Abstract

Bifurcation Equation: $\lambda x = Lx + R(\lambda, x), \quad \lambda \in \mathbb{R}, \; x \in X, \quad$ (BEqn)

X a Banach space, L compact and linear, R compact and higher order in x

A celebrated result of Paul Rabinowitz 1971 is that
global connected sets of non-trivial solutions of (BEqn) bifurcate from trivial solutions at non-zero eigenvalues λ_0 *of odd algebraic multiplicity of L.*

To understand what this intuitively powerful statement means in practice a simple example is constructed to show that the global bifurcation theorem does not imply the existence of paths, other than singletons, of non-trivial solutions.

This has obvious implications for applications.

Similarly, when the operators are infinitely differentiable and classical bifurcation occurs locally at a simple eigenvalue, the global connected sets which bifurcate may not have any paths outside a small neighbourhood of the bifurcation point.

Along the same lines, the set of non-trivial solutions which by variational theory bifurcate from eigenvalues of any multiplicity when the problem has gradient structure may not be connected and may not contain any paths except singletons.

In his PhD thesis (Cambridge 1972) Norman Dancer developed from the theory of real-analytic varieties a global bifurcation theory for real-analytic-operator equations that leads to global piecewise-smooth paths of solutions. Today's talk illustrates the importance of his hypothesis but leaves open the question ”is there an alternative?”