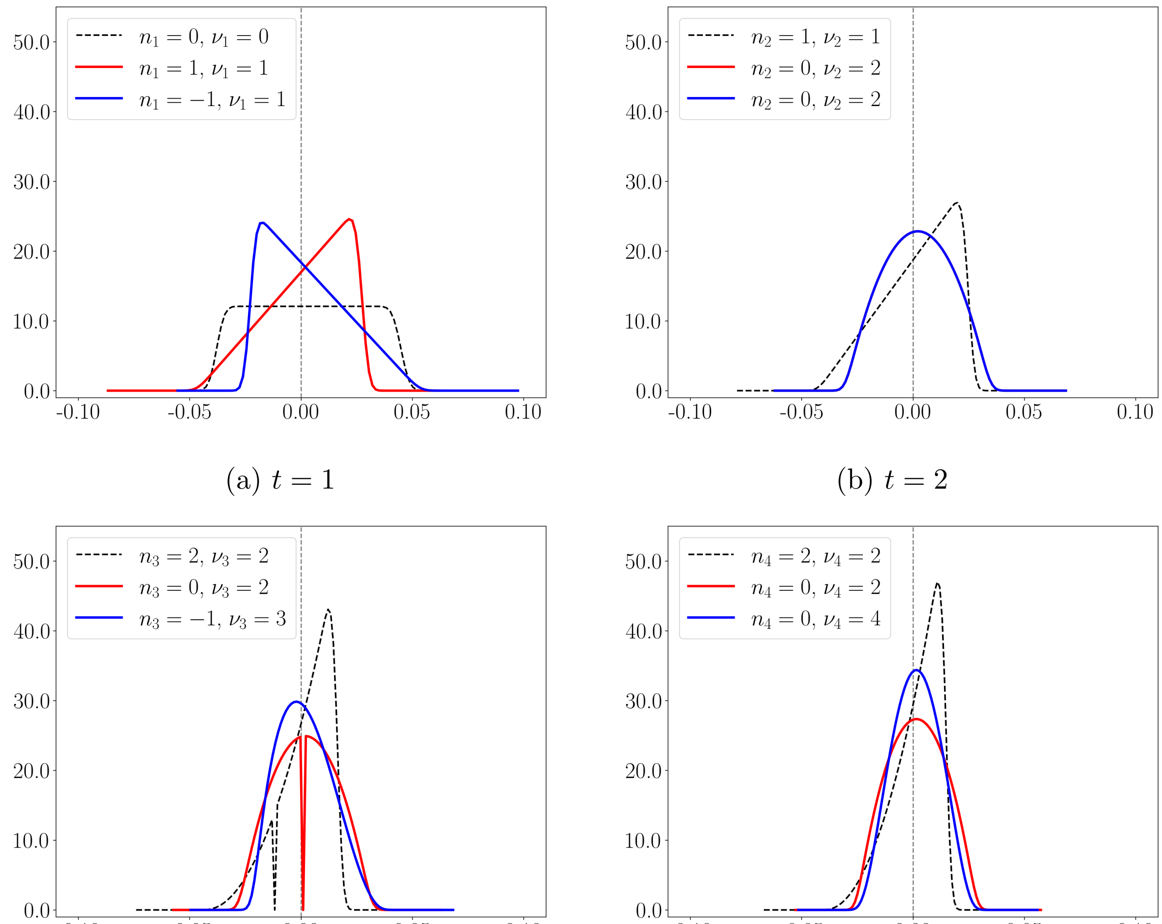
(a) $t = 1$ (b) $t = 2$

(c) $t = 3$ (d) $t = 4$

Figure I.A.9: **Gamma exposure across r**

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 1, 1)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.

I.A.3.2 Low disagreement on v

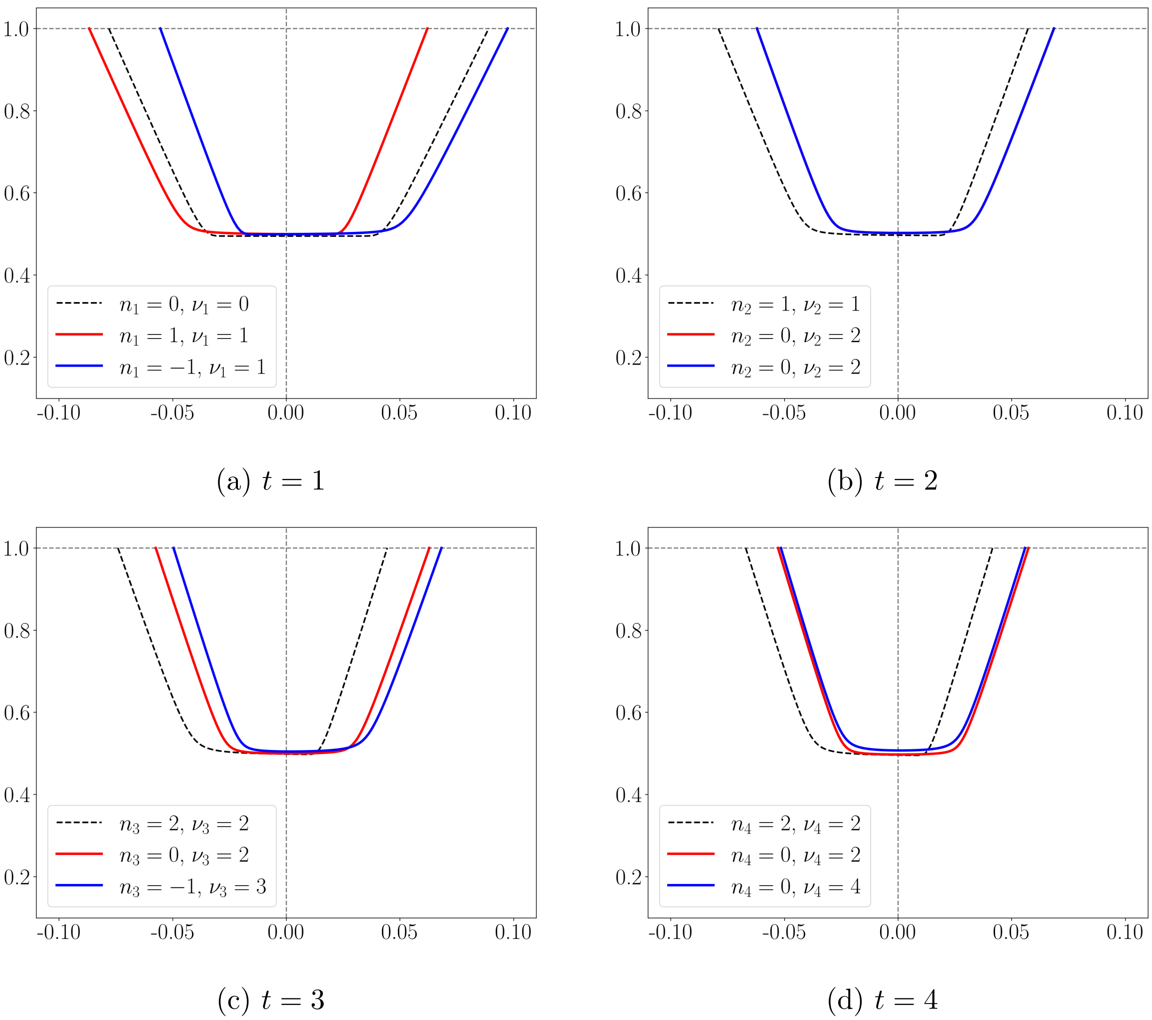


(a) $t = 1$ (b) $t = 2$

(c) $t = 3$ (d) $t = 4$

Figure I.A.10: **Wealth distribution across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



(a) $t = 1$

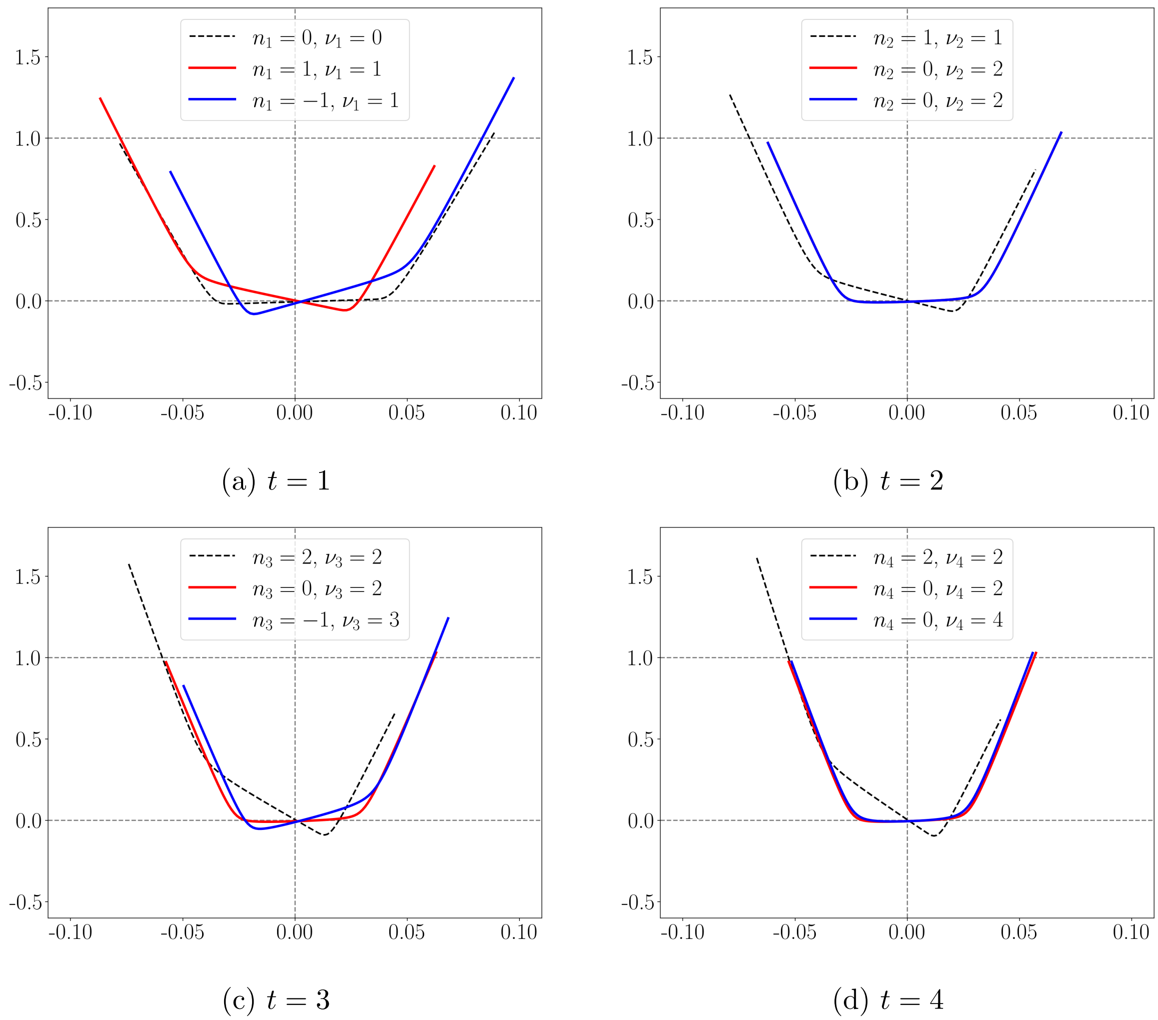
(b) $t = 2$

(c) $t = 3$

(d) $t = 4$

Figure I.A.11: **Wealth weighted beliefs of v across r**

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



(a) $t = 1$

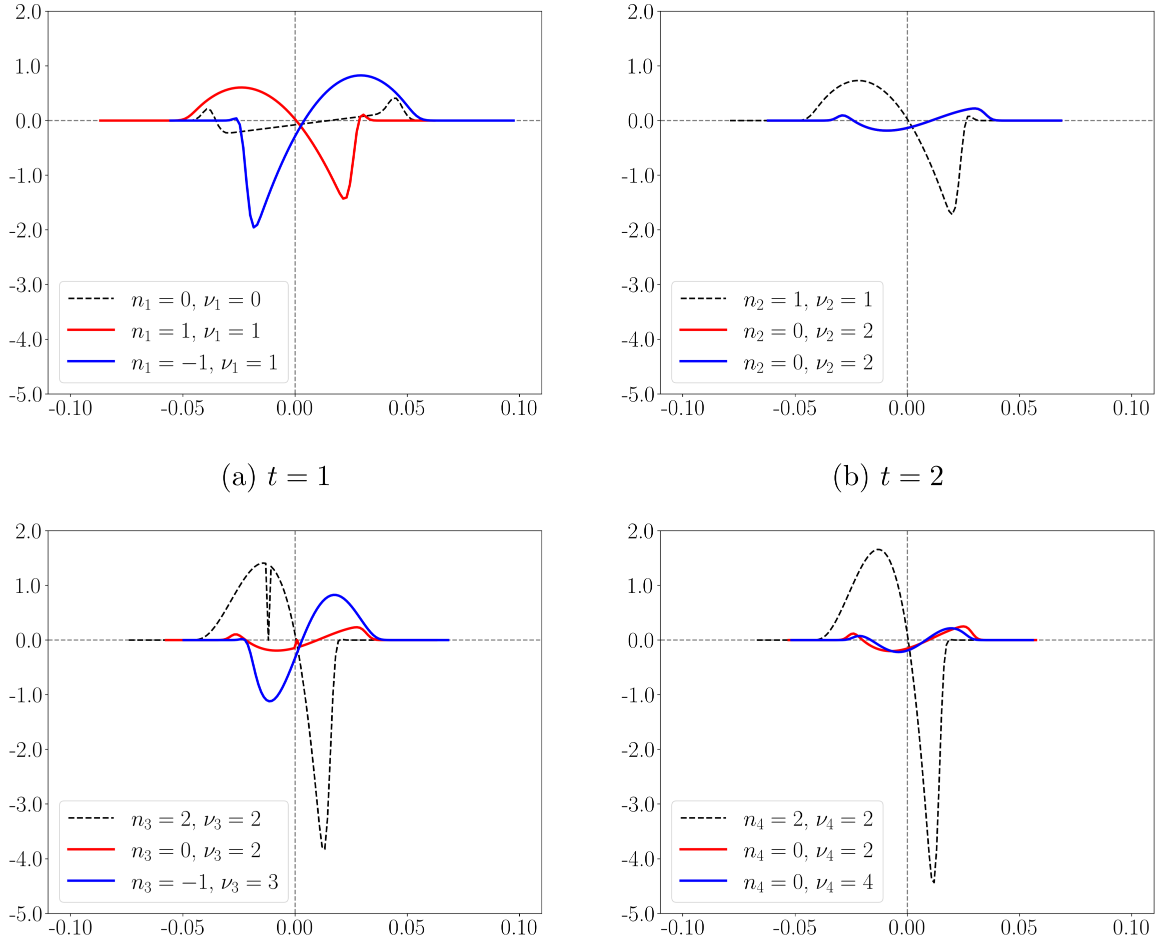
(b) $t = 2$

(c) $t = 3$

(d) $t = 4$

Figure I.A.12: **Portfolio weights in 'straddle' across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.



(a) $t = 1$

(b) $t = 2$

(c) $t = 3$

(d) $t = 4$

Figure I.A.13: **Gamma exposure across** r

The initial wealth across agents are assumed to be uniformly distributed, i.e. $(\alpha_{u0}, \beta_{u0}, \alpha_{v0}, \beta_{v0}) = (1, 1, 100, 100)$. The three colored lines corresponds to the three sample paths in Figure 2. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05 n_T}$.